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Research Article

A Note on Fuzzy Soft Ditopological Spaces

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Abstract

In this study, we define fuzzy soft ditopological spaces as the generalization of soft ditopological spaces defined by Simsekler et. al. (2016). Fuzzy soft ditopological space is a combination of fuzzy soft topological and fuzzy soft cotopological spaces which are defined by two independent structures fuzzy soft open and fuzzy soft closed sets. Also we give fuzzy soft τ-continuity in fuzzy soft topology , fuzzy soft κ-continuity in fuzzy soft cotopology and by using these two types of continuity we finally define the fuzzy soft continuity in a fuzzy soft ditopology.

Keywords: Fuzzy soft sets, fuzzy soft ditopology, fuzzy soft continuity.

Bulanık Esnek Ditopolojiler Üzerine Bir Çalışma

Öz

Bu çalışmada, Simsekler Dizman ve arkadaşları tarafından (2016) tanımlanan esnek ditopolojik uzayların bir genelleştirilmesi olan bulanık esnek ditopolojik uzaylar tanımlanmıştır. Bulanık esnek ditopolojik uzaylar birbirinden bağımsız olarak tanımladığımız bulanık esnek açık ve bulanık esnek kapalı kümeler kullanılarak tanımlanmış bulanık esnek topolojiler ve bulanık esnek kotopolojilerin bir kombinasyonudur. Ayrıca bulanık esnek topolojilerde tanımladığımız τ-süreklilik ve bulanık esnek kotopolojilerde κ-sürekiliği kullanarak bulanık esnek ditopolojilerde sürekliliği verdik.

Anahtar Kelimeler: Bulanık esnek kümeler, bulanık esnek ditopoloji, bulanık esnek süreklilik

1. Introduction

Since the real life problems in several areas are more objective, solving these problems by classical mathematics rules is not appropriate many times. Fuzzy set theory defined by Zadeh (1965) gives us an opportunity to define a set and its elements by a different way than the well-known approach ‘’Black or White’’ and presents us a new angle to sight the connections between the set and its elements. The main opinion of the theory is defining a fuzzy set by the fuzzy membership function and hence state that which ‘’degree’’ an element pertion to a fuzzy set. Chang (1968) was first investigated fuzzy set topology. The soft set was described in 1999 by Molodtsov (1999) as a new approach for uncertainity and the theory is based on defining a soft set by a parameter set of objects in the universe with a mapping. It is easily seen that fuzzy and soft sets are interdependent (Aktas and Cagman, 2007). Maji et al. (2001) first studied the hybrid

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model of fuzzy and soft sets and defined fuzzy soft set (briefly fs-set) as a new type of vague sets. Ahmad and Kharal (2012) contributed to the fuzzy soft theory and described the concept of a mapping between fs-sets. Aktas and Cagman (2007) defined the notion of soft groups and some of their properties. Feng et al. (2008) studied the soft semirings and investigated several related properties. Nazmul and Samanta (2010) contributed the algebraic structures of soft sets. Shabir and Naz (2011) described the soft topological spaces and separation axioms of soft topological spaces. Tanay and Kandemir (2011) searched the topology of fs-sets.

Soft and fs-sets and the structures on these sets were observed by several researchers (Ahmad and Hussain, 2012, Atmaca and Zorlutuna, 2014, Aygunoglu and Aygun, 2011, Aygunoglu et. al., 2014, Bera et.al., 2017, Cagman and Aktas, 2011, Cagman et. al., 2011, Karaaslan et. al., 2012, Karasalan 2016, Karaslan et. al., 2013, Min, 2011, Ozturk, 2016, Pazar and Aygun, 2012, Roy and Samanta, 2012, Roy and Samanta, 2013, Simsekler ve Yuksel, 2013, Zorlutuna et. al., 2012).

Ditopology was first given by L.M. Brown and investigated by L.M. Brown and co-authors (1998). Actually ditopology is connected with the notion of the bitopology described by J.L. Kelly (1963). The ditopology is based on two structures, openness and closedness, which are defined as independent concepts from each other.
In this paper, we interpret the parameters E and A, which are used in the definition of fs-sets, different from other papers written on fs-topology. We assume that if a parameter does not belong to A, the value for this parameter is not defined. Considering fs-sets by this idea makes differences while defining the fs-topologies. This way leads to define fs-open sets and fs-closed sets as independent structures. We can summarize our study in the following way. First, we define fs-topological spaces by fs-open sets and we investigate the properties of this space. Then we define fs-cotopological spaces by using fs-closed sets. We define the concept of fs-remote neighborhood of a fs-point and observe the properties of fs-cotopological spaces. Finally, we define the fs-ditopological spaces by combining the fs-topology and cotopology and sum up the results of this paper.

2. Preliminaries

Throughout the paper U will denote the universe and E will denote the parameter set and I^U will denote the all fuzzy sets on U. Let A and B be nonempty sets of E.
In this section we give the main definitions of fs-set theory which can be found several papers we cited in introduction. As we state in the previous section our Fuzzy soft set definition is different from this works in the meaning of parameter set.

Definition 1. f_A is called the fuzzy soft set (briefly fs-set over) U if $f_A\colon A \to I^U$ is a mapping defined by $f_A(e) = {\mu_{f_A}}^e$ where ${\mu_{f_A}}^e(u) \neq 0$ for each $u \in U$ otherwise i.e, $e \notin A$, $f_A(e)$ will not be considered.
Definition 2. The complement of a fs-set f_A is a fs-set denoted by ${f_A}^c$ where ${f_A}^c\colon A \to I^U$ is a mapping defined by $\mu^e_{{f_A}^c}(u) = 1 - \mu^e_{f_A}(u)$ for all $e \in A, u \in U$.
Definition 3. Let f_A and g_B be two fs-sets over U. f_A is said to be a fs-subset of g_B if $A \subseteq B$ and ${\mu_{f_A}}^e(u) \leq {\mu_{g_B}}^e(u)$ for each $e \in A$ and $u \in U$.
Definition 4. Let f_A and g_B be two fs-sets over U. The intersection of f_A and g_B denoted by

$f_A \sqcap g_B$ is a fs-set h_C where $C = A \cap B$ and is defined by

$$\mu_{H_C}{}^{e}(u) = min\{\mu_{f_A}{}^{e}(u), \mu_{g_B}{}^{e}(u)\}, \forall e \in C, \forall u \in U.$$

Definition 5. Let f_A and g_B be two fs-sets over U. The union of f_A and g_B denoted by $f_A \sqcup g_B$ is a fs-set h_C where $C = A \cup B$ and is defined by

$$\mu_{H_C}{}^{e}(u) = \begin{cases} \mu_{f_A}{}^{e}(u), if\ e \in A - B \\ \mu_{g_B}{}^{e}(u), \text{if } e \in B - A \\ max\{\mu_{f_A}{}^{e}(u), \mu_{g_B}{}^{e}(u)\}, \text{if } e \in A \cap B \end{cases}, \forall u \in U.$$

Definition 6. The fs-set f_A over U is defined to be null fs-set and is denoted by Φ where $f_A(e) = 0^{-}, \forall e \in A, \forall u \in U$.

Definition 7. The fs-set f_A over U is defined to be absolute fs-set and is denoted by $U_{\tilde{E}}$ where $f_A(e) = I^{-}, \forall e \in A, \forall u \in U$.

Theorem 1. Let f_A and g_B be two fs-sets over U. Then the followings hold:

i) $(f_A \sqcap g_B)^c \sqsubseteq f_A{}^c \sqcup g_B{}^c$.

ii) $f_A{}^c \sqcap g_B{}^c \sqsubseteq (f_A \sqcup g_B)^c$.

Proof. i) Let $(f_A \sqcap g_B)^c = h_C, C = A \cap B$. Then, $\mu_{h_C}^{e}(u) = max\{\mu_{f_A^c}^{e}(u), \mu_{g_B^c}^{e}(u)\}$, for all $e \in A \cap B$ and for all $u \in U$. On the other hand, let $f_A{}^c \sqcup g_B{}^c = k_M, M = A \cup B$.

$$\mu_{k_M}^{e}(u) = \begin{cases} \mu_{f_A^c}^{e}(u), e \in A - B \\ \mu_{g_B^c}^{e}(u), e \in B - A \\ \max\left\{\mu_{f_A^c}^{e}(u), \mu_{g_B^c}^{e}(u)\right\}, e \in A \cap B \end{cases}$$

Hence the proof is completed.

ii) Let $(f_A \sqcup g_B)^c = h_C$. Then

$$\mu_{h_C}^{e}(u) = \begin{cases} \mu_{f_A^c}^{e}(u), e \in A - B \\ \mu_{g_B^c}^{e}(u), e \in B - A \\ \min\left\{\mu_{f_A^c}^{e}(u), \mu_{g_B^c}^{e}(u)\right\}, e \in A \cap B \end{cases}$$

On the other hand let $f_A{}^c \sqcap g_B{}^c = k_M, M = A \cap B$. Then, $\mu_{k_M}^{e}(u) = \min\left\{\mu_{f_A^c}^{e}(u), \mu_{g_B^c}^{e}(u)\right\}, e \in A \cap B$.

Hence the proof is completed.

Remark 1. The inverse inclusions do not hold generally.

Example 1. Let $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\}$, $B = \{e_1, e_3\}$. Let f_A, g_B be two fs-sets as defined by,

$$f_A = \left\{e_1 = \{u_1^{0.5}, u_2^{0.7}\}, e_2 = \{u_2^{0.5}, u_3^{0.6}\}\right\},$$

$$g_B = \left\{e_1 = \{u_1^{0.3}, u_2^{0.4}, u_3^{0.6}\}, e_3 = \{u_3^{0.2}\}\right\}.$$

i) We obtain the union of $f_A{}^c, g_B{}^c$.

$$f_A{}^c \sqcup g_B{}^c = \left\{e_1 = \{u_1^{0.7}, u_2^{0.6}, u_3^{1}\}, e_2 = \{u_1^{1}, u_2^{0.5}, u_3^{0.4}\}, e_3 = \{u_1^{1}, u_2^{1}, u_3^{0.8}\}\right\}.$$

On the other side,

$$(f_A \sqcap g_B)^c = \{e_1 = \{u_1^{0.7}, u_2^{0.6}, u_3^{1}\}\}.$$

It can be easily seen that $f_A{}^c \sqcup g_B{}^c$ is not a subset of $(f_A \sqcap g_B)^c$.

$$\text{ii) } (f_A \sqcup g_B)^c = \left\{e_1 = \{u_1^{0.5}, u_2^{0.3}, u_3^{0.4}\}, e_2 = \{u_1^{1}, u_2^{0.5}, u_3^{0.4}\}, e_3 = \{u_1^{1}, u_2^{1}, u_3^{0.8}\}\right\}.$$

Now we obtain the intersection of $f_A{}^c, g_B{}^c$.

$$f_A{}^c \sqcap g_B{}^c = \{e_1 = \{u_1^{0.5}, u_2^{0.3}, u_3^{0.4}\}\}.$$

It can be easily seen that $(f_A \sqcup g_B)^c$ is not a subset of $f_A{}^c \sqcap g_B{}^c$.

3. Fuzzy Soft Topological Spaces

Fs-topological spaces were defined and searched in several papers. (Aygunoglu et. al 2014, Atmaca and Zorlutuna 2014, Pazar and Aygun 2012, Roy and Samanta 2012, Roy and Samanta 2013, Simsekler and Yuksel 2013). In this work the interpretation of fs-sets is different from the published papers and this leads different results while defining fs-topological spaces and the properties of this spaces. Also in this section we use only fs-openness as an independent structure from closedness.

Definition 7. Let U be the universe and E be a parameter set. The fs-topological space is a pair $(U_E^{\sim}, \tau_f)$ where τ_f is a family of fs-sets over U satisfying the following conditions:

i) $\Phi, U_E^{\sim} \in \tau_f$,
ii) If $f_A, g_B \in \tau_f$ then $f_A \sqcap g_B \in \tau_f$,
iii) If $f_{A_i} \in \tau_f$ for all $i \in I$ then $\sqcup_{i\in I} f_{A_i} \in \tau_f$.

τ_f is called a fuzzy soft topology (briefly fs-topology) of fs-sets over U. The pair $(U_E^{\sim}, \tau_f)$ is called fs-topological space . The elements of τ are defined to be fs-open sets.

Definition 8. (Atmaca and Zorlutuna, 2013) The fs-set f_A is called a fs-point and denoted by e_x^{λ} if there exists the parameter e and the fuzzy point x_λ such that $f_A^e(x) = \lambda$ where $x \in U$ and $\lambda \in (0,1]$.

Definition 9. (Atmaca and Zorlutuna,2013) e_x^{λ} is called an element of f_A denoted by $e_x^{\lambda} \in^{\sim} f_A$ if $\lambda \le \mu_{f_A}{}^e(x)$.

Lemma 1. If $e_x^{\lambda} \sqcap f_A = \Phi$ then $e_x^{\lambda} \notin^{\sim} f_A$.

Proof. Let $e_x^{\lambda} \sqcap f_A = \Phi$. Then $\min\{\lambda, \mu_{f_A}{}^e(x)\} = 0$. This shows that $\mu_{f_A}{}^e(x) = 0$ and hence $e_x^{\lambda} \notin^{\sim} f_A$.

Remark 2. The inverse inclusion of theorem does not satisfy generally.

Example 2. Let $U = \{x, y\}$,$E = \{k, l, m\}$. Let $f_A = \{l = \{x_{0.7}, y_{0.6}\}, m = \{x_{0.3}, y_{0.2}\}\}$ be a fs-set over U and $m_y^{0.6}$ be a fs-point. It can be easily shown that $m_y^{0.6} \notin^{\sim} f_A$ but $m_y^{0.6} \sqcap f_A = \{m = \{y_{0.2}\}\}$.

Definition 10. Let $(U_E^{\sim}, \tau_f)$ be a fs-topological space, e_x^{λ} be a fs-point and f_A be a fs-set. f_A is called fs-τ-neighborhood of e_x^{λ} if there exists a fs-open set g_B such that $e_x^{\lambda} \in^{\sim} g_B \sqsubseteq f_A$. The family of all fs-neighborhoods of e_x^{λ} is denoted by $\aleph(e_x^{\lambda})$.

Theorem 2. Let $(U_E^{\sim}, \tau_f)$ be a fs-topological space, e_x^{λ} be a fs-point and f_A, g_B be two fs-sets. The followings hold for $\aleph(e_x^{\lambda})$:

i) If $f_A \in \aleph(e_x^{\lambda})$ then $e_x^{\lambda} \in^{\sim} f_A$.
ii) If $f_A, g_B \in \aleph(e_x^{\lambda})$ then $f_A \sqcap g_B \in \aleph(e_x^{\lambda})$.
iii) If $f_A \in \aleph(e_x^{\lambda})$ and $f_A \sqsubseteq g_B$ then $g_B \in \aleph(e_x^{\lambda})$.

Proof. The proof can be done similarly to the analogous statements in Simsekler and Yuksel (2013).

Definition 11. Let $(U_E^{\sim}, \tau_f)$ be a fs-topological space, e_x^{λ} be a fs-point and f_A be a fs-set e_x^{λ} is called fs-interior point of f_A if there exists a fs-open set g_B such that $e_x^{\lambda} \in^{\sim} g_B \sqsubseteq f_A$.

$$int f_A = \sqcup_{i\in I}\{g_{B_i} \sqsubseteq U_E^{\sim} : g_{B_i} \in \tau_f, g_{B_i} \sqsubseteq f_A, i \in I\}$$

Theorem 3. Let $(U_E^{\sim}, \tau_f)$ be a fs-topological space and f_A be a fs-set. Then the followings hold:

i) $int f_A \sqsubseteq f_A$.
ii) $int f_A$ is a fs-open set.
iii) $int f_A$ is the biggest fs-open set contained in f_A.
iv) f_A is a fs-open set iff $int f_A = f_A$.

Proof. The proof can be done similarly to the analogous statements in Simsekler and Yuksel (2013).

Definition 12. (Atmaca and Zorlutuna, 2013) e_x^λ is called quasi with f_A and denoted by $e_x^\lambda q f_A$ if $\lambda + \mu_{f_A}^e(x) > 1$.

Definition 13. (Atmaca and Zorlutuna, 2013) f_A is called quasi with g_B and denoted by $f_A q g_B$ if there exists $e \in A \cap B$, $u \in U$ such that $\mu_{f_A}^e(u) + \mu_{g_B}^e(u) > 1$.

Definition 14. (Atmaca and Zorlutuna, 2013) Let $(U_E^{\sim}, \tau_f)$ be a fs-topological space, e_x^λ be a fs-point and v_A, w_B be fs-sets. v_A is called a fs-Q-neighborhood of e_x^λ if there exists $w_B \in \tau_f$ such that $e_x^\lambda q w_B \sqsubseteq v_A$. The all fs-Q-neighborhoods of e_x^λ is denoted by Q$\aleph(e_x^\lambda)$.

Theorem 4. Let $(U_E^{\sim}, \tau_f)$ be a fs-topological space, e_x^λ be a fs-point and v_A, w_B be fs-sets. The followings hold for Q$\aleph(e_x^\lambda)$:

i) If $v_A \in$ Q$\aleph(e_x^\lambda)$ then $e_x^\lambda q v_A$.

ii) If $v_A, w_B \in$ Q$\aleph(e_x^\lambda)$ then $v_A \sqcap w_B \in$ Q$\aleph(e_x^\lambda)$.

iii) If $v_A \in$ Q$\aleph(e_x^\lambda)$ and $v_A \sqsubseteq w_B$ then $w_B \in$ Q$\aleph(e_x^\lambda)$.

Proof. The proof can be done similarly to the analogous statements in Simsekler and Yuksel (2013).

Proposition 1. (Atmaca and Zorlutuna, 2013) Let f_A, g_B be two fs-sets. If $f_A \sqsubseteq g_B$ then f_A is not quasi coincident with g_B^c.

Definition 15. (Pazar and Aygun, 2012) Let U,V be universe sets, E and P the paramer sets,FS(U,E) and FS(V,P) be the families of fs-sets over U and V respectively. Let $\varphi: U \to V, \psi: E \to P$ be mappings. Then the pair (φ, ψ) is called fs-mapping and is denoted by $\varphi_\psi = (\varphi, \psi): FS(U,E) \to FS(V,P)$.

Let f_A be a fs-set of FS(U,E). The image of f_A under the fs-mapping φ_ψ is a fs-set of FS(V,P) and defined by:

$$\varphi_\psi(f_A)^p(v) = \begin{cases} \bigvee_{\varphi(u)=v} \bigvee_{\psi(e)=p} f_A^e(u), if\ u \in \varphi^{-1}(v), \\ 0^-, otherwise \end{cases}$$

or $\forall p \in \psi(e)$ and $\forall v \in V$.

Let g_B be a fs-set of FS(V,P). The preimage of g_B under the fs-mapping φ_ψ is a fs-set of FS(U,E) and defined by:

$$\varphi_\psi{}^{-1}(g_B)^e(u) = (g_B)^{\psi(e)}(\varphi(u))$$

If φ, ψ are injective then φ_ψ is injective, if φ, ψ are surjective then φ_ψ is surjective.

Definition 16. Let $(U_E^{\sim}, \tau_f), (V_P^{\sim}, \tau_f{}^*)$ be two fs-topological spaces. $\varphi: U \to V, \psi: E \to P$ be mappings and e_x^λ be a fs-point. $\varphi_\psi = (\varphi, \psi): FS(U,E) \to FS(V,P)$ is called fs-τ-continuous at e_x^λ if for any τ- neighborhood g_B of $\varphi_\psi(e_x^\lambda)$, there exists a fs-$\tau$-neighborhood f_A of e_x^λ such that $\varphi_\psi(f_A) \sqsubseteq g_B$.

Theorem 5. Let $(U_E^{\sim}, \tau_f), (V_P^{\sim}, \tau_f{}^*)$ be two fs-topological spaces. $\varphi: U \to V, \psi: E \to P$ be mappings and e_x^λ be a fs-point. Then the followings are equivalent:

i) $\varphi_\psi = (\varphi, \psi): FS(U,E) \to FS(V,P)$ is fs-τ-continuous mapping at e_x^λ.

ii) For any fs-τ- neighborhood g_B of $\varphi_\psi(e_x^\lambda)$ there exists a fs-τ- neighborhood f_A of e_x^λ such that $f_A \sqsubseteq \varphi_\psi{}^{-1}(g_B)$.

iii.) The inverse image of every fs-τ-neighborhood of $\varphi_\psi(e_x^\lambda)$ is a fs-τ-neighborhood of e_x^λ.

Proof. The proof can be done similarly to the analogous statements in Atmaca and Zorlutuna (2013).

Example 2. Let $U = \{x, y, z\}$, $V = \{a, b, c\}$ be the universe sets, $E = \{e_1, e_2\}$, $P = \{p_1, p_2\}$ be the parameter sets, $(U_E^{\sim}, \tau_f), (V_P^{\sim}, \tau_f{}^*)$ be two fs-topological spaces where

$\tau_f = \{\Phi, U_E^{\sim}, f_A = \{e_1 = \{x_{0.3}, y_{0.8}, z_{0.5}\}, e_2 = \{x_{0.7}, y_{0.8}, z_{0.4}\}\}$,

$g_B = \{e_2 = \{x_{0.3}, y_{0.2}, z_{0.8}\}, e_3 = \{x_{0.4}, y_{0.6}, z_{0.5}\}\}, h_C = \{e_2 = \{x_{0.3}, y_{0.2}, z_{0.4}\}\}$,

$s_E = \{e_1 = \{x_{0.3}, y_{0.8}, z_{0.5}\}, e_2 = \{x_{0.7}, y_{0.8}, z_{0.8}\}, \{e_3 = \{x_{0.4}, y_{0.6}, z_{0.5}\}\}$.

$\tau_f{}^* = \{\Phi, V_P^{\sim}, k_D = \{p_1 = \{a_{0.5}, b_{0.7}, c_{0.8}\}\}, m_F = \{p_2 = \{a_{0.3}, b_{0.4}, c_{0.8}\}\}$,

$n_P = \{p_1 = \{a_{0.5}, b_{0.7}, c_{0.8}\}, \{p_2 = \{a_{0.3}, b_{0.4}, c_{0.8}\}\}$.

Let $\varphi: U \to V, \psi: E \to P$ be two mappings where $\varphi(x) = \varphi(y) = a, \varphi(z) = b, \psi(e_1) = p_2, \psi(e_2) = p_1$. Then the fs-mapping φ_ψ is is fs-τ-continuous at ${e_2}_x^{0.3}$.

4. Fuzzy Soft Cotopological Spaces

Definition 17. Let U be the universe and E be the parameter set. A fs-cotopological space is a pair $(U_E^{\sim}, \kappa_f)$ where κ_f is a family of fs-sets over U which holds the following conditions:

i) $\Phi, U_E^{\sim} \in \kappa_f$,

ii) If $k_A, m_B \in \kappa_f$ then $k_A \sqcup m_B \in \kappa_f$,

iii) If $(k_A)_{i\in I} \in \kappa_f$ then $\sqcap_{i\in I} (k_A)_{i\in I} \in \kappa_f$.

κ_f is called a fs-cotopology of fs-sets over U. The pair $(U_E^{\sim}, \kappa_f)$ is called fs-cotopological space. The elements of κ_f are called fs-closed sets.

Definition 18. Let r_A be a fs-set over U. r_A is calledthe fs-remote neighborhood of e_x^λ if there exists a fs-closed set k_B such that $e_x^\lambda \notin^{\sim} k_B \sqsupseteq r_A$. The family of all fs-remote neighborhhods of e_x^λ is denoted by $\mathcal{R}\aleph(e_x^\lambda)$.

Example 3. Let $E = \{e_1, e_2, e_3\}$, $U = \{x, y\}$ and

$\kappa_f = \{\Phi, U_E^{\sim}, k_A = \{e_1 = \{x_{0.3}, y_{0.5}, z_{0.7}\}, e_2 = \{x_{0.7}, y_{0.6}, z_{0.3}\}\}$,

$m_B = \{e_1 = \{x_{0.2}, y_{0.3}, z_{0.8}\}, \quad e_3 = \{x_{0.9}, y_{0.3}, z_{0.6}\}\}, n_C = \{e_1 = \{x_{0.2}, y_{0.3}, z_{0.7}\}\}$,

$s_E = \{e_1 = \{x_{0.3}, y_{0.5}, z_{0.8}\}, e_2 = \{x_{0.7}, y_{0.6}, z_{0.3}\}, e_3 = \{x_{0.9}, y_{0.3}, z_{0.6}\}\}$.

Then $r_C = \{e_1 = \{x_{0.1}, y_{0.2}, z_{0.5}\}\} \in \mathcal{R}\aleph({e_1}_x^{0.5})$ since ${e_1}_x^{0.5} \notin^{\sim} m_B \sqsupseteq r_C$.

Theorem 6. Let $(U_E^{\sim}, \kappa_f)$ be a fs-cotopological space, e_x^λ be a fs-point, r_A, s_B be two fs-sets over U. Then the followings hold for $\mathcal{R}\aleph(e_x^\lambda)$:

i) If $r_A \in \mathcal{R}\aleph(e_x^\lambda)$ then $e_x^\lambda \notin^\sim r_A$.
ii) If $r_A, s_B \in \mathcal{R}\aleph(e_x^\lambda)$ then $r_A \sqcup s_B \in \mathcal{R}\aleph(e_x^\lambda)$.
iii) If $r_A \sqsubseteq s_B$ and $s_B \in \mathcal{R}\aleph(e_x^\lambda)$ then $r_A \in \mathcal{R}\aleph(e_x^\lambda)$.

Proof. 1. It is clear from the definition18.

2. Let $r_A, s_B \in \mathcal{R}\aleph(e_x^\lambda)$. Then there exist fs-closed sets k_{A_1}, m_{B_1} such that

$e_x^\lambda \notin^\sim k_{A_1} \sqsupseteq r_A$ and $e_x^\lambda \notin^\sim m_{B_1} \sqsupseteq s_B$.

It is seen that $\lambda > k_{A_1}{}^e(x) > r_A{}^e(x)$ and $\lambda > m_{B_1}{}^e(x) > s_B{}^e(x)$ and hence $\lambda > \max\{k_{A_1}{}^e(x), m_{B_1}{}^e(x)\} >$ $\max\{r_A{}^e(x), s_B{}^e(x)\}$.

This shows that $r_A \sqcup s_B \in \mathcal{R}\aleph(e_x^\lambda)$.

3. Let $s_B \in \mathcal{R}\aleph(e_x^\lambda)$. Then there exists a fs-closed set m_{B_1} such that $e_x^\lambda \notin^\sim m_{B_1} \sqsupseteq s_B$. Since $r_A \sqsubseteq s_B$, $e_x^\lambda \notin^\sim m_{B_1} \sqsupseteq r_A$. It shows that $r_A \in \mathcal{R}\aleph(e_x^\lambda)$.

Theorem 7. Let $(U_E^\sim, \kappa_f)$ be a fs-cotopological space, e_x^λ be a fs-point. The family of all sets of the fs-remote neighborhood of e_x^λ is a σ-ideal, i.e,is hereditary and additive.

Proof. The proof is obvious from Theorem 6.

Definition 19. Let $(U_E^\sim, \kappa_f)$ be a fs-cotopological space, f_A be a fs-set over U. The intersection of all fs-closed sets containing f_A is called the closure of f_A and is denoted by clf_A.

$$clf_A = \sqcap_{i\in I}\{v_{B_i} \sqsubseteq U_E^\sim : v_{B_i} \in \kappa_f, f_A \sqsubseteq v_{B_i}, i \in I\}$$

Theorem 8. Let $(U_E^\sim, \kappa_f)$ be a fs-cotopological space and f_A be a fs-set over U. The followings hold:

i) clf_A is a fs-closed set.
ii) $f_A \sqsubseteq clf_A$.
iii) clf_A is the smallest fs-closed set containing f_A.
iv) $f_A \in \kappa_f$ iff $f_A = clf_A$.

Proof. The proof can be done similarly to the analogous statements Simsekler and Yuksel (2013).

Definition 20. Let $(U_E^\sim, \kappa_f)$ be a fs-cotopological space v_A be a fs-set over U. v_A is called fs-Q-coneighborhood of e_x^λ if there exists a fs-closed set k_B such that $e_x^{1-\lambda}$ is not quasi coincident with k_B and $k_B \sqsupseteq v_A$.

Theorem 9. Let $(U_E^\sim, \kappa_f)$ be a fs-cotopological space and v_A, w_B be two fs-sets over U. Then the followings hold:

i)If v_A is a fs-Q-coneighborhood of e_x^λ then $e_x^{1-\lambda}$ is not quasi coincident with v_A.

ii) If v_A, w_B are fs-Q-coneighborhood of e_x^λ then $v_A \sqcup w_B$ is a fs-Q-coneighborhood of e_x^λ.

iii) If v_A is a fs-Q-coneighborhood of e_x^λ and $w_B \sqsubseteq v_A$ then w_B is a fuzzuy soft Q-coneighborhood of e_x^λ.

Definition 21. Let $(U_E^\sim, \kappa_f), (V_P^\sim, \kappa_f{}^*)$ be two fs-topological spaces, $\varphi: U \to V, \psi: E \to P$ be mappings and e_x^λ be a fs-point. $\varphi_\psi = (\varphi, \psi): FS(U,E) \to FS(V,P)$ is called fs-κ-continuous at e_x^λ if for any fs-remote neighborhood v_A of $\varphi_\psi(e_x^\lambda)$, there exists a fs-remote neighborhood w_B of e_x^λ such that $\varphi_\psi(w_B) \sqsubseteq v_A$.

Theorem 10. Let $(U_E^\sim, \kappa_f), (V_P^\sim, \kappa_f{}^*)$ be two fs-topological spaces, $\varphi: U \to V, \psi: E \to P$ be mappings, e_x^λ be a fs-point and $\varphi_\psi =$

$(\varphi, \psi): FS(U, E) \to FS(V, P)$ be a mapping. Then the followings are equivalent:

i)$\varphi_\psi = (\varphi, \psi): FS(U, E) \to FS(V, P)$ is fs-κ-continuous mapping at e_x^λ.
ii) For any fs-remote neighborhood v_A of $\varphi_\psi(e_x^\lambda)$ there exists a fs-remote neighborhood w_B of e_x^λ such that $w_B \sqsupseteq {\varphi_\psi}^{-1}(v_A)$.

iii) The inverse image of every fs-remote neighborhood of $\varphi_\psi(e_x^\lambda)$ is a fs-remote neighborhood of e_x^λ.

5. Fuzzy Soft Ditopological Spaces

After the definitions and theorems mentioned in the previous sections we can give the main purpose of this paper- a fs-ditopological space, which is a synthesis of a fs-topology, connected to the property of opennes in the space and a fs-cotopology, relaying on the property of closedness in the space.

Definition 22. The triple $(U_E^{\sim}, \tau_f, \kappa_f)$ is said to be a fs-ditopological space if $U_E^{\sim}$ is a fs-set, τ_f is a fs-topology and κ_f is a fs-cotopology on $U_E^{\sim}$. A pair $\delta = (\tau_f, \kappa_f)$ is called a fs-ditopology on $U_E^{\sim}$.

Definition 23. Let $(U_E^{\sim}, \delta)$ be a fs-ditopological space, f_B, m_C be two fs-sets on U and e_x^λ be a fs-point. A pair (f_B, m_C) is called a fs-neighborhood of e_x^λ if f_B is a fs-τ-neighborhood and m_C is a fs-remote neighborhood of e_x^λ.

Fs-interior and fs-closure of a fs-set f_A in a fs-ditopological space $(U_E^{\sim}, \delta)$ are defined respectively by:

$$int f_A = \sqcup_{i \in I}\{g_{B_i} \sqsubseteq U_E^{\sim}: g_{B_i} \in \tau_f, g_{B_i} \sqsubseteq f_A, i \in I\}$$

$$cl f_A = \sqcap_{i \in I}\{v_{B_i} \sqsubseteq U_E^{\sim}: v_{B_i} \in \kappa_f, f_A \sqsubseteq v_{B_i}, i \in I\}$$

Definition 24. Let $(U_E^{\sim},\ \delta_1)$, $(V_P^{\sim},\ \delta_2)$ be two fs-ditopological spaces. A mapping $\varphi_\psi = (\varphi, \psi): (U_E^{\sim},\ \delta_1) \to (V_P^{\sim},\ \delta_2)$ is called fs-continuous if the preimage of any fs-set from τ_2 is in τ_1 and the preimage of any fs-set from κ_2 is in κ_1.

Theorem 11. A mapping

$\varphi_\psi = (\varphi, \psi): (U_E^{\sim},\ \delta_1) \to (V_P^{\sim},\ \delta_2)$ be a mapping Then the followings are equivalent:

i) φ_ψ is fs-continuous at e_x^λ.

ii) For any fs-τ- neighborhood f_A and fs-remote neighborhood v_A of $\varphi_\psi(e_x^\lambda)$ there exist a fs-τ- neighborhood g_B and a fs-remote neighborhood w_B of e_x^λ such that $g_B \sqsubseteq {\varphi_\psi}^{-1}(f_A)$ and $w_B \sqsupseteq {\varphi_\psi}^{-1}(v_A)$ respectively.

iii) The preimage of every fs-τ- neighborhood of $\varphi_\psi(e_x^\lambda)$ is a fs-τ- neighborhood of e_x^λ and the preimage of every fs-remote neighborhood of $\varphi_\psi(e_x^\lambda)$ is a fs-remote neighborhood of e_x^λ.

Proof. The proof is obvious by Theorem 5. and Theorem 10.

6. References

Ahmad, B. and Hussain, S., 2012. On some structures of soft topology. Mathematical Sciences, DOI: 10.1186/2251-7456-6-64.

Aktas, H. and Cagman, N. 2007. Soft sets and soft groups. Inform. Sci., 77, 2726-2735.

Atmaca, S. and Zorlutuna, I. 2013. On fuzzy soft topological spaces. Annals of Fuzzy Mathematics and Informatics, 5(2), 377-386.

Atmaca, S. and Zorlutuna, I. 2014. On Topological Structures of Fuzzy Parametrized Soft Sets. The Scientific World Journal, Article ID164176.

Aygunoglu, A. and Aygun, H. 2011. Some notes on soft topological spaces. Neural Computing and Applications, 21(1),113-119.

Aygunoglu, A., Cetkin, V. and Aygun, H. 2014. An introduction to fs-topological spaces. Hacettepe Journal of Mathematics and Statistics,43(2),197-208.

Bera, S, Roy, S.K, Karaaslan, F, and Cağman, N. 2017. Soft congruence relation over lattices, Hacettepe Journal of Mathematics and Statistics, 46(6), 1035 – 1042.

Brown, L.M. and Diker, M., 1998. Ditopological texture spaces and intuitionistic sets. Fuzzy Sets and Systems, 98, 217-224.

Cagman, N., Karatas, S., and Enginoglu, S. 2011. Soft topology. Computers and Mathematics with Applications, 62, 351-358.

Chang, C.L.1968. Fuzzy topological spaces. J. Math. Appl., 24, 182-193.

Feng, F., Jun, Y.B. and Zhao, X. 2008. Soft semirings. Computers and Mathematics with Applications, 56, 2621-2628.

Karaaslan, F, Cagman, N, and Enginoglu, S. 2012. Soft Lattices, Journal of New Results in Science (1), 5-17.

Karaaslan, F. 2016. Soft Classes and Soft Rough Classes with Applications in Decision Making. Mathematical Problems in Engineering, 1-11. Doi: 10.1155/2016/1584528.

Karaaslan, F, Çagman N. 2013. Fuzzy Soft Lattice Theory, ARPN Journal of Science and Technology, 3(3).

Kelly, J.L. 1963. Bitopological spaces. Proc. Lond. Math.Soc. III Ser, 13, 71-89.

Maji, P.K. Biswas, R. and Roy, A.R. 2001. Fuzzy soft sets. J. Fuzzy Math. 9(3), 589-602.

Min, W.K. 2011. A note on soft topological spaces. Comput. Math. Appl., 62, 3524-3528.

Molodtsov, D. 1999. Soft set theory-First results. Computers Math. Appl.,37(4/5), 19-31.

Nazmul, S. and Samanta, S.K. 2010. Soft topological groups. Kochi J. Math., 5,151-161.

Ozturk, T. 2016. A new approach to soft uniform spaces,Turkish Journal of Mathematics, 40, 1071-1084.

Pazar Varol, B. and Aygun, H. 2012. Fuzzy soft topology. Hacettepe Journal of Mathematics and Statistics, 41(3), 407-419.

Roy, S. and Samanta, T.K. 2012. A note on fuzzy soft topological spaces. Annals of Fuzzy Mathematics and Informatics, 3, 305-311.

Roy, S. and Samanta, T.K. 2013. An introduction to open and closed sets on fuzzy soft topological spaces. Annals of Fuzzy Math. and Inf., 6(2), 425-431.

Shabir, M. and Naz, M. 2011. On soft topological spaces. Comput. Math. Appl.,61, 1786-1799.

Simsekler, T. and Yuksel, S. 2013. Fuzzy soft topological spaces. Annals of Fuzzy Mathematics and Informatics, 5(1), 87-96.

Simsekler Dizman, T., Sostak, A. and Yuksel, S. 2016. Soft Ditopological Spaces. Filomat, 30(1), 209-222.

Tanay, B. and Kandemir, M.B. 2011. Topological structures of fuzzy soft sets. Computers and Mathematics with Applications, 61, 412-418.

Zadeh, L.A. 1965. Fuzzy sets. Information and Control, 8, 338-353.

Zorlutuna, I, Akdag, M., Min, W.K. and Atmaca, S. 2012. Remarks on soft topological spaces. Annals of Fuzzy Mathematics and Informatics, 3(2), 171-185.