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BATTERY CAPACITY ESTIMATION WITH INVERSE DISTANCE WEIGHTING

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ABSTRACT

This work presents a battery capacity estimation method based on inverse distance weighting multivariate interpolation. The proposed method uses a parametric approach based on a generic rechargeable battery model. Battery model parameters are estimated with an Extended Kalman Filtering based algorithm. Battery capacity estimation is performed with an inverse distance weighting multivariate interpolation model based upon the estimated battery parameters. The proposed method is tested against a set of Ni-Mh batteries and it is concluded that this method is feasible for practical applications.

KEYWORDS: Rechargeable batteries, extended Kalman filtering, inverse distance weighting, estimation.

INTRODUCTION

Today, rechargeable batteries become more important for both consumer electronics and industrial applications as the usage of modern portable electronic devices increases. As a consequence of this, battery state-of-health (SOH) estimation becomes necessary to get the maximum performance from batteries. Battery state-of-health can be defined as the maximum battery capacity, which a battery can deliver during discharge.

To estimate the battery SOH, a battery model should be used. There are various types of battery models in the literature. Electrochemical models have a complex structure with a large number of parameters [1,2]. In electrical circuit models, battery parameters are defined as circuit parameters [3-11]. These models have the capability of the analytical insight. In mathematical models, battery terminal voltage is defined as a mathematical equation and the battery parameters are the variables of this equation, which are obtained from battery tests [12]. In impedance-based models, battery is modeled as an impedance circuit [13].

There are several studies for the battery SOH estimation in the literature. Some of them use the impedance measurement method [13]. In some studies, battery capacity is estimated as a state variable of a dynamic system [11,12].

In this work, battery SOH is estimated with a multivariate interpolation method. In particular, inverse distance weighting method is used to estimate the battery capacity. Battery model parameters are obtained via a Kalman filtering based estimation algorithm. These estimated parameters are used as a feature vector for each test battery. The proposed battery capacity estimation method is tested on a set of Ni-Mh batteries. Estimation results are compared with the measurements.

BATTERY MODEL

In this work, a generic electrical circuit model [14,15] shown in Figure 1 is used as a rechargeable battery model to estimate the battery SOH. This model captures the basic structure and dynamics of rechargeable batteries.

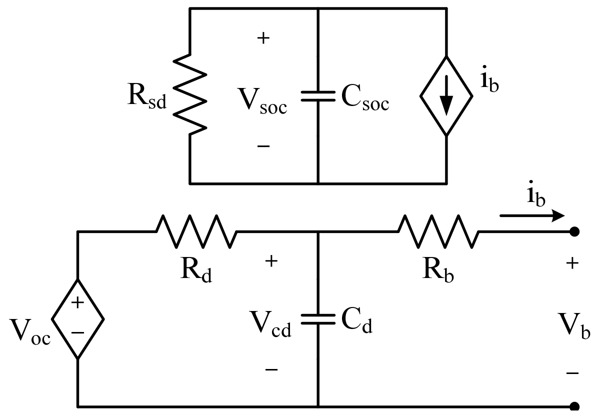

Figure 1: Generic rechargeable battery model

Here, V_{soc} is the voltage drop on the capacitor C_{soc} and will be assumed to take values between 0V and 1V. 0V will indicate that the battery is empty and 1V shows that the battery is 100% full. The value of C_{soc} is the battery capacity in terms of ampere-seconds. R_{sd} is the battery self-discharge resistance. R_b is the battery resistance. Diffusion time constant T_d is calculated from the product of the diffusion capacitance C_d with the diffusion resistance R_d. V_b and i_b are the battery terminal voltage and the battery terminal current respectively. V_{oc} represents the battery open-circuit voltage. There is a relation between open-circuit voltage and battery state-of-charge (SOC) [3] as seen in Eq 1.

$$V_{oc} = mV_{soc} + n \tag{1}$$

Here, constants m and n are nominal values depending on battery type.

MODEL PARAMETER ESTIMATION

In this work, parameters of the battery model shown in Figure 1 are estimated with an Extended Kalman Filter based estimation algorithm. The state space equations of the battery model are written as follows:

$$\dot{V}_{cd} = -\frac{1}{R_d C_d} V_{cd} + \frac{m}{R_d C_d} V_{soc} + \frac{n}{R_d C_d} - \frac{1}{C_d} i_b \tag{2}$$

$$\dot{V}_{soc} = -\frac{1}{R_{sd} C_{soc}} V_{soc} - \frac{1}{C_{soc}} i_b \tag{3}$$

Here, self-discharge resistance R_{sd} is assumed to be very large and will be ignored. If the state variables are chosen as follows:

$$x_1 = V_{cd} \tag{4}$$

$$x_2 = V_{sc} \tag{5}$$

$$x_3 = \frac{1}{C_d} \tag{6}$$

$$x_4 = R_b \tag{7}$$

state equations of the dynamic model can be written as below:

$$\dot{x}_1 = -\frac{1}{T_d}x_1 + \frac{m}{T_d}x_2 + \frac{n}{T_d} - x_3 i_b \tag{8}$$

$$\dot{x}_2 = -\frac{1}{C_{soc}} i_b \tag{9}$$

$$\dot{x}_3 = 0 \tag{10}$$

$$\dot{x}_4 = 0 \tag{11}$$

Here, T_d is the time constant associated with the battery voltage V_b when the battery is open-circuited and equals to the product of C_d and R_d. T_d can be estimated via a simple open circuit test [14, 15]. C_{soc} is the nominal capacity of the battery in terms of ampere-seconds. The output equation of the model is given as:

$$y = V_b = x_1 - x_4 i_b \tag{12}$$

Because of Eq 8 and Eq 12, state space model is not linear. An Extended Kalman Filter is applied to the dynamic battery model (Eqs 8-12) whose input is the battery current i_b and the output is the battery terminal voltage V_b. Note that, this dynamical model allows us identify battery parameters C_d and R_b provided other battery parameters n, m and T_d are given. In this work, n and m parameters are taken as the nominal values associated with Ni-Mh batteries while the diffusion time constant T_d is determined experimentally by an open circuit test [14, 15].

The first part of the Extended Kalman Filtering method [16,17] is the time update:

$$\hat{x}_{k+1}^- = f(x) \tag{13}$$

$$P_{k+1}^- = A_k P_k A_k' + Q_k \tag{14}$$

$$A_k = \left.\frac{\partial f(x_k)}{\partial x_k}\right|_{x_k = \hat{x}_k} \tag{15}$$

where, *f* denotes the dynamic of the state space model, *P* is the error covariance matrix, and *Q* is the process noise covariance matrix. In this work, process noise covariance matrix is chosen as a positive definite matrix in order to have the flexibility to estimate C_d and R_b as time varying parameters.

The second part of the Extended Kalman Filtering method is the measurement update:

$$K_{k+1} = P_{k+1}^{-} C'_{k+1} (C_{k+1} P_{k+1}^{-} C'_{k+1} + R)^{-1} \tag{16}$$

$$P_{k+1} = (I - K_{k+1} C_{k+1}) P_{k+1}^{-} \tag{17}$$

$$\hat{x}_{k+1} = \hat{x}_{k+1}^{-} + K_{k+1} (y_{k+1} - g(\hat{x}_{k+1}^{-})) \tag{18}$$

$$C_{k+1} = \left. \frac{\partial g(x_{k+1})}{\partial x_{k+1}} \right|_{x_{k+1} = \hat{x}_{k+1}^{-}} \tag{19}$$

where, g denotes the output equation, K is the Kalman gain matrix, and R is the covariance matrix of the measurement noise. In this work, covariance parameter is chosen in accordance with the measurement resolution of our experimental set up.

In order to determine battery model parameters, R_b and C_d for each of the 2.1Ah Ni-Mh batteries in our test set, the extended Kalman filtering algorithm summarized above is run under 2.1 ampere constant loading. Typical examples of these parameter estimation experiment are shown in Figure 2 and Figure 3 respectively.

As seen in Figure 2 and Figure 3; while battery resistance is almost constant during the full load current test, diffusion capacitance displays a time-varying behavior under the same loading condition.

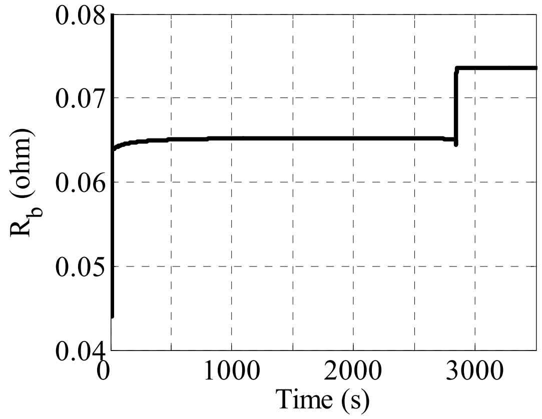

Figure 2: Estimated model parameter: R_b

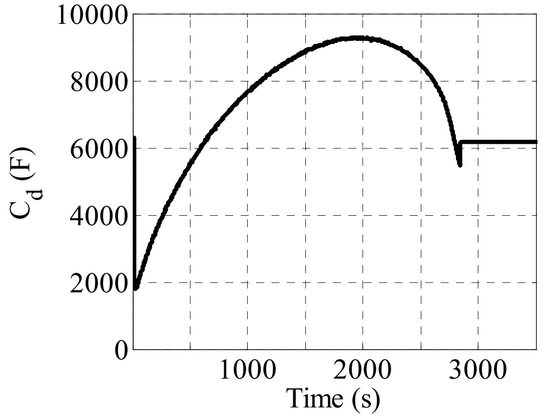

Figure 3: Estimated model parameter: C_d

The measured and estimated battery terminal voltages are given in Figure 4.

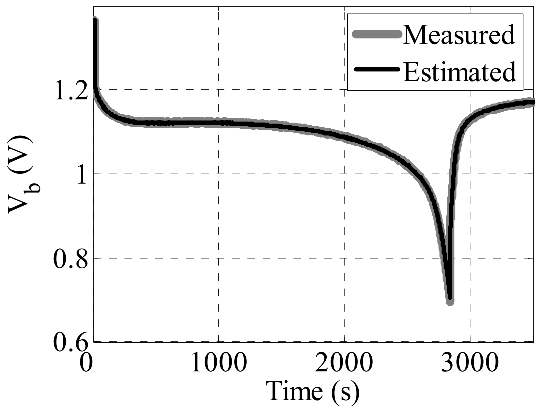

Figure 4: Measured and estimated V_b

As seen in Figure 4, difference between the measured and the estimated battery voltages is extremely small; we also provide the relative absolute estimation error in Figure 5.

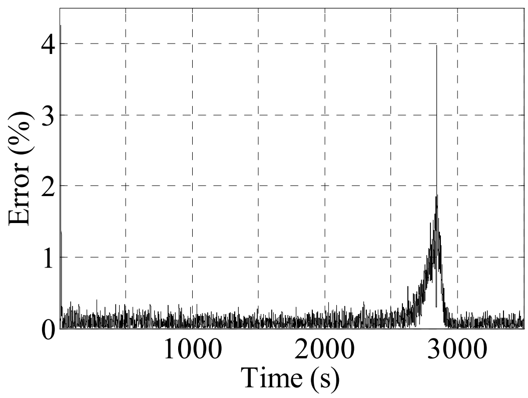

Figure 5: Relative absolute error in battery voltage estimation

The mean of the relative absolute error between the measured and estimated battery terminal voltages is 0.12%. This small error ratio implies that the battery model used in this work can model rechargeable battery dynamics properly. Indeed, this experimental result is the unique experimental verification of the proposed battery parameters estimation framework; because the battery terminal voltage is the only variable that can be measured externally.

INVERSE DISTANCE WEIGHTING METHOD

Inverse distance weighting can be used as a multivariate interpolation method. We take function u as a scalar function on a linear n-dimensional space:

$$u:\ \Re^n \rightarrow \Re\ ,\quad x \in \Re^n\ ,\quad u(x) \in \Re \tag{20}$$

At point x_k, the value of the function u is u_k.

$$u_k = u(x_k)\ ,\quad x_k \in \Re^n\ ,\quad k=1,2,3,\ldots,N \tag{21}$$

Let $d(x,y)$ be a metric on $\Re^n$, which characterizes the distance between point x and point y. If the function $u(x)$ is defined as:

$$u(x) = \frac{\sum_{k=1}^{N} w_k(x) u_k}{\sum_{k=1}^{N} w_k(x)} \tag{22}$$

where,

$$w_k(x) = d^{-1}(x,x_k) \tag{23}$$

then $u(x)$ is an inverse distance weighted multivariate function [18]. As an example, if N=2, then:

$$u(x) = \frac{d^{-1}(x,x_1)}{d^{-1}(x,x_1)+d^{-1}(x,x_2)} u_1 + \frac{d^{-1}(x,x_2)}{d^{-1}(x,x_1)+d^{-1}(x,x_2)} u_2 \tag{24}$$

Any distance function can be used for inverse distance weighting. In this work, in addition to the inverse distance weighting, a modified inverse distance weighting is also used as an interpolation function as given in Eq 25.

$$u(x) = \frac{\sum_{k=1}^{N} w_k(x) u_k}{\sum_{k=1}^{N} w_k(x)} \tag{25}$$

where,

$$w_k(x)=d_k^{-1}(x,x_k) \tag{26}$$

BATTERY CAPACITY ESTIMATION

To estimate the battery capacity with inverse distance weighting multivariate interpolation, 16 Ni-Mh batteries with 2.1Ah capacity were purchased and each battery in the set is labeled from 1 to 16. The battery group labeled from 1 to 8 is called Group 0. An aging procedure was applied to batteries labeled from 9 to 16. This new set was named as Group 2, which can be interpreted as "heavily used" batteries [15]. Finally, battery capacities were measured and estimated for all 16 batteries in two different groups by using the identical parameter identification methodology given in section above.

For the battery capacity estimation, battery capacity C is taken as a multivariate function of three battery parameters (R_b ,C_d and T_d).

$$C=C(R_b,\ C_d,\ T_d)=C(x) \tag{27}$$

The mean (average) measured capacity of 8 unused batteries (C_1) and the mean measured capacity of 8 heavily used batteries (C_2) is calculated and taken as the two support points (μ_0 and μ_2) of the interpolation.

$$C_1=C(x_1)=C(\mu_0)=5920A.s \tag{28}$$

$$C_2=C(x_2)=C(\mu_2)=5713A.s \tag{29}$$

If the distance function is chosen as

$$d(x,y) = (x\text{-}y)^T\ P^{-1}\ (x\text{-}y) \tag{30}$$

where,

$$P=(P_0+P_2)/2 \tag{31}$$

Then the interpolation function with inverse distance weighting is

$$C(x) = \frac{d^{-1}(x,\mu_0)}{d^{-1}(x,\mu_0) + d^{-1}(x,\mu_2)} C_1 + \frac{d^{-1}(x,\mu_2)}{d^{-1}(x,\mu_0) + d^{-1}(x,\mu_2)} C_2 \quad (32)$$

Here, μ_0 and μ_2 are the mean values and P_0 and P_2 are the covariance matrices of Group 0 and Group 2 respectively. Battery capacity estimation is carried out with this interpolation function. Estimated values are compared with the measured battery capacity in Table 1.

The mean absolute error for battery capacity estimations given in Table 1 is 1.48%. Battery labeled 09 has a large absolute error. If this battery is taken out as an outlier; then the mean absolute error for 15 test batteries drops to 0.81%.

In this study, battery capacities are also estimated with a modified inverse distance weighting method. For this modified method, the distance functions in Eqs 24-25 are chosen as Mahalanobis distance functions:

$$d_1(x,x_1) = (x - \mu_0)^T P_0^{-1} (x - \mu_0) \quad (33)$$

$$d_2(x,x_2) = (x - \mu_2)^T P_2^{-1} (x - \mu_2) \quad (34)$$

And, multivariate interpolation function is defined as:

$$C(x) = \frac{d_1^{-1}(x,\mu_0)}{d_1^{-1}(x,\mu_0) + d_2^{-1}(x,\mu_2)} C_1 + \frac{d_2^{-1}(x,\mu_2)}{d_1^{-1}(x,\mu_0) + d_2^{-1}(x,\mu_2)} C_2 \quad (35)$$

When battery capacity estimation is carried out with this modified interpolation function, the mean absolute error is calculated as 2.23%. Results are given in Table 2. If the battery labeled 09 is ignored; then the mean absolute error for 15 test batteries drops to 1.35%.

Table 1: Battery capacity estimation with inverse distance weighting

Battery	Measured C, (A-s)	Estimated C, (A-s)	Absolute error, (%)
01	5898	5883	0.25
02	5927	5903	0.41
03	5953	5895	0.97
04	5944	5908	0.61
05	5902	5917	0.26
06	5912	5914	0.03
07	5905	5906	0.02
08	5918	5917	0.02
09	5230	5735	9.67
10	5681	5779	1.72
11	5814	5754	1.02
12	5512	5763	4.55
13	5842	5812	0.51
14	5847	5799	0.82
15	5855	5749	1.81
16	5927	5864	1.07

Table 2: Battery capacity estimation results of modified inverse distance weighting (distance function is a Mahalanobis function)

Battery	Measured C, (A-s)	Estimated C, (A-s)	Absolute error, (%)
01	5898	5809	1.51
02	5927	5854	1.22
03	5953	5835	1.97
04	5944	5862	1.38
05	5902	5846	0.95
06	5912	5872	0.68
07	5905	5783	2.06
08	5918	5873	0.76
09	5230	5714	9.25
10	5681	5714	0.58
11	5814	5714	1.73
12	5512	5715	3.68
13	5842	5739	1.77
14	5847	5721	2.15
15	5855	5714	2.41
16	5927	5715	3.57

As a third alternative, distance function is chosen as the negative of the quadratic discriminant function for modified inverse distance weighting interpolation where d_1 and d_2 can be given as follows:

$$d_1(x,\mu_0) = \frac{1}{2}\log\left|P_0\right| + \frac{1}{2}(x-\mu_0)^T P_0^{-1}(x-\mu_0) - \log \pi_0 \tag{36}$$

$$d_2(x,\mu_2) = \frac{1}{2}log|P_2| + \frac{1}{2}(x-\mu_2)^T P_2^{-1}(x-\mu_2) - log\,\pi_2 \quad (37)$$

Here, π_0 and π_2 are a priori probabilities of Group 0 and Group 2 respectively which both are taken as 0.5. When battery capacity estimation is performed; the mean absolute error is calculated as 1.86%. Results are given in Table 3.

Table 3: Battery capacity estimation results of modified inverse distance weighting (distance function is a quadratic discriminant function)

Battery	Measured C, (A-s)	Estimated C, (A-s)	Absolute error, (%)
01	5898	5859	0.65
02	5927	5883	0.74
03	5953	5871	1.38
04	5944	5884	1.01
05	5902	5879	0.39
06	5912	5889	0.38
07	5905	5854	0.87
08	5918	5890	0.48
09	5230	5715	9.27
10	5681	5715	0.60
11	5814	5714	1.72
12	5512	5720	3.77
13	5842	5784	0.99
14	5847	5742	1.79
15	5855	5716	2.38
16	5927	5727	3.37

If the battery labeled 09 is taken out again, then the mean absolute error for 15 test batteries drops to 1.05%.

CONCLUSIONS

In this work, a battery capacity estimation methodology based on inverse distance weighting multivariate interpolation is proposed. A generic electrical circuit model is proposed and used as a rechargeable battery model. Aside from some parameters taken at their nominal values depending on type of the batteries; the batteries are characterized with three measurable parameters. These three battery parameters are measured with an Extended Kalman Filtering based estimation algorithm. In order to estimate the battery capacity, battery capacity is taken as a multivariate function of measured battery parameters and an inverse distance weighting interpolation model is applied to test batteries. Absolute errors between estimated and measured battery capacities are calculated and the mean absolute error is found reasonably small. While the classical inverse distance weighting interpolation methodology gives the best results; we also observed that a modified methodology, which makes use of different distance metrics for interpolation support points, is also of some merit. We conclude that, the proposed methodology of estimating rechargeable battery capacity as a multivariate function of battery parameters is a feasible one. By means of experimental results, it is also shown that battery capacity functions can be interpolated via inverse distance weighted functions based upon battery parameters estimated by extended Kalman filtering.

ÖZET

Bu çalışmada, ters uzaklık ağırlıklı çok değişkenli aradeğerlemeye dayanarak, yeniden doldurulabilir bataryaların kapasite kestirimine yönelik bir yöntem sunulmaktadır. Önerilen yöntem, yeniden doldurulabilen bataryalar için genel bir batarya modelini esas alan parametrik bir yaklaşım kullanmaktadır. Batarya model parametreleri, Genişletilmiş Kalman Filtre tabanlı bir algoritma ile kestirilmektedir. Kestirilen batarya parametrelerine dayanarak batarya kapasite kestirimi çok değişkenli bir aradeğerleme modeli olan ters uzaklık ağırlıklandırma ile gerçekleştirilmektedir. Önerilen metodoloji, Ni-Mh bataryalar üzerinden sınanmıştır ve pratik uygulamalarda uygulanabilir olarak değerlendirilmektedir.

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