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THE SYSTEM STABILITY SOFTWARE TOOL BASED ON ROUTH-HURWITZ CRITERION

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Abstract: Stability is one of the most important parameters / factors in the field of system analysis and design. For this reason, stability analysis should be well learned and understood in engineering education as well as need to be performed perfectly in practice. In this study; a software tool has been developed that can perform absolute, relative and conditional stability analysis for single input single output linear systems based on the Routh-Hurwitz criterion. Stability analysis of the systems defined by the user can be performed easy, fast and efficiently - including all possible general and special conditions - step-by-step in detail with the designed software tool, and the results can be obtained both numerically and graphically with many parameters.

Keywords: Stability, Routh-Hurwitz criterion, software tool, engineering education.

Routh-Hurwitz Kriteri Tabanlı Sistem Kararlılık Yazılım Aracı

Öz: Kararlılık, sistem analiz ve tasarımı alanındaki en önemli parametre / etkenlerden birisidir. Bu nedenle kararlılık analizinin hem mühendislik eğitiminde çok iyi öğrenilip kavranması hem de uygulamada kusursuz bir şekilde yapılması gerekmektedir. Gerçekleştirilen çalışmada; doğrusal zamanla değişmeyen tek girişli tek çıkışlı sistemler için Routh-Hurwitz kriterine göre mutlak, göreceli ve şartlı kararlılık analizi yapabilen bir yazılım aracı geliştirilmiştir. Tasarlanan yazılım aracı ile kullanıcının tanımladığı sistemlerin kararlılık analizleri; kolay, hızlı ve etkin bir biçimde - tüm olası genel ve özel durumları içererek - ayrıntılı bir şekilde adım adım gerçekleştirebilmekte ve birçok parametreleriyle birlikte sonuçlar hem sayısal hem de grafiksel olarak elde edilebilmektedir.

Anahtar Kelimeler: Kararlılık, Routh-Hurwitz kriteri, yazılım aracı, mühendislik eğitimi.

1. INTRODUCTION

The most important feature of the systems is stability. For this reason, stability analysis is at the center of system analysis and design. While stability analysis of linear time-invariant (LTI) systems is easier and simpler; the analysis is very difficult and complex in non-linear and time-varying systems. There are several methods for performing the relevant analysis (Bennett, 1996; Dorf and Bishop, 2010; Franklin et al., 2014; Golnaraghi and Kuo, 2009; Nise, 2015; Ogata, 2009). However, some of these methods - especially in large systems - are difficult to apply manually, so they must be performed by using computer systems. Several applications related to the stability of systems have been developed for both practice and education (Chen et al., 2013;

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Emami and Benin, 2016; Kheir et al., 1996; MathWorks, 2018; Vatansever and Hatun, 2014; Vatansever and Yalcin, 2017).

In this study, a software tool which performs stability analysis according to the Routh-Hurwitz criterion was designed for the LTI systems. By using the software tool having user-friendly interface; absolute, relative and conditional stability analysis of the system defined by the user/student by entering the coefficients of the characteristic equation can be performed. Stability analyzes can be performed step-by-step in the software tool, which takes all possible situations into account, in the way that the user has chosen for special cases. Along with stable, marginally stable and unstable results of the system, stability limits can also be obtained for the systems defined in terms of symbolic parameters. Thus, the software tool can be used very easily, effectively and efficiently in both practice and education purposes.

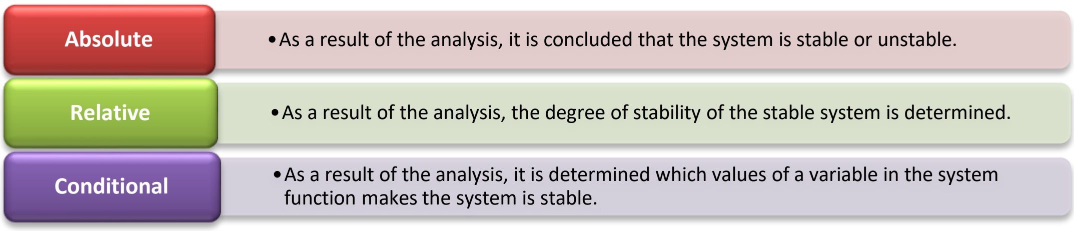
This paper is organized as follows: Routh-Hurwitz stability criterion is briefly described in Section 2. The designed software tool and applications (results) are given in Section 3-4 and finally, the study conclusions are detailed in Section 5.

2. ROUTH-HURWITZ STABILITY CRITERION

Although there are many types and definitions of stability, dynamic systems that produce a limited output for the limited input are considered as a stable system. The stability definitions in a linear time-invariant system are summarized in Table 1, associated with the system's total response and characteristic equation. In addition, stability analysis is classified as in Figure1 (Dorf and Bishop, 2010; Franklin et al., 2014; Golnaraghi and Kuo, 2009; Nise, 2015; Ogata, 2009).

Table 1. Stability definitions using total response or characteristic equation

Stability	The total response of a system	Roots of characteristic equation
	$y(t) = \underbrace{y_{natural}(t)}_{zero-input\ response} + \underbrace{y_{forced}(t)}_{zero-state\ response}$	$\Delta(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$
Stable (simply stable, asymptotically stable)	When the time goes to infinity, the answer goes to zero	All roots are in the left half s-plane
Marginally stable	When the time goes to infinity, the answer remains constant or oscillated	There are at least one simple root on the $j\omega$-axis that is not a complex multiple root pair
Unstable	When the time goes to infinity, the answer grows unlimited	At least one simple root on the right half s-plane or at least one complex multiple root pair on the $j\omega$-axis



- As a result of the analysis, it is concluded that the system is stable or unstable.
- As a result of the analysis, the degree of stability of the stable system is determined.
- As a result of the analysis, it is determined which values of a variable in the system function makes the system is stable.

Figure 1:
Types of stability analysis

The Routh-Hurwitz criterion is a method of stability analysis without calculating the roots of the characteristic equation. For the system to be stable according to this criteria, in other words, in order to have all the roots of the characteristic equation in the left half of the s-plane, first of all, it must satisfy the following necessity and sufficiency conditions:

- *Necessity condition:* All coefficients of the characteristic equation are present (different from zero) and have the same sign.
- *Sufficiency condition:* No change of signs in the first column of the obtained Routh table.

From the coefficients of the characteristic equation (Eq. 1), the Routh table is formed as follows.

$$\Delta(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0 \tag{1}$$

However, alternative solutions used in some specific cases are summarized in Table 2 (Dorf and Bishop, 2010; Franklin et al., 2014; Golnaraghi and Kuo,2009; Nise, 2015; Ogata, 2009). If there is no change of signs in the first column of this table, the system is stable; The system is unstable if there is a change of sign, and the number of these changes of signs also gives the number of roots in the right half of the s-plane of the characteristic equation.

s^n	a_n	a_{n-2}	a_{n-4}	a_{n-6}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	a_{n-7}	$\cdots$
s^{n-2}	$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}$	$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$	$b_3 = \frac{a_{n-1}a_{n-6} - a_n a_{n-7}}{a_{n-1}}$	$\cdots$	
s^{n-3}	$c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1}$	$c_2 = \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1}$	$\cdots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$		
s^2	d_1	d_2	0		
s^1	e_1	0			
s^0	$f_1 = \frac{e_1 d_2 - d_1 0}{e_1}$				

Table 2. Special cases in Routh table and theirs alternative solutions

Special Cases	Solution
The first element in any line is 0	Writing a symbolic very small positive ε instead of a coefficient which is zero
	Using the reciprocal of the characteristic polynomial
	Addition of a stable pole to the characteristic equation
All elements in any row have 0	Using auxiliary polynomial

3. THE DESIGNED SOFTWARE TOOL

The software tool designed using MATLAB (MathWorks, 2018) performs absolute, relative and conditional stability analyzes according to the Routh-Hurwitz stability criterion taking into account all possible situations. The basic flow diagram, main screen, fundamental menus and tool palette of the software tool are shown in Fig. 2-3 and Table 3-4. In the main screen, the coefficients of the characteristic equation of the system are entered manually and the analysis type is selected and stability analysis is performed. Using the right-click menu of the mouse, the roots of the characteristic equation can be seen and the map of the roots can be also drawn.

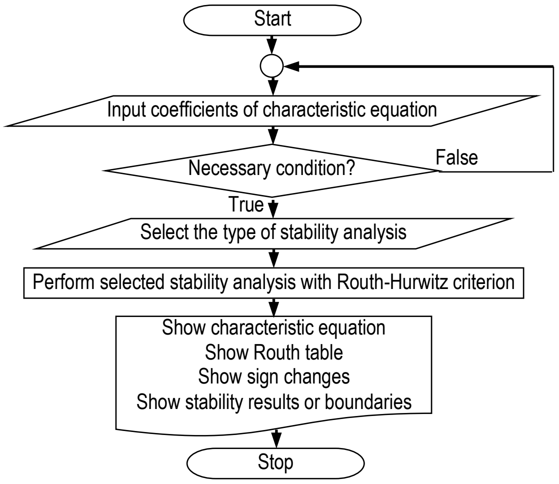


Figure 2:
Basic flow-chart of the designed software tool

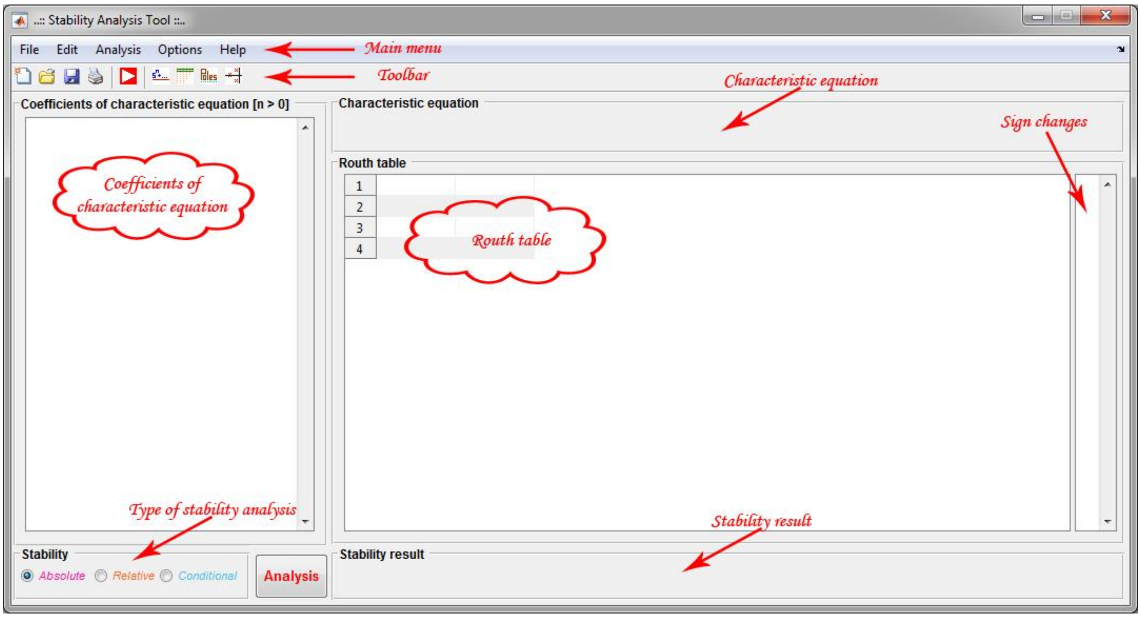


Figure 3:

The main screenshot of the designed software tool

Table 3. The main menu items and their tasks

Menu	File	Edit	Analysis		Options	Help
	New Ctrl+N Open Ctrl+O Save Ctrl+S Print Ctrl+P Exit Ctrl+0	Undo Ctrl+Z Redo Cut Ctrl+X Copy Ctrl+C Paste Ctrl+V	Stability analysis Ctrl+A Show	Characteristic equation Routh-Hurwitz table Poles Pole's map	Preferences	Help Ctrl+H About
Task						

Table 4. The toolbar items and their tasks

New	Open	Save	Print	Analysis	Characteristic equation	Routh table	Poles	Map of poles

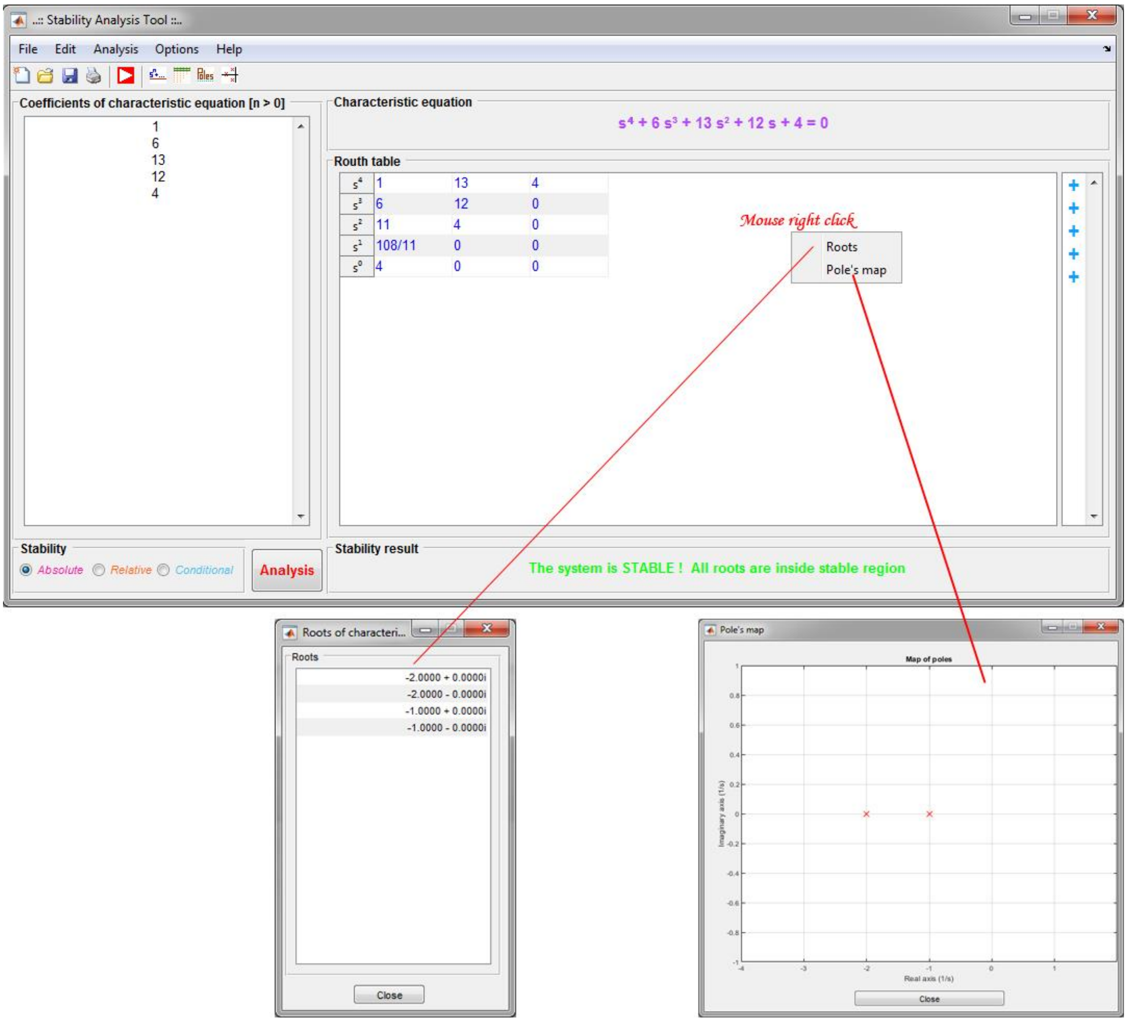
4. APPLICATIONS

In this paper, to present a sample application for each subject is adopted principally. As a first application, absolute stability analysis of the system having the following characteristic equation is presented.

$$s^4 + 6s^3 + 13s^2 + 12s + 4 = 0 \tag{2}$$

The screenshot of the first application obtained is given in Fig. 4. As seen from the screenshot in the Fig. 4, Routh table is formed according to the entered coefficients and the sign changes of the first column of the table are printed, the characteristic equation is written in the appropriate format and stability status is reported. In addition, by clicking on the right mouse button on the

Routh table, the sub-menu options can be used to show the roots of the characteristic equation or the mapping of the roots in separate windows. Thus, the user / student can also confirm the result of the stability analysis.



Coefficients of characteristic equation [n > 0]

1
6
13
12
4

Characteristic equation

$s^4 + 6s^3 + 13s^2 + 12s + 4 = 0$

Routh table

s^4	1	13	4
s^3	6	12	0
s^2	11	4	0
s^1	108/11	0	0
s^0	4	0	0

Stability

● Absolute ○ Relative ○ Conditional

Stability result

The system is STABLE ! All roots are inside stable region

Roots

-2.0000 + 0.0000i
-2.0000 - 0.0000i
-1.0000 + 0.0000i
-1.0000 - 0.0000i

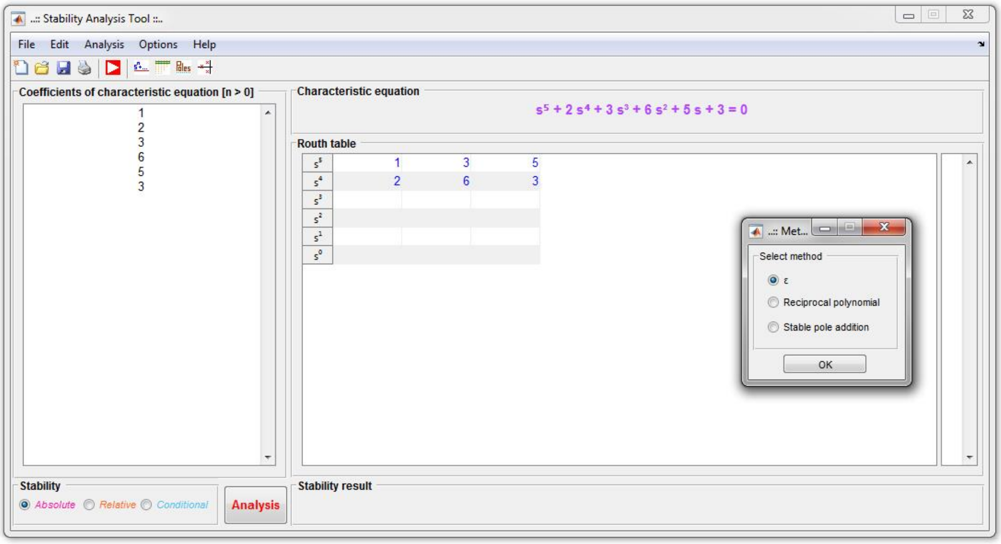
Figure 4:

The screenshots for first application

In the second application, in the absolute stability analysis of the system having the following characteristic equation,

$$s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3 = 0 \tag{3}$$

zero value is obtained in the first column of the Routh table (Nise, 2015). In this case, the designed software tool offers the user/student to choose one of the solution methods in Table 2 (Fig. 5a). The screenshots of the solutions obtained in all three methods are given Fig. 5b-d.



Coefficients of characteristic equation [n > 0]

1
2
3
6
5
3

Characteristic equation

$s^5 + 2 s^4 + 3 s^3 + 6 s^2 + 5 s + 3 = 0$

Routh table

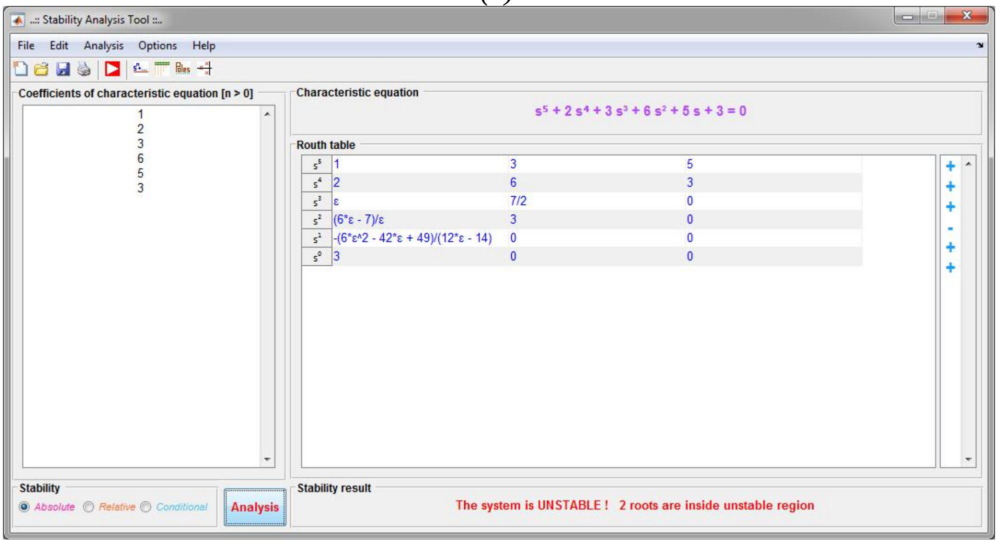
s^5	1	3	5
s^4	2	6	3
s^3			
s^2			
s^1			
s^0			

Select method
● ε
○ Reciprocal polynomial
○ Stable pole addition

Stability
● Absolute ○ Relative ○ Conditional

Stability result

(a)



Coefficients of characteristic equation [n > 0]

1
2
3
6
5
3

Characteristic equation

$s^5 + 2 s^4 + 3 s^3 + 6 s^2 + 5 s + 3 = 0$

Routh table

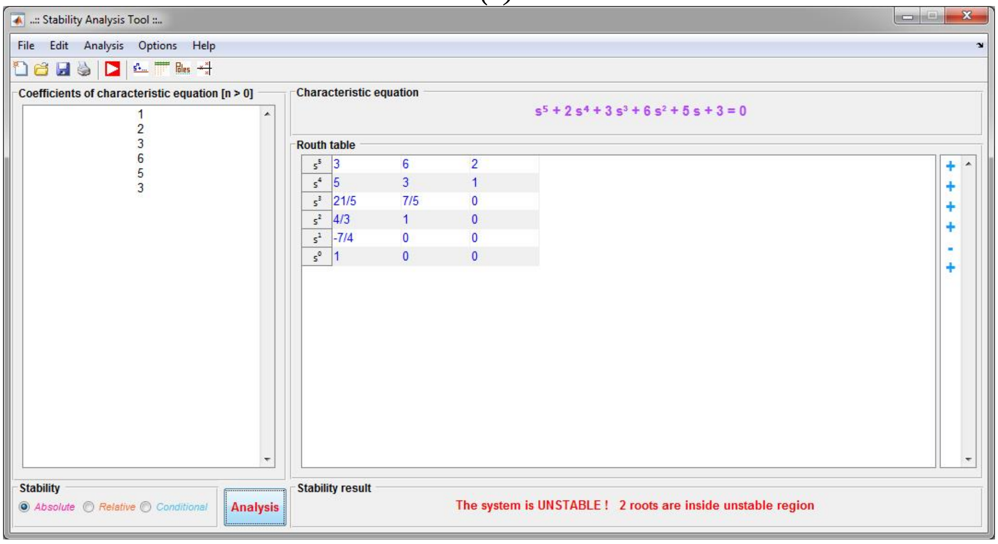
s^5	1	3	5	+
s^4	2	6	3	+
s^3	ε	7/2	0	+
s^2	(6*ε - 7)/ε	3	0	-
s^1	-(6*ε^2 - 42*ε + 49)/(12*ε - 14)	0	0	+
s^0	3	0	0	+

Stability
● Absolute ○ Relative ○ Conditional

Stability result

The system is UNSTABLE ! 2 roots are inside unstable region

(b)



Coefficients of characteristic equation [n > 0]

1
2
3
6
5
3

Characteristic equation

$s^5 + 2 s^4 + 3 s^3 + 6 s^2 + 5 s + 3 = 0$

Routh table

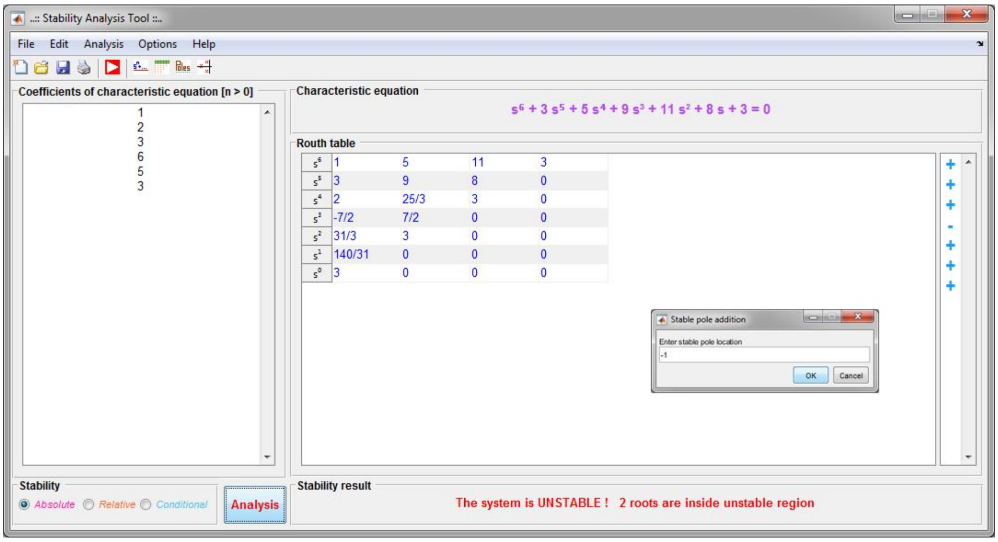
s^5	3	6	2	+
s^4	5	3	1	+
s^3	21/5	7/5	0	+
s^2	4/3	1	0	+
s^1	-7/4	0	0	-
s^0	1	0	0	+

Stability
● Absolute ○ Relative ○ Conditional

Stability result

The system is UNSTABLE ! 2 roots are inside unstable region

(c)



Coefficients of characteristic equation [n > 0]
1
2
3
6
5
3

Characteristic equation: $s^6 + 3s^5 + 5s^4 + 9s^3 + 11s^2 + 8s + 3 = 0$

Routh table

s^6	1	5	11	3
s^5	3	9	8	0
s^4	2	25/3	3	0
s^3	-7/2	7/2	0	0
s^2	31/3	3	0	0
s^1	140/31	0	0	0
s^0	3	0	0	0

Stable pole addition
Enter stable pole location: -1

Stability: ● Absolute ○ Relative ○ Conditional

Stability result: The system is UNSTABLE ! 2 roots are inside unstable region

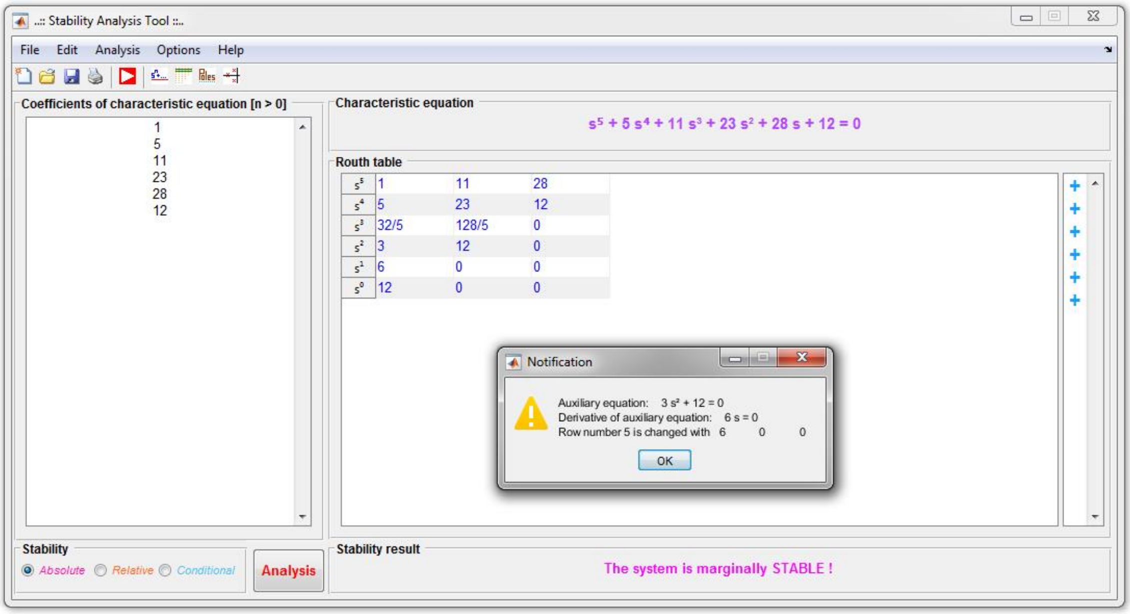
(d)

Figure 5:
The screenshots for second application

In the third application, in the absolute stability analysis of the system having the following characteristic equation,

$$s^5 + 5s^4 + 11s^3 + 23s^2 + 28s + 12 = 0 \tag{4}$$

All values in a row of the Routh table are obtained as zero (Franklin et al., 2014). The designed software obtain a solution by using an auxiliary polynomial and informing in this case (Fig. 6).



Coefficients of characteristic equation [n > 0]
1
5
11
23
28
12

Characteristic equation: $s^5 + 5s^4 + 11s^3 + 23s^2 + 28s + 12 = 0$

Routh table

s^5	1	11	28
s^4	5	23	12
s^3	32/5	128/5	0
s^2	3	12	0
s^1	6	0	0
s^0	12	0	0

Notification
Auxiliary equation: $3s^2 + 12 = 0$
Derivative of auxiliary equation: $6s = 0$
Row number 5 is changed with 6 0 0

Stability: ● Absolute ○ Relative ○ Conditional

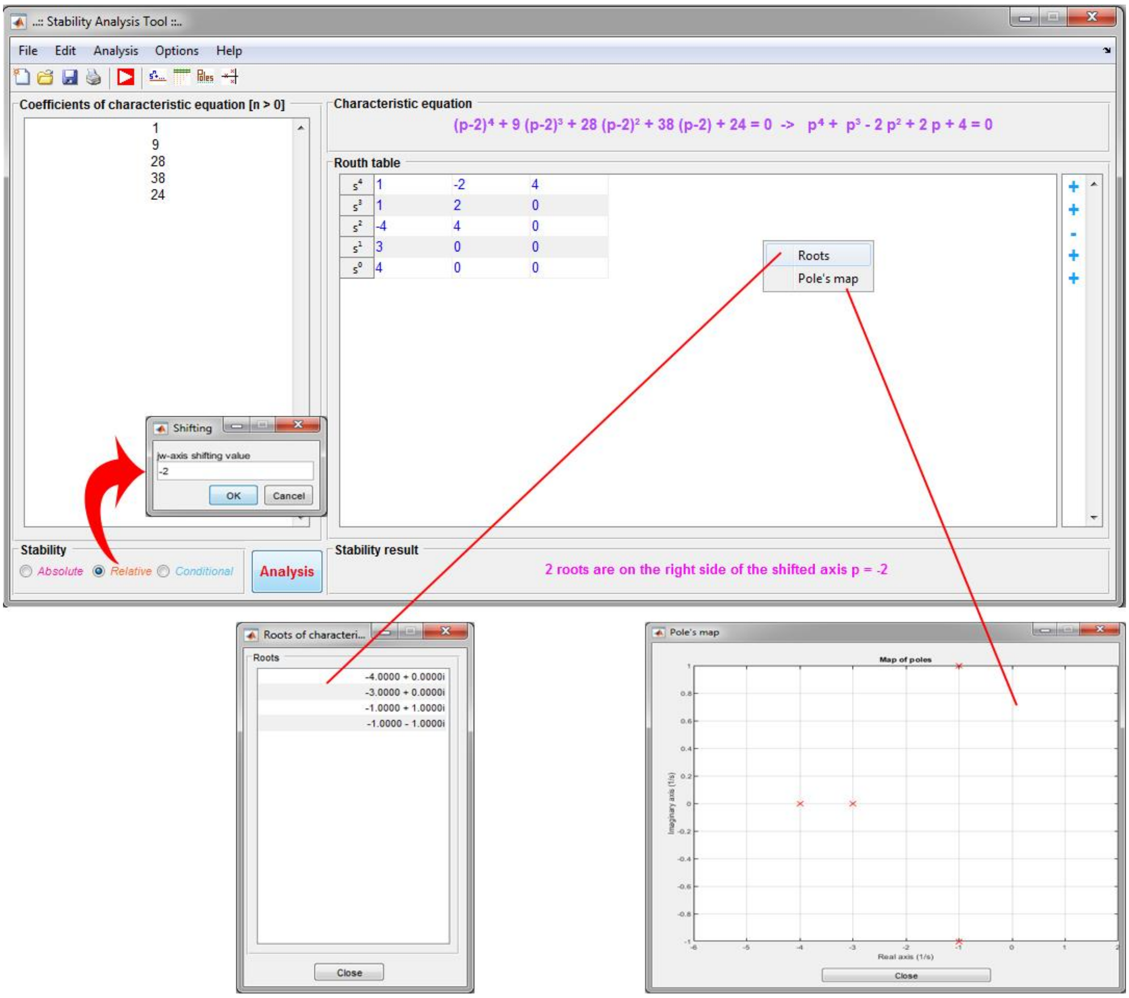
Stability result: The system is marginally STABLE !

Figure 6:
The screenshot for third application

In the fourth application, in the relative stability analysis of the system having the following characteristic equation,

$$s^4 + 9s^3 + 28s^2 + 38s + 24 = 0 \tag{5}$$

a shifting value is requested from the user and the stability analysis is performed. Fig. 7 shows the shifted/reconstructed characteristic equation, the corresponding Routh table, the sign changes of the first column in the Routh table. Below the screenshot, a map of roots and the values of the roots related to the results obtained from the performed stability analysis are seen.



Coefficients of characteristic equation [n > 0]
1
9
28
38
24

jw-axis shifting value: -2

Characteristic equation
(p-2)⁴ + 9 (p-2)³ + 28 (p-2)² + 38 (p-2) + 24 = 0 -> p⁴ + p³ - 2 p² + 2 p + 4 = 0

Routh table

s^4	1	-2	4
s^3	1	2	0
s^2	-4	4	0
s^1	3	0	0
s^0	4	0	0

Stability: ○ Absolute ● Relative ○ Conditional

Stability result
2 roots are on the right side of the shifted axis p = -2

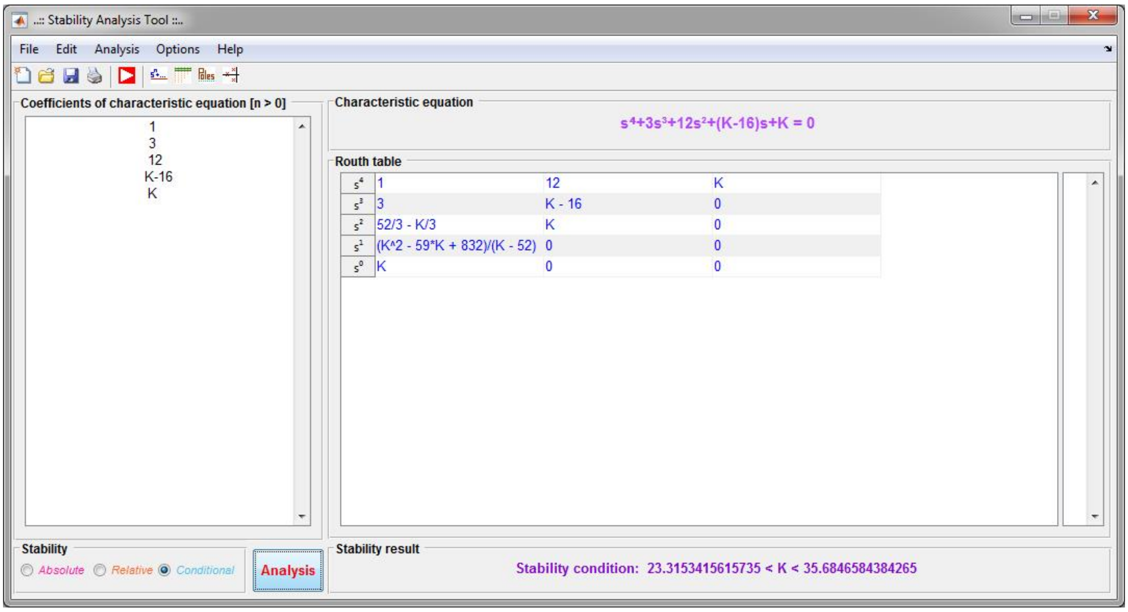
Roots
-4.0000 + 0.0000i
-3.0000 + 0.0000i
-1.0000 + 1.0000i
-1.0000 - 1.0000i

Figure 7:
The screenshots for fourth application

In the last application, conditional stability analysis of the system having the following characteristic equation is presented (Yüksel, 2016).

$$s^4 + 3s^3 + 12s^2 + (K - 16)s + K = 0 \tag{6}$$

Fig. 8 shows the symbolic characteristic equation used in the analysis, the corresponding Routh table and the obtained stability conditions.



Coefficients of characteristic equation [n > 0]

1
3
12
K-16
K

Characteristic equation

$s^4+3s^3+12s^2+(K-16)s+K = 0$

Routh table

s^4	1	12	K
s^3	3	K - 16	0
s^2	52/3 - K/3	K	0
s^1	(K^2 - 59*K + 832)/(K - 52)	0	0
s^0	K	0	0

Stability

○ Absolute ○ Relative ● Conditional

Stability result

Stability condition: 23.3153415615735 < K < 35.6846584384265

Figure 8:
The screenshot for fifth application

5. CONCLUSION

The stability is very important for the systems. Therefore, the analysis should be performed with high accuracy. In the study, a software tool, which is not an existing any package on MATLAB or another program, was designed for the stability analysis of linear time invariant systems according to the Routh-Hurwitz criterion. By using this software tool, the absolute, relative and conditional stability analyzes of the systems defined by entering the coefficients of the characteristic equation by the user can be performed easily and quickly; and, step-by-step with explanations and using different solutions, numerical, symbolic and graphical results can be obtained.

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