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Mathematical Modeling of a Diesel Electrical Locomotive

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Abstract

Railway transportation has a huge share of energy consumption in the industry. The efficient power consumption is the most important issue for the minimization of the cost of railway transportation. The control strategy for efficient power consumption is varying due to the type of locomotive. Each type of locomotive needs its specific power model for this purpose.
The range of the locomotives used in Turkey are full of diversity according to the type and size of the motor. In this research, DE 24000 type of locomotive is chosen for mathematically modeling of its required power under various load, speed, ramp, and curve conditions. Then a mathematical equation is produced according to these values with Least Square Method. The optimum efficient working point is found as 735.5-ton load and 38km/h speed with the aim of this produced equation.

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1. INTRODUCTION

The cost components of a railway like other transportation projects can be classified to:

- The supplier costs. (Planning, Design, and Administrative; Right of way; Construction; Operating; Maintenance)
- The user costs. (Access time; Waiting time; Travel time in the train; Accident; Comfortable)
- The out of system costs. (Environmental, Social)

This study belongs to the second item, user costs. Because the goal is to help the user for efficient speeds and loads at a certain route. So, this will ensure efficient driving. Efficient driving is based on train resistance according to load, speed, and route information.

When it comes to the resistance of the train sets, and to examine the subject in more detail, the optimization process can be traced back to the oldest periods of development in the railways. Many researchers have made investigations in this field and have reached the empirical equations shared in the findings section. Studies in literature are mainly based on cruise resistance and their prediction models. Travel resistance was reported by Davis (1926) as follows:

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$$Rs = AV^2 + BV + C$$

A is a constant which changes proportionally to the square of the speed and represents aerodynamic resistance caused by air pressure and friction.

B is a constant which is responsible for mechanical resistances and HVAC (Heating, Ventilating, and Air Conditioning).

C is a constant which is not fixed to the vehicle speed but is a function of weight.

In the past, detailed tests have been conducted to determine these constants. The cruise resistance coefficients of different trains were found for the Shinkansen. (Hara et al., 1967) Besides, different tests and cruise resistance tests were applied to the passenger cars and locomotives of Eurofim (Report ERRI C, 1993) However, since the tests are very costly, different empirical equations have been developed in the past for estimating the resistance of certain trains. An overview of the methods adopted by the main national railways (up to 2000) and a calculation tool, to calculate cruise resistance, in which the effects of various characteristics of the train's architecture can be taken into account, are presented; these results are compared with the results of other equations for calculating train resistance (Rochard & Schmid, 2000).

More recently, Lukaszewicz has proposed a method that allows the determination of train resistance coefficients by measuring only train speed and position from full-scale cruise tests (Lukaszewicz, 2007a). In this study, resistance was determined by the change in kinetic and potential energy of a train traveling between successful measurement points. Using this method (Lukaszewicz, 2007b), the same authors shared experimental results to determine the travel resistance of different trains and the effect of variables such as speed, number of axles, number of wagons, axle load, road type, and train length. Since 2005, a CEN (European Committee for Standardization) standard (EN 14067-4:2005+A1:2009) has described methodologies for evaluating the coefficients of Davis's formula starting from a predictive formula, numerical simulations, and reduced-scale tests from full-scale test measurements, but no strictly accepted methods have been obtained.

Cruise tests are performed to determine the speed-dependent terms (A and B) according to CEN Standards. There is a need for a special test for term C, which means that the train is traveling at a very low speed. To find the coefficients A and B in the CEN standard, the regression method and the velocity history identification method were used. The first cruise test is based on the combination of all available experimental data and the second is based on the combination of the equation of motion. Both methods require a very good knowledge of the test section properties (slopes and curve radiuses).

In another study in the literature, the standard methods for determining Davis's coefficients were compared to new methods. In particular, it has been shown that the three coefficients of the Davis's formula can be estimated by two tests only, the first is a very low-speed test on a high altitude slope section (without having to perform a traction test), and the second is a travel test starting at the train's maximum speed. It also proposed a regression method, which is a new method to define the A and B coefficients in the Davis equation. The main advantage of this method is that it does not need to know the characteristics and coefficient C of the railway line. Starting from experimental full-scale tests (characterized by a mass of 450 tonnes) scaled for a general ideal train; the entire procedure for determining travel resistance coefficients is described. The comparison of the results obtained by different methods for estimating the coefficients A and B of the Davis equation are presented and analyzed (Somaschini et al., 2016).

In this study, different from the above-mentioned studies, a single mathematical model was used to determine total resistance by using empirical equations that were accepted for all resistances. For this purpose, the Least Square Method (LSM) is used. Then, it was determined the optimum working point of the train according to load and speed with the aim of this mathematical model.

2. MATERIAL AND METHOD

The locomotive which is used in this study is DE 24000 locomotive. This locomotive has a diesel-electric motor on it, which is used in a loaded train. The approximate weight of the locomotive is 113 tonnes. It uses a 72V alternator and $600V_{RMS}$ generator. At the diesel part, the locomotive uses the Diesel V16 motor. The fuel capacity of the locomotive is 4.960 liters. The maximum speed of the locomotive can reach 144 km/h. The output power can reach 1760 kW. The tractive effort can reach 394-kilo newtons however, at the maximum speed it falls to 48-kilo newtons (Anonymous, 2016). These performance values of the locomotive can be obtained from the technical catalog of the locomotive.

On the other hand, the proposed mathematical model provides to obtain necessary load, speed, ramp, and curve values for any level of the power, which can not be obtained directly from the technical catalog. If the required power is taken as constant, the maximum speed is changing in the inverse direction of the change in the load and the ramp. Furthermore, speed has a more complex relationship with the curve. These relations can be expressed with a nonlinear mathematical model. The obtained values from the proposed model were used to present the performance curves of the locomotive in a railway in Turkey. These curves were also used to demonstrate the power efficiency of the locomotive.

The mathematical model is created using empirical equations shown below. The Pcon value in the formula stands for the power for the continuous regime, GL stands for the load of the locomotive, and the GV stands for the load to be carried, the P-value is axle press, N is axle numbers and A is the front viewing area that is 12 m^2 in all calculations. rl is cruising resistance of locomotive, rv is cruising resistance of cars, rk is the resistance force due to curve and rr is the resistance force due to the gradient. R is the total resistance which a locomotive must overcome to move (Urlu, 1999; Anonymous, 2018). The other resistances are neglected. (Other resistances that arise due to wind velocity, tunnels, or train marshaling and the acceleration resistance will not be discussed here)

This cruising resistance formula of locomotives is known as the Davis formula:

$$\mathrm{rl} = 0{,}65 + \frac{13{,}15}{\mathrm{P}} + 0{,}00932 * \mathrm{V} + \frac{0{,}004526 * \mathrm{A} * \mathrm{V}^2}{\mathrm{P} * \mathrm{N}} \, (\mathrm{daN/ton}) \tag{1}$$

This cruising resistance formula of cars is known as the Strahi formula:

$$\mathrm{rv} = 2 + 0{,}057 \frac{\mathrm{V}^2}{100} \, (\mathrm{daN/ton}) \tag{2}$$

This curve resistance formula is known as the Röcki formula:

$$\mathrm{rk} = \frac{650}{\mathrm{k} - 55} \, (\mathrm{daN/ton}) \tag{3}$$

r is the ramp value in ‰. This value is taken as positive when climbing the ramp and negative when descending:

$$\mathrm{rr} = \mathrm{r} \tag{4}$$

All the resistances must be multiplied with train and carried load:

$$\mathrm{RL} = \mathrm{rl} * \mathrm{GL} \tag{5}$$

$$\mathrm{RV} = \mathrm{rv} * \mathrm{GV} \tag{6}$$

$$\mathrm{RK} = \mathrm{rk} * (\mathrm{GV} + \mathrm{GL}) \tag{7}$$

$$\mathrm{RR} = \mathrm{rr} * (\mathrm{GV} + \mathrm{GL}) \tag{8}$$

$$\mathrm{R} = \mathrm{RL} + \mathrm{RV} + \mathrm{RK} + \mathrm{RR} \tag{9}$$

$$\mathrm{Pcon} = \frac{\mathrm{R} * \mathrm{V}}{360} (\mathrm{daN}) \tag{10}$$

The proposed mathematical model of the locomotive is constructed by the application of the least-squares method. The least-squares method is a standard approach in regression analysis to approximate the solution of overdetermined systems, i.e., sets of equations in which there are more equations than unknowns. "Least squares" means that the overall solution minimizes the sum of the squares of the residuals made in the results of every single equation.

The most important application is in data fitting as this study. The best fit in the least-squares sense minimizes the sum of squared residuals (a residual being: the difference between an observed value, and the fitted value provided by a model). The details of the method are given below:

2.1. Least Square Method (LSM)

Y is the dependent variable, X_1 is the independent variable, β_1 is the unknown parameter of the X_1 variable and εi is error terms that can not be observed. It is possible to express one variable linear regression equation:

$$\mathrm{Y} = \beta_0 + \beta_0 \mathrm{X}_{1\mathrm{i}} + \epsilon_{\mathrm{i}}, \mathrm{i} = 1,2, \ldots, \mathrm{n} \tag{11}$$

One of the most commonly used methods to find $\beta_0 + \beta_1$ variables is the Least Squares Method. The finding values for $\beta + \beta_1$ are $\hat{\beta}_0$and $\hat{\beta}_1$. In this case, the new equation can be written as follows:

$$\mathrm{Y} = \hat{\beta}_0 + \hat{\beta}_0 \mathrm{X}_{1\mathrm{i}} + \epsilon_{\mathrm{i}}, \mathrm{i} = 1,2, \ldots, \mathrm{n} \tag{12}$$

The basis of the used LSM to find the values of the are $\hat{\beta}_0$and $\hat{\beta}_1$ terms in the equation rely on finding the values that will make the smallest sum of the squares of the total deviations. The error terms are the differences between the $\mathrm{Y_i}$ (observed values) and the $\hat{\mathrm{Y}}_{\mathrm{i}}$ (expected values) (Ryan, 2008). This difference can be expressed as follows:

$$\hat{\epsilon}_{\mathrm{i}} = \mathrm{Y_i} - \hat{\mathrm{Y}}_{\mathrm{i}} \tag{13}$$

The sum of the differences between the value of estimation and that should be:

$$\sum_{\mathrm{i}}^{\mathrm{n}} \hat{\epsilon}_{\mathrm{i}} = \sum_{\mathrm{i}}^{\mathrm{n}} \left(\mathrm{Y_i} - \hat{\mathrm{Y}}_{\mathrm{i}}\right) = 0 \tag{14}$$

LSM determines the difference of $\hat{\beta}_0$and $\hat{\beta}_1$ (the estimation parameters of $\hat{\beta}_0$and $\hat{\beta}_1$) which will be smallest:

$$\text{the smallest} \sum_{\mathrm{i}=1}^{\mathrm{n}} \hat{\epsilon}_{\mathrm{i}}^{\,2} = \text{the smallest} \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(\mathrm{Y_i} - \hat{\mathrm{Y}}_{\mathrm{i}}\right)^2 \tag{15}$$

To obtain the LSM estimations of the regression coefficients, when the partial derivatives according to $\hat{\beta}_0$ $\hat{\beta}_1$ equation 6 are equalized to zero, Equation 17 and 18 normal equations are obtained. When the necessary analyzes are made through these equations, $\hat{\beta}_0$and $\hat{\beta}_1$ which are the estimations of the β_0 and β_1 can be obtained from equations 19 and 20 (Gürünlü Alma & Vupa, 2008)

$$\sum_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{Y_i} - (\hat{\beta}_0 - \hat{\beta}_1 \mathrm{X}_{1\mathrm{i}}))^2 = \mathrm{L} \tag{16}$$

$$\sum_{i=1}^{n} Y_i = \hat{\beta}_0 n + \hat{\beta}_1 \sum_{i=1}^{n} X_i \tag{17}$$

$$\sum_{i=1}^{n} X_{1i} Y_i = \hat{\beta}_0 \sum_{i=1}^{n} X_{1i} + \hat{\beta}_1 \sum_{i=1}^{n} {X_{1i}}^2 \tag{18}$$

$\hat{\beta}_0$, $\hat{\beta}_1$ and the calculation of the regression coefficient is as follows:

$$\hat{\beta}_1 = \frac{(n[\sum_{i=1}^{n} X_{1i} Y_i] - (\sum_{i=1}^{n} X_{1i})(\sum_{i=1}^{n} Y_{1i})}{(\sum_{i=1}^{n} X_{1i}^2) - (\sum_{i=1}^{n} X_{1i})^2} = \frac{\sum_{i=1}^{n} (X_{1i} - \bar{X}) - (Y_{1i} - \bar{Y})}{\sum_{i=1}^{n} (X_{1i} - \bar{X})^2} \tag{19}$$

$$\hat{\beta}_0 = \frac{\sum_{i=1}^{n} Y_i - \hat{\beta}_1 \sum_{i=1}^{n} X_{1i})}{n} = \bar{Y} - \hat{\beta}_1 \bar{X} \tag{20}$$

$$R^2 = \frac{\sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \tag{21}$$

3. CASE STUDY

Data to build a mathematical model are obtained from the Tülomsaş (locomotive producer company in Turkey). Then Matlab for LSM is used to find the optimum working point of locomotive according to a specific railway route. The railway line is a part of Konya (Turkey) conventional line between 282+969,130 km and 283+443,920 km. The mathematical model is used to find the optimum load and speed values for this route.

Mathematical model of DE 24000:

$$\begin{aligned} Pcon = {} & (10^{-4} + 1.6x10^{-6}L)V^3 + 0.0029V^2 \\ & + \Big(\big((0.031 + 0.224x10^{-3}L)k^2 - (31 + 22.4x10^{-3}L)k \\ & + (6.34 + 5.66x10^{-3}L)\big) + (0.2 + 3x10^{-4}L)(r-1)\Big)V \end{aligned} \tag{22}$$

Constraints:

10 km/h ≤ V ≤140 km/h

130 m ≤ k ≤430 m

0 ‰ ≤ r

L ≤ 1400 ton (include locomotive weight)

Pcon ≤ 1760 kW

Pcon: Power (kW)

L: Load (ton)

k: curve (meter)

r: ramp level (‰ ramp)

RMSE is about 0 for this novelty model for DE24000 locomotive. Then according to this model, some curves are obtained in this specific railway route. This route is shown in Table 1, the yellow part is used in this study (k=0).

Table 1. *Line Data*

Konya Conventional Line Between 277-284								
S.NO	***Start***	***Level***	***Finish***	***Level***	***Distance***	***Level***	***Ramp %***	**Ramp ‰**
1	277+521,61	795,293	278+149,320	794,660	627,710	-0,633	-0,00101	-1,01
2	278+149,320	794,660	279+140,000	794,496	990,680	-0,164	-0,00017	-0,17
3	279+140,000	794,496	279+669,790	792,516	529,790	-1,980	-0,00374	-3,74
4	279+669,790	792,516	281+190,830	791,333	1521,040	-1,183	-0,00078	-0,78
5	281+190,830	791,333	281+737,380	782,581	546,550	-8,752	-0,01601	-16,01
6	281+737,380	782,581	282+969,130	781,511	1231,750	-1,070	-0,00087	-0,87
7	282+969,130	781,511	283+443,920	789,108	474,790	7,597	0,01600	16,00
8	283+443,920	789,108	284+490,680	788,692	1046,760	-0,416	-0,00040	-0,40

3D figure can be drawn for power, load, and speed. Figure 1 is below.

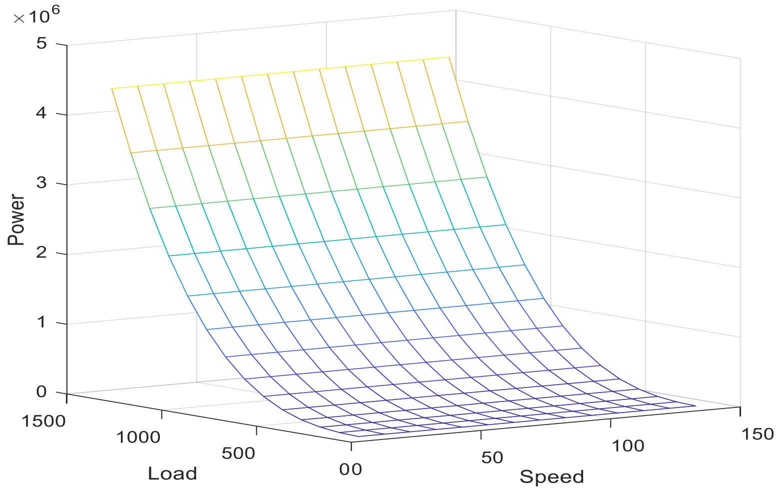

Figure 1. *Power-Load-Speed Figure*

L-S diagrams are very useful to obtain speed and load values. These values are obtained according to max power. Figure 2 is the Load-Speed diagram of the DE24000 below.

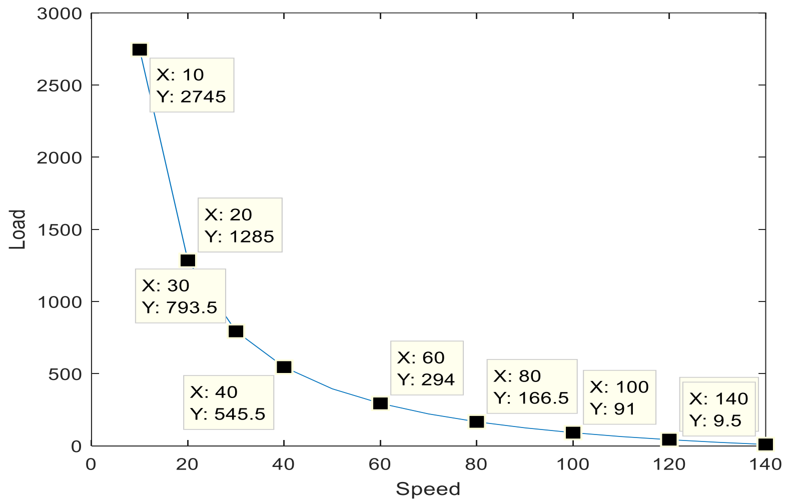

Figure 2. *Load-Speed Diagram of The DE24000*

There is an optimization problem here. In a certain power, load and speed have an intersection point. This point is the best working point for DE24000. Figure 3 shows this point.

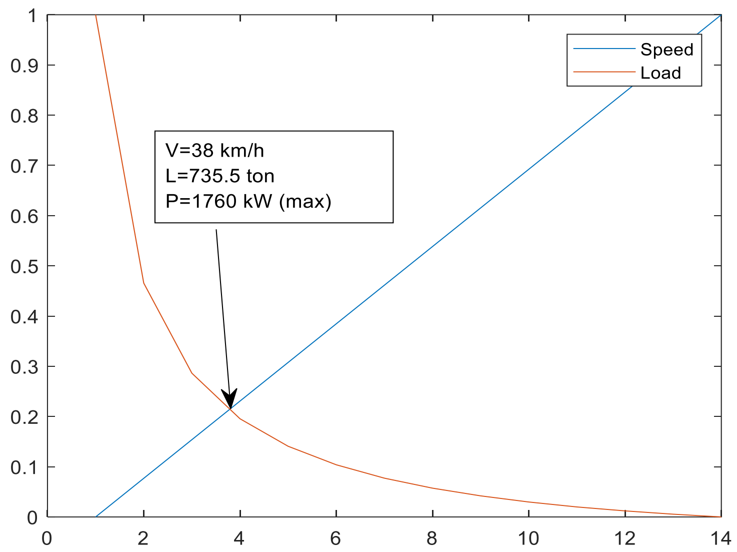

Figure 3. *Optimum Speed and Load Values of The DE24000*

The optimum point is found as 38 km/h and 735.5 tons for this railway route. This point is the value when both speed and load are equally important. If the arrival time is more important or the load to be carried is more important for the user, these points will be changed and these values can get easily from this novelty mathematical model. If the route will change these values will be changed too.

Previous studies generally are created with controlling dwell time, running time, and departure state for energy efficiency in railway systems. These systems are used in both regenerative braking and power supply systems but sometimes only power supply systems are used. (Albrecht, 2004; Chen et al., 2005; Kim & Oh, 2009).

4. RESULT

Differently previous studies, optimum speed, and load points are tried to be found in this study. There is no direct study similar to this study in the literature that can be given as an example.

The optimum point is found as 38 km/h and 735.5 tons according to the chosen route as previously mentioned. In this case, this calculation has been made considering that load and speed are the same effects on energy consumption (the locomotive was operated at full power).

There are two contributions to this study. The first contribution is that the power requirement of the locomotive can find easily with changing load, speed, curve, and ramp level of the railway. In this study, only one part of the railway is used but DE24000 can be used on all the railway's aims of this model. The customer of this locomotive will be able to see if the locomotive will work properly on the railway which is desired before planning.

Secondly, one more contribution is that the user can find the optimum working point of the locomotive according to a certain railway anywhere. Because efficiency speed and load values can change with the railway route and locomotive type. Some warning systems can be added to ensure that machinist can go on optimum energy conditions.

4. CONCLUSION

It is quite easy to set up warning systems by implementing some algorithms for energy efficiency in the railways. These systems can be prepared for locomotives and sold together with them. Ramp level, curve, length of the railway, and the load information are entered into the system. These systems will provide the maximum speed information to the user when the maximum power value is limited. When this speed is exceeded, the system will alert. Such an application will help to use energy very economically.

Also, unhandled in this work, adding time knowledge to the model, the effect of time-saving on energy can be searched too. According to all information (ramp level, curve, length of the railway, and the load), how long time takes the journey and energy consumption can be found. This study is among the studies planned to be done in the future.

CONFLICT OF INTEREST

The authors declare that, there is no conflict of interest regarding the publication of this paper.

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