EVALUATION ON VISUALIZATION AND CONCRETIZATION IN MATHEMATICS EDUCATION

Matematik Eğitiminde Görselleştirme ve Somutlaştırma Üzerine Bir Değerlendirme

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Abstract

This study aims to determine to what extent visualization and concretization gain acceptance by means of comparison of national and international literature about visualization and concretization in mathematics education. In the study, 38 publications from the publication of "Principles and Standards for School Mathematics" by NTCM (1989) to 2013 and international literature about visualization and concretization in mathematics education that are in Turkish and English were analyzed by means of determined databases. Examining the databases based on literature review, 38 publications regarding visualization and concretization in mathematics education, being nationally or internationally published, being in article, paper or thesis formats, having a qualitative, quantitative or mixed method, the distribution of research studies in terms of years and the distribution of them in terms of subject matters were analyzed. As a result of the findings, it was revealed that there are not enough publications in order to say that the subjects regarding the visualization and concretization in mathematics education become widespread but there is a continuous increase in terms of the research studies on visualization in mathematics education. It was seen that the number of the publications on visualization in mathematics education is lower than those conducted in abroad.

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Furthermore, what matters in the research studies on visualization in mathematics education was also revealed in the study.

Key words: *visualization, concretization, mathematics education*

Öz

Yapılan bu çalışmada, matematik eğitiminde görselleştirme ve somutlaştırma konularında ulusal ve uluslararası alan yazımının taranıp karşılaştırılmasıyla matematik eğitiminde görselleştirme ve somutlaştırmanın ne derece kabul gördüğünün ortaya konulması amaçlanmaktadır. Çalışmada, belirlenen veri tabanlar kullanılarak ulusal ve uluslararası alanlarda matematik eğitiminde görselleştirme ve somutlaştırmayla ilgili yayın dili Türkçe ve İngilizce olanlar ve NCTM'nin "Okul Matematiği için Prensipler ve Standartlar" adlı belgesinin yayınlanmasından (1989 yılı) 2013 yılına kadar yayınlanan 38 yayın incelemeye alınmıştır. Literatür taraması sonucunda matematik eğitiminde görselleştirme ve somutlaştırma konularıyla ilgili ulaşılan 38 yayın, çalışmanın yurtiçinde ve yurtdışında yayınlanmış olması; makale, bildiri ve tez formatında olması, yönteminin nicel, nitel ya da karma yöntem olması, yapılan çalışmaların yıllara göre dağılımı ve çalışmaların konulara göre dağılımları incelenmiştir. Elde edilen bulgular sonucunda, matematik eğitiminde görselleştirme ve somutlaştırma konularının yaygınlaştığını söyleyebilmek için yeterli yayın sayısının olmadığı, fakat matematik eğitiminde görselleştirmeyle ilgili yapılan çalışmaların sürekli olarak artış gösterdiği sonucuna ulaşılmıştır. Ülkemizde matematik eğitiminde görselleştirmeyle ilgili yapılan yayın sayısının yurt dışına göre az olduğu görülmüştür. Ayrıca Matematik eğitiminde görselleştirmeyle ilgili yapılan çalışmaların hangi konularda olduğu da çalışmada ortaya konulmuştur.

Anahtar kelimeler: *görselleştirme, somutlaştırma, matematik eğitimi*

Introduction

There has been an increase in scientific research on the necessity to provide education and training with modern methods. The diversity of the research conducted in this field has been analysed from a broad perspective; besides, the effect of the use of visual materials which is one of the requirements of a modern learning environment, and

consequently the effect of visualization seem to increase (Uysal Koğ, 2012). There are various description of visualization in the literature. According to these descriptions, there are different defining ways such as the process of creating images (pictures, illustrations, charts, diagrams, etc.) that serve for the development of visual thinking, concretization of concepts or thoughts considered as abstract (Krutetskii, 1976), the journey from the outside world to the minds of students (Nemirovsky & Noble, 1997), structuring visual mental images and transformation process (Presmeg, 1997). Accordingly, the process of visualization is concluded to be conducted in two different ways. One of them is to envision an event that occurs in the outside world or an object whereas the other is to transfer a structure that is envisioned into physical environment by means of tools such as computer, blackboards, etc. (Zazkis, Dubinsky, & Dautermann, 1996).

The discipline of mathematics has been said to be a science formed by the combination of three different communication resources since the early times. These communication resources are natural (spoken) language (Hieroglyphics, Greek, Turkish, German, English, etc.), symbolic language (signs +, -, =, symbols,: x, y, z, etc.) and shapes. Teacher firstly utilizes verbal expressions while defining or explaining mathematical concepts, and then uses a universal language of mathematical symbols (Alsina & Nelsen, 2006). Symbols are hard to understand and abstract; therefore, it is often difficult to understand them by themselves. Hence, it seems not enough to explain a mathematical subject only verbally or express it by symbols. In that sense, visual objects are in question. Visual objects can enable to transform concepts or expressions considered or accepted as abstract

into a concrete entity. That is, mathematical visualization can be defined as the use of potential concrete illustrations of objects in order to achieve a more effective approach to the abstract relations of mathematics (Guzman, 2002). Visualization in mathematics education is considered as envisioning during the solution of a mathematical problem and the mathematical understanding and process of discovering by means of envisioning (Zimmermann & Cunningham, 1991) whereas it can be defined as reasoning during the process of problem solving by means of visual or spatial skills (Gutierrez, 1996). Therefore, visualization can be considered as an important source in improving mathematical thinking skills, in the process of problem solving and establishing connections between these concepts in this process, teaching abstract concepts and enabling them to become concrete concepts in the light of these definitions.

When the historical development of mathematical visualization analysed, there are three eras that are not separate from each other by certain boundaries but demonstrate the general opinion of that time (Şan, 2008). The Pythagoreans that can be regarded as the first mathematicians in modern sense, proved the relations between numbers and sets of numbers by means of tools such as pebble stones, etc. and thus used a kind of visualization method. Descartes discussed several rules related to the process of visualization in his Regula ad direction ingerie, and these rules highlight the importance of shapes and images on mathematical thinking. In general, these rules can be summarized as follows: the visualization of concepts with the objects known to be present in the human mind while learning facilitates understanding; there is a necessity to reflect abstract or abstruse concepts in people's

imagination by means of pure shapes, which can enable people to gather attention for a longer time (Şan, 2008).

The dominance of a quite strong visual content seems to be present in 17th century mathematics. However, Gauss called mathematics a science of the eye, which shows how much he gives importance to visualization in mathematics. While the emergence of non-Euclidean geometries decreased the effect of visualization in 19th century, the view that visualization can be addressed as a significant research area of mathematics education has been dominant since the beginning of 1990s (Şan, 2008; Uysal Koğ, 2012). As can be seen from the history of mathematics, visualization can be said to be important for mathematics and mathematics education. In that sense, Dufour-Janvier et al. (1987) expressed that visualization is unavoidable for mathematics, and summarized the relation between visualization and mathematics in four key items (as cited in Tekin, 2010: 28). These are "(1) Visual representations are in the nature of mathematics; (2) Visual representations enable the concretization of a concept in various ways; (3) Visual representations are used in dealing with some of the challenges in mathematics; (4) Visual representations aim to make mathematics more interesting and engaging." Alsina & Nelsen (2006) stated that there are two important objectives of using visualization in mathematics courses, and expressed that they are to shorten long lectures by making an appropriate drawing and to facilitate the mental reasoning of students related to graphical intuition by means of these drawings.

In the literature, visualization approach is stated to be carried out in three ways as graphics or shapes, animations, and computer software

programs (Dundar, Gökkurt & Soylu, 2012; İpek, 2003). The effect of visualization approach on mathematics has been discussed for several times; some researchers put forward that visualization can increase the level of mathematical understanding of students, facilitate problem solving and enable students to try different ways in problem solving, establish the relation between mathematics and art, develop the aesthetic feelings of students, create positive attitude towards mathematics and accordingly increase the self-confidence of students (Şan, 2008; Tall & Thomas, 1989; Thompson & Dreyfus, 1988). Taşova (2011) stated that the most critical advantage of visualization in mathematics is to transform an abstract situation into semi-abstract or concrete form and this advantage can be beneficial for the students that have difficulty in understanding some of abstract mathematics subjects. Some researchers reflect that visualization approach in mathematics might prevent the extensive mathematical thinking skills of students on the subject because of the view that it aims to convert the mathematical thinking of students into certain visual or figural patterns (Presmeg, 1986; Touger, 1986). On the other hand, Tall (1991) advocated that visualization in mathematics cannot be certainly accepted or denied, but expressed that visualization has an important place in the development process of mathematics due to its nature (as cited in İpek, 2003: 21).

In this study, the question of "What is the role of visualization and concretization in mathematics education in the studies of national and international publications?" has been aimed to be answered by means of a literature review. Accordingly, this study aims to determine to what extent visualization and concretization gain acceptance by means of comparison of national and international literature about

visualization and concretization in mathematics education and objectives to ascertain the methods used in these studies.

Method

This study that aimed to examine the publications of visualization and concretization in mathematics education, utilized descriptive model because of the objective of demonstrating the existing situation as it appears. The collection, analysis and interpretation of the data of the study were carried out by means of qualitative research methods. In the study, the distributions of the studies on visualization and concretization in mathematics education as a result of the literature review were analysed in terms of being nationally or internationally published, and their formats of article, paper, and thesis, their methods of quantitative, qualitative or mixed, the years, as well as the subjects of studies.

The studies reviewed

This study is a literature review study and utilized the method of document review of theses and articles about visualization and concretization in mathematics education. The document review analyses the written materials that have information about the situation to be researched (Yıldırım & Şimşek, 2011). This study analysed the publications with the publication language of Turkish and English in the databases examined about visualization and concretization in mathematics education in international and national fields as well as 38 publications dating from the publication of NCTM's "Principles and Standards for School Mathematics" (1989) to 2013.

In the review process, the analysis consisted of the articles published in the printed refereed journals, presentation works carried

out scientific meetings such as congress-conference symposium etc. and the books in which these works are printed, the articles in the refereed journals in electronic format on internet, master's and doctoral theses. The relevant literature review was carefully carried out; the resources obtained in the literature review to gather data were evaluated. In that process, the subsequent publications (if any) were not included to the review.

Data collection tools

In this study the method of document analysis was conducted as data collection technique. The document analysis involves the analysis of written materials that provide information about intended case or cases to be investigated. The document analysis can generally be performed in five stages including accessing documents, checking the authenticity of the documents, understanding documents, analysis of data and use of data (Yıldırım & Şimşek, 2011).

The data required for this study were collected by means of literature review. The studies on visualization and concretization in mathematics education in international and national fields between the years of 1989 – 2014 were analysed by means of ULAKBIM, PUBMED, WEB OF SCIENCE, JSTORE, EBSCHO and Higher Education Council (YOK) Thesis databases. This review was carried out by using the keywords of "visualization", "concretization" and "math education" together. The literature review by means of these keywords found 38 resources.

Data analysis

As a result of the literature review, 38 studies on visualization and concretization in mathematics education in international and national fields were found. In the study research studies regarding visualization and concretization in mathematics education as a result of literature review were analyzed in 6 categories; some of these categories

were obtained through the literature and some of them were formed by researchers. Çiltaş, Güler, & Sözbilir (2012) conducted a content analysis of the articles in mathematics education; within this regard, they analyzed how the articles have been distributed in terms of years, fields and research methods. Yenilmez & Sölpük (2014) analyzed the dissertations between 2004 and 2013 in terms of research method, degree level, subjects and concepts associated with mathematics program. Nevertheless, Yalçınkaya & Özkan (2012) analyzed the articles regarding alternative methods of mathematics teaching between 2000 and 2011, in terms of faculties of education, the distribution of them according to years and subjects, research methods and data collection techniques. Four of the categories among six categories in data analysis were formed through benefiting from literature and those are the categories that analyze the studies in terms of year, subject matter, research method and degree level. Nevertheless, the other two categories were added by researchers. The first one is the categorie that analyzes the studies in terms of their formats as article, thesis or paper and the other one is the categorie that analyzes the studies in terms of being published as national or international.

Findings

This study, in accordance with above mentioned criteria, obtained data of the existing studies in the relevant literature about visualization and concretization in mathematics education.

In terms of the place of the publications in the literature

The distribution of the publications, found as a result of the review, in terms of publishing places is shown in Table 1.

Table 1. *Distribution of the studies in terms of the publishing places*

Notionally		Internationally	
n	%	n	%
9	23.6	29	76.3

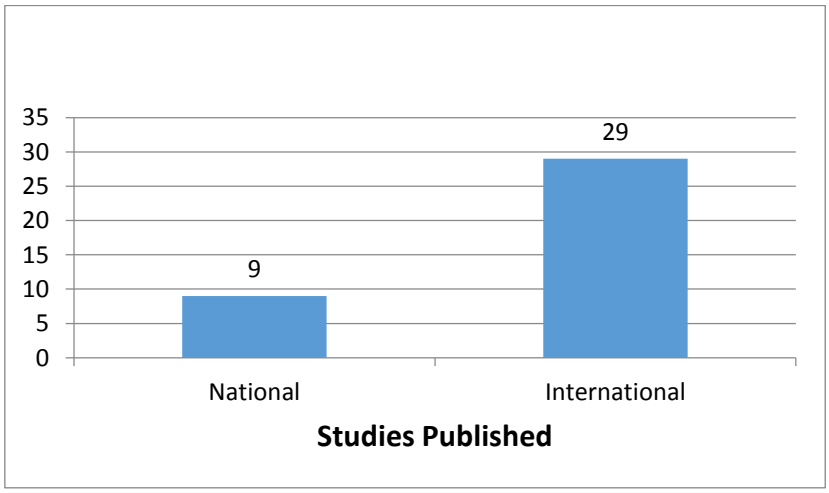

Figure 1. *Distribution of the studies in terms of the publishing places*

When 38 studies were analysed in terms of being internationally or nationally published, 29 of them were found to be internationally published whereas 9 of them were nationally published.

In terms of publishing format of the publications in the literature

The distribution of the publications, found as a result of the review, in terms of publishing format is shown in Table 2.

Table 2. *The Shape of the studies published*

	Nationally	Internationally	n	%
Thesis	6	20	26	68.4
Article	3	7	10	26.3
Report	-	2	2	5.3

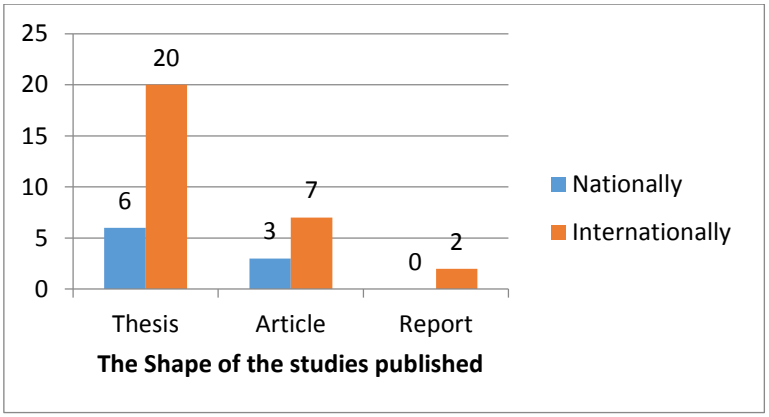

Figure 2. *The Shape of the studies published*

6 studies of a total of 9 studies nationally published are in the format of thesis whereas 3 studies are published as article. 20 studies of a total of 27 studies internationally published are in the format of thesis; 7 of them are published as article, and 2 studies are found to be published as report. In general, 26 of 38 studies are found to be theses whereas 10 of them are articles and 2 of them are in the format of report.

In terms of methods of the publications in the literature

The distribution of the publications, found as a result of the review, in terms of methods is shown in Table 3.

Table 3. *In terms of Method*

	Nationally	Internationally	n	%
Quantitative	7	14	21	55.3
Qualitative	1	8	9	23.6
Mixed	1	7	8	21

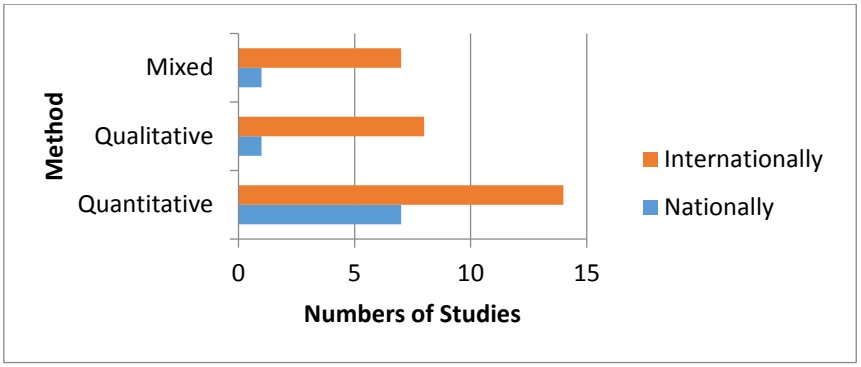

Figure 3. *In terms of Method*

The studies about visualization in mathematics education were researched about their methods that they were carried out. 7 of the studies carried out in Turkey utilized quantitative method; 1 of them used qualitative method while 1 of them utilized mixed method.

14 of the studies carried out abroad utilized quantitative method; 8 of them used qualitative method whereas 7 of them used mixed method. In total, 21 of the studies used quantitative method whereas 9 of them used qualitative method, and 8 of them utilized mixed method. In terms of qualitative method, the techniques of observation, interview, and content analysis were generally used whereas experimental patterns were dominantly used in terms of quantitative methods. However, some of the studies were found to utilize screening methods.

Table 4. *Methods in terms of studies type*

	Quantitative		Qualitative		Mixed		Total
	n	%	n	%	n	%	
Thesis	13	50	5	19.2	8	30.7	26
Article	8	80	2	20	-		10
Report	-	-	2	100	-		2
Total	21		9		8		38

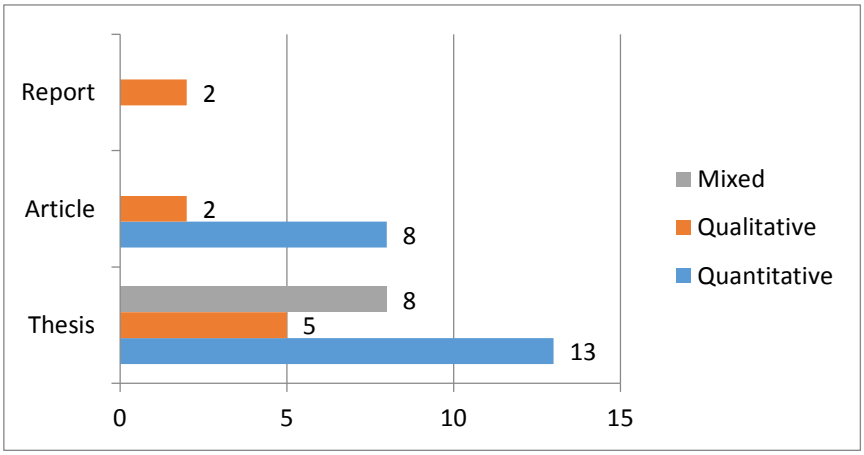

Figure 4. *Methods in terms of studies type*

13 of 26 thesis studies about visualization and concretization in mathematics education used quantitative methods whereas 5 of them used qualitative methods, and 8 of them used mixed methods. 8 of 10 articles were conducted in quantitative methods whereas 2 were carried out in qualitative methods, and no study was found to be conducted in mixed method. The two reports were found to have utilized qualitative methods.

Distribution in terms of sampling and publication years of the publications in the literature

The results of the publications, found as a result of the review, in terms of sampling is shown in Table 5.

Table 5. *Classification of the studies in terms of sampling groups*

	Elementary School	Secondary School	High School	University
Number of Studies (f)	4	5	6	14
%	13.8	17.2	20.6	48.2

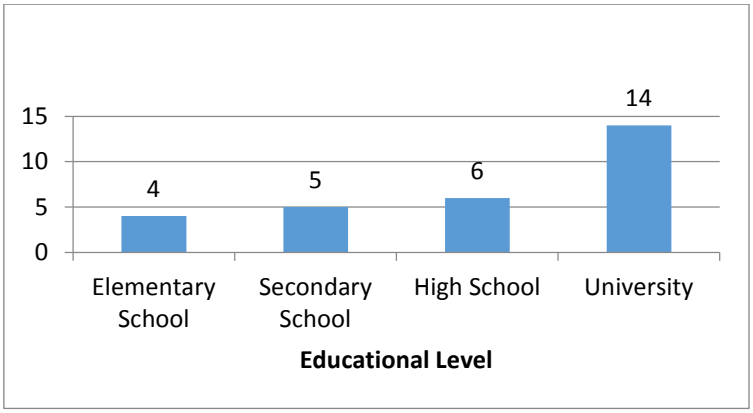

Figure 5. *Classification of the studies in terms of sampling groups*

In the studies with quantitative and mixed methods, the class level of the students that the researchers used as sampling was examined. 4 of 29 studies with quantitative and mixed methods consisted elementary school level students; 5 of them, secondary school level students; 6 of them, high school level students; and, 14 of them university level students as participants, and the studies about visualization on mathematics education were conducted in mathematics subjects specific to their own levels.

Table 6. *Distribution of study in terms of years*

	1989-1993	1994-1998	1999-2003	2004-2008	2009-2013
Number of Studies	2	5	6	11	14
%	5.2	13.1	15.7	28.9	36.8

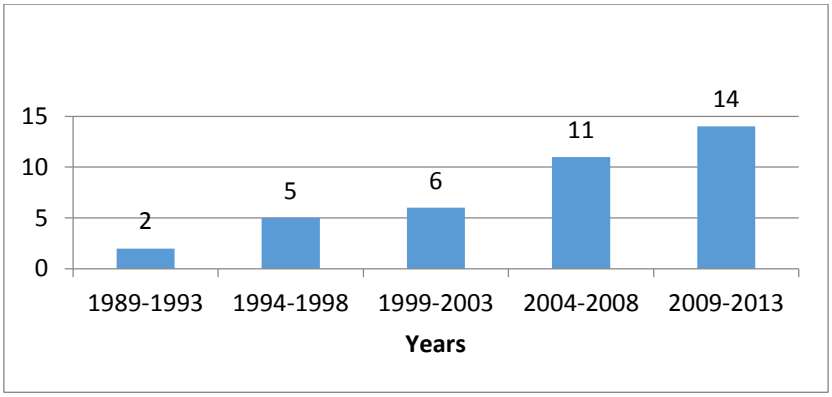

Figure 6. *Distribution of study in terms of years*

Analyzing the table 6 regarding the distribution of the research studies in terms of visualization in mathematics education between 1989-2013, it is seen that they increase perpetually. It was revealed that while there were two research studies between 1989-1993, 14 research studies were conducted between 2009-2013.

Distribution in terms of subjects of the publications in the literature

The distribution of the publications, found as a result of the review, in terms of subject is demonstrated in Table 7.

Table 7. *Distribution in terms of subjects of the publications in the literature*

Learning Branch	Subject	Number of studies	Total
Numbers and Algebra	Problem Solving	6	31
	Trigonometry	4	
	Complex Numbers	2	
	Modular Arithmetic	1	
	Algebraic Expressions	2	
	Functions	3	
	Limit	2	
	Derivative	1	
	Integral	1	
	Algebra	9	
Geometry	Geometry	5	5
Data, Counting and Probability	Graphics	2	2

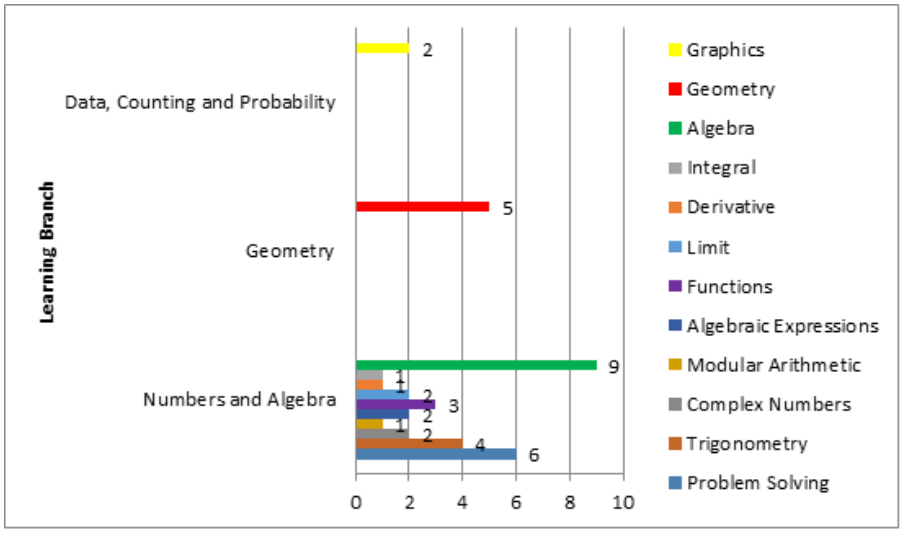

Figure 7. *Distribution in terms of subjects of the publications in the literature*

The studies included to the study were also analysed in terms of mathematical subjects. When these subjects were analysed, the subjects such as complex numbers, trigonometry, modular arithmetic, algebraic expressions, equations, functions, limit, geometry, graphs, place values of non-decimal numbers, group, metric spaces, ring, field, vector, vector spaces, and identities were found to be dealt with.

Discussion and Conclusion

This study aims to put forward the role of visualization and concretization in mathematics education in national and international fields by means of literature review. Accordingly, 38 studies considered to be related to visualization and concretization in mathematics education were found by screening national and international databases.

Furthermore, these studies were evaluated in terms of being internationally or nationally published, the formats of article, paper, and

thesis, the language of the study as Turkish and English, and the methods of quantitative, qualitative or mixed.

The conclusion that the major part of the studies on visualization in mathematics education was conducted abroad. It can be concluded that the number of the studies conducted in our country about visualization in mathematics education have been quite low compared to abroad. The reason of this situation can be that the studies related to visualization are primarily conducted abroad. However, given the overall number of the studies related to visualization and concretization in mathematics education, these studies can conclude that visualization has not become widespread in mathematics education.

As Şan (2008) discussed in his research, the reason of this situation may be due to the decreasing effect of visualization after the emergence of non-Euclidean geometry in the 19th century. Given the number of the studies that are found on visualization in mathematics education, it can be concluded there is a necessity to increase the studies on this subject.

It was found that most of the studies found related to the research subject are theses, and the major part of the rest of the articles was comprised of a section of thesis studies. That most of the studies are generally theses supports the idea that this field is a new study field. Given that thesis studies were conducted on unstudied subjects, the high number of theses, articles and reports in this field may conclude that visualization in mathematics education is starting to become a popular field. Furthermore, quite lesser number of articles and reports compared to the number of theses obtained in the study can be considered as an

indicator of insufficient studies on visualization in mathematics education.

When the studies found were analyzed in terms of research method, it was found that the number of the studies with quantitative method were higher. When the studies with quantitative method were analyzed, experimental patterns were found to be generally dominant. The reason of this finding may be due to the fact that researchers aim much more to demonstrate the efficiency of visualization approach and tend to compare visualization approach with other approaches. When the studies with qualitative were analyzed, it was found that the studies were in the form of literature review and analysis of visualization studies in general. This may result from that visualization subject has not been studied much and accordingly researchers tend to conduct studies to explain what visualization is and to demonstrate which studies have been carried out. Besides, all of the studies with mixed method were found to be in the form of thesis. This may result from that thesis studies make a detailed research and analyze data from various aspects.

When the class levels of the students that researchers deal with in their studies were analyzed, the university level of students was generally found to be studied with. In the studies, the participant levels were revealed to decrease from university level towards elementary school level. The reason of this situation may result from a more convenient access to university-level participators by researchers. The reason why visualization subjects are conducted more in university level may be the idea that teacher candidates should be more aware of the visualization and that accordingly there are more studies conducted with teacher candidates.

When the studies were analyzed in terms of year, it was revealed that the studies on visualization in mathematics education have been constantly increasing and most of the studies were conducted in the recent years (2009-2013). This result can be stated to be an idea that supports that this field is a newly studied field. The reason of the gradual increase in the studies in years is reflected to be that there is a widespread use of visual tools due to technological developments and visualization accordingly is started to be included to education much more. The emergence of programs such as geogebra, sketchpath, mathtype can be stated to enable the studies on visualization in mathematics education to increase. Furthermore, it can concluded that there will be an increase in studies on visualization and concretization in mathematics education in subsequent years since the studies have been constant increasing.

According to the results about on which subjects the studies on visualization in mathematics education are conducted, a more research on algebra was found. The reason of this finding may be result from that the field of geometry and graphic subject are more visual than the field of algebra and that the need for visualization and concretization is thought to be higher due to view that the subjects of the field of algebra are more abstract. Karakaya (2011) found out that graphics are the most widely used visual objects in the course books that were analyzed. The view of Karakaya (2011) can be stated to support the view that the field of geometry and graphic subject are more visual than the field of algebra. Therefore, the reason of more studies conducted in the field of algebra may result from the view that this field needs to be supported by visual aspects. Besides, it was observed that the studies related to

visualization in the fields of geometry and data, counting and probability are quite lesser compared to those in the field of algebra. It can be concluded that there is a necessity to increase the studies on this field.

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