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Research Article

Some Applications of the Concircular Mappings on the Weyl Manifolds

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Abstract

In this paper, two applications of concircular mappings on the Weyl manifolds are given: Firstly, a necessary and sufficient condition for an Einstein-Weyl manifold to be concircularly Ricci-flat is obtained. Secondly, after defining a special type of semi-symmetric non-metric connection which is called S -concircular, some properties of the Weyl manifold with a vanishing curvature tensor with respect to such a connection are examined.
Keywords: Weyl manifold, semi-symmetric non-metric S -concircular connection, vanishing curvature tensor.

Weyl Manifoldları Üzerindeki Concircular Tasvirlerin Bazı Uygulamaları

Öz

Bu çalışmada, Weyl manifoldları üzerindeki concircular tasvirlerin iki uygulaması verilmiştir: İlk olarak, Einstein-Weyl manifoldunun concircular Ricci düz olabilmesi için bir gerek-yeter şart elde edilmiştir. Daha sonra da, S -concircular olarak adlandırılan özel tipteki bir yarı simetrik non-metrik konneksiyon tanımlanarak, böyle bir konneksiyona göre sıfırlanan eğrilik tensörüne sahip Weyl manifoldunun bazı özellikleri incelenmiştir.
Anahtar Kelimeler: Weyl manifoldu, yarı simetrik non-metrik S -concircular konneksiyon, sıfırlanan eğrilik tensörü.

1. Introduction

Scholz widely reviewed the physical concepts in Weyl geometry, (Scholz, 2008). Herman Weyl first introduced a gauge invariant theory to unify gravity and electromagnetic theories in 1918. This theory is not acceptible as a unified theory since the electromagnetic potential does not couple to spinor being essential for the electromagnetic theory. This does not mean that Weyl geometry has a physical meaning as well in different theories and in the second part of the 20th century, the Weyl geometry has been studied in some research fields of physics such as quantum mechanics, particle physics, gravity and cosmology. Despite the unified theory of Weyl not being acceptable as physical theory, it introduced a useful theory in differential geometry. The mathematics of the theory is a generalization of the Riemannian geometry and the connection is an instructive example of non-metric connections.

A Weylian metric on a differentiable manifold M can be given by pairs (g,φ) of a non-degenerate symmetric differential 2-form g and a differential 1-form φ. The Weylian metric consists of the equivalance class of such pairs, with $(\tilde{g},\tilde{\varphi})\sim(g,\varphi)$ if and only if

(i) $\tilde{g} = \lambda^2 g$,
(ii) $\tilde{\varphi} = \varphi - d(log\lambda)$ (1)

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for a strictly positive real function $\lambda > 0$ on M. Choosing a representative means to gauge the Weylian metric; g is then the Riemannian component and φ the scale connection of the gauge. A change of representative (1) is called a Weyl(scale) transformation; it consists of a rescaling (i) and a scale gauge transformation (ii). A manifold with a Weylian metric (M,g,φ) will be called a Weyl manifold.

In the recent mathematical literature, a Weyl structure on a differentiable manifold M is spesified by a pair (g,φ) consisting of a conformal metric g and an affine, which is torsion-free, connection Γ , respectively its covariant derivative ∇. For any conformal metric g, there is a differential 1-form φ_g such that

$$\nabla g + 2\varphi_g \otimes g = 0 \quad (2)$$

which is called weak compatibility of the affine connection with the metric. (2) can be expressed in local coordinates by

which is called *weak compatibility* of the affine connection with the metric. (2) can be expressed in local coordinates by

$$\nabla_k g_{ij} - 2 g_{ij} T_k = 0 \quad (3)$$

or equivalently,

$$\nabla_k g^{ij} + 2 g^{ij} T_k = 0 \quad (4)$$

where T_k is a complementary covariant vector field, (Norden, 1976). Such a Weyl manifold will be denoted by W_n (g_{ij}, T_k). If $T_k = 0$ or T_k is gradient, a Riemannian manifold is obtained.

One could also formulate above compatibility condition by

$$\Gamma - {}_g\Gamma = 1 \otimes \varphi_g + \varphi_g \otimes 1 - g \otimes \varphi_g^* \quad (5)$$

where 1 denotes identity in Hom (V;V) for every $V = T_x M$, ${\varphi_g}^*$ is the dual of φ_g with respect to g and g^Γ is the Levi-Civita connection of g. In local coordinates, (5) is given by

$$\Gamma_{jk}^i = \left\{ {i \atop jk} \right\} - \delta_j^i T_k - \delta_k^i T_j + g_{jk} T^i \quad (6)$$

where $\left\{ {i \atop jk} \right\}$'s are the coefficients of the Levi-Civita connection, (Norden, 1976).

The curvature tensor R_{ijk}^h, the Ricci tensor R_{ij} and the scalar curvature R of the symmetric connection Γ on the Weyl manifold are defined by

$$R_{ijk}^h = \partial_j \Gamma_{ik}^h - \partial_k \Gamma_{ij}^h + \Gamma_{rj}^h \Gamma_{ik}^r - \Gamma_{rk}^h \Gamma_{ij}^r$$

$$R_{ij} = R_{ijr}^r \quad (7)$$

$$R = R_{ij} g^{ij}$$

The coefficients Γ_{jk}^i and the curvature tensor R_{ijk}^h of the symmetric connection Γ change by

$${\Gamma_{jk}^i}^* = \Gamma_{jk}^i + \delta_j^i P_k + \delta_k^i P_j - g_{jk} P^i \quad (8)$$

and

$${R_{ijk}^h}^* = R_{ijk}^h + 2\delta_i^h \nabla_{[j} P_{k]} + \delta_k^h P_{ij} - \delta_j^h P_{ik} + g_{ij} g^{hr} P_{rk} - g_{ik} g^{hr} P_{rj} \quad (9)$$

respectively, where

$$T_i - {T_i}^* = P_i$$

and

$$P_{ij} = \nabla_j P_i - P_i P_j + \frac{1}{2} g_{ij} g^{kr} P_k P_r$$

Under conformal mapping ${g_{ij}}^* = g_{ij}$, (Norden, 1976).

The conformal curvature tensor C_{mijk} and the concircular curvature tensor $\tilde{C}_{mijk}$ of the symmetric connection Γ on the Weyl manifold are given by

$$C_{mijk} = R_{mijk} - \frac{1}{n} g_{mi} R^r_{rjk} + \frac{1}{n-2}\left(g_{mj}R_{ik} - g_{mk}R_{ij} - g_{ij}R_{mk} + g_{ik}R_{mj}\right) - \frac{1}{n(n-2)}\left\{g_{mj}R^r_{rki} - g_{mk}R^r_{rji} - g_{ij}R^r_{rkm} + g_{ik}R^r_{rjm}\right\} - \frac{R}{(n-1)(n-2)}\left(g_{mj}g_{ik} - g_{mk}g_{ij}\right) \quad (10)$$

and

$$\tilde{C}_{mijk} = R_{mijk} - \frac{R}{n(n-1)}\left(g_{mk}g_{ij} - g_{mj}g_{ik}\right) \quad (11)$$

where R^h_{ijk}, R_{ij} and R denote the curvature tensor, Ricci tensor and scalar curvature tensor of Γ , respectively, (Miron, 1968 & Özdeğer, 2002).

The projective curvature tensor P_{mijk} of Γ is obtained as

$$P_{mijk} = R_{mijk} + \frac{g_{mi}}{n+1}\left(R_{jk} - R_{kj}\right) + \frac{1}{n^2-1}\left[g_{mj}\{nR_{ik} + R_{ki}\} - g_{mk}\{nR_{ij} + R_{ji}\}\right] \quad (12)$$

(Eisenhart, 1927).

By using (10), (11) and (12), the conformal curvature tensor C_{mijk} and the projective curvature tensor P_{mijk} of the connection Γ are expressed in terms of the concircular curvature tensor $\tilde{C}_{mijk}$ by the following equations:

$$C_{mijk} = \tilde{C}_{mijk} - \frac{2}{n} g_{mi}\tilde{C}_{[kj]} + \frac{1}{n-2}\left(g_{mj}\tilde{C}_{ik} - g_{mk}\tilde{C}_{ij} + g_{ik}\tilde{C}_{mj} - g_{ij}\tilde{C}_{mk}\right) - \frac{2}{n(n-2)}\left\{g_{mj}\tilde{C}_{[ik]} - g_{mk}\tilde{C}_{[ij]} + g_{ik}\tilde{C}_{[mj]} - g_{ij}\tilde{C}_{[mk]}\right\} \quad (13)$$

$$P_{mijk} = \tilde{C}_{mijk} + \frac{g_{mi}}{n+1}\left(\tilde{C}_{jk} - \tilde{C}_{kj}\right) + \frac{1}{n^2-1}\left[g_{mj}\{n\tilde{C}_{ik} + \tilde{C}_{ki}\} - g_{mk}\{n\tilde{C}_{ij} + \tilde{C}_{ji}\}\right] \quad (14)$$

2. Preliminaries

V.Murgescu defined the coefficients $\overline{\Gamma}^i_{jk}$ of a generalized connection $\overline{\Gamma}$ on the Weyl manifold by

$$\overline{\Gamma}^i_{jk} = \Gamma^i_{jk} + a_{jkh}g^{hi}$$

where

$$a_{jkh} = g_{jr}\Omega^r_{kh} + g_{rk}\Omega^r_{jh} + g_{rh}\Omega^r_{jk} \quad (15)$$

and Γ^i_{jk} 's are the coefficients of the symmetric connection Γ, (Murgescu, 1968).

By choosing

$$\Omega^i_{jk} = \delta^i_j a_k - \delta^i_k a_j$$

in (15), the coefficients $\overline{\Gamma}^i_{jk}$'s of a *semi symmetric connection* $\overline{\Gamma}$ on the Weyl manifold are obtained by

$$\overline{\Gamma}^i_{jk} = \Gamma^i_{jk} + \delta^i_k S_j - \delta^i_j S_k \quad (16)$$

(Unal, 2005). In (11), $S_i = -2a_i$ where a_i is an arbitrary covariant vector field.

The torsion tensor T^i_{jk} with respect to the semi symmetric connection $\overline{\Gamma}$ is

$$T^i_{jk} = \delta^i_k S_j - \delta^i_j S_k \quad (17)$$

and the compatibility condition of the Weyl manifold admitting $\overline{\Gamma}$ is in the form of

$$\overline{\nabla}_k g_{ij} = 2 g_{ij} T_k$$

which means that semi symmetric connection is also *non-metric* or *recurrent metric.*

In local coordinates, the curvature tensor of $\overline{\Gamma}$ is defined by

$$\overline{R}_{ijk}^{h} = \partial_j \overline{\Gamma}_{ik}^{h} - \partial_k \overline{\Gamma}_{ij}^{h} + \overline{\Gamma}_{rj}^{h} \overline{\Gamma}_{ik}^{r} - \overline{\Gamma}_{rk}^{h} \overline{\Gamma}_{ij}^{r} \quad (18)$$

By means of (16) and (18), the relation between the curvature tensors R_{ijk}^{h} and $\overline{R}_{ijk}^{h}$ of Γ and $\overline{\Gamma}$, respectively, is obtained as

$$\overline{R}_{ijk}^{h} = R_{ijk}^{h} + \delta_k^h S_{ij} - \delta_j^h S_{ik} + g_{ij} g^{hr} S_{rk} - g_{ik} g^{hr} S_{rj} \quad (19)$$

where

$$S_{ij} = S_{i,j} - S_i S_j + \frac{1}{2} g_{ij} g^{kr} S_k S_r \quad (20)$$

and $S_{i,j}$ denotes the covariant derivative of S_i with respect to the symmetric connection Γ, (Unal, 2005).

Transvecting (19) by g_{mh} and contracting on the indices h and k in the same equation give

$$\overline{R}_{mijk} = R_{mijk} + g_{mk} S_{ij} - g_{mj} S_{ik} + g_{ij} S_{mk} - g_{ik} S_{mj} \quad (21)$$

and

$$\overline{R}_{ij} = R_{ij} + (n-2) S_{ij} + S g_{ij} \quad (22)$$

where $S = g_{mk} S^{mk}$, respectively.

The scalar curvatures R and $\overline{R}$ of the connections Γ and $\overline{\Gamma}$, respectively, are related by

$$\overline{R} = R + 2(n-1) S \quad (23)$$

The concircular curvature tensor $\overline{\tilde{C}}_{ijk}^{h}$ of the semi symmetric non-metric connection $\overline{\Gamma}$ is defined by

$$\overline{\tilde{C}}_{ijk}^{h} = \overline{R}_{ijk}^{h} - \frac{\overline{R}}{n(n-1)} \left(\delta_k^h g_{ij} - \delta_j^h g_{ik} \right) \quad (24)$$

(Nurcan, 2014).

First transvecting (24) by g_{mh} and then contracting on the indices h and k in (24), the equations

$$\overline{\tilde{C}}_{mijk} = \overline{R}_{mijk} - \frac{\overline{R}}{n(n-1)} \left(g_{mk} g_{ij} - g_{mj} g_{ik} \right) (25)$$

and

$$\overline{\tilde{C}}_{ij} = \overline{R}_{ij} - \frac{\overline{R}}{n} g_{ij} \quad (26)$$

are obtained.

Lemma 2.1: The concircular curvature tensor of the semi symmetric non-metric connection $\overline{\Gamma}$ has the following properties:

(a) $\overline{\tilde{C}}_{mijk} + \overline{\tilde{C}}_{mikj} = 0$;

(b) $\overline{\tilde{C}}_{mijk} + \overline{\tilde{C}}_{imjk} = 4 g_{mi} \nabla_{[k} T_{j]}$;

(c) $\overline{\tilde{C}}_{rjk}^{r} = \overline{R}_{rjk}^{r}$;

(d) $\overline{\tilde{C}}_{mijk} + \overline{\tilde{C}}_{mjki} + \overline{\tilde{C}}_{mkij} = 0.$

The concircular curvature tensors $\tilde{C}_{ijk}^{h}$ and $\overline{\tilde{C}}_{ijk}^{h}$ of Γ and $\overline{\Gamma}$, respectively, are related by

$$\overline{\tilde{C}}_{ijk}^{h} = \tilde{C}_{ijk}^{h} + \delta_k^h S_{ij} - \delta_j^h S_{ik} + g_{ij} g^{mh} S_{mk} - g_{ik} g^{mh} S_{mj} - \frac{2}{n} S \left(\delta_k^h g_{ij} - \delta_j^h g_{ik} \right) (27)$$

by substituting (11), (19) and (23) in (24).

Transvecting (27) by g_{mh} and contracting on h and k in the same equation give

$$\overline{\tilde{C}}_{mijk} = \tilde{C}_{mijk} + g_{mk}S_{ij} - g_{mj}S_{ik} + g_{ij}S_{mk} - g_{ik}S_{mj} - \frac{2}{n}S\left(g_{mk}g_{ij} - g_{mj}g_{ik}\right) \quad (28)$$

and

$$\overline{\tilde{C}}_{ij} = \tilde{C}_{ij} + (n-2)S_{ij} - \frac{(n-2)}{n}g_{ij}S. \quad (29)$$

3. Main Theorems

3.1. *A Special condition for an Einstein-Weyl manifold to be concircularly Ricci-flat*

It is well known that a Riemannian manifold is called Einstein manifold if

$$K_{ij} = \frac{K}{n}g_{ij}$$

where K_{ij} and K denote the Ricci tensor and the scalar curvature, respectively, (Besse, 1987). Similar to this definition, Einstein-Weyl manifold is defined as follows:

Definition 3.1.1: If the symmetric part of the Ricci tensor R_{ij} of the connection Γ satisfies

$$R_{(ij)} = \frac{R}{n}g_{ij}$$

on any Weyl manifold, then it is called *Einstein-Weyl manifold* and denoted by $(EW)_n$.

The condition in the above definition implies

$$\tilde{C}_{(ij)} = R_{(ij)} - \frac{R}{n}g_{ij} = 0 \Leftrightarrow \tilde{C}_{ij} = \tilde{C}_{[ij]} \quad (30)$$

which means that the concircular Ricci tensor $\tilde{C}_{ij}$ is skew-symmetric on $(EW)_n$.

By using (30) in (13) and (14), the conformal curvature tensor C_{mijk} and the projective curvature tensor P_{mijk} of $(EW)_n$ are given as in the following corollary:

Corollary 3.1.1: The conformal curvature tensor C_{mijk} and the projective curvature tensor P_{mijk} of Γ are defined by

$$C_{mijk} = \tilde{C}_{mijk} - \frac{2}{n}g_{mi}\tilde{C}_{kj} + \frac{1}{n}\left(g_{mj}\tilde{C}_{ik} - g_{mk}\tilde{C}_{ij} + g_{ik}\tilde{C}_{mj} - g_{ij}\tilde{C}_{mk}\right) \quad (31)$$

and

$$P_{mijk} = \tilde{C}_{mijk} - \frac{2}{n+1}g_{mi}\tilde{C}_{kj} + \frac{1}{n+1}\left[g_{mj}\tilde{C}_{ik} - g_{mk}\tilde{C}_{ij}\right] \quad (32)$$

on an Einstein-Weyl manifold $(EW)_n$.

Einstein manifolds are concircularly Ricci flat in the Riemannian geometry, but this case is different in Weyl geometry. So we will give a condition for any $(EW)_n$ to be concircularly Ricci flat after giving the following definition:

Definition 3.1.2: $(EW)_n$ is called *concircularly Ricci flat*, if the concircularly Ricci tensor $\tilde{C}_{ij}$ satisfies

$$\tilde{C}_{ij} = 0 \quad (33)$$

Suppose that the tensors C_{mijk}, P_{mijk} and $\tilde{C}_{mijk}$ coincide on $(EW)_n$. Firstly,by equaling C_{mijk} and $\tilde{C}_{mijk}$, it is obtained that

$$\frac{2}{n}g_{mi}\tilde{C}_{kj} - \frac{1}{n}\left(g_{mj}\tilde{C}_{ik} - g_{mk}\tilde{C}_{ij} + g_{ik}\tilde{C}_{mj} - g_{ij}\tilde{C}_{mk}\right) = 0 \quad (34)$$

Transvecting (34) by g^{mi} yields (33). Then, by equaling P_{mijk} and $\tilde{C}_{mijk}$, the resulted equation

$$\frac{2}{n+1}g_{mi}\tilde{C}_{kj} - \frac{1}{n+1}\left[g_{mj}\tilde{C}_{ik} - g_{mk}\tilde{C}_{ij}\right] = 0 \quad (35)$$

implies (33) which states that $(EW)_n$ is concircularly Ricci flat.

Conversely, let $(\boldsymbol{EW})_{\boldsymbol{n}}$ be concircularly Ricci flat. Then, substituting (33) in (31) and (32) implies

$$C_{mijk} = P_{mijk} = \tilde{C}_{mijk}.$$

So, it can be given the following theorem:

Theorem 3.1.1: The necessary and sufficient condition for an Einstein-Weyl manifold $(\boldsymbol{EW})_{\boldsymbol{n}}$ to be concircularly Ricci flat is that conformal curvature tensor, concircular curvature tensor and projective curvature tensor coincide.

3.2. ***Vanishing curvature tensor on the Weyl manifolds admitting a semi- symmetric non-metric S-concircular connection***

Liang defined a semi symmetric recurrent metric connection which is S -concircular on the Riemannian manifolds, (Liang, 1994). In this paper, a semi symmetric non-metric S -concircular connection on the Weyl manifold is defined as follows:

Definition 3.2.1: If the semi symmetric non-metric connection $\overline{\Gamma}$ satisfies the condition given by

$$S_{ij} = \nabla_j S_i - S_i S_j + \frac{1}{2} g_{ij} g^{rs} S_r S_s = \beta g_{ij}$$

where β is a smooth function on the Weyl manifold, then it is called S *-concircular*.

By relating to the above definition, the following theorem is presented in the 12. National Geometry Symposium in Bilecik University:

Theorem 3.2.1 [8]**:** A necessary and sufficient condition for the semi symmetric non-metric connection $\overline{\Gamma}$ to be S -concircular is that the concircular curvature tensors $\tilde{C}_{mijk}$ and $\overline{\tilde{C}}_{mijk}$ of the connections Γ and $\overline{\Gamma}$, respectively, coincide.

If the semi symmetric non-metric connection is S -concircular, we know that

$$\overline{\tilde{C}}_{mijk} = \tilde{C}_{mijk}$$

which means that

$$\overline{R}_{mijk} - \frac{\overline{R}}{n(n-1)}\left(g_{mk}g_{ij} - g_{mj}g_{ik}\right) = R_{mijk} - \frac{R}{n(n-1)}\left(g_{mk}g_{ij} - g_{mj}g_{ik}\right).$$

Suppose that the Weyl manifold has a vanishing curvature tensor with respect to the semi symmetric non-metric S -concircular connection, that is,

$$\overline{R}_{mijk} = 0.$$

Then, $\overline{R}_{ij} = 0$ and $\overline{R} = 0$. In this case,

$$R_{mijk} = \frac{R}{n(n-1)}\left(g_{mk}g_{ij} - g_{mj}g_{ik}\right) \quad (36)$$

By transvecting (36) by g_{mh},

$$R^h_{ijk} = \frac{R}{n(n-1)}\left(\delta^h_k g_{ij} - \delta^h_j g_{ik}\right)$$

and by contracting on the indices h and i,

$$R^h_{hjk} = 0$$

which means that Ricci tensor R_{ij} is symmetric. By transvecting (36) by g^{mk},

$$R_{ij} = \frac{R}{n} g_{ij} \quad (37)$$

is obtained.

Hence we have the following:

Theorem 3.2.2: The Weyl manifold which has a vanishing curvature tensor with respect to the semi symmetric non-metric S -

concircular connection is an Einstein-Weyl manifold.

By substituting (37) in (10), (11) and (12), respectively, the conformal curvature tensor, concircularly curvature tensor and projective curvature tensor are in the form of

$$C_{mijk} = 0, \quad \tilde{C}_{mijk} = 0, \; P_{mijk} = 0.$$

Hence it can be given the following corollary:

Corollary 3.2.1: The Weyl manifold with a vanishing curvature tensor with respect to the semi symmetric non-metric S -concircular connection is conformally flat, concircularly flat and projectively flat.

So, considering conformally symmetry, concircularly symmetry and projectively symmetry for the Weyl manifolds mentioned above are illogical. But what can we say about locally symmetry for the Weyl manifold?

Firstly, suppose that the Weyl manifold which has a vanishing curvature tensor with respect to the semi symmetric non-metric S -concircular connection is locally symmetric, i.e. $R_{\;mijk,l} = 0$. In this case, curvature tensor R_{mijk} is in the form of (36). By taking covariant derivative of that equation and using the definition of locally symmetry,

$$\frac{R_{,l}}{n(n-1)}\left(g_{mk}g_{ij} - g_{mj}g_{ik}\right) + \frac{R}{n(n-1)}\left(g_{mk,l}g_{ij} + g_{mk}g_{ij,l} - g_{mj,l}g_{ik} - g_{mj}g_{ik,l}\right)=0 \quad (38)$$

is obtained. (38) is rewritten as follows by using (3) in (38):

$$\frac{(R_{,l}+4T_lR)}{n(n-1)}\left(g_{mk}g_{ij} - g_{mj}g_{ik}\right) = 0 \quad (39)$$

From (39), this means that $R_{,l} + 4T_lR = 0$. If the covariant derivative of $R = R_{ij}g^{ij}$ given by

$$R_{,l} = R_{ij,l}g^{ij} + R_{ij}\left(-2g^{ij}T_l\right)$$

with the aid of (4) is substituted in the last equation,

$$\left(R_{ij,l} + 2T_lR_{ij}\right)g^{ij} = 0$$

meaning

$$R_{ij,l} + 2T_lR_{ij} = 0 \Leftrightarrow R_{ij,l} = -2T_lR_{ij}$$

is obtained.

Conversely, assume that the mentioned Weyl manifold is Ricci-recurrent with the recurrency vector of $-2T_l$. By taking the covariant derivative of $R = R_{ij}g^{ij}$ and using $R_{ij,l} = -2T_lR_{ij}$, it is obtained that

$$R_{,l} = \left(-2T_lR_{ij}\right)g^{ij} + R_{ij}\left(-2T_lg^{ij}\right)$$

which leads us

$$R_{,l} = -4T_lR \quad . \quad (40)$$

If (40) is substituted in (39), then

$$R_{mijk,l} = 0.$$

In the view of the above results, we can state the following theorem:

Theorem 3.2.3: The necessary and sufficient condition for the Weyl manifold with a vanishing curvature tensor with respect to a semi symmetric non-metric S -concircular connection to be locally symmetric is that it is Ricci-recurrent with the recurrency vector of $-2T_l$.

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