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Bessel ve Airy Denklemlerini Çözmekte Uygulanan Yeni Metot

New Method of Solving Bessel and Airy Equations

Almazbek İsaev [1*]

[1] Kırgızistan Türkiye Manas Üniversitesi,Mühendislik Fakültesi,Çevre Mühendisliği Bölümü,720038,Bişkek,KIRGIZİSTAN
Sorumlu Yazar / Corresponding Author *: almazbek.isaev68@gmail.com

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Öz

Makalede,Bessel ve Airy denklemlerinin yeni metodun uygulanmasından gelen çözümleri ile Bessel fonksyonları çözümlerinin yakınsaklıkları incelenmektedir.

Anahtar Kelimeler: *Riccati, Bessel ve Airy denklemleri*

Abstract

In this article, Bessel and Airy equations are analyzed with the solution of the new method and the convergence of Bessel functions.

Keywords: *Riccati, Bessel and Airy equations*

1. Giriş

Önceden yapılmış klasik dönüşümü ve [1-3] çalışmalarına dayanarak Bessel denklemin daha genişletilmiş bir şekile getirerek, özel hali olarak Airy denklemi çıkartılacaktır.Çözümünün yakınsaklığı Mathcad 14 aracı ile kurulmuş grafiklerde gösterilecektir.

2. Materyal ve Metot

2.1. Normal Bessel diferansiyel denklemi

$$t^2 \cdot \frac{d^2X(t)}{dt^2} + t \cdot \frac{dX(t)}{dt} + (t^2 - \nu^2) \cdot X(t) = 0$$

Burada ν- parametredir [3]

Hatırlandığında $t \neq 0 \rightarrow$ (1)

$$t \cdot \frac{d^2X(t)}{dt^2} + \frac{dX(t)}{dt} + \left(t - \frac{\nu^2}{t}\right) \cdot X(t) = 0$$

Yerine koyarak $X(t) = \frac{Z(t)}{\sqrt{t}} \rightarrow$

$$\frac{d^2Z(t)}{dt^2} + \left(1 - \frac{\nu^2 - \frac{1}{4}}{t^2}\right) \cdot Z(t) = 0 \qquad (2)$$

Yeni metodun (Almaz Beyin metodu) uygulanması ile [1-3]

$$Z(t) = e^{ln\left|\frac{1}{cos(\theta)} + tan(\theta)\right|}$$

$$W(\theta) = ln\left|\frac{1}{cos(\theta)} + tan(\theta)\right|; \ \theta = \theta(t)$$

$$\frac{S(t)}{dt} + \frac{x(t)}{dt} = \frac{1}{cos(\theta)} + tan(\theta) = tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right) = e^{W(\theta)}$$

$$\frac{S(t)}{dt} - \frac{x(t)}{dt} = \frac{1}{cos(\theta)} - tan(\theta) = e^{-W(\theta)} = \frac{1}{Z(t)};$$

$$-W(\theta) = ln\left|\frac{1}{cos(\theta)} - \frac{sin(\theta)}{cos(\theta)}\right|$$

$$\frac{1}{cos(\theta)} = cosh\big(W(\theta)\big) = \frac{e^{W(\theta)} + e^{-W(\theta)}}{2}$$

$$\tan(\theta) = \sinh(W(\theta)) = \frac{e^{W(\theta)} - e^{-W(\theta)}}{2}$$

$$sin(\theta) = tanh(W(\theta)) = \frac{sinh(W(\theta))}{cosh(W(\theta))}$$

$$\frac{\frac{e^{W(\theta)} - e^{-W(\theta)}}{2}}{\frac{e^{W(\theta)} + e^{-W(\theta)}}{2}} = \frac{e^{2\cdot W(\theta)} - 1}{e^{2\cdot W(\theta)} + 1}$$

$$S(t) + x(t) = \int e^{W(\theta)}\, dt + C_{S+x}$$

$$S(t) - x(t) = \int e^{-W(\theta)}\, dt + C_{S-x}$$

$$Z(t) = e^{G(t)} = e^{W_0(t)+S(t)+x(t)}$$

$$e^{G(t)} \cdot \left[\frac{d^2G(t)}{dt^2} + \left(\frac{dG(t)}{dt}\right)^2\right] + e^{G(t)} \cdot \left[1 - \frac{\nu^2-\frac{1}{4}}{t^2}\right] = 0 \qquad (3)$$

$$\underbrace{\frac{d^2W_0(t)}{dt^2} + \left(\frac{dW_0(t)}{dt}\right)^2 + 1}_{0} + \frac{d^2(S(t)+x(t))}{dt^2} + 2 \cdot \frac{dW_0(t)}{dt} \cdot \frac{d(S(t)+x(t))}{dt} + \left(\frac{d(S(t)+x(t))}{dt}\right)^2 - \frac{\nu^2-\frac{1}{4}}{t^2} = 0$$

$$\frac{d^2W_0(t)}{dt^2} + \left(\frac{dW_0(t)}{dt}\right)^2 + 1 = 0 \rightarrow \frac{d\left(\frac{dW_0(t)}{dt}\right)}{\left(\frac{dW_0(t)}{dt}\right)^2+1} = -dt$$

$$arctan\left(\frac{dW_0(t)}{dt}\right) = C_t - t \rightarrow \frac{dW_0(t)}{dt} = tan(C_t - t) = \frac{sin(C_t-t)}{cos(C_t-t)}$$

$$W_0(t) = ln\left|C_{W_0} \cdot cos(C_t - t)\right|; \quad e^{W_0(t)} = C_{W_0} \cdot cos(C_t - t)$$

$$C_t = \frac{\pi}{4}\ ; \ \ C_{W_0} = \frac{\sqrt{2}}{2}$$

$$\frac{d(S(t)+x(t))}{dt} = e^{W(t)} \rightarrow \frac{d^2(S(t)+x(t))}{dt^2} + 2 \cdot \frac{dW_0(t)}{dt} \cdot \frac{d(S(t)+x(t))}{dt} + \left(\frac{d(S(t)+x(t))}{dt}\right)^2 - \frac{\nu^2-\frac{1}{4}}{t^2} = 0$$

$$\frac{de^{W(t)}}{dt} + 2 \cdot \frac{dW_0(t)}{dt} \cdot e^{W(t)} + e^{2\cdot W(t)} - \frac{\nu^2-\frac{1}{4}}{t^2} = 0$$

Riccati denklemine götürmektedir.

$$\frac{de^{W(t)}}{dt} = \underbrace{-1}_{a(t)} \cdot e^{2\cdot W(t)} \underbrace{-2 \cdot \frac{dW_0(t)}{dt}}_{b(t)} \cdot e^{W(t)} \underbrace{+\frac{\nu^2-\frac{1}{4}}{t^2}}_{c(t)} = 0$$

$$e^{2\cdot W(t)} + \left(2 \cdot \frac{dW_0(t)}{dt} + \frac{dW(t)}{dt}\right) \cdot e^{W(t)} - \frac{\nu^2-\frac{1}{4}}{t^2} = 0$$

$$\sin(\theta(t))^* = -\int \left(2 \cdot \frac{dW_0(t)}{dt} + 1 - \frac{\nu^2 - \frac{1}{4}}{t^2}\right) dt + C_\theta$$

$W(t)^* = arctan\left[\sin(\theta(t))^*\right]$ veya

$$W(t)^* = \sum_{n=0}^{N} \frac{\sin(\theta(t))^{*2\cdot n+1}}{2\cdot n+1} + C_W$$

$$D(t) = \left(2 \cdot \frac{dW_0(t)}{dt} + \frac{dW(t)^*}{dt}\right)^2 + 4 \cdot 1 \cdot \left(\frac{\nu^2-\frac{1}{4}}{t^2}\right)$$

$$e^{W(t)_{1,2}} = \frac{-\left(2\cdot\frac{dW_0(t)}{dt}+\frac{dW(t)^*}{dt}\right) \pm \sqrt{\left(2\cdot\frac{dW_0(t)}{dt}+\frac{dW(t)^*}{dt}\right)^2 + 4\cdot 1\cdot\left(\frac{\nu^2-\frac{1}{4}}{t^2}\right)}}{2}$$

[1,3] göz önüne alarak.

$$N = 0 \rightarrow W(t)^* = \sum_{n=0}^{0} \frac{\sin(\theta(t))^{*2\cdot 0+1}}{2\cdot n+1} + C_W = \sin(\theta(t))^*$$

$$\left(S(t) + x(t)\right)_{1,2} = \int \overbrace{e^{W(t)_{1,2}}}^{e^{sin\theta(t)_{1,2}}} \cdot dt + C_{S+x}$$

$$Z(t) = C_{W_0} \cdot cos(C_t - t) \cdot e^{\left(S(t)+x(t)\right)_{1,2}}$$

$$e^{sin\theta(t)_1} = \frac{c(t)}{a(t)} = \frac{\frac{\nu^2-\frac{1}{4}}{t^2}}{-1}\ ;\ e^{sin\theta(t)_2} = 1$$

2.1.1. Bulgular

$$\left(S(t) + x(t)\right)_1 = -\int \frac{\nu^2-\frac{1}{4}}{t^2} \cdot dt + C_{S+x} \qquad (4)$$

$$\left(S(t) + x(t)\right)_2 = \int 1 \cdot dt + C_{S+x}$$

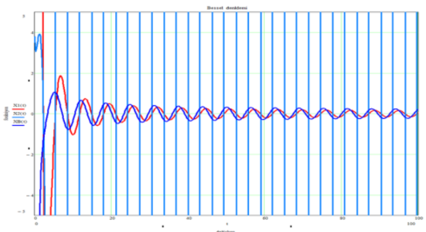
$$X_1(t) = \frac{C_{W_0} \cdot cos(C_t - t)}{\sqrt{t}} \cdot e^{\left(S(t)+x(t)\right)_1} \qquad (5)$$

$$X_2(t) = \frac{C_{W_0} \cdot cos(C_t - t)}{\sqrt{t}} \cdot e^{\left(S(t)+x(t)\right)_2}$$

$$X_B(t) = C_1 \cdot J_n(m,t) + C_2 \cdot Y_n(m,t)$$

Burada C_1, C_2 −sabitler ve · $J_n(m,t), Y_n(m,t)$ −Besselin özel fonksiyonlarıdır [6].

Bessel denkleminin genel çözümleri.



Şekil 1. $\nu = 3, n = 3, n = m$

2.2. Bessel denkleminin genişletilmiş şekle dönüştürülmesi ve özel hali olarak Airy denklemi

$$t^2 \cdot \frac{d^2X(t)}{dt^2} + t \cdot \frac{dX(t)}{dt} + (t^2 - \nu^2) \cdot X(t) = 0$$

Klasik dönüşümleri kullandığımızda.

$$X(t) = \xi^{\alpha} \cdot U(\xi);\ t = \gamma \cdot \xi^{\beta} \qquad (6)$$

$$\frac{dt}{d\xi} = \gamma \cdot \beta \cdot \xi^{\beta-1} \rightarrow \frac{d\xi}{dt} = \frac{1}{\gamma \cdot \beta} \cdot \xi^{1-\beta}$$

$$\frac{dX(t)}{dt} = \frac{X(t)}{d\xi} \cdot \frac{d\xi}{dt} = \frac{1}{\gamma \cdot \beta} \cdot \xi^{1-\beta} \cdot \frac{d\left(\xi^{\alpha} \cdot U(\xi)\right)}{d\xi} = \frac{1}{\gamma \cdot \beta} \cdot \xi^{1-\beta} \cdot \left[\alpha \cdot \xi^{\alpha-1} \cdot U(\xi) + \xi^{\alpha} \cdot \frac{dU(\xi)}{d\xi}\right]$$

$$\frac{dX(t)}{dt} = \frac{\alpha}{\gamma \cdot \beta} \cdot \xi^{\alpha-\beta} \cdot U(\xi) + \frac{1}{\gamma \cdot \beta} \cdot \xi^{\alpha-\beta+1} \cdot \frac{dU(\xi)}{d\xi}$$

$$\frac{d\frac{X(t)}{dt}}{dt} = \frac{d\frac{X(t)}{dt}}{d\xi} \cdot \frac{d\xi}{dt} = \frac{\alpha}{\gamma \cdot \beta} \cdot \xi^{1-\beta} \cdot \frac{d\left[\frac{\alpha}{\gamma \cdot \beta} \cdot \xi^{\alpha-\beta} \cdot U(\xi) + \frac{1}{\gamma \cdot \beta} \cdot \xi^{\alpha-\beta+1} \cdot \frac{dU(\xi)}{d\xi}\right]}{d\xi}$$

$$\frac{d^2X(t)}{dt^2} = \frac{\alpha}{\gamma \cdot \beta} \cdot \xi^{1-\beta} \cdot \left[\frac{\alpha}{\gamma \cdot \beta} \cdot (\alpha - \beta) \cdot \xi^{\alpha-\beta-1} \cdot U(\xi) + \frac{\alpha}{\gamma \cdot \beta} \cdot \xi^{\alpha-\beta} \cdot \frac{dU(\xi)}{d\xi} + \frac{\alpha}{\gamma \cdot \beta} \cdot (\alpha - \beta + 1) \cdot \xi^{\alpha-\beta} \cdot \frac{dU(\xi)}{d\xi} + \frac{\alpha}{\gamma \cdot \beta} \cdot \xi^{\alpha-\beta+1} \cdot \frac{d^2U(\xi)}{d\xi^2}\right]$$

$$\frac{d^2X(t)}{dt^2} = \frac{\alpha}{\gamma^2 \cdot \beta^2} \cdot (\alpha - \beta) \cdot \xi^{\alpha-2\cdot\beta} \cdot U(\xi) + \frac{\alpha}{\gamma^2 \cdot \beta^2} \cdot \xi^{\alpha-2\cdot\beta+1} \cdot \frac{dU(\xi)}{d\xi} + \frac{(\alpha-\beta+1)}{\gamma^2 \cdot \beta^2} \cdot \xi^{\alpha-2\cdot\beta+1} \cdot \frac{dU(\xi)}{d\xi} + \frac{1}{\gamma^2 \cdot \beta^2} \cdot \xi^{\alpha-2\cdot\beta+2} \cdot \frac{d^2U(\xi)}{d\xi^2}$$

denkleme koyduğumuzda:

$$\gamma^2 \cdot \xi^{2\cdot\beta} \cdot \left[\frac{\alpha}{\gamma \cdot \beta} \cdot (\alpha - \beta) \cdot \xi^{\alpha-2\cdot\beta} \cdot U(\xi) + \frac{(2\cdot\alpha-\beta+1)}{\gamma^2 \cdot \beta^2} \cdot \xi^{\alpha-2\cdot\beta+1} \cdot \frac{dU(\xi)}{d\xi} + \frac{1}{\gamma^2 \cdot \beta^2} \cdot \xi^{\alpha-2\cdot\beta+2} \cdot \frac{d^2U(\xi)}{d\xi^2}\right] + \gamma \cdot \xi^{\beta} \cdot \left[\frac{\alpha}{\gamma \cdot \beta} \cdot \xi^{\alpha-\beta} \cdot U(\xi) + \frac{1}{\gamma \cdot \beta} \cdot \xi^{\alpha-\beta+1} \cdot \frac{dU(\xi)}{d\xi}\right] + \left[\gamma^2 \cdot \xi^{2\cdot\beta} - \nu^2\right] \cdot \xi^{\alpha} \cdot U(\xi) = 0$$

$$\frac{\alpha \cdot (\alpha-\beta)}{\beta^2} \cdot \xi^{\alpha} \cdot U(\xi) + \frac{(2\cdot\alpha-\beta+1)}{\beta^2} \cdot \xi^{\alpha+1} \cdot \frac{dU(\xi)}{d\xi} + \frac{1}{\beta^2} \cdot \xi^{\alpha+2} \cdot \frac{d^2U(\xi)}{d\xi^2} + \frac{\alpha}{\beta} \cdot \xi^{\alpha} \cdot U(\xi) + \frac{1}{\beta} \cdot \xi^{\alpha+1} \cdot \frac{dU(\xi)}{d\xi} + \left[\gamma^2 \cdot \xi^{2\cdot\beta} - \nu^2\right] \cdot \xi^{\alpha} \cdot U(\xi) = 0$$

$$\frac{1}{\beta^2} \cdot \xi^2 \cdot \frac{d^2U(\xi)}{d\xi^2} + \left[\frac{2\cdot\alpha-\beta+1+\beta}{\beta^2}\right] \cdot \xi \cdot \frac{dU(\xi)}{d\xi} + \left[\gamma^2 \cdot \xi^{2\cdot\beta} - \nu^2 + \frac{\alpha \cdot (\alpha-\beta)}{\beta^2} + \frac{\alpha}{\beta}\right] \cdot U(\xi) = 0$$

$$\frac{1}{\beta^2} \cdot \xi^2 \cdot \frac{d^2U(\xi)}{d\xi^2} + \left[\frac{2\cdot\alpha+1}{\beta^2}\right] \cdot \xi \cdot \frac{dU(\xi)}{d\xi} + \left[\gamma^2 \cdot \xi^{2\cdot\beta} - \nu^2 + \frac{\alpha^2-\alpha\cdot\beta+\alpha\cdot\beta}{\beta^2}\right] \cdot U(\xi) = 0$$

$$\xi^2 \cdot \frac{d^2U(\xi)}{d\xi^2} + \underbrace{(2 \cdot \alpha + 1)}_{b} \cdot \xi \cdot \frac{dU(\xi)}{d\xi} + \left[\underbrace{\gamma^2 \cdot \beta^2}_{\omega_0^2} \cdot \xi^{\overbrace{2\cdot\beta}^{m}} + \underbrace{\alpha^2 - \beta^2 \cdot \nu^2}_{k^2}\right] \cdot U(\xi) = 0$$

$$\xi^2 \cdot \frac{d^2U(\xi)}{d\xi^2} + b \cdot \xi \cdot \frac{dU(\xi)}{d\xi} + \left[\omega_0^2 \cdot \xi^{m} + k^2\right] \cdot U(\xi) = 0$$

$$2 \cdot a + 1 = b\ ;\ \alpha^2 - \beta^2 \cdot \nu^2 = k^2;\ \ \beta^2 \cdot \gamma^2 = \omega_0^2\ ;\ 2 \cdot \beta = m \qquad (7)$$

Genişletilmiş şeklindeki Bessel diferansiyel denklemi

eğer $b = 0\ ;\ k^2 = 0;\ \ \omega_0^2 = -1\ ;\ m = 3$

$$\frac{d^2U(\xi)}{d\xi^2} - \xi \cdot U(\xi) = 0 \qquad (8)$$

Airy diferansiyel denklemi

Burada $\alpha = -\frac{1}{2}\ ;\ \nu = \frac{1}{3};\ \beta = \frac{3}{2}\ ;\ \gamma = \frac{2}{3} \cdot i$

$$X(t) = \xi^{-\frac{1}{2}} \cdot U(\xi);\ \ t = \frac{2}{3} \cdot i\xi^{-\frac{3}{2}}\ \rightarrow\ U(\xi) = \xi^{-\frac{1}{2}} \cdot X(t) \qquad (9)$$

Bessel dekleminin çözümün değiştirerek [3]

$$X_1(t) = \frac{C_{W_0} \cdot \cos(C_t - t)}{\sqrt{t}} \cdot e^{\left(\nu^2 - \frac{1}{4}\right) \cdot \frac{1}{t} + C_{S+x}} \rightarrow X_1(\xi) = \frac{C_{W_0} \cdot \cos\left(C_t - \frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}\right)}{\sqrt{\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}}} \cdot e^{\left(\nu^2 - \frac{1}{4}\right) \cdot \frac{1}{\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}} + C_{S+x}} \qquad (10)$$

$$U_1(\xi) = \xi^{\frac{1}{2}} \cdot X_1(\xi) = \xi^{\frac{1}{2}} \cdot \frac{C_{W_0} \cdot cos\left(C_t - \frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}\right)}{\sqrt{\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}}} \cdot e^{\left(\nu^2 - \frac{1}{4}\right) \cdot \frac{1}{\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}} + C_{S+x}}$$

$$X_2(t) = \frac{C_{W_0} \cdot \cos(C_t - t)}{\sqrt{t}} \cdot e^{t + C_{S+x}} \rightarrow X_2(\xi) = \frac{C_{W_0} \cdot \cos\left(C_t - \frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}\right)}{\sqrt{\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}}} \cdot e^{\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}} + C_{S+x}}$$

$$U_2(\xi) = \xi^{\frac{1}{2}} \cdot X_2(\xi) = \xi^{\frac{1}{2}} \cdot \frac{C_{W_0} \cdot \cos\left(C_t - \frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}\right)}{\sqrt{\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}}} \cdot e^{\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}} + C_{S+x}}$$

$U_B(\xi) = C_1 \cdot Ai(\xi) + C_2 \cdot Bi(\xi) + \left(Im\left(Ai(\xi)\right) + Im\left(Bi(\xi)\right)\right) \cdot i$ [6]

2.2.1. Bulgular

Airy denkleminin çözümünü bulmak için aşağıdaki ifade kullanılmaktadır.

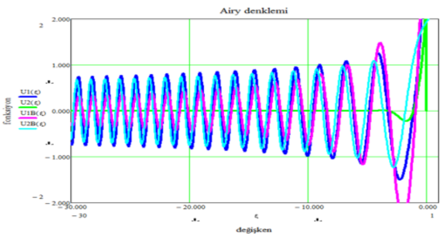
$t = \frac{2}{3} \cdot i \cdot \xi^{-\frac{3}{2}}$ yerine $t = -\frac{2}{3} \cdot i \cdot \xi^{-\frac{3}{2}}$ burada $i = \sqrt{-1}$ ve

$$U_1(\xi) = \xi^{\frac{1}{2}} \cdot \frac{C_{W_0} \cdot \cos\left(C_t - \frac{(-2)}{3} \cdot i \cdot \xi^{\frac{3}{2}}\right)}{\sqrt{-\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}}} \cdot e^{\left(\frac{1}{9} - \frac{1}{4}\right) \cdot \frac{1}{-\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}} + C_{S+x}}$$

$$U_2(\xi) = \xi^{\frac{1}{2}} \cdot \frac{C_{W_0} \cdot \cos\left(C_t - \frac{(-2)}{3} \cdot i \cdot \xi^{\frac{3}{2}}\right)}{\sqrt{-\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}}} \cdot e^{-\frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}} + C_{S+x}}$$

$$U_{1B}(\xi) = C_1 \cdot Ai(\xi) + C_2 \cdot Bi(\xi) + \left(Im\left(Ai(\xi)\right) + Im\left(Bi(\xi)\right)\right) \cdot i$$

$$U_{2B}(\xi) = C_1 \cdot J_n\left(\frac{1}{3}, \frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}\right) + C_2 \cdot Y_n\left(\frac{1}{3}, \frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}\right)$$



Şekil 2. $\nu = \frac{1}{3}, m = \frac{1}{3}, n = m$

2.3. Modifiye Bessel denkleminin genişletilmiş şekline dönüştürülmesi ve onun özel hali olarak Airy denklemi

$$t^2 \cdot \frac{d^2X(t)}{dt^2} + t \cdot \frac{dX(t)}{dt} + (t^2 - \nu^2) \cdot X(t) = 0$$

yerine $t = -i \cdot t$, burada $t = \sqrt{-1} \rightarrow \iota^2 = -1$

$$(-i)^2 \cdot t^2 \cdot \frac{d\frac{dX(t)}{dt}}{(-i)^2 \cdot dt} + (-i) \cdot t \cdot \frac{dX(t)}{(-i) \cdot dt} + ((-i)^2 \cdot t^2 - \nu^2) \cdot X(t) = 0$$

$$t^2 \cdot \frac{d^2X(t)}{dt^2} + t \cdot \frac{dX(t)}{dt} - (t^2 + \nu^2) \cdot X(t) = 0 \quad (11)$$

Modifiye Bessel denklemi benzeri bir şekilde dönüştürmelerini yaparsak

$$X(t) = \xi^{\alpha} \cdot U(\xi); \; t = \gamma \cdot \xi^{\beta}$$

$$\frac{1}{\beta^2} \cdot \xi^2 \cdot \frac{d^2U(\xi)}{d\xi^2} + \left[\frac{2 \cdot \alpha - \beta + 1 + \beta}{\beta^2}\right] \cdot \xi \cdot \frac{dU(\xi)}{d\xi} + \left[\gamma^2 \cdot \xi^{2 \cdot \beta} + \nu^2 - \frac{\alpha \cdot (\alpha - \beta)}{\beta^2} - \frac{\alpha}{\beta}\right] \cdot U(\xi) = 0$$

$$\frac{1}{\beta^2} \cdot \xi^2 \cdot \frac{d^2U(\xi)}{d\xi^2} + \left[\frac{2 \cdot \alpha + 1}{\beta^2}\right] \cdot \xi \cdot \frac{dU(\xi)}{d\xi} + \left[\gamma^2 \cdot \xi^{2 \cdot \beta} + \nu^2 - \frac{\alpha^2}{\beta^2}\right] \cdot U(\xi) = 0$$

$$\xi^2 \cdot \frac{d^2U(\xi)}{d\xi^2} + \underbrace{(2 \cdot \alpha + 1)}_{b} \cdot \xi \cdot \frac{dU(\xi)}{d\xi} - \left[\underbrace{\gamma^2 \cdot \beta^2}_{\omega_0^2} \cdot \xi^{\overbrace{2 \cdot \beta}^{m}} + \underbrace{\beta^2 \cdot \nu^2 - \alpha^2}_{k^2}\right] \cdot U(\xi) = 0$$

$$\xi^2 \cdot \frac{d^2U(\xi)}{d\xi^2} + b \cdot \xi \cdot \frac{dU(\xi)}{d\xi} - [\omega_0^2 \cdot \xi^m + k^2] \cdot U(\xi) = 0$$

yine $b = 0$; $k^2 = 0$; $\omega_0^2 = 1$; $m = 3$

$$\frac{d^2U(\xi)}{d\xi^2} - \xi \cdot U(\xi) = 0$$

ama $\alpha = -\frac{1}{2}$; $\nu = \frac{1}{3}$; $\beta = \frac{3}{2}$; $\gamma = \frac{2}{3}$ (12)

Kontrol edildiğinde:

$$X(t) = \xi^{-\frac{1}{2}} \cdot U(\xi); \; t = \frac{2}{3} \cdot \xi^{\frac{3}{2}} \rightarrow \xi = \left(\frac{3}{2} \cdot t\right)^{\frac{2}{3}}; \; U(\xi) = \xi^{\frac{1}{2}} \cdot X(t) = \left(\frac{3}{2} \cdot t\right)^{\frac{1}{3}} \cdot X(t)$$

$$\frac{dt}{d\xi} = \gamma \cdot \beta \cdot \xi^{\beta - 1} \rightarrow \frac{dt}{d\xi} = \left(\frac{2}{3} \cdot \frac{3}{2}\right) \cdot \left(\frac{3}{2} \cdot t\right)^{\frac{2}{3} \cdot \left(\frac{3}{2} - 1\right)}; \; \frac{dt}{d\xi} = \left(\frac{3}{2} \cdot t\right)^{\frac{1}{3}}$$

$$\frac{dU(\xi)}{d\xi} = \frac{dU(\xi)}{dt} \cdot \frac{dt}{d\xi} = \frac{d\left[\left(\frac{3}{2} \cdot t\right)^{\frac{1}{3}} \cdot X(t)\right]}{dt} \cdot \left(\frac{3}{2} \cdot t\right)^{\frac{1}{3}} = \left(\frac{3}{2}\right)^{\frac{1}{3}} \cdot \left(\frac{3}{2}\right)^{\frac{1}{3}} \cdot t^{\frac{1}{3}} \cdot \left[\frac{1}{3} \cdot t^{\frac{1}{3} - 1} \cdot X(t) + t^{\frac{1}{3}} \cdot \frac{dX(t)}{dt}\right] = \frac{1}{3} \cdot \left(\frac{3}{2}\right)^{\frac{2}{3}} \cdot t^{\frac{1}{3} - 1} \cdot X(t) + \left(\frac{3}{2}\right)^{\frac{2}{3}} \cdot t^{\frac{2}{3}} \cdot \frac{dX(t)}{dt}$$

$$\frac{d\frac{dU(\xi)}{d\xi}}{dt} \cdot \frac{dt}{d\xi} = \left(\frac{3}{2}\right)^{\frac{1}{3}} \cdot t^{\frac{1}{3}} \cdot \frac{d\left[\frac{1}{3} \cdot \left(\frac{3}{2}\right)^{\frac{2}{3}} \cdot t^{-\frac{1}{3}} \cdot X(t) + \left(\frac{3}{2}\right)^{\frac{2}{3}} \cdot t^{\frac{2}{3}} \cdot \frac{dX(t)}{dt}\right]}{dt} = \left(\frac{3}{2}\right)^{\frac{1}{3}} \cdot t^{\frac{1}{3}} \cdot \left[\frac{1}{3} \cdot \left(-\frac{1}{3}\right) \cdot \left(\frac{3}{2}\right)^{\frac{2}{3}} \cdot t^{-\frac{1}{3} - 1} \cdot X(t) + \frac{1}{3} \cdot \left(\frac{3}{2}\right)^{\frac{2}{3}} \cdot t^{-\frac{1}{3}} \cdot \frac{dX(t)}{dt} + \left(\frac{3}{2}\right)^{\frac{2}{3}} \cdot \left(\frac{2}{3}\right) \cdot t^{\frac{2}{3} - 1} \cdot \frac{dX(t)}{dt} + \left(\frac{3}{2}\right)^{\frac{2}{3}} \cdot t^{\frac{2}{3}} \cdot \frac{d^2X(t)}{dt^2}\right]$$

$$\frac{d^2U(\xi)}{d\xi^2} = -\frac{1}{6} \cdot t^{-1} \cdot X(t) + \frac{1}{2} \cdot \frac{dX(t)}{dt} + \frac{dX(t)}{dt} + \frac{3}{2} \cdot t \cdot \frac{d^2X(t)}{dt^2}$$

$$\xi \cdot U(\xi) = \left(\frac{3}{2}\right)^{\frac{2}{3}} \cdot t^{\frac{2}{3}} \cdot \left(\frac{3}{2}\right)^{\frac{1}{3}} \cdot t^{\frac{1}{3}} \cdot X(t)$$

$$\frac{3}{2} \cdot t \cdot \frac{d^2X(t)}{dt^2} + \frac{3}{2} \cdot \frac{dX(t)}{dt} - \frac{1}{6} \cdot t^{-1} \cdot X(t) - \frac{3}{2} \cdot t \cdot X(t) = 0$$

$$t^2 \cdot \frac{d^2X(t)}{dt^2} + t \cdot \frac{dX(t)}{dt} - \left(t^2 + \underbrace{\frac{1}{9}}_{\nu^2}\right) \cdot X(t) = 0$$

değiştirmeleri ile

$$t = -i \cdot t \rightarrow \frac{t}{-i} = t \rightarrow \frac{i \cdot t}{-i^2} = i \cdot t$$

$$(i \cdot t)^2 \cdot \frac{d\left(\frac{dX(t)}{d(i \cdot t)}\right)}{d(i \cdot t)} + (i \cdot t) \cdot \frac{dX(t)}{d(i \cdot t)} - \left((i \cdot t)^2 + \frac{1}{9}\right) \cdot X(t) = 0$$

$$i^2 \cdot t^2 \cdot \frac{d\left(\frac{dX(t)}{dt}\right)}{i^2 \cdot dt} + i \cdot t \cdot \frac{dX(t)}{i \cdot dt} - \left(i^2 \cdot t^2 + \frac{1}{9}\right) \cdot X(t) = 0$$

$$t^2 \cdot \frac{d\left(\frac{dX(t)}{dt}\right)}{dt} + t \cdot \frac{dX(t)}{dt} + \left(\cdot t^2 - \underbrace{\frac{1}{9}}_{\nu^2}\right) \cdot X(t) = 0; \; i^2 = -1$$

Normal Bessel denklemine gelinmektedir.

$$(\mathrm{S(t)} + \mathrm{x(t)})_{1,2} = \int \overbrace{e^{W(t)_{1,2}}}^{e^{sin\theta(t)_{1,2}}} \cdot dt + C_{S+x}$$

$$Z(t) = C_{W_0} \cdot cos(C_t - t) \cdot e^{(S(t)+x(t))_{1,2}}$$

$$e^{sin\theta(t)_1} = \frac{c(t)}{a(t)} = \frac{-\frac{\frac{1}{9}-\frac{1}{4}}{t^2}}{1}; \; e^{sin\theta(t)_2} = 1$$

2.3.1. Bulgular

$$(S(t) + x(t))_1 = -\int \frac{\frac{1}{9}-\frac{1}{4}}{t^2} \cdot dt + C_{S+x} = \left(\frac{1}{9} - \frac{1}{4}\right) \cdot \frac{1}{t} + C_{S+x} \quad (13)$$

$$(S(t) + x(t))_2 = \int 1 \cdot dt + C_{S+x} = t + C_{S+x}$$

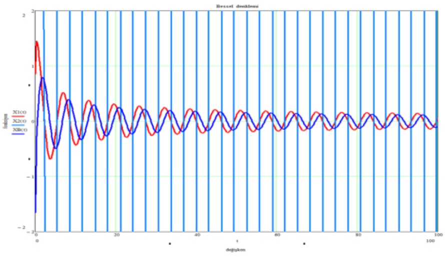
$$X_1(t) = \frac{C_{W_0} \cdot cos(C_t - t)}{\sqrt{t}} \cdot e^{(S(t)+x(t))_1};$$

$$X_2(t) = \frac{C_{W_0} \cdot cos(C_t - t)}{\sqrt{t}} \cdot e^{(S(t)+x(t))_2}$$

$$X_B(t) = C_1 \cdot J_n(m,t) + C_2 \cdot Y_n(m,t)$$

Burada C_1, C_2 –sabitler ve $J_n(m,t), Y_n(m,t)$ –Besselin özel fonksiyonları [6].

Bessel denkleminin genel çözümleri.



Şekil 3. $\nu = \frac{1}{3}, \mathrm{m} = \frac{1}{3}, \mathrm{n} = \mathrm{m}$

$$U_1(\mathrm{t}) = \left(\frac{3}{2} \cdot t\right)^{\frac{1}{3}} \cdot \frac{C_{W_0} \cdot \cos(C_t - t)}{\sqrt{t}} \cdot e^{\int e^{W_1(t)} dt + C_{S+x}}$$

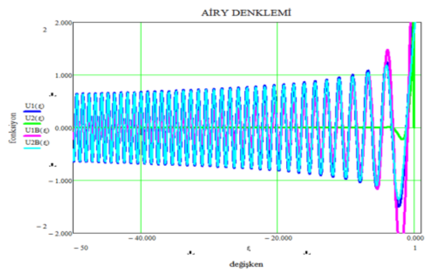
$$U_2(\mathrm{t}) = \left(\frac{3}{2} \cdot t\right)^{\frac{1}{3}} \cdot \frac{C_{W_0} \cdot \cos(C_t - t)}{\sqrt{t}} \cdot e^{\int e^{W_2(t)} dt + C_{S+x}}$$

yerine koyarak $t = \frac{2}{3} \cdot i \cdot \xi^{\frac{3}{2}}$ Airy denkleminin çözümü bulunmaktadir.

$$U_1(\xi) = \left[\frac{3}{2} \cdot \left(\frac{2}{3} \cdot \sqrt{-1} \cdot \xi^{\frac{3}{2}}\right)\right]^{\frac{1}{3}} \cdot \frac{C_{W_0} \cdot cos\left(C_t - \frac{2}{3} \cdot \sqrt{-1} \cdot \xi^{\frac{3}{2}}\right)}{\sqrt{\frac{2}{3} \cdot \sqrt{-1} \cdot \xi^{\frac{3}{2}}}} \cdot e^{\frac{5}{36} \cdot \frac{1}{\sqrt{\frac{2}{3} \cdot \sqrt{-1} \cdot \xi^{\frac{3}{2}}}} + C_{S+x}}$$

$$U_2(\xi) = \left[\frac{3}{2} \cdot \left(\frac{2}{3} \cdot \sqrt{-1} \cdot \xi^{\frac{3}{2}}\right)\right]^{\frac{1}{3}} \cdot \frac{C_{W_0} \cdot cos\left(C_t - \frac{2}{3} \cdot \sqrt{-1} \cdot \xi^{\frac{3}{2}}\right)}{\sqrt{\frac{2}{3} \cdot \sqrt{-1} \cdot \xi^{\frac{3}{2}}}} \cdot e^{\frac{2}{3} \cdot \sqrt{-1} \cdot \xi^{\frac{3}{2}} + C_{S+x}}$$

$$U_B(\xi) = C_1 \cdot Ai(\xi) + Bi(\xi) + \left(Im(Ai(\xi)) + Im(Bi(\xi))\right) \cdot i$$



Şekil 4. $\nu = \frac{1}{3}, \mathrm{m} = \frac{1}{3}, \mathrm{n} = \mathrm{m}$

3. Tartışma ve Sonuç. Riccati denklemlerinin çözümünün integralı [1-3] olduğuna rağmen Bessel ve Airy denklemlerinin çözümleri ile Bessel ve Airy fonksiyonları yardımı ile kurulmuş çözümlerinin yakınsaklığı Mathcad 14 [6] grafiklerinde gösterilmektedir. Onların analizi de iyi bir sonuç alındığından söz etmektedir. Bundan sonraki çalışmalarda yöntemin uygulanmasının çeşitli varyasyonları incelenecektir.

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