CYCLIC PRESENTATIONS OF TORUS KNOTS WITH DUNWOODY PARAMETERS

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Abstract

We obtained a different cyclic presentation of torus knots of type $(3k+2,\ m(3k+2)+3)$ for Dunwoody parameters $(a,b,c,r)=(k+1,k,m(2k+1)(2k+2)+k+1,(2k+1)(2k+2)-k)$ which is $\alpha^{m(3k+2)+3}(\gamma^{-1}\alpha^{-m(k+1)-1})^2[\alpha^{m(3k+2)+3}(\gamma^{-1}\alpha^{-m(k+1)-1})^3]^k$ when $k \geq 1$ and $m > 0$.

DUNWOODY PARAMETRELERİ İLE TOR DÜĞÜMLERİ'NİN DEVİRLİ TEMSİLLERİ

Özet

$k \geq 1$ ve $m > 0$ iken $(a,b,c,r)=(k+1,k,m(2k+1)(2k+2)+k+1,(2k+1)(2k+2)-k)$ Dunwoody parametreleri için $(3k+2,\ m(3k+2)+3)$ tipinden tor düğümlerinin $\alpha^{m(3k+2)+3}(\gamma^{-1}\alpha^{-m(k+1)-1})^2[\alpha^{m(3k+2)+3}(\gamma^{-1}\alpha^{-m(k+1)-1})^3]^k$ şeklinde farklı bir devirli temsilini elde ettik

Keywords: Torus Knots, Heegaard splitings, Dunwoody parameters, cyclically presented groups.

1. Introduction

The cyclically presented groups comprise a rich source of groups which are interesting from a topological point of view. The connection between cyclically presented groups and cyclic branched coverings of knots and links was studied in particularly [1] [2] [4] and [5].

L.Neuwirth describes an algorithm for deciding is a group presentation with generators and relations corresponds to spine of closed compact 3-manifold in [8]. The nice approach to determine cyclically presented groups arise fundamental groups of 3 manifold was developed and an algorithm to enumerate Heegaard diagrams with the necessary cyclic symmetry was described by Dunwoody in [2]. Details on knot theory, cyclically branched of knots and details on cyclic presentation of groups can be found in [3] and [7] respectively

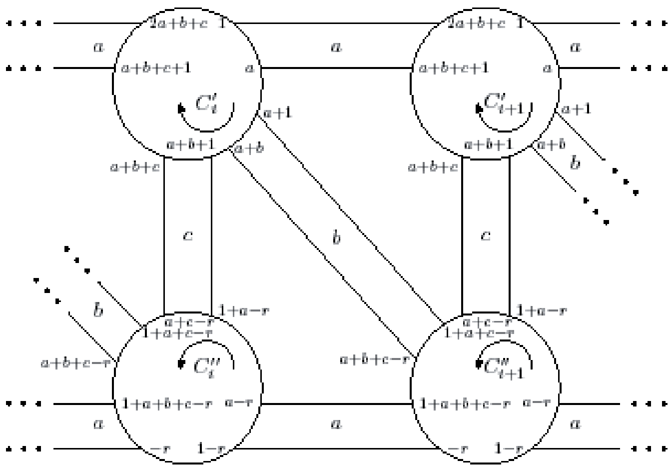


Figure 1

The family of Dunwoody manifold has been introduced in [2] by a class of trivalent regular planar graphs (called Dunwoody diagrams) with cyclic symmetry, depending on six integers a,b,c,n,r,s such that $n>0, a,b,c \geq 0$ and $a+b+c>0$. For certain values of the parameters, called admissible, the Dunwoody diagrams $D(a,b,c,n,r,s)$ turn out to be Heegaard diagrams, so defining a wide class of closed oriantable 3-manifolds $M(a,b,c,n,r,s)$ with cyclically presented fundemental groups.

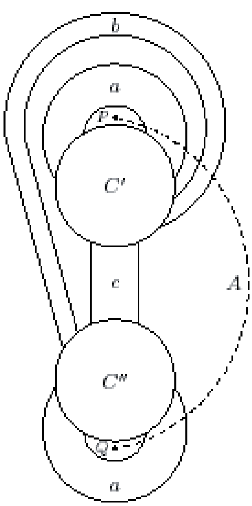


Figure 2

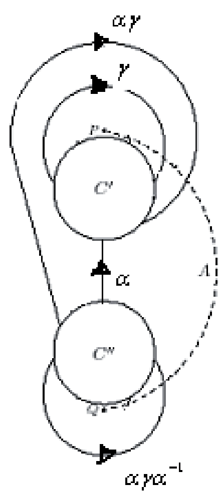
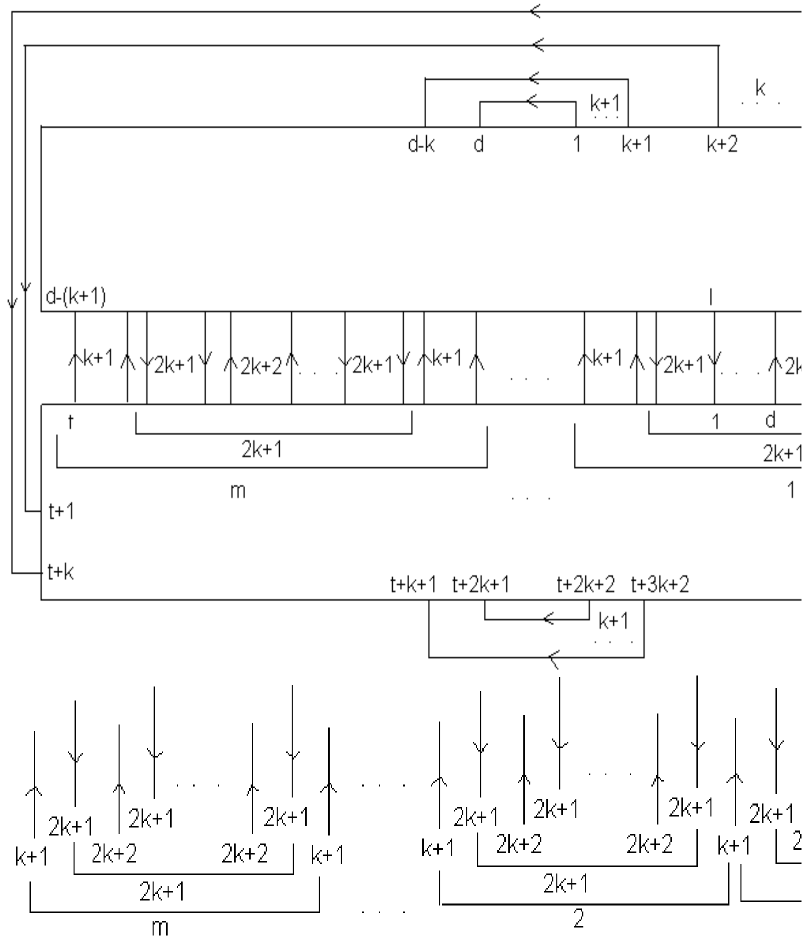


Figure 3


k
k+1
d-k
d
1
k+1
k+2
d-(k+1)
l
k+1
2k+1
2k+2
2k+1
k+1
k+1
2k+1
t
2k+1
1
d
2k+1
m
1
t+1
t+k
t+k+1
t+2k+1
t+2k+2
t+3k+2
k+1
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2k+2
k+1
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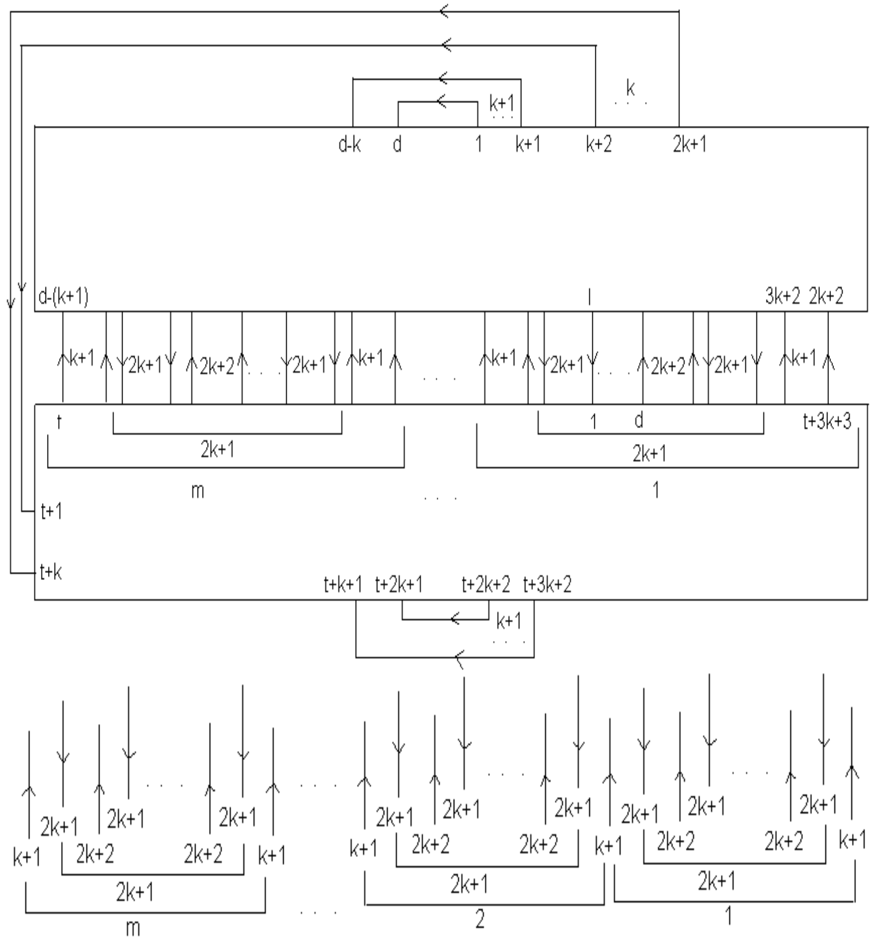
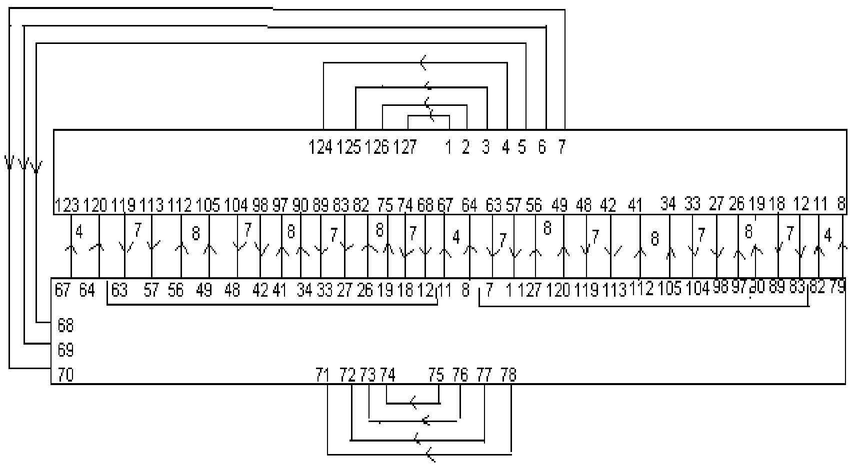
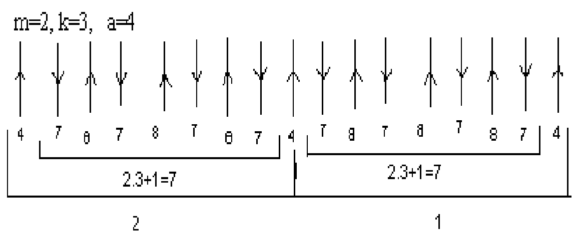
2



For example i) For (4,3,116,53) and k=3, m=2 Heegard diagram is



Direction situation of arrow towards up and down for c



and

$$w = \alpha^{25}(\gamma^{-1}\alpha^{-9})^2[\alpha^{25}(\gamma^{-1}\alpha^{-9})^3]^3$$

$t(11,25)$

Table for $c \le 284$ is in below

k	m	(a,b,c,r)	w	$t(3k+2,\ m(3k+2)+$
1	1	(2,1,14,11)	$\alpha^8(\gamma^{-1}\alpha^{-3})^2\alpha^8(\gamma^{-1}\alpha^{-3})^3$	$t(5,8)$
1	2	(2,1,26,11)	$\alpha^{13}(\gamma^{-1}\alpha^{-5})^2\alpha^{13}(\gamma^{-1}\alpha^{-5})^3$	$t(5,13)$
1	3	(2,1,38,11)	$\alpha^{18}(\gamma^{-1}\alpha^{-7})^2\alpha^{18}(\gamma^{-1}\alpha^{-7})^3$	$t(5,18)$
1	4	(2,1,50,11)	$\alpha^{23}(\gamma^{-1}\alpha^{-9})^2\alpha^{23}(\gamma^{-1}\alpha^{-9})^3$	$t(5,23)$
1	5	(2,1,62,11)	$\alpha^{28}(\gamma^{-1}\alpha^{-11})^2\alpha^{28}(\gamma^{-1}\alpha^{-11})^3$	$t(5,28)$
2	1	(3,2,33,28)	$\alpha^{11}(\gamma^{-1}\alpha^{-4})^2[\alpha^{11}(\gamma^{-1}\alpha^{-4})^3]^2$	$t(8,11)$

2	2	$(3,2,63,28)$	$\alpha^{19}(\gamma^{-1}\alpha^{-7})^2[\alpha^{19}(\gamma^{-1}\alpha^{-7})^3]^2$	$t(8,19)$
2	3	$(3,2,93,28)$	$\alpha^{27}(\gamma^{-1}\alpha^{-10})^2[\alpha^{27}(\gamma^{-1}\alpha^{-10})^3]^2$	$t(8,27)$
2	4	$(3,2,123,28)$	$\alpha^{35}(\gamma^{-1}\alpha^{-13})^2[\alpha^{35}(\gamma^{-1}\alpha^{-13})^3]^2$	$t(8,35)$
2	5	$(3,2,153,28)$	$\alpha^{43}(\gamma^{-1}\alpha^{-16})^2[\alpha^{43}(\gamma^{-1}\alpha^{-16})^3]^2$	$t(8,43)$
3	1	$(4,3,60,53)$	$\alpha^{14}(\gamma^{-1}\alpha^{-5})^2[\alpha^{14}(\gamma^{-1}\alpha^{-5})^3]^3$	$t(11,14)$
3	2	$(4,3,116,53)$	$\alpha^{25}(\gamma^{-1}\alpha^{-9})^2[\alpha^{25}(\gamma^{-1}\alpha^{-9})^3]^3$	$t(11,25)$
3	3	$(4,3,172,53)$	$\alpha^{36}(\gamma^{-1}\alpha^{-13})^2[\alpha^{36}(\gamma^{-1}\alpha^{-13})^3]^3$	$t(11,36)$
3	4	$(4,3,228,53)$	$\alpha^{47}(\gamma^{-1}\alpha^{-17})^2[\alpha^{47}(\gamma^{-1}\alpha^{-17})^3]^3$	$t(11,47)$
3	5	$(4,3,284,53)$	$\alpha^{58}(\gamma^{-1}\alpha^{-21})^2[\alpha^{58}(\gamma^{-1}\alpha^{-21})^3]^3$	$t(11,58)$

3. References

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