

## ON THE “SPOOKY ACTION AT A DISTANCE” IN THE CPHD FILTER

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### ABSTRACT

Using the pair correlation function, spooky interaction between distant targets in the cardinalized probability hypothesis density (CPHD) filter is investigated. It is shown that the spooky interaction between distant targets in the CPHD filter is a direct result of its independent identically distributed cluster process (IIDCP) target model. That is, the spooky effect in the CPHD filter is not due to the indistinguishable nature of the elements in the unlabeled random finite set formulation, but due to the particular approximation of target process assumed by the CPHD filter.

**KEYWORDS:** Independent identically distributed cluster process, IIDCP, CPHD filter, spooky action at a distance, pair correlation function.

### 1. INTRODUCTION

The probability hypothesis density (PHD) filter [1,2] is an efficient tracking algorithm which has a linear algorithmic complexity with respect to the number of targets as well as the number of measurements. Unfortunately, due to its Poisson point process (PPP) target model, the canonical number estimate of the PHD filter is well known to be of high variance [3]. In particular, target death problem [4] is a known cause for such variability. To stabilize the canonical number estimates, CPHD filter, which propagates not only the intensity function of the approximating target process, but also its canonical number probability mass function (pmf) was introduced in [5]. While stabilizing the canonical number estimates, [6] showed that the CPHD filter's local target estimators “*exhibit a peculiar, counter-intuitive behavior: upon a missed detection, PHD weight is shifted from the undetected part of the PHD to the detected part, no matter how far apart the parts are. The amount of the shifted weight and, hence, the remaining weight of the undetected part both depend on the total target number*”. This phenomenon is named as *spooky action at a distance* in the CPHD filter due to its analogy to a similar phenomenon in quantum entanglement. To investigate whether the spooky action at a distance in the CPHD filter is an artifact of the random finite set formulation of the multitarget filtering problem, [7] used paranormal implementation of the labeled multitarget

Bayes filter and showed that it did not exhibit such “spooky” interactions between distant targets. Therefore, it was concluded that “*the spooky effect is not an artifact of the random finite set formulation of the multi-target tracking problem*”. On the other hand, [7] raised the following important questions “*In view of this conclusion, the interesting question is what actually causes the spooky effect? Is it the indistinguishable nature of the elements in unlabeled random finite set? Or is it merely artifacts of the particular approximations used in the PHD, CPHD and multi-Bernoulli filters?*” in its concluding remarks.

In this work, it is shown that the spooky interaction between distant targets in the CPHD filter is a direct result of its IIDCP target model. To show this fact, in Section 2, the probability generating functional (PGFL) of the IIDCP is given and its reduced Palm intensity function [8-12] is derived. It is shown that, given that a target exists at a known state, the ratio of conditional intensity function and the (unconditional) intensity function at an arbitrary target state depends on the canonical number pmf. This ratio of conditional intensity function and the intensity functions is known as the pair correlation function in the greater point process literature [13]. Therefore, even though target spatial distribution is iid (by assumption) for the IIDCP target model, an arbitrary canonical number pmf does lead to correlations between arbitrary points. That is, conditioning that a target exists at a known state changes the intensity function at an arbitrary state, unless the canonical number pmf is Poisson [11]. Moreover, assuming a truncated canonical number pmf which permits the possibility of at most two targets, pair correlation function of the Bayes posterior process in the CPHD filter is derived in Section 3. The inseparable structure of the resulting pair correlation function shows that second factorial moment of the Bayes posterior process is in fact coupled. That is, the second factorial moment cannot be written as the product of first moments even for target pairs at a distance. This result for the CPHD filter is in contrast to our previous results reported in [9] and [11] which show for the PHD filter that interaction term vanishes with respect to measurement likelihood function for distant targets. Therefore, correlations between distant targets vanish for the PHD filter, but not in the CPHD filter. That is, the *spooky effect* in the CPHD filter is not due to *indistinguishable nature of the elements in unlabeled random finite set*, but due to the particular target process approximation used by the CPHD filter.

## 2. IIDCP PGFL AND ITS PAIR CORRELATION FUNCTION

IID cluster process is the finite point process model of the measurement and target processes assumed by the CPHD filter [2]. Let  $p_{n, s_1, \dots, s_n}$  denote the probability density function (pdf) of the ordered  $n$ -tuple  $s_1, \dots, s_n$  which involves  $n$  points at states  $s_1$  to  $s_n$ , and let  $p^N_n$  denote the canonical number pmf of the underlying point process. The IIDCP model assumes that for a given canonical number  $n$ , which is governed by  $p^N_n$ , the spatial distribution of  $n$  points is

independent and identically distributed (iid), that is  $p(n, s_1, \dots, s_n) = \prod_{i=1}^n p(s_i)$

where  $p(\cdot)$  is the spatial probability distribution function of a single point. Then, the PGFL of IID cluster process is given by

$$G[h] = \sum_{n=0}^{\infty} p^n \int_S h(s) p(s) ds^n = G^N \int_S h(s) p(s) ds \quad (1.1)$$

where  $G^N(\cdot)$  is the ordinary probability generating function (PGF) of the canonical number,  $N$ , of the process. Let  $G'^N(\cdot)$  denote its ordinary derivative. The PGFL of the reduced Palm distribution for the IID cluster process conditioned on the existence of a point at  $x$  is given by [11]

$$G[h | x] \equiv \frac{\frac{\partial G}{\partial x} h}{\frac{\partial G}{\partial x} 1} = \frac{p(x) \sum_{n=1}^{\infty} p^n \int_S h(s) p(s) ds^{n-1}}{p(x) \sum_{n=1}^{\infty} p^n n} \quad (1.2)$$

The term  $p(x) \neq 0$  can be cancelled. The sum in the numerator is the ordinary derivative of the PGF of canonical number evaluated at  $\int_S h(s) p(s) ds$ , while the sum in the denominator is the derivative the PGF evaluated at one. Thus,

$$G[h | x] = \frac{G'^N \int_S h(s) p(s) ds}{G'^N(1)} \quad (1.3)$$

This gives the conditional intensity function at  $x_1$  as

$$\begin{aligned} m_{(1)}(x_1 | x) &\equiv \frac{\partial}{\partial x_1} G[1 | x] = \frac{\frac{\partial}{\partial x_1} G'^N \int_S h(s) p(s) ds \Big|_{h=1}}{G'^N(1)} \\ &= \frac{p(x_1) G'^{nN}(1)}{G'^N(1)} = \frac{p(x_1) \sum_{n=2}^{\infty} p^n \int_S h(s) p(s) ds^{n-1}}{\sum_{n=1}^{\infty} p^n n} \end{aligned} \quad (1.4)$$

where  $G'^{nN}(\cdot)$  is the second derivative of  $G^N(\cdot)$ . The ratio of conditional intensity and the intensity functions at  $x_1$  is then given by

$$\frac{m_1(x_1 | x)}{m_1(x_1)} = \frac{m_2(x_1, x)}{m_1(x_1) m_1(x)} = \rho(x_1, x) = \frac{\sum_{n=2}^{\infty} p^{N,n,n-1}}{\left( \sum_{n=1}^{\infty} p^{N,n,n} \right)^2} \quad (1.5)$$

where  $\rho(x_1, x)$  denotes the pair correlation function between  $x_1$  and  $x$  [11].

For the Poisson distributed canonical number pmf, it is easy to show that the pair correlation function, that is the ratio of the conditional intensity and the intensity functions, is equal to one. For such case, which is the target and measurement process model for the PHD filter, the IID cluster process is a PPP. As shown in [11], for an arbitrary canonical number pmf however, the ratio can be larger or smaller than one. For target tracking applications, this means that the arbitrary canonical number pmf leads to a coupled second factorial moment for all pairs in the CPHD filter's assumed target model.

### 3. PAIR CORRELATION FUNCTION FOR THE CPHD FILTER'S BAYES POSTERIOR PROCESS

This paper is concerned with the correlations in the CPHD filter's approximate corrector step. The predicted target process of the CPHD filter is approximated with an IIDCP with appropriate intensity function and canonical number pmf. The measurement model of the CPHD filter presumes the following assumptions [5]:

- 1) a single target with state  $x$  generates, with probability  $P^D(x)$ , at most one observation
- 2) any observation is generated by a single target
- 3) the false alarm process is an IID cluster process.

Under these assumptions, the PGFL of the predicted target process,  $G^{\Xi}(h)$  and the false alarm process  $G^{Y_c}(g)$  are defined in (2.1)

$$\begin{aligned} G^{\Xi}(h) &= \sum_{n=0}^{\infty} p^{\Xi,n} \int_S h(s) \psi_1^n(s) ds = F^{\Xi} \int_S h(s) \psi_1^1(s) ds \\ \psi_1^1(s) &= \frac{f^{\Xi}(s)}{\int_S f^{\Xi}(s) ds} = \frac{f^{\Xi}(s)}{\mu^{\Xi}} \\ G^{Y_c}(g) &= F^Y \int_S g(y) \psi_1^2(y) dy \\ \psi_1^2(y) &= \frac{\lambda(y)}{\int_S \lambda(y) dy} = \frac{\lambda(y)}{\mu^Y} \end{aligned} \quad (2.1)$$

where  $f^{\Xi} s$  is the intensity function of the predicted target process and  $\lambda y$  denotes the intensity function of the false alarm process.

Using the branching process form of the target – target generated measurement process [14], the joint PGFL for target and target generated measurement process is defined in (2.2)

$$\begin{aligned} G^{Y\Xi} g, h &= F^{\Xi} \left( \int_S h s \left[ 1 - P^D s + \int_Y P^D s g y p y | s dy \right] \frac{f^{\Xi} s}{\mu^{\Xi}} ds \right) \\ &\times F^Y \left( \int_S g y \frac{\lambda y}{\mu^Y} dy \right) \end{aligned} \quad (2.2)$$

where  $p y | s$  is the measurement likelihood function. Assuming that the false alarm process is PPP, then the joint PGFL of predicted target process and the measurement process is defined in (2.3).

$$\begin{aligned} G^{Y\Xi} g, h &= F^{\Xi} T g, h \exp \left[ \int_Y \lambda y g y - 1 dy \right] \\ T g, h &= \int_S h s \left[ 1 - P^D s + \int_Y P^D s g y p y | s dy \right] \frac{f^{\Xi} s}{\mu^{\Xi}} ds \end{aligned} \quad (2.3)$$

Taking the functional derivative of the joint PGFL with respect to an impulse at a measurement with state  $z_1$  gives (2.4).

$$\begin{aligned} \frac{\partial G^{Y\Xi}}{\partial z_1} g, h &= \exp \left[ \int_Y \lambda y g y - 1 dy \right] \lambda z_1 \\ &\times \left[ F T g, h + \frac{F^{(1)} T g, h}{\lambda z_1} \int_S h s P^D s p z_1 | s \frac{f^{\Xi} s}{\mu^{\Xi}} ds \right] \\ &= \exp \left[ \int_Y \lambda y g y - 1 dy \right] \lambda z_1 \left[ F T g, h + F^{(1)} T g, h \frac{U_1 h}{\lambda z_1} \right] \end{aligned} \quad (2.4)$$

In (2.4),  $U_{\sigma_1, \dots, \sigma_k} h$  is defined as:

$$U_{\sigma_1, \dots, \sigma_k} h = \prod_{j=1}^k \frac{\partial T}{\partial z_{\sigma_j}} 0, h = \prod_{j=1}^k \int_S h s P^D s p z_{\sigma_j} | s \frac{f^{\Xi} s}{\mu^{\Xi}} ds \quad (2.5)$$

In (2.5),  $\sigma_j$  represents the index of measurement due to which the functional derivative of  $T(g, h)$  is taken (i.e.  $U_{1,3} h = \frac{\partial T}{\partial z_1}(0, h) \frac{\partial T}{\partial z_3}(0, h)$ ).  $F^{(1)} z$  represents the *normal* derivative of the probability generating function  $F(z)$  :

$$F^{(n)} z = \frac{d^n F(z)}{dz^n} \quad (2.6)$$

Taking the second functional derivative with respect to an impulse at  $z_2$  gives (2.7).

$$\begin{aligned} \frac{\partial^2 G^{Y\Xi}}{\partial z_1 \partial z_2}(g, h) &= \exp \left[ \int_Y \lambda(y) g(y) - 1 dy \right] \lambda(z_1) \lambda(z_2) \\ &\times \left[ \begin{aligned} &F T(g, h) + F^{(1)} T(g, h) \frac{U_1 h}{\lambda(z_1)} \\ &+ F^{(1)} T(g, h) \frac{U_2 h}{\lambda(z_2)} + F^{(2)} T(g, h) \frac{U_{1,2} h}{\lambda(z_1) \lambda(z_2)} \end{aligned} \right] \end{aligned} \quad (2.7)$$

Following this pattern, it is easy to show that the  $k$ . order functional derivative of the joint PGFL with respect to impulses at  $z_1, \dots, z_k$  is given by (2.8).

$$\begin{aligned} \frac{\partial^k G^{Y\Xi}}{\partial z_1 \dots \partial z_k}(g, h) &= A(g, h) \exp \left[ \int_Y \lambda(y) g(y) - 1 dy \right] \prod_{i=1}^k \lambda(z_i) \\ A(g, h) &= \\ &\left[ \begin{aligned} &F T(g, h) + F^{(1)} T(g, h) \sum_{i=1}^k \frac{U_i h}{\lambda(z_i)} + \\ &+ F^{(2)} T(g, h) \sum_{i=1}^k \sum_{\substack{j=1 \\ j \neq i}}^k \frac{U_{i,j} h}{\lambda(z_i) \lambda(z_j)} + \dots + F^{(k)} T(g, h) \frac{U_{1, \dots, k} h}{\prod_{i=1}^k \lambda(z_i)} \end{aligned} \right] \end{aligned} \quad (2.8)$$

Therefore the Bayes posterior target process is defined by (2.9) [14].

$$G^{\Xi|Y}(h) = \frac{\frac{\partial^k G^{Y\Xi}}{\partial z_1 \dots \partial z_k}(0, h)}{\frac{\partial^k G^{Y\Xi}}{\partial z_1 \dots \partial z_k}(0, 1)} = \frac{A(0, h)}{A(0, 1)} \quad (2.9)$$

The CPHD filter approximates Bayes posterior process with an IIDCP which has the same intensity function and canonical number pmf. The probability generating function for the canonical number of the Bayes posterior process is given by (2.10).

$$F_{z|v} = \frac{1}{A_{0,1}} \left[ F_{T=0,h} + F^{(1)}_{T=0,h} \sum_{i=1}^k \frac{U_i h}{\lambda z_i} + \right. \\ \left. \times \left[ +F^{(2)}_{T=0,h} \sum_{i=1}^k \sum_{j \neq i}^k \frac{U_{i,j} h}{\lambda z_i \lambda z_j} + \dots + F^{(k)}_{T=0,h} \frac{U_{1,\dots,k} h}{\prod_{i=1}^k \lambda z_i} \right] \right]_{h.=z} \quad (2.10)$$

Putting  $h . = z$  in (2.10) gives (2.11)

$$T_{0,h} \Big|_{h.=z} = z \int_S 1 - P^D s \frac{f^{\Xi} s}{\mu^{\Xi}} ds = z \mu_{miss} \quad (2.11)$$

and (2.12).

$$U_{\sigma_1, \dots, \sigma_k} h \Big|_{h.=z} = z^k \prod_{j=1}^k \int_S P^D s p_{z_{\sigma_j}} | s \frac{f^{\Xi} s}{\mu^{\Xi}} ds \\ = z^k \prod_{j=1}^k \mu_{detect \sigma_j} \quad (2.12)$$

The intensity function of the Bayes posterior process is given by (2.13) [14].

$$m_1 x | v = \frac{\partial G^{\Xi|Y}}{\partial x} 1 = \frac{\partial A_{0,1}}{\partial x} \quad (2.13)$$

Equations (2.8-2.13) describe the CPHD filter approximate corrector step. For the sake of convenience, let us assume that the canonical number pmf of the predicted target process is truncated such that  $p^N n = 0$  for  $n > 2$ . Then the probability generating function of the predicted target process is given by (2.14).

$$F_z = p^N 0 + p^N 1 z + p^N 2 z^2 \quad (2.14)$$

For the pgf defined in (2.14), the functional  $A_{0,h}$  is given by (2.15).

$$\begin{aligned}
A(0, h) &= F(T(0, h) + F^{(1)}(T(0, h) \sum_{i=1}^k \frac{U_i h}{\lambda z_i} + F^{(2)}(T(0, h) \sum_{i=1}^k \sum_{j=1, j \neq i}^k \frac{U_{i,j} h}{\lambda z_i \lambda z_j})) \\
&= p(0, \cdot) + p(1, \cdot) T(0, h) + p(2, \cdot) T(0, h)^2 + \sum_{i=1}^k \frac{U_i h}{\lambda z_i} [p(1, \cdot) + 2p(2, \cdot) T(0, h)] + 2p(2, \cdot) \sum_{i=1}^k \sum_{j=1, j \neq i}^k \frac{U_{i,j} h}{\lambda z_i \lambda z_j} \quad (2.15) \\
&= p(0, \cdot) + p(1, \cdot) \left[ T(0, h) + \sum_{i=1}^k \frac{U_i h}{\lambda z_i} \right] + p(2, \cdot) \left[ T(0, h)^2 + 2 \sum_{i=1}^k \frac{U_i h}{\lambda z_i} T(0, h) + 2 \sum_{i=1}^k \sum_{j=1, j \neq i}^k \frac{U_{i,j} h}{\lambda z_i \lambda z_j} \right]
\end{aligned}$$

Direct calculation gives the normalizing factor  $A(0, 1)$ .

$$\begin{aligned}
A(0, 1) &= p(0, \cdot) + p(1, \cdot) \left[ \mu_{miss} + \sum_{i=1}^k \frac{\mu_{detect i}}{\lambda z_i} \right] \quad (2.16) \\
&+ p(2, \cdot) \left[ \mu_{miss}^2 + 2 \sum_{i=1}^k \frac{\mu_{miss} \mu_{detect i}}{\lambda z_i} + 2 \sum_{i=1}^k \sum_{j=1, j \neq i}^k \frac{\mu_{detect i} \mu_{detect j}}{\lambda z_i \lambda z_j} \right]
\end{aligned}$$

Then the intensity function of the Bayes posterior process at  $x_1$  is calculated with (2.17).

$$\frac{\partial G^{\Xi|Y}}{\partial x_1}(1) = \frac{\frac{\partial A}{\partial x_1}(0, 1)}{A(0, 1)} = \frac{C_1(x_1)}{A(0, 1)} \quad (2.17)$$

where  $C_1(x_1)$  is defined in (2.18).

$$\begin{aligned}
C_1(x_1) &= p(1, \cdot) \left[ \frac{\partial T}{\partial x_1}(0, 1) + \sum_{i=1}^k \frac{\frac{\partial U_i}{\partial x_1}(1)}{\lambda z_i} \right] \\
&+ p(2, \cdot) \left[ 2 \frac{\partial T}{\partial x_1}(0, 1) \mu_{miss} + 2 \sum_{i=1}^k \frac{\frac{\partial U_i}{\partial x_1}(1) \mu_{miss} + \mu_{detect i} \frac{\partial T}{\partial x_1}(0, 1)}{\lambda z_i} \right. \\
&\left. + 2 \sum_{i=1}^k \sum_{j=1, j \neq i}^k \frac{\frac{\partial U_i}{\partial x_1}(1) \mu_{detect j} + \frac{\partial U_j}{\partial x_1}(1) \mu_{detect i}}{\lambda z_i \lambda z_j} \right] \quad (2.18)
\end{aligned}$$



In (2.18), the previously undefined term  $\frac{\partial T}{\partial x_1} 0,1$  is given by (2.19)

$$\frac{\partial T}{\partial x_1} 0,1 = 1 - P^D x_1 \frac{f^\Xi x_1}{\mu^\Xi} \quad (2.19)$$

and  $\frac{\partial U_i}{\partial x_1} 1$  is defined in (2.20).

$$\frac{\partial U_i}{\partial x_1} 1 = P^D x_1 p z_i | x_1 \frac{f^\Xi x_1}{\mu^\Xi} \quad (2.20)$$

Similarly, the second factorial moment with respect to two impulses at  $x_1$  and  $x_2$  is given by (2.21)

$$\frac{\partial^2 G^{\Xi \vee Y}}{\partial x_1 \partial x_2} 1 = \frac{\partial^2 A}{\partial x_1 \partial x_2} 0,1 = \frac{C_2 x_1, x_2}{A 0,1} \quad (2.21)$$

where  $C_2 x_1, x_2$  is defined in (2.22)

$$C_2 x_1, x_2 = p 2, \cdot \left[ \begin{array}{l} \left[ 2 \frac{\partial T}{\partial x_1} 0,1 \frac{\partial T}{\partial x_2} 0,1 + 2 \sum_{i=1}^k \frac{\frac{\partial U_i}{\partial x_1} 1}{\lambda z_i} \frac{\partial T}{\partial x_2} 0,1 + \frac{\partial U_i}{\partial x_2} 1 \frac{\partial T}{\partial x_1} 0,1 \right] \\ + 2 \sum_{i=1}^k \sum_{\substack{j=1 \\ j \neq i}}^k \frac{\frac{\partial U_i}{\partial x_1} 1}{\lambda z_i} \frac{\frac{\partial U_j}{\partial x_2} 1}{\lambda z_j} + \frac{\partial U_j}{\partial x_1} 1 \frac{\partial U_i}{\partial x_2} 1 \end{array} \right] \quad (2.22)$$

Using (1.5), the pair correlation function  $\rho x_1, x_2$  for the CPHD filter with the truncated canonical number pmf is given by (2.23).

$$\rho x_1, x_2 = \frac{m_2 x_1, x_2}{m_1 x_1 m_1 x_2} = \frac{A 0,1 C_2 x_1, x_2}{C_1 x_1 C_1 x_2} \quad (2.23)$$

The non-separable structure of the pair correlation function of (2.23) shows that second factorial moment of the Bayes posterior process is coupled, that is the second factorial moment cannot be written as the product of first moments even for distant target pairs. This result indicates that there exist interactions that do not

vanish with increasing inter-target distance in the Bayes posterior target process of the CPHD filter.

#### 4. CONCLUSIONS

In this paper, the spooky interaction between distant targets in the CPHD filter is investigated using the pair correlation function. It is shown that the spooky interaction between distant targets in the CPHD filter is a direct result of its independent identical cluster process (IIDCP) target model. In contrast to the PHD filter, pair correlation in the CPHD filter does not vanish for distant targets. Therefore, the *spooky effect* in the CPHD filter is not due to *indistinguishable nature of the elements in unlabeled random finite set*, but due to the target process approximation of the CPHD filter.

#### ÖZET

Sayal olasılık hipotez yoğunluk (cardinalized probability hypothesis density, CPHD) filtresinde gözlenen uzak hedefler arasındaki tuhaf etkileşim ikili korelasyon fonksiyonu ile incelenmiştir. Uzak hedefler arasındaki tuhaf etkileşimin CPHD filtresi tarafından kullanılan bağımsız özdeşçe dağılmış öbek hedef süreci (independent identically distributed cluster process) modeli kaynaklı olduğu gösterilmiştir. CPHD filtresinde gözlenen uzak hedefler arasındaki tuhaf etkileşim, işaretlenmemiş rastgele sonlu küme formülasyonundaki elemanlarının ayır edilemez yapısından değil, CPHD filtresinin kullandığı özel hedef süreci yaklaşımından kaynaklanmaktadır.

**ANAHTAR KELİMELER:** Bağımsız özdeşçe dağılmış öbek süreci, IIDCP, CPHD filtresi, uzakta tuhaf etkileşim, ikili korelasyon fonksiyonu

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