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Improving Student Achievement in Mathematics Courses Taught in Foundation Year at University

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Abstract: This paper presents how student achievement in foundation year mathematics courses may be improved by planning learning experiences which are responsive both to the students' learning needs and to the discipline of mathematics. A case study was conducted involving 372 students enrolled at Manchester Metropolitan University (MMU), United Kingdom, who completed an initial test for preliminary assessment of their mathematical knowledge. The results of quantitative and qualitative analysis of their answers, together with the I-Cube model, were used for planning and delivery of the mathematics learning experiences included in lectures, the aim being to enable students to build on their existing interests, proficiencies, experiences and competencies. The students then completed a second test which aimed to assess the learners' conceptual development. The results of quantitative and qualitative analysis of these answers showed an improvement in student achievement. The paper also contains suggestions for improvement of I-Cube model implementation through the design and application of online versions of the two tests. This would enable greater personalisation of learning and assessment and allow feedback to be given in real-time, thus making the mathematics lectures more enjoyable and effective in developing students' knowledge and skills. In addition, the development of online tutorials for students to study at home before attending face-to-face tutorials (blended learning approach) would enable the students to develop positive mathematical identities and become strong mathematical learners.

Keywords: Mathematics, Fractions, Teaching, Foundation and Manchester Metropolitan university

Introduction

Understanding of fractions is dependent on the middle years of students' education. During the foundation year at university, all relevant knowledge will be refreshed and tested, and a poor understanding of fractions may be revealed. This learning gap is especially problematic because fractions are a critical aspect of mathematical scholarship, essential for algebra applications and other more advanced areas of the field (NMAP, 2008).

Students' understanding of relational concepts are necessary not only for deeper mathematical understanding, but also to support daily activities. However, fractions are especially difficult for students to learn and present ongoing pedagogical challenges to mathematics teachers (Siemon et al., 2015); these difficulties are often observed across all levels of education, beginning from the early primary years (Gupta & Wilkerson, 2015). The reasons for such difficulties, particularly in primary school, are often underpinned by issues with larger cognitive processes, including proportional reasoning and spatial reasoning (Artin, 1958). In relation to having different notions of fractions, Aksu (2012) conducted a study which explored *"Differences in student*

- Selection and peer-review under responsibility of the Organizing Committee of the Conference

performance when fractions were presented in the contexts of understanding the meaning of fractions, computations with fractions, and solving word problems involving fractions…"; her study showed that limited understanding of the different meanings of fractions affects students' ability to generalise and work with fractional concepts. Similarly, Siemon et al. (2015) indicates that learning fractions is difficult because students are commonly. Firmender et al. (2014) add that the concept of fractions is perceived as one of the most difficult areas in school mathematics to learn and teach. The most frequently mentioned factor contributing to the complexity is that fractions have five interrelated constructs, namely, the part-whole, ratio, operator, quotient and measure.

A study of practices in the classrooms of the foundation year in MMU showed that students frequently initiated unexpected uses of fractions as operator and quotient and drew on a part-whole understanding when solving fractional problems. Also, the students' background knowledge varied according to their capabilities, home environment, perspective, language and different ways of solving fractions (Eichler & Erens, 2014). The problem which confronts teachers and lecturers in the foundation year of university is therefore how to enable students to overcome and solve the fraction problem, in order to look at more complex issues in mathematics or other relevant courses.

This study considers numerous strategies for improving the teaching of fractions in higher education institutions in general and universities in particular. Various frameworks and models have been investigated using this study sample.

Problems Faced by Students and Teachers When Teaching Fractions

Why do so many students still struggle with fractions? This question commonly affects lecturers, particularly those teaching first-year students in universities. The psychologist Roberts and many other researchers have outlined the reasons why rational number arithmetic is so difficult. This includes not only fractions expressed in the form a/b, but also those in decimal and percentage formats. Although fractions of the a/b form present the most difficulty for children and adults, decimal notation and percentages pose significant problems of their own. In expressions such as '130% on real performance', for example, many students do not completely understand that percentages are fractions (Siegler & Lortie-Forgues, 2017).

Many research studies have looked at the performance of rational number arithmetic among international students and teachers. These studies have identified several reasons why rational number arithmetic is so difficult, which can be divided into two classes:

A. Inherent sources of difficulty, and
B. Cultural sources of difficulty.

As rational numbers are more complex than whole numbers, they are naturally more difficult to understand and use. Inherent sources of difficulty are universal, and even students who famously outperform their peers throughout the world in mathematics still have difficulty with fractions.

Two inherent sources of difficulty

1. It is difficult to understand what rational numbers mean. Each whole number is represented by a single symbol (1, 2, 3, 37, 996 and so on). However, rational numbers can be expressed as fractions, decimals or percentages. It is not at all obvious why 1/4, 0.25 and 25% all refer to the same quantity. Even more confusing, any rational number can be represented by an infinite number of different fractional expressions. The numbers in the series 1/2, 2/4, 3/6, 4/8 and 5/10 appear to be getting larger; after all, both the numerators and the denominators are increasing, yet they all represent the same quantity.

2. Arithmetic operations with rational numbers are far more complex than they are with whole numbers. With whole numbers, the methods that students have learned for adding, subtracting, multiplying and dividing are straightforward to perform. Furthermore, the reasons why they work are easy to demonstrate with objects. However, the methods for fractional arithmetic are complex, and play different roles. For example, it is difficult to understand why the lowest common denominator is used when adding or subtracting fractions but not when multiplying them, or why the second fraction should be inverted and multiplied when dividing. With such a

shallow understanding of rational number arithmetic, it is no wonder so many students have difficulty with it (Ludden, 2017).

Although these inherent complexities plague most students, other sources of difficulty with rational number arithmetic are cultural in origin. That accounts for the better performance among some students compared with their counterparts. Specifically, these cultural difficulties include the following.

Four cultural sources of difficulty

1. Lecturer knowledge. Mathematics teachers who have received the best instruction in rational number arithmetic are, when asked what is meant by something, able to provide the most suitable explanation (Carayannis, 2015); it is hard to provide quality instruction to students when the lecturer only half understands the concepts.

2. Textbook quality. A comparison of primary school mathematics textbooks internationally shows that some devote far more space to rational number arithmetic, and provide more practice problems, than others (NCOTOM; 2007). To the extent that practice makes perfect, students from schools using these textbooks have a decided advantage over their peers.

3. Language. Fractional vocabulary may be much easier to learn in some countries' educational systems than others. For example, mathematical language in China is easier than in other countries in Asia, as the meaning of the fraction is more explicit.

4. Relevance of mathematics to students' future. One reason why so many students experience a phobia for maths is that they do not see its relevance to their future daily lives (Masters, 2017). There is ongoing debate in higher education about whether algebra or elementary statistics is the most useful mathematics course, since in some subjects, students will never need to solve a quadratic equation, but they will need to deal with statistical information such as polls, surveys, census data and economic reports. Yet these kinds of data are frequently presented as rational numbers, fractions and percentages.

Strategies for Delivering Information to Higher Education Students

In fact, there are various different strategies for tackling and solving students' problems with fractions in higher education, and many issues to consider for each one. As an international university, MMU is trying to update its teaching and learning methodology in order to raise students' levels and improve lecturer performance at the same time. The team at MMU has considered strategies such as the following.

Bloom's Taxonomy

The Taxonomy of Educational Objectives was introduced by Professor Benjamin Bloom at the University of Chicago in the 1950s. The main objective of this taxonomy was to structure a system for categorising and quantifying learning behaviour which would assist in the development and assessment of educational learning (Bloom & Krathwohl, 1956). Firstly, Bloom identified the cognitive domain with six learning levels: data recall, understanding, applying, analysing, synthesising and evaluating. The original taxonomy was used to classify and test learning objectives across the six learning levels. Then, an adjusted version of the cognitive domain was produced: levels five and six from 'synthesis' and 'evaluation' were replaced by 'evaluation' and 'creation', while the psychomotor domain addressed skills related to practical applications (Anderson & Krathwohl, 2001).

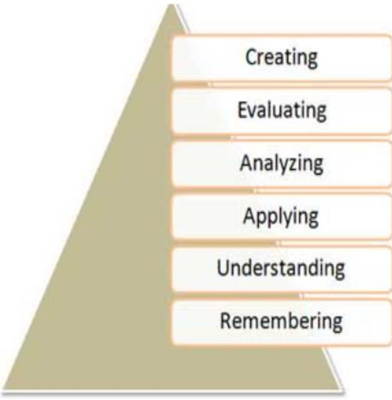


Figure 1. The six levels of Bloom's Taxonomy

Bloom's Taxonomy has developed and changed its verbs many times to provide a more specific representation of learning outcomes.

Salmon model

Salmon designed a model in 2000 that has proven its success in relation to e-learning theories, as shown in Figure 2 below. This model includes five stages, each of which requires participants to master certain technical skills and calls for different e-moderating skills.

The 'interactivity bar' running along the right of the flight of steps in the model suggests the intensity of interactivity that can be expected between participants at each stage. At the first stage, they interact only with one or two others. After stage two, the number of others with whom they interact gradually increases, although stage five often results in a return to more individual pursuits. This paper will present an active application of this model.

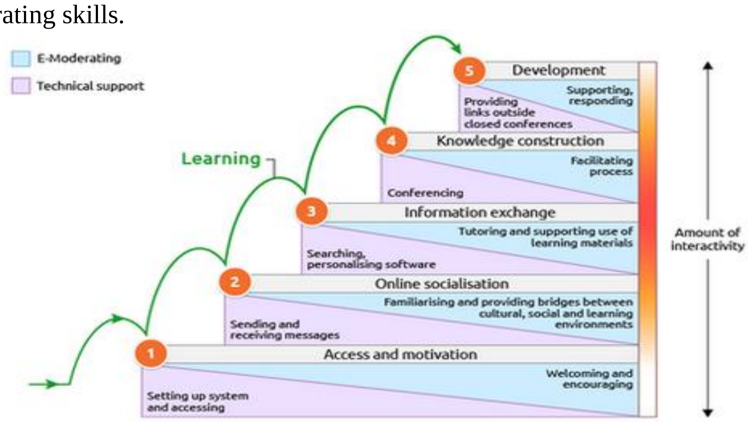


Figure 2. The Five stages of the Salmon model

I-Cube model

Kenan (2015) used interconnected lines to create a new cube-shaped model. Each edge in this cube represents an element which is essential to the success of an e-learning strategy. The cube is called the I-Cube because all elements begin with the letter 'i', as shown in Figures 3A and 3B. This is not a general solution to be implemented in every HEI, but it contains suggestions about aspects which should be considered when improving the quality of teaching and learning processes in the digital era.

The I-Cube can be considered a base or cornerstone in the creation of a strategy to improve e-learning implementation. I-cube activities will help in the development of suitable approaches to introducing blended learning, which is the basic stage in ensuring successful e-learning in the future.

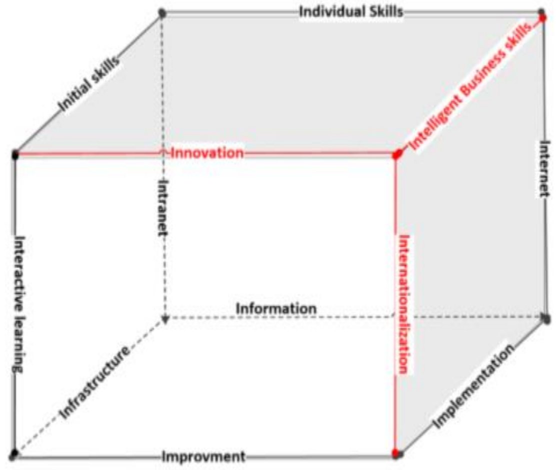


Figure 3A. The I-Cube model

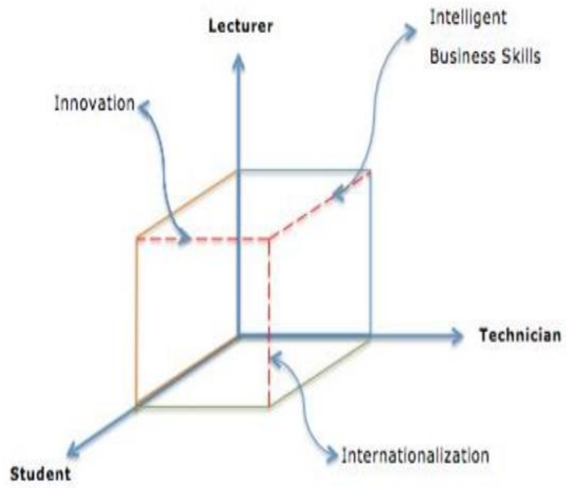


Figure 3B. The I-Cube dimensions

Methodology

The following evaluation method was adopted with a sample of foundation year students from the 2017/2018 academic year at Manchester Metropolitan University (MMU) in the United Kingdom. The sample comprised 372 students registered on the mathematics course, who would be assessed by two tests before their final examination. Test 1, at the start of the academic year, was a general assessment to diagnose the mathematical level of the students, who had come from different colleges and a variety of backgrounds. MMU receives a wide range of international students every year, and the test was designed to take into account the many different strategies they may have adopted, some of which are mentioned above. The questions were thus formulated by considering the disparity in background knowledge resulting from the students' different capabilities, home environment, perspective and language. Test 2 took place halfway through the course (in January 2018) to evaluate the level of change as a result of following the teaching and learning strategy.

The students' test papers were collected after an hour and a half (the length of the test), then marked to determine the students' different levels and identify any weaknesses that should be focused on for additional consideration in the teaching and learning plan before lessons started. The results were analysed using the Excel program from the Microsoft Office package, which enabled quick and easy feedback of averages and other related functions.

Test 1– Preliminary Assessment of Student Knowledge

Test 1 was a general assessment comprising many subjects. The teachers' intention was simply to test the students' levels and to refresh their memories of general mathematical basics, including fractions; according to Bloom's Taxonomy, remembering is considered the foundation of knowledge. There were twenty questions in the test, and those related to fractions were numbers 6, 9, 10, 16 and 20, as illustrated in Figure 4 below.

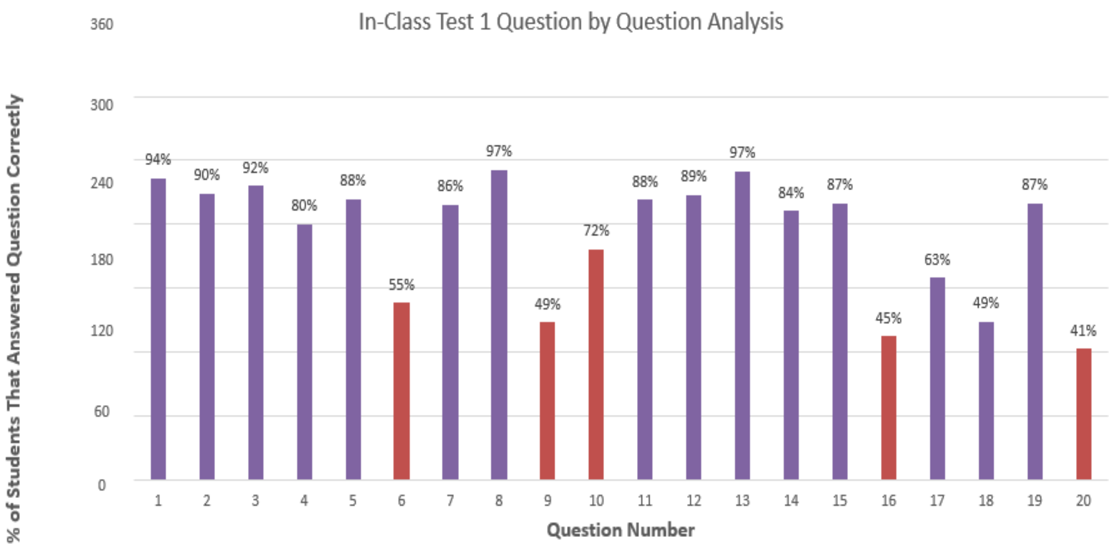


Figure 4. Results of Test 1 questions

- Q6 in Test 1

The results of question 6 are shown in Figure 5. The question was clearly and directly related to fractions and most students should have been able to answer it from their background knowledge (secondary school or college lessons). However, although the question was simple, only about 55% of the students gave the right answer (see Figure 4).

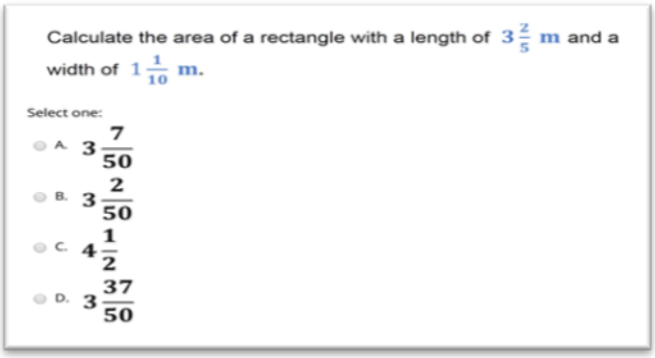


Figure 5. Question 6 of Test 1

- Q9 in Test 1

The results of question 9 are shown in Figure 6. This question involved fractions only indirectly, but students should have been able to understand the role of fractions in enabling them to find the percentages easily. Only 49% of the students were able to answer this question correctly (see Figure 4).

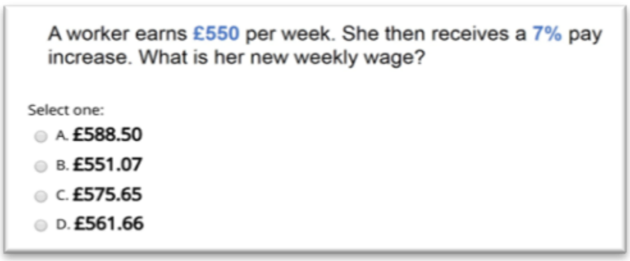


Figure 6. Question 9 of Test 1

- Q10 in Test 1

The results of question 10 are shown in Figure 7. This was similar to question 9; it was about how percentages are divided by the total or multiplied to find the total. 72% of the students got this question correct, so this can be considered the most successful of the fraction-related questions (see Figure 4).

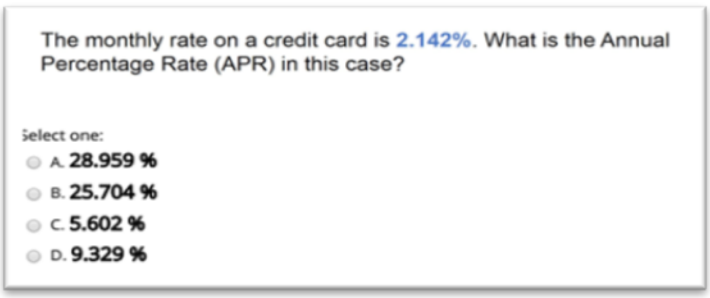
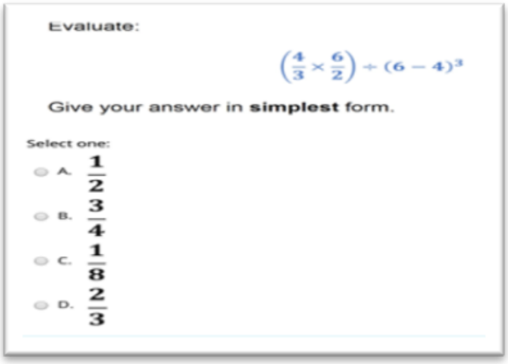


Figure 7. Question 10 of Test 1

- **Q16 in Test 1**

The results of question 16 are shown in Figure 8. This question was designed on the basis of Bloom's Taxonomy and aimed to refresh the students' memories regarding the use of fractions, but only about 45% were able to answer correctly (see Figure 4).

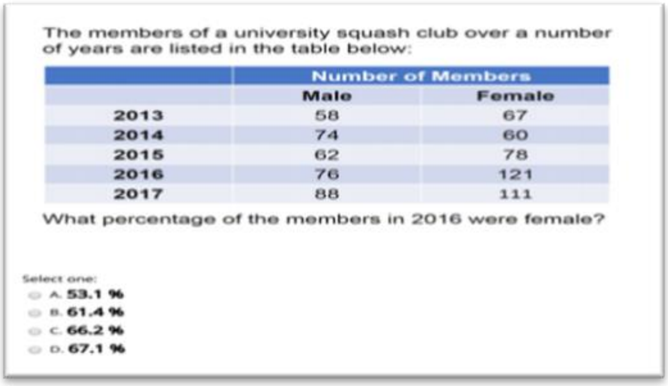

Evaluate:

$$\left(\frac{4}{3}\times\frac{6}{2}\right)\div(6-4)^3$$

Give your answer in **simplest** form.

Select one:
- A. $\frac{1}{2}$
- B. $\frac{3}{4}$
- C. $\frac{1}{8}$
- D. $\frac{2}{3}$

Figure 8. Question 16 of Test 1


The members of a university squash club over a number of years are listed in the table below:

	Number of Members	
	Male	Female
2013	58	67
2014	74	60
2015	62	78
2016	76	121
2017	88	111

What percentage of the members in 2016 were female?

Select one:
- A. 53.1 %
- B. 61.4 %
- C. 66.2 %
- D. 67.1 %

Figure 9. Question 20 of Test 1

- **Q20 in Test 1**

The results of question 20 are shown in Figure 9. The question was about percentages and how to calculate them, which can be considered as indirectly related to fractions. This should have been a straightforward question for any student in the mathematics department but unfortunately, just 41% of the students produced the correct answer (see Figure 4).

The Teaching and Learning Plan

The strategy of the MMU is to help both lecturer and students by providing prior preparation for lessons from a lesson booklet and a website with relevant links. In addition, the mathematics learning experiences included in the lectures were planned with the aim of enabling students to build on their existing interests, proficiencies, experiences and competencies (reading and listening skills, language, mathematical reasoning, ability to cope with complexity, etc.).

The steps of Bloom's Taxonomy (see Figure 1 above) were considered in choosing the test questions. For example, in Test 1, Q6 and Q16 were directly related to fractions, while Q9 and Q20 were indirectly related to them *('Remember' step).* Then, in the lectures, mathematical learning experiences were planned which would be responsive to both the students' learning needs and the discipline of mathematics *('Understand' and 'Apply' steps).* By providing appropriate challenges, effective teachers can signal their high but realistic expectations. This means building on students' existing thinking and, often, modifying tasks to provide alternative pathways to understanding *('Analyse' step).*

For low-achieving students, teachers need to find ways to reduce the complexity of tasks without falling back on repetition and without compromising the mathematical integrity of the activity. Modifications include using prompts, reducing the number of steps or variables, simplifying how results are to be represented, reducing the amount of written recording and using extra thinking tools *('Evaluate' step).* Similarly, by putting obstacles in the way of solutions, removing some information, requiring the use of representations or asking for generalisations, teachers can increase the challenge for academically advanced students *('Creative' step).*

The Salmon model (see Figure 2) was also used for the design and delivery of the mathematics sessions, and to create a relationship between the lecturers and their students. The first stage in motivating access to and use of any software system is to provide encouragement, along with helpful guidance regarding where to find technical support. The second stage is online socialisation, which involves providing openings for sending, receiving and exchanging information, and the establishment of ground rules. The third stage relates to information exchange and moving activities out of the classroom; this includes reporting and discussion of findings or results between the lecturer and students to support the use of learning materials. The fourth stage is knowledge construction to build connections between models and work-based learning experiences; open activities involving discussions and questions are used to encourage reflection. The fifth stage involves development and consideration of the learning processes. Here, students are able to become critical of the medium, and support and response is provided only when required (Salmon, 2011).

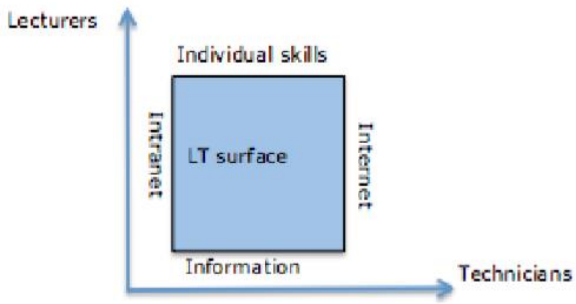
Use of the I-Cube model to design mathematics lectures

In order to include and increase the use of technology in the design of future maths lessons, it was necessary to develop and implement e-learning tutorials which could be used by students before going to face-to-face tutorial sessions in the period between the November and January tests. For any such educational strategy to succeed, three important stakeholders should be considered, namely, the lecturers, students and technicians (Kenan et al., 2017). These are the three dimensions of the I-Cube (see Figure 3B), and this model was utilised alongside the strategies referred to above, as well as others not discussed in this paper.

The structure of the I-Cube model is built on relationships between the key stakeholders as follows:

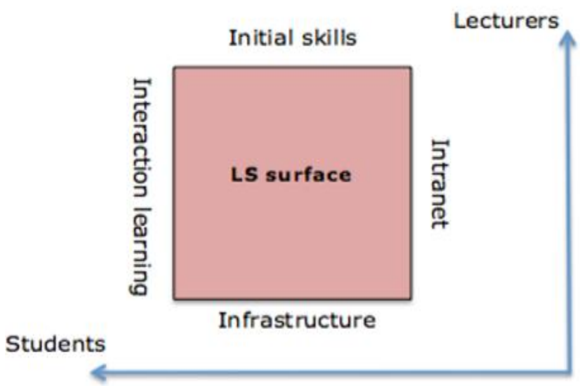
1. Analysis of the correlation between lecturers' and technicians' points of view (see Figure 10) indicates that there are four factors which are important to the successful development of e-learning in general. These four factors can be drawn as a square, with the four edges being information, the Internet, individual skills and the intranet. Each edge plays a key role in the success of e-leaning performance.

Figure 10. Relationship between lecturers and technicians



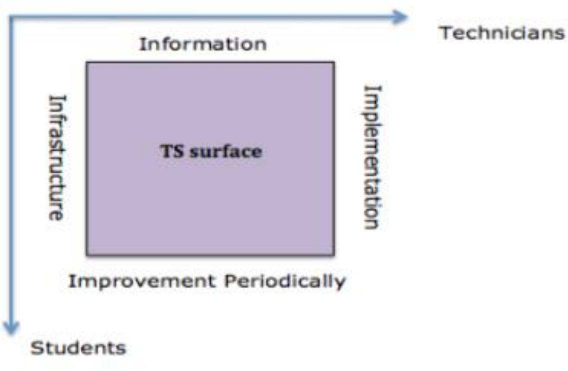
2. Pedagogical theories have presented different models and frameworks regarding the relationship between lecturers and their students. Four factors arise from this relationship which may be considered essential to success in the development of e-learning in general. These four factors can also be drawn as a square, the edges being the intranet, the infrastructure, interactive learning and initial skills (see Figure 11).

Figure 11. Relationship between lecturers and students



3. The relationship between technicians and students is complementary to the two previous relationships or surfaces. This aspect presents a further surface with four edges, representing four factors which can be drawn as a square (see Figure 12), namely, information, implementation, periodic improvement and infrastructure. There are two edges in this square which work jointly with the other surfaces (information and infrastructure) and these play an especially important role in the success of e-leaning performance.

Figure 12. Relationship between technicians and students



To summarise, the three relationships discussed above are between the main groups of stakeholders. As these are linked by joint elements (see Figures 3A and 3B), none of the surfaces can be completed without the others.

Thus, the factors to be considered after combining the three surfaces are information, intranet, the Internet, individual skills, infrastructure, interactive learning, initial skills, periodic improvement and implementation. Two further factors taken from theories of innovation, namely internationalisation and intelligent business, will have a great effect on the development of this teaching and learning process, and therefore these are applied to complete the I-Cube of skills that should be included in new strategies for higher education systems (Kenan, 2015).

Test 2 – Assessing Learners' Conceptual Development

In January 2018, the same number of students (372) who had taken the first test were assessed again using Test 2. This test was formulated differently, due to the inclusion of other mathematical subjects. There were 20 questions, testing students' knowledge and understanding of fractions, square-roots, exponents and equations for probability. Figure 13 below shows the results of Test 2:

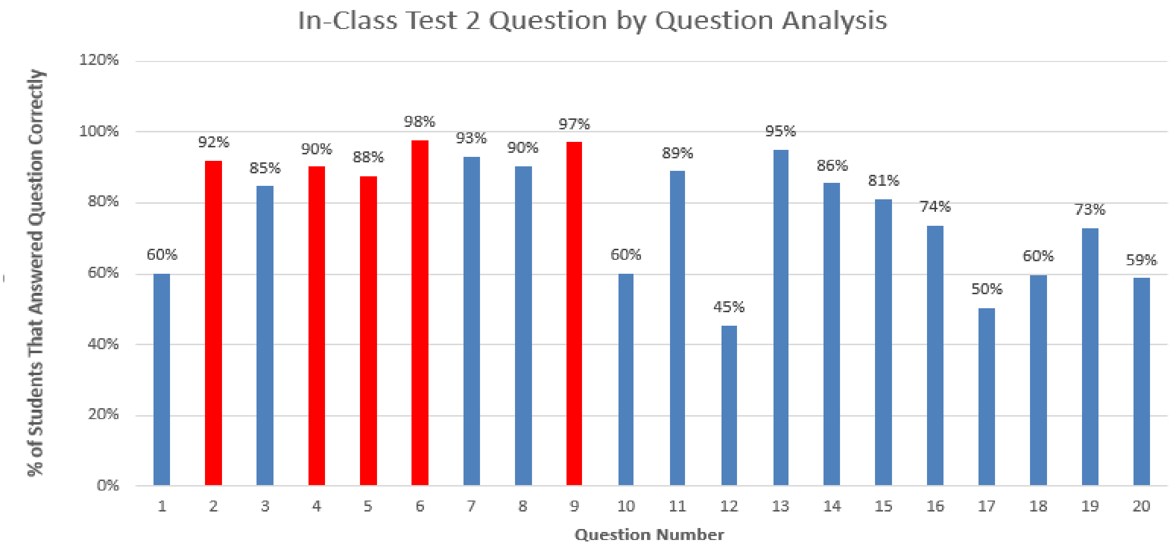


Figure 13. Results of Test 2 questions

The red columns indicate the questions involving fractions, which relate to this paper's aim of showing the students' progress in this aspect through the academic year. Questions 2, 4, 5, 6 and 9 are directly or indirectly concerned with fractions.

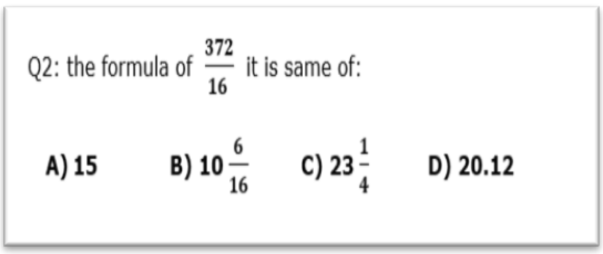


Figure 14. Question 2 of Test 2

- **Q2 in Test 2**

The results of question 2 are shown in Figure 14. This question was purely concerned with fractions, and 92% of the students' answers were correct. This means there had been considerable improvement in the students' ability to solve equations related to fractions (see Figure 13).

- **Q4 in Test 2**

The results of question 4 are shown in Figure 15. This question directly involved fractions in relation to probability. Here, 90% of the answers were correct. This improvement demonstrates the positive effects of the teaching strategy (see Figure 13).

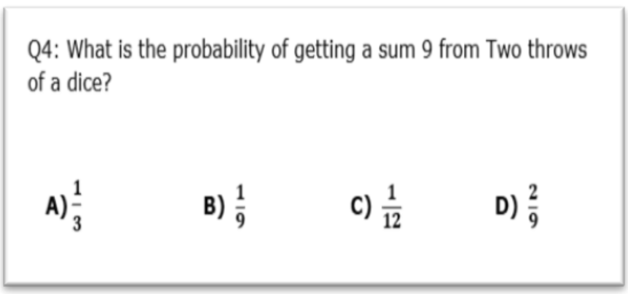


Figure 15. Question 4 of Test 2

Q6: In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither blue nor green?

A) $\frac{2}{3}$ B) $\frac{8}{21}$ C) $\frac{3}{7}$ D) $\frac{9}{22}$

Figure 16. Question 4 of Test 2

Q9: The following table shows the results of a test given to class of students at MMU:

Test Range	Number of Students
80 or below	0
81-90	0
91-100	0
101-110	43
111-120	56
121-130	79
131 and above	13
Total	**191**

The probability that a student has got less than 121 is:

A) 0.52 B) $\frac{92}{191}$ C) 0.41 D) $\frac{93}{100}$

Figure 17. Question 4 of Test 2

- **Q6 in Test 2**

The results of question 6 are shown in Figure 16. This question was again about probability, and here 98% of the answers were correct, showing clear evidence of development and improvement (see Figure 13).

- **Q9 in Test 2**

The answers to question 8 are shown in Figure 17. This question was indirectly related to fractions, and 97% of the answers were correct (see Figure 13).

Comparison of Statistical Analysis of Students' Answers

After application of the three strategies to the case study (Bloom's Taxonomy, Salmon model and I-cube), there were evident improvements in the students' levels of understanding, memory and deep knowledge. This is demonstrated by the answers to the probability and fractions questions, as shown in Figure 13. The results were analysed using the Excel program from the Microsoft Office package, the functions of which provide quick and easy feedback. The averages for Test 1 and Test 2 were calculated using the AVERAGE function, based on the following equations.

Average of marks for all questions in Test 1 = $\frac{\sum_{j=1}^{20} Answer1_j}{20} = 76.65\%$ ……………………………..…. (Eq. 1)

Average of marks for all questions in Test 2 = $\frac{\sum_{j=1}^{20} Answer2_j}{20} = 78.25\%$ ……………………..………….. (Eq. 2)

Then, the average marks for Q6, Q9, Q10, Q16 and Q20 from Test 1 were calculated:

Average of marks for questions about fractions in Test 1 = $\frac{A_6+A_9+A_{10}+A_{16}+A_{20}}{5} = 52.4\%$ ……………….. (Eq. 3)

Average of marks for questions about fractions in Test 2 = $\frac{A_2+A_4+A_5+A_6+A_9}{5} = 93\%$ ………………….… (Eq. 4)

These results indicate that the students' knowledge and understanding were generally improved by using Bloom's Taxonomy and the I-cube model to design and develop mathematics lessons.

In general, the difference in students' levels resulting from the online assessments available to schools is highly variable (Alcock & Simpson, 2011). Some online assessments are designed primarily to achieve more efficient test delivery, while others appear to be shaped by what is technologically possible, rather than educationally desirable (Code et al., 2014). However, instructionally useful assessments draw on an empirically-based recognition of how knowledge, skills and understanding develop in an area of learning. They are aligned with well-constructed learning progressions that describe the nature of student progress. They are designed with an appreciation of how learning builds on to earlier learning and lays the foundations for future learning; the crucial role of prerequisite skills and knowledge in learning success; the kinds of misunderstandings students commonly develop; and the common errors that students make (Viirman, 2014).

Interactive resources are also essential for practice and these have been available for several years at a range of levels. For example, the Guardian Teacher Network of 2018 describes activities on its site in which "Key stage 2 students can learn to recognise and understand unit fractions, such as 1/2, 1/3, 1/4, with online shading activities" (Guardian, 2018). Students need to be able to practise comparing and ordering simple fractions, then move onto relating fractions to division; relating fractions to their decimal representations will help learners make important links between fractions, decimals and percentages and ratio (Getenet & Callingham, 2017). These subjects may have been taught as different topics and to avoid problems later, it is important for students to be able to convert between mixed numbers and improper fractions, understand equivalent fractions or rewrite a fraction.

In the case study presented in this paper, the development of problem-solving and reflective skills alongside subject-based educational skills helped to overcome the difficulties of teaching fractions. This has been demonstrated by analysis of the students' grades for Tests 1 & 2, and is supported by the personal observations of the authors who taught this group of students (see Eqs. 3 and 4). Although the students' overall marks for Test 1 were high, their results were lower when questions about fractions were introduced. The proposed solution was to use the I-cube model and Bloom's taxonomy to develop new strategies for the design of future lectures.

Conclusions

The paper has demonstrated the improvement in student achievement on mathematics courses taught in foundation year which can be achieved by planning mathematics learning experiences that are responsive both to the students' learning needs and to the discipline of mathematics. 372 students who were enrolled at Manchester Metropolitan University (MMU), United Kingdom, completed an initial test (Test 1) for the preliminary assessment of their mathematical knowledge. The average mark for all questions was 76.65% and the average mark for the questions related to fractions was 52.4%. This was an excellent pedagogical approach to check the initial level of student understanding of fractions.

The lecturers decided to plan and deliver mathematics learning experiences based on Bloom's Taxonomy, the five-stage model of online learning by Salmon and the I-Cube model. Afterwards, the students completed a second test (Test 2) aiming to assess the learners' conceptual development. The results of quantitative and qualitative analysis of their answers showed an improvement in student achievement on this mathematics course (the average mark for all questions in Test 2 was 78.25%, while the average mark for questions related to fractions had risen to 93%). Therefore, it was obvious that the students' level of knowledge and understanding of fractions had improved considerably (the average mark for these questions having increased from 52.4% to 93%).

The authors intend to perform future work including improvement of I-Cube model implementation through the design and application of online versions of the two tests (Test 1 and Test 2). The aim of this is to enable personalised learning and assessment of students and allow feedback to be given in real-time, making the mathematics lectures more enjoyable and effective in developing students' knowledge and skills. The intention is also to develop online tutorials for students to study at home before going to face-to-face tutorials (blended learning approach), which will enable the students to develop positive mathematical identities and become powerful mathematical learners.

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