Araştırma Makalesi / *Research Article* Iğdır Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 9(3): 1633-1645, 2019
Matematik / Mathematics *Journal of the Institute of Science and Technology*, 9(3): 1633-1645, 2019

DOI: 10.21597/jist.512911 ISSN: 2146-0574, eISSN: 2536-4618

New Exact Solutions of Fractional Fitzhugh-Nagumo Equation

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ABSTRACT: The main aim of this article is obtaining the travelling wave, solitary wave and periodic wave solutions for time fractional Fitzhugh-Nagumo equation which used as a model for reaction–diffusion, transmission of nerve impulses, circuit theory, biology and population genetics. The new extended direct algebraic method is employed for this aim. The fractional derivative is in conformable sense which is an applicable, well behaved and understandable definition.

Keywords: Conformable fractional derivative, New extended direct algebraic method, Fitzhugh-Nagumo equation

Kesirli Fitzhugh-Nagumo Denkleminin Yeni Tam Çözümleri

ÖZET: Bu makaledeki ana amaç, reaksiyon difüzyon, sinir sinyallerinin yayılımında, devre teorisi, biyoloji ve popülasyon genetiği modeli olarak kullanılan zaman kesirli Fitzhugh-Nagumo denkleminin hareketli dalga, soliter dalga ve periyodik dalga çözümlerini elde etmektir. Bu amaç için yeni genişletilmiş direkt cebirsel yöntem kullanılmıştır. Kesirli türev ifadesi uygulanabilir, iyi tanımlı ve anlaşılabilir bir tanım olan conformable türündendir.

Anahtar Kelimeler: Conformable Kesirli Türev, Yeni Genişletilmiş Direk Cebirsel Yöntem, Fitzhugh-Nagumo equation

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Geliş tarihi / *Received:*15.01.2019
Kabul tarihi / *Accepted:*14.06.2019

INTRODUCTION

In this article authors obtained the new travelling and solitary wave solutions of time fractional Fitzhugh-Nagumo equation

$$D_t^{\gamma} u - D_x^2 u = u(u-s)(1-u) \qquad (1)$$

which is an important nonlinear reaction–diffusion equation and generally handled for modeling the transmission of nerve impulses (Fitzhugh, 1961; Nagumo et al., 1962) also handled in circuit theory, biology and the area of population genetics (Aronson et al., 1978) as mathematical models with the aid of computer software called Mathematica. There many studies including different numerical or analytical methods for obtaining the solutions of Fitzhugh-Nagumo model. For instance, Li and Guo (Li and Guo, 2006) used first integral method for acquiring the exact solutions of Fitzhugh-Nagumo model. Abbasbandy used homotopy analysis method (Abbasbandy, 2008) to get the approximate analytical solutions of Fitzhugh-Nagumo equation. Hariharan and Kannan employed haar wavelet method to get the numerical solutions of Fitzhugh-Nagumo equation (Hariharan and Kannan, 2010). Also, Kumar *et al.* presented a new method which is combination of q-homotopy analysis approach and Laplace transform approach to evaluate the numerical results for the Fitzhugh–Nagumo equation of fractional order (Kumar *et al.*, 2018).

We comprehensively obtained the exact solutions of Fitzhugh-Nagumo equation by applying wave transform and new extended direct algebraic method. By this transform the nonlinear fractional partial differential equation turns into nonlinear differential equation with integer order. After then the new extended direct algebraic method is employed to acquire new wave solutions.

Our work reported here is a first step towards understanding structural and physical behavior of reaction–diffusion, transmission of nerve impulses models and also circuit theory, biology and the area of population genetics. We hope that our work will be very useful in better understanding the models that the Fitzhugh-Nagumo equation corresponds. To the best of our knowledge all the obtained results are new and firstly seen in the literature.

Fractional calculus which is known as derivation and integration with arbitrary order is as old as known calculus. At the beginning the fractional calculus is seen as a great paradox. The proof of this expression is implicit in the letter L'Hospital to Leibniz which can be considered as the beginning of the adventure of this attractive subject. Fractional calculus remained dormant from the 17th century to the 20th century. Then during the last three decades fractional calculus has seen the value it deserves. Studies on the complex and nonlinear nature models made this subject valuable. Because scientists understood that using the fractional derivation or integration is the best way for establishing a model of nonlinear and complex natural phenomena (Kurt et al., 2017; Tasbozan et al., 2018b; Tasbozan et al., 2017). While the scientists were modeling the real world events, they used some definitions of fractional derivative and integral as tools (Cenesiz et al., 2017; Tasbozan et al., 2018a). The most popular ones are Riemann-Lioville and Caputo type fractional derivatives and integrals (Atangana, 2015). Caputo type definitions uses integer order derivative in the initial conditions instead of fractional order. This property makes Caputo type definitions one step ahead. But recently some deficiencies of Caputo and Riemann-Liouville type derivative definitions are declared by Atangana (Atangana, 2015). Atangana declared some criterias (Atangana, 2015) that need to be satisfied for a given operator to be called fractional derivative. Some of them do not satisfied by Caputo and Riemann-Liouville type fractional derivatives. For instance

- Both Riemann-Liouville and Caputo fractional operators do not satisfy chain rule.
- Both Riemann-Liouville and Caputo fractional operators do not describe the rate of change of the function near the input value.
- Both Riemann-Liouville and Caputo fractional operators do not satisfy the reciprocal rule.
- Riemann-Liouville and Caputo derivative operators do not satisfy quotient rule.

The rest of article is organized as follows. In section 2 basic definitions and theorems for conformable fractional calculus are expressed. In section 3 description of the implemented method called new extended direct algebraic method is given. In section 4 the travelling and solitary wave solutions of time fractional Fitzhugh-Nagumo are obtained by employing the new extended direct algebraic method.

BASIC DEFINITIONS

To get rid of these deficiencies scientists studied to express a new definition. Khalil et al. (Khalil et al., 2014) presented a new, applicable and understandable definition of derivative and integral with fractional order called “conformable fractional derivative and integral” which overcomes above mentioned deficiencies.

Definition 1. Let, $f:(0,\infty)\to\mathbb{R}$ be a function. Then, γ th order conformable fractional derivative of f is defined as

$$D_t^{\gamma} f(t)=\lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\gamma})-f(t)}{\varepsilon}, \tag{1}$$

for all $t>0,\ \gamma\in(0,1)$ (Khalil et al., 2014).

Definition 2. Suppose that $a\geq 0$ and $t\geq a$. Also, let f be a function defined on $(a,t]$. Then, the $\gamma-$ fractional integral of f is defined by,

$${}_tI_a^{\gamma} f(t)=\int_a^t \frac{f(x)}{x^{1-\gamma}}dx$$

where the Riemann improper integral exist (Khalil et al., 2014). Some basic properties of conformable fractional derivative is given below (Khalil et al., 2014; Abdeljawad, 2015).

1. $\frac{d^{\gamma}}{dt^{\gamma}}(t^{\eta})=\eta t^{\eta-\gamma}.$

2. $\frac{d^{\gamma}}{dt^{\gamma}}\left(f(t)g(t)\right)=f(t)\frac{d^{\gamma}}{dt^{\gamma}}\left(g(t)\right)+g(t)\frac{d^{\gamma}}{dt^{\gamma}}\left(f(t)\right).$

3. $\frac{d^{\gamma}}{dt^{\gamma}}\left(\frac{f(t)}{g(t)}\right)=\frac{g(t)\frac{d^{\gamma}}{dt^{\gamma}}\left(f(t)\right)-f\frac{d^{\gamma}}{dt^{\gamma}}\left(g(t)\right)}{g^2(t)}.$

4. $\frac{d^{\gamma}}{dt^{\gamma}}(c)=0$ for all constant functions $f(t)=c.$

where $\gamma \in (0,1)$.

Theorem 1. ***(Chain Rule)*** (Abdelhawad, 2015) Assume $f, g:(a,\infty) \to \mathbb{R}$ be $\gamma-$differentiable functions, where $0<\gamma \le 1$. Let $h(t)=f\left(g(t)\right)$. Then $h(t)$ is $\gamma-$differentiable and for all t with $t \ne a$ and $g(t) \ne 0$ we have

$$\left(D_\gamma^a h\right)(t)=\left(D_\gamma^a f\right)\left(g(t)\right)\left(D_\gamma^a g\right)(t)\, g(t)^{\gamma-1}.$$

MATERIALS AND METHODS

Now lets represent the new extended direct algebraic method (Rezazadeh *et al.*, 2019). The method is used several times for obtaining the different types of partial differential equations (Rezazadeh, 2018a, Rezazadeh *et al.*, 2018b). Consider the nonlinear time fractional partial differential equation of the form

$$T\left(u, D_t^\gamma u, D_x u, D_t^{(2\gamma)} u, D_x^2 u, \ldots\right)=0 \tag{2}$$

where u is an unknown function and $D_t^{(m\gamma)}$ means $m(m \in \mathbb{Z}^+)$ times conformable fractional derivative of the function u. Regarding the wave transformation, as

$$u(x,t)=U(\xi), \qquad \xi = kx + w\frac{t^\gamma}{\gamma}, \tag{3}$$

where k and w are arbitrary constants to be evaluated later. Using the chain rule (Abdeljawad, 2015) and wave transform (3) in Eq. (2), led to following nonlinear ordinary differential equation

$$G\left(U, U', U'', \ldots\right)=0, \tag{4}$$

where prime denotes the integer order derivative of function U with respect to ξ. Suppose that Eq. the solution of Eq. (4) can be expressed as the following form

$$U(\xi)=\sum_{i=0}^{M} a_i Q^i(\xi), \quad a_M \ne 0, \tag{5}$$

where $a_i\left(0 \le i \le M\right)$ are constants coefficients to be examined onwards, M is a positive integer which is evaluated by balancing procedure in Eq. (4) and $Q(\xi)$ ensures the ODE

$$Q'\left(\xi\right)=Ln(A)\left(\alpha+\beta Q(\xi)+\sigma Q^2(\xi)\right), \quad A \ne 0,1, \tag{6}$$

where α, β and σ are constants. The solution set of Eq. (6) are given as follows.

1) Assume that $\beta^2 - 4\alpha\sigma < 0$ and $\sigma \ne 0$,

$$Q_1(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\tan_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2}\xi\right),$$

$$Q_2(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\cot_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2}\xi\right),$$

$$Q_3(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\left(\tan_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\xi\right)\pm\sqrt{pq}\sec_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\xi\right)\right),$$

$$Q_4(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\left(-\cot_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\xi\right)\pm\sqrt{pq}\csc_A\left(\sqrt{-\left(\beta^2-4\alpha\sigma\right)}\xi\right)\right),$$

$$Q_5(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{4\sigma}\left(\tan_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{4}\xi\right)-\cot_A\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{4}\xi\right)\right).$$

2) Suppose that $\beta^2-4\alpha\sigma>0$ and $\sigma\neq 0$,

$$Q_6(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\tanh_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{2}\xi\right),$$

$$Q_7(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\coth_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{2}\xi\right),$$

$$Q_8(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\left(-\tanh_A\left(\sqrt{\beta^2-4\alpha\sigma}\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(\sqrt{\beta^2-4\alpha\sigma}\xi\right)\right),$$

$$Q_9(\xi)=-\frac{\beta}{2\sigma}+\frac{\sqrt{\beta^2-4\alpha\sigma}}{2\sigma}\left(-\coth_A\left(\sqrt{\beta^2-4\alpha\sigma}\xi\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(\sqrt{\beta^2-4\alpha\sigma}\xi\right)\right),$$

$$Q_{10}(\xi)=-\frac{\beta}{2\sigma}-\frac{\sqrt{\beta^2-4\alpha\sigma}}{4\sigma}\left(\tanh_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{4}\xi\right)+\coth_A\left(\frac{\sqrt{\beta^2-4\alpha\sigma}}{4}\xi\right)\right).$$

3) Consider that $\alpha\sigma>0$ and $\beta=0$,

$$Q_{11}(\xi)=\sqrt{\frac{\alpha}{\sigma}}\tan_A\left(\sqrt{\alpha\sigma}\xi\right),$$

$$Q_{12}(\xi)=-\sqrt{\frac{\alpha}{\sigma}}\cot_A\left(\sqrt{\alpha\sigma}\xi\right),$$

$$Q_{13}(\xi)=\sqrt{\frac{\alpha}{\sigma}}\left(\tan_A\left(2\sqrt{\alpha\sigma}\xi\right)\pm\sqrt{pq}\sec_A\left(2\sqrt{\alpha\sigma}\xi\right)\right),$$

$$Q_{14}(\xi)=\sqrt{\frac{\alpha}{\sigma}}\left(-\cot_A\left(2\sqrt{\alpha\sigma}\xi\right)\pm\sqrt{pq}\csc_A\left(2\sqrt{\alpha\sigma}\xi\right)\right),$$

$$Q_{15}(\xi)=\frac{1}{2}\sqrt{\frac{\alpha}{\sigma}}\left(\tan_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)-\cot_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)\right).$$

4) Regard that $\alpha\sigma<0$ and $\beta=0$,

$$Q_{16}(\xi)=-\sqrt{-\frac{\alpha}{\sigma}}\tanh_A\left(\sqrt{-\alpha\sigma}\xi\right),$$

$$Q_{17}(\xi)=-\sqrt{-\frac{\alpha}{\sigma}}\coth_A\left(\sqrt{-\alpha\sigma}\xi\right),$$

$$Q_{18}(\xi)=\sqrt{-\frac{\alpha}{\sigma}}\left(-\tanh_A\left(2\sqrt{-\alpha\sigma}\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\sqrt{-\alpha\sigma}\xi\right)\right),$$

$$Q_{19}(\xi)=\sqrt{-\frac{\alpha}{\sigma}}\left(-\coth_A\left(2\sqrt{-\alpha\sigma}\xi\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(2\sqrt{-\alpha\sigma}\xi\right)\right),$$

$$Q_{20}(\xi)=-\frac{1}{2}\sqrt{-\frac{\alpha}{\sigma}}\left(\tanh_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)+\coth_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)\right).$$

5) When $\beta=0$ and $\sigma=\alpha$,

$$Q_{21}(\xi)=\tan_A\left(\alpha\xi\right),$$

$$Q_{22}(\xi)=-\cot_A\left(\alpha\xi\right),$$

$$Q_{23}(\xi)=\tan_A\left(2\alpha\xi\right)\pm\sqrt{pq}\sec_A\left(2\alpha\xi\right),$$

$$Q_{24}(\xi)=-\cot_A\left(2\alpha\xi\right)\pm\sqrt{pq}\csc_A\left(2\alpha\xi\right),$$

$$Q_{25}(\xi)=\frac{1}{2}\left(\tan_A\left(\frac{\alpha}{2}\xi\right)-\cot_A\left(\frac{\alpha}{2}\xi\right)\right).$$

6) If $\beta=0$ and $\sigma=-\alpha$, chosen

$$Q_{26}(\xi)=-\tanh_A\left(\alpha\xi\right),$$

$$Q_{27}(\xi)=-\coth_A\left(\alpha\xi\right),$$

$$Q_{28}(\xi) = -\tanh_A\left(2\alpha\xi\right) \pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\alpha\xi\right),$$

$$Q_{29}(\xi) = -\coth_A\left(2\alpha\xi\right) \pm \sqrt{pq}\,\mathrm{csch}_A\left(2\alpha\xi\right),$$

$$Q_{30}(\xi) = -\frac{1}{2}\left(\tanh_A\left(\frac{\alpha}{2}\xi\right) + \coth_A\left(\frac{\alpha}{2}\xi\right)\right).$$

7) While $\beta^2 = 4\alpha\sigma$,

$$Q_{31}(\xi) = \frac{-2\alpha(\beta\xi LnA + 2)}{\beta^2\xi LnA}.$$

8) When $\beta = k$,$\alpha = mk(m \neq 0)$ and $\sigma = 0$,

$$Q_{32}(\xi) = A^{k\xi} - m.$$

9) When $\beta = \sigma = 0$,

$$Q_{33}(\xi) = \alpha\xi LnA.$$

10) When $\beta = \alpha = 0$,

$$Q_{34}(\xi) = \frac{-1}{\sigma\xi LnA}.$$

11) When $\alpha = 0$ and $\beta \neq 0$,

$$Q_{35}(\xi) = -\frac{p\beta}{\sigma\left(\cosh_A(\beta\xi) - \sinh_A(\beta\xi) + p\right)},$$

$$Q_{36}(\xi) = -\frac{q\beta}{\sigma\left(\cosh_A(\beta\xi) - \sinh_A(\beta\xi) + q\right)},$$

$$Q_{37}(\xi) = -\frac{\beta\left(\sinh_A(\beta\xi) + \cosh_A(\beta\xi)\right)}{\sigma\left(\sinh_A(\beta\xi) + \cosh_A(\beta\xi) + q\right)},$$

12) When $\beta = k$,$\sigma = mk(m \neq 0)$ and $\alpha = 0$,

$$Q_{38}(\xi) = \frac{pA^{k\xi}}{p - mqA^{k\xi}}.$$

where ξ is an independent variable, p and q are arbitrary constants greater than zero and named as deformation parameters. Subrogating Eqs. (5) and (6) into Eq. (4) and zeroing the coefficients of $Q^i(\xi)$, we acquire an algebraic equation system with respect to $a_i\,(i = 0,1,\ldots,M)$ and k and c. Then substituting the obtained results of constants and solution set of Eq. (6) into Eq. (5) by using the wave transform (3), we obtain the exact wave solutions for Eq. (2).

Remark 1. The generalized hyperbolic and triangular functions are defined as

$$\sinh_A(\xi)=\frac{pA^{\xi}-qA^{-\xi}}{2},\ \cosh_A(\xi)=\frac{pA^{\xi}+qA^{-\xi}}{2},$$

$$\tanh_A(\xi)=\frac{pA^{\xi}-qA^{-\xi}}{pA^{\xi}+qA^{-\xi}},\ \coth_A(\xi)=\frac{pA^{\xi}+qA^{-\xi}}{pA^{\xi}-qA^{-\xi}},$$

$$\operatorname{sech}_A(\xi)=\frac{2}{pA^{\xi}+qA^{-\xi}},\ \operatorname{csch}_A(\xi)=\frac{2}{pA^{\xi}-qA^{-\xi}},$$

$$\sin_A(\xi)=\frac{pA^{i\xi}-qA^{-i\xi}}{2i},\ \cos_A(\xi)=\frac{pA^{i\xi}+qA^{-i\xi}}{2},$$

$$\tan_A(\xi)=-i\frac{pA^{i\xi}-qA^{-i\xi}}{pA^{i\xi}+qA^{-i\xi}},\ \cot_A(\xi)=i\frac{pA^{i\xi}+qA^{-i\xi}}{pA^{i\xi}-qA^{-i\xi}},$$

$$\sec_A(\xi)=\frac{2}{pA^{i\xi}+qA^{-i\xi}},\ \csc_A(\xi)=\frac{2i}{pA^{i\xi}-qA^{-i\xi}},$$

RESULTS AND DISCUSSION

Considering the time fractional Fitzhugh–Nagumo equation (1) where $0<\gamma<1$ and D_t^{γ} denotes the conformable derivative operator with fractional order. Employing the chain rule (Abdeljawad, 2015) and wave transform Eq. (1) turns into nonlinear ordinary differential equation

$$wU_{\xi}-k^2U_{\xi\xi}-U(U-s)(1-U)=0. \tag{7}$$

Supposing that the Eq. (7) has the solution in the form (5). By the balancing procedure one can find $M=1$. So the solution becomes as follows.

$$u(\xi)=a_0+a_1Q(\xi). \tag{8}$$

Substituting Eq. (8) into (7), collecting the coefficients of $Q^i(\xi)$ together and equating them to zero led to a set of algebraic equations with respect to a_0, a_1, k and w. Solving these algebraic equations with the help of computer software, we acquire

$$\begin{aligned} a_0 &= \frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right), \\ a_1 &= \sqrt{2}k\sigma LnA, \\ w &= \frac{1-2s}{2\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}, \\ k &= -\frac{1}{\sqrt{2Ln^2A(\beta^2-4\alpha\sigma)}}. \end{aligned} \tag{9}$$

So for the given conditions expressed below, the solutions can be obtained as follows

When $\beta^2-4\alpha\sigma<0$ and $\sigma\neq 0$,

$$U_1(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}+\frac{\sqrt{-\Delta}}{2\sigma}\tan_A\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right),$$

$$U_2(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}-\frac{\sqrt{-\Delta}}{2\sigma}\cot_A\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right),$$

$$U_3(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}+\frac{\sqrt{-\Delta}}{2\sigma}\left(\tan_A\left(\sqrt{-\Delta}\xi\right)\pm\sqrt{pq}\sec_A\left(\sqrt{-\Delta}\xi\right)\right)\right),$$

$$U_4(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}+\frac{\sqrt{-\Delta}}{2\sigma}\left(-\cot_A\left(\sqrt{-\Delta}\xi\right)\pm\sqrt{pq}\csc_A\left(\sqrt{-\Delta}\xi\right)\right)\right),$$

$$U_5(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}+\frac{\sqrt{-\Delta}}{4\sigma}\left(\tan_A\left(\frac{\sqrt{-\Delta}}{4}\xi\right)-\cot_A\left(\frac{\sqrt{-\Delta}}{4}\xi\right)\right)\right)$$

where $\Delta=\beta^2-4\alpha\sigma$ and $\xi=-\frac{x}{\sqrt{2}\sqrt{\Delta Ln^2A}}+\frac{(1-2s)t^{\gamma}}{2\gamma\sqrt{\Delta Ln^2A}}$.

When $\beta^2-4\alpha\sigma>0$ and $\sigma\neq 0$,

$$U_6(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}-\frac{\sqrt{\Delta}}{2\sigma}\tanh_A\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right),$$

$$U_7(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}-\frac{\sqrt{\Delta}}{2\sigma}\coth_A\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right),$$

$$U_8(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}+\frac{\sqrt{\Delta}}{2\sigma}\left(-\tanh_A\left(\sqrt{\Delta}\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(\sqrt{\Delta}\xi\right)\right)\right),$$

$$U_9(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}+\frac{\sqrt{\Delta}}{2\sigma}\left(-\coth_A\left(\sqrt{\Delta}\xi\right)\pm\sqrt{pq}\,\mathrm{csch}_A\left(\sqrt{\Delta}\xi\right)\right)\right),$$

$$U_{10}(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{Ln^2A(\beta^2-4\alpha\sigma)}}\right)+\sqrt{2}k\sigma LnA\left(-\frac{\beta}{2\sigma}-\frac{\sqrt{\Delta}}{4\sigma}\left(\tanh_A\left(\frac{\sqrt{\Delta}}{4}\xi\right)+\coth_A\left(\frac{\sqrt{\Delta}}{4}\xi\right)\right)\right)$$

where $\Delta=\beta^2-4\alpha\sigma$ and $\xi=-\frac{x}{\sqrt{2}\sqrt{\Delta Ln^2A}}+\frac{(1-2s)t^{\gamma}}{2\gamma\sqrt{\Delta Ln^2A}}$.

When $\alpha\sigma>0$ and $\beta=0$,

$$U_{11}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(\sqrt{\frac{\alpha}{\sigma}}\tan_A\left(\sqrt{\alpha\sigma}\xi\right)\right),$$

$$U_{12}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(-\sqrt{\frac{\alpha}{\sigma}}\cot_A\left(\sqrt{\alpha\sigma}\xi\right)\right),$$

$$U_{13}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(\sqrt{\frac{\alpha}{\sigma}}\left(\tan_A\left(2\sqrt{\alpha\sigma}\xi\right)\pm\sqrt{pq}\sec_A\left(2\sqrt{\alpha\sigma}\xi\right)\right)\right),$$

$$U_{14}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(\sqrt{\frac{\alpha}{\sigma}}\left(-\cot_A\left(2\sqrt{\alpha\sigma}\xi\right)\pm\sqrt{pq}\csc_A\left(2\sqrt{\alpha\sigma}\xi\right)\right)\right),$$

$$U_{15}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(\frac{1}{2}\sqrt{\frac{\alpha}{\sigma}}\left(\tan_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)-\cot_A\left(\frac{\sqrt{\alpha\sigma}}{2}\xi\right)\right)\right)$$

where $\Delta=-4\alpha\sigma$ and $\xi=-\frac{x}{\sqrt{2}\sqrt{\Delta Log(A)^2}}+\frac{(1-2s)t^{\gamma}}{2\gamma\sqrt{\Delta Log(A)^2}}$.

When $\alpha\sigma<0$ and $\beta=0$,

$$U_{16}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(-\sqrt{-\frac{\alpha}{\sigma}}\tanh_A\left(\sqrt{-\alpha\sigma}\xi\right)\right),$$

$$U_{17}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(-\sqrt{-\frac{\alpha}{\sigma}}\coth_A\left(\sqrt{-\alpha\sigma}\xi\right)\right),$$

$$U_{18}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(\sqrt{-\frac{\alpha}{\sigma}}\left(-\tanh_A\left(2\sqrt{-\alpha\sigma}\xi\right)\pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\sqrt{-\alpha\sigma}\xi\right)\right)\right),$$

$$U_{19}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(\sqrt{-\frac{\alpha}{\sigma}}\left(-\coth_A\left(2\sqrt{-\alpha\sigma}\xi\right)\pm \sqrt{pq}\,\mathrm{csch}_A\left(2\sqrt{-\alpha\sigma}\xi\right)\right)\right),$$

$$U_{20}(\xi)=\frac{1}{2}+\sqrt{2}k\sigma LnA\left(-\frac{1}{2}\sqrt{-\frac{\alpha}{\sigma}}\left(\tanh_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)+\coth_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\xi\right)\right)\right).$$

where $\Delta=-4\alpha\sigma$ and $\xi=-\dfrac{x}{\sqrt{2}\sqrt{\Delta Ln^2A}}+\dfrac{(1-2s)t^{\gamma}}{2\gamma\sqrt{\Delta Ln^2A}}$.

When $\beta=0$ and $\sigma=\alpha$,

$$U_{21}(\xi)=\frac{1}{2}+\sqrt{2}k\alpha LnA\left(\tan_A\left(\alpha\xi\right)\right),$$

$$U_{22}(\xi)=\frac{1}{2}+\sqrt{2}k\alpha LnA\left(-\cot_A\left(\alpha\xi\right)\right),$$

$$U_{23}(\xi)=\frac{1}{2}+\sqrt{2}k\alpha LnA\left(\tan_A\left(2\alpha\xi\right)\pm\sqrt{pq}\sec_A\left(2\alpha\xi\right)\right),$$

$$U_{24}(\xi)=\frac{1}{2}+\sqrt{2}k\alpha LnA\left(-\cot_A\left(2\alpha\xi\right)\pm\sqrt{pq}\csc_A\left(2\alpha\xi\right)\right),$$

$$U_{25}(\xi)=\frac{1}{2}+\sqrt{2}k\alpha LnA\left(\frac{1}{2}\left(\tan_A\left(\frac{\alpha}{2}\xi\right)-\cot_A\left(\frac{\alpha}{2}\xi\right)\right)\right)$$

where $\Delta=-4\alpha^2$ and $\xi=-\dfrac{x}{\sqrt{2}\sqrt{\Delta Ln^2A}}+\dfrac{(1-2s)t^{\gamma}}{2\gamma\sqrt{\Delta Ln^2A}}$.

When $\beta=0$ and $\sigma=-\alpha$,

$$U_{26}(\xi)=\frac{1}{2}-\sqrt{2}k\alpha LnA\left(\tanh_A\left(\alpha\xi\right)\right),$$

$$U_{27}(\xi)=\frac{1}{2}-\sqrt{2}k\alpha LnA\coth_A\left(\alpha\xi\right),$$

$$U_{28}(\xi)=\frac{1}{2}-\sqrt{2}k\alpha LnA\left(\tanh_A\left(2\alpha\xi\right)\mp i\sqrt{pq}\,\mathrm{sech}_A\left(2\alpha\xi\right)\right),$$

$$U_{29}(\xi)=\frac{1}{2}-\sqrt{2}k\alpha LnA\left(\coth_A\left(2\alpha\xi\right)\mp\sqrt{pq}\,\mathrm{csch}_A\left(2\alpha\xi\right)\right),$$

$$U_{30}(\xi)=\frac{1}{2}-\sqrt{2}k\alpha LnA\left(\tanh_A\left(\frac{\alpha}{2}\xi\right)+\coth_A\left(\frac{\alpha}{2}\xi\right)\right).$$

where $\Delta=4\alpha^2$ and $\xi=-\frac{x}{\sqrt{2}\sqrt{\Delta Ln^2A}}+\frac{(1-2s)t^\gamma}{2\gamma\sqrt{\Delta Ln^2A}}$.

When $\alpha=0$ and $\beta\neq0$,

$$U_{31}(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{\beta^2Ln^2A}}\right)-\sqrt{2}k\sigma LnA\left(\frac{p\beta}{\sigma\left(\cosh_A(\beta\xi)-\sinh_A(\beta\xi)+p\right)}\right),$$

$$U_{32}(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{\beta^2Ln^2A}}\right)-\sqrt{2}k\sigma LnA\left(\frac{q\beta}{\sigma\left(\cosh_A(\beta\xi)-\sinh_A(\beta\xi)+q\right)}\right),$$

$$U_{33}(\xi)=\frac{1}{2}\left(1-\frac{\beta LnA}{\sqrt{\beta^2Ln^2A}}\right)-\sqrt{2}k\sigma LnA\left(\frac{\beta\left(\sinh_A(\beta\xi)+\cosh_A(\beta\xi)\right)}{\sigma\left(\sinh_A(\beta\xi)+\cosh_A(\beta\xi)+q\right)}\right),$$

where $\xi=-\frac{x}{\sqrt{2}\sqrt{\beta^2Ln^2A}}+\frac{(1-2s)t^\gamma}{2\gamma\sqrt{\beta^2Ln^2A}}$.

When $\beta=l$, $\sigma=ml(m\neq0)$, $p=q$, $\alpha=0$,

$$U_{36}(\xi)=\frac{1}{2}\left(1-\frac{l\,LnA}{\sqrt{l^2Ln^2A}}\right)+\sqrt{2}klmLnA\left(\frac{A^{l\xi}}{1-mA^{l\xi}}\right)$$

where $\Delta=l^2$ and $\xi=-\frac{x}{\sqrt{2}\sqrt{\Delta Ln^2A}}+\frac{(1-2s)t^\gamma}{2\gamma\sqrt{l^2Ln^2A}}$.

CONCLUSION

In this article authors employed new direct algebraic method to obtain the travelling and solitary wave solution of fractional Fitzhugh-Nagumo equation arising in nonlinear reaction–diffusion equation, transmission of nerve, circuit theory, biology and the area of population genetics. All the obtained results can be useful for the scientists who are going to make further studies on this subject.

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