A MATHEMATICAL MODEL FOR PREDICTING THE TEMPERATURE PROFILES DURING MICROWAVE HEATING OF CYLINDRICAL FOODS

Coskan ILICALI
Kyrgyzstan Turkey Manas University, Faculty of Engineering,
Department of Food Engineering,
Bishkek/Kyrgyzstan
E-posta: coskan.ilicali@gmail.com

Filiz ICIER
Ege University, Faculty of Engineering, Department of Food Engineering,
Izmir/Turkey
E-posta: filiz.icier@ege.edu.tr

Abstract

An analytical equation for uniform, normal surface microwave power for cylindrical objects has been proposed. The surface microwave power for cylindrical objects undergoing unidirectional energy transfer was a function of average power absorbed, dimensions of the cylinder and the attenuation constant. A finite difference model has been developed to predict the temperature profiles. The numerical model has been tested against well-known analytical solutions and excellent agreement was observed. This model has been used to simulate literature temperature profiles. Satisfactory agreements were obtained for the interior points. However, the predicted surface temperatures were higher than the experimental temperatures. It is believed that the proposed expression for the surface power may be used to check the uniformity and normalcy of the surface microwave.

Key words: microwave, modeling, food, temperature

1. Introduction

Mathematical models simulating microwave heating of foods will yield useful results if they are based on realistic assumptions and use input model parameters characterizing the heating process. Maxwell equations and Lambert's law may be used to describe the microwave energy absorption. However, Maxwell equations are of complex nature. [1] have compared the predictions of a numerical model using these two equations to represent microwave energy absorption. They have concluded that the predictions of the two formulations depend on a critical thickness which is a function of the penetration depth.

According to Lambert's law, during microwave heating, the decay of the incident, normal surface microwave power P_o with distance x from the surface may be expressed as [2]

$$P(x) = P_0 e^{-2\delta x} \tag{1}$$

where δ is the attenuation constant, 1/m.

A penetration depth, δ_p has been defined as δ_p=1/(2δ) where $P(x)/P_0 = 1/e$ [2]. [1] have shown that if the thickness of a slab is greater than 2.7 times the penetration depth, the two formulations yield identical results. Therefore, for sufficiently thick samples Lambert's law may be used for microwave absorption. Accurate determination of P_0 is very important in microwave model testing.

[3] have used a calorimetric method to determine the power absorbed. The technique consists of the calculation of the absorbed power by different water volumes submitted to heating, under the same operating conditions (position, power and container size) as used in the experiments [4]. The surface power was taken to be equal to the power absorbed and the surface power absorbed was expressed as a function of the weight of the sample. Equations proposed by [3] and [4] are shown in Eqs. (2) and (3) respectively, as examples to this approach for cylindrical samples.

$$P_0 = 453.2 + 59.8\ln(weight) \qquad 25\ \text{g} \le \text{weight} \le 2500\ \text{g} \tag{2}$$

$$P_0 = \frac{1246.5W}{1.45W + 79.53} \qquad 50\ \text{g} \le \text{weight} \le 400\ \text{g} \tag{3}$$

However, the powers calculated by the calorimetric model are the average absorbed powers rather than the surface powers.

[5] has defined the absorbed power as

$$P_{abs} = \int_V P(x)dV = \int_0^H \int_0^{2\pi R} \int P_0\, e^{-2\delta x} dx d\theta dz \tag{4}$$

and upon integration has obtained the following equation relating the power absorbed to the surface power :

$$P_0 = \frac{P_{abs}\,\delta}{(1 - e^{-2\delta R})\pi H} \tag{5}$$

[6] and [7] have used Eq.(5) in their microwave heat transfer models. However, in a cylindrical body a volume element is not ($dxd\theta dz$) and Eq.(5) has inconsistent dimensions on the left and right hand sides.

The object of the present work was to use the approach by [5] to relate the power absorbed and surface power and the use the evaluated surface powers in a finite difference model to predict experimental data.

2. Materials and Methods

2.1. Experimental data

The experimental data of [5] were used to test the present model. [5]has measured the temperature profiles of 2% agar gels cylinders containing 0%, 40% and 60% sucrose in a 1500 W microwave oven operating at 2450 MHz. To ensure one dimensional energy transfer, the bottom and the top of the cylinders were shielded with Aluminum foils. Average temperature rise of known volumes of water and sucrose solutions were measured in the same experimental set-up with the gels under identical conditions with the gels. Temperatures were measured with fiber optic sensors at the mid–plane at four different positions. The height of the cylinders was 0.086 m. Two different radii were used; 0.03 m and 0.035 m.

2.2. Mathematical model

The mass average power absorbed within the body can be evaluated from the following equation;

$$P_{av} = \frac{\int P(x)\, dm}{m} \tag{6}$$

where m is the mass of the body. For a constant density system, $P_{average}$ reduces to

$$P_{av} = \frac{\int_V P(x)\, dV}{V} \tag{7}$$

where V is the volume of the cylindrical body, and dV is the volume of the control element. The average power absorbed becomes

$$P_{av} = \frac{\int_0^H \int_0^{2\pi} \int_0^R P_0\, e^{-2\delta x} r\, dr\, d\theta\, dz}{V} \tag{8}$$

Assuming P_0 is constant and expressing x as a function of r and integrating, one obtains

$$P_{av} = \frac{P_0 H\pi}{V\delta^2}\left[R\delta + \frac{1}{2}\left(e^{-2\delta R} - 1\right)\right] \tag{9}$$

Eq.(9) was used to calculate the surface power.

P_{av} can be estimated by measuring the average temperature rise in a body for a given time as

$$P_{av} = \frac{\rho V Cp \Delta T_{av}}{t} \tag{10}$$

where ΔT_{av} is the mass average temperature rise after a given microwave heating time t. Experimental values of [5] for t and ΔT_{av} *were used.*

For microwave heating in a long cylindrical body, the unsteady state heat conduction equation for solids with constant physical properties can be modified by the addition of a microwave power absorption term;

$$\rho Cp \frac{\partial T}{\partial t} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{P}{V} \tag{11}$$

where P/V is the volumetric heat generation term.

Using Eq.(11) or making an energy balance for a cylindrical shell around node i, the following finite difference equation (FDE) can be obtained for the interior nodes;

$$-\gamma\frac{(M-I+1.5)}{(M-I+1)}T_{I-1}{}' + (1+2\gamma)T_I{}' - \gamma\frac{(M-I+0.5)}{(M-I+1)}T_{I+1}{}' = \tag{12}$$
$$T_I + \frac{P_0}{V}(e^{-2\delta(I-1.5)\Delta r} - e^{-2\delta(I-0.5)\Delta r})\frac{M^2\Delta t}{2(M-I+1)\rho Cp}$$

Where $\gamma = \frac{\alpha\Delta t}{\Delta r^2}$; α is the thermal diffusivity; *M* is the number of radial intervals; $\Delta r = \frac{R}{M}$; Δt is the time increment. The superscript ‘denotes the temperature after the time increment Δ*t*.

For the surface of the cylinder (*I* = 1) convection boundary condition was used and the following FDE was obtained;

$$(1+2\gamma+2\gamma\frac{(M+0.5)}{M}\frac{h\Delta r}{k})T_1{}' - 2\gamma T_2{}' = T_1 + 2\gamma\frac{(M+0.5)}{M}\frac{h\Delta r}{k}T_a \tag{13}$$
$$+\frac{P_0}{V}(1-e^{-\delta\Delta r})\frac{M\Delta t}{\rho Cp}$$

For the symmetry axis of the cylinder (*I* = *MP1*), adiabatic axis was assumed and the FDE was

$$-4\gamma T_M{}' + (1+4\gamma)T_{MP1}{}' = T_{MP1} + \frac{P_0}{V}(e^{-2\delta(M-0.5)\Delta r} - e^{-2\delta(M)\Delta r})\frac{4M^2\Delta t}{\rho Cp} \tag{14}$$

A FORTRAN 95 program was prepared to solve the FDEs derived. *M* and Δ*t* were taken as 15 and 0.1 seconds, respectively. The computer programs prepared were checked against the well-known analytical solutions by assuming no microwave heat generation. Excellent agreements were observed. Therefore, these programs were used in predicting the experimental temperature profiles.

3. Results and Discussion

Input data to the numerical model are shown in Table 1. Absorbed power calculated from Eq. (10) using the physical property data in Table 1, heating time of 15 sec and average temperature rises as reported by [5] and the surface powers calculated by using Eq. (9) are

shown in Table 2. It is interesting to note that for samples with identical dimensions (Runs 1 to 3) placed at the same position in the microwave oven, surface powers calculated from Eq.(9) are different for each run and increase as the attenuation constants increase. Since the surface power for a definite geometry and load at a given position in the microwave oven should be independent of the samples heated, this behavior was attributed to the inaccuracies in the attenuation constants, and non-uniformity in the microwave field and lack of conformity to the normal power transmission assumption. The ratio of absorbed power to the surface power was found to vary between 0.34 and 0.53. These values suggest that using the calorimetric method to evaluate the surface power may lead to under estimation of the predicted temperature profiles.

Table 1. Input data used in the numerical model for 2 % agar gels containing different percentages of sucrose.

Run #	% sucrose	R, m	H, m	δ, 1/m	ρ, kg/m^3	Cp, J/kgK	k, W/mK	h, W/m^{2}K	T_a, ^{o}C
1	0[a]	0.030[a]	0.086[a]	38[a]	1000	4184	0.628	39.4[c]	25[d]
2	40[a]	0.030[a]	0.086[a]	67[a]	1176[b]	3091[b]	0.510[b]	39.4[c]	25[d]
3	60[a]	0.030[a]	0.086[a]	78[a]	1286[b]	2544[b]	0.433[b]	39.4[c]	25[d]
4	0[a]	0.035[a]	0.086[a]	38[a]	1000	4184	0.628	39.4[c]	25[d]

[a] Padua (1993)
[b] Calculated from compositions
[c] Lin et al. (1995)
[d] assumed

Table 2. Experimental absorbed powers (Padua, 1993) and calculated surface powers

Run#	$P_{absorbed}$[a] ,W	P_o[b],W	$P_{absorbed}/P_o$
1	298.4	561.2	0.53
2	471.4	1253.8	0.38
3	477.3	1416.9	0.34
4	369.3	755.2	0.49

[a] Padua (1993); calculated from Eq. (10)
[b] Present work ; calculated from Eq. (9)

When the surface powers were calculated from Eq.(9), relatively satisfactory agreement between the experimental center temperatures of [5] and numerical predictions for agar gel solutions containing 0 % sucrose (Run #1 and 4) were observed

(Figs.(1) and (4)). However, the surface temperatures predicted by the numerical model were higher than the experimental temperatures, indicating that the actual surface powers were lower than the surface powers calculated from Eq. (9).

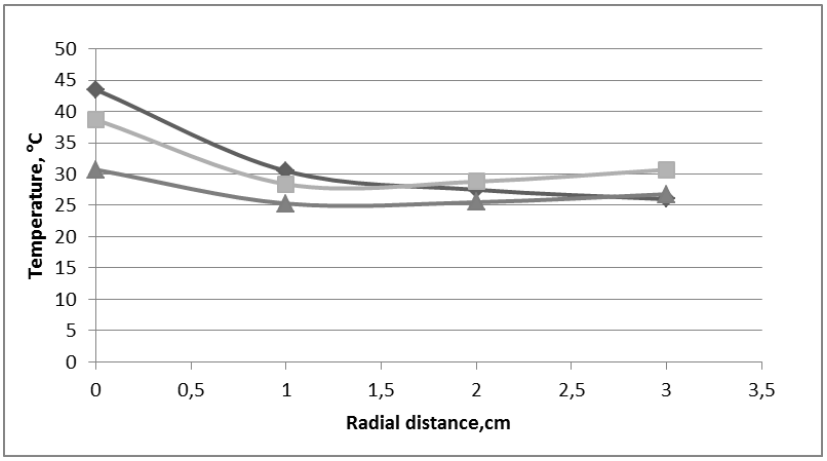


Fig.1. Comparison of the predictions of the present model with the experimental data of Padua (1993) for 2 % agar gel cylinders (R = 0.03 m, H= 0.086 m)

When the surface powers calculated from Eq. (9) were used in the numerical model for agar gels containing 40 % and 60 % sucrose (Runs 2 and 3), very high numerical temperatures were predicted by the numerical model. Therefore, surface power for these two runs, considering that the physical properties for water are well established, were taken to be equal to the surface power predicted for agar gel containing no sucrose (P_0 = 561.2 W). Numerical predictions by the numerical model were in satisfactory agreement with the inner nodes (Figs. 2 and 3). However, the predicted surface temperatures were higher than the experimental temperatures of [5] Conflicting results were observed for the center temperatures.

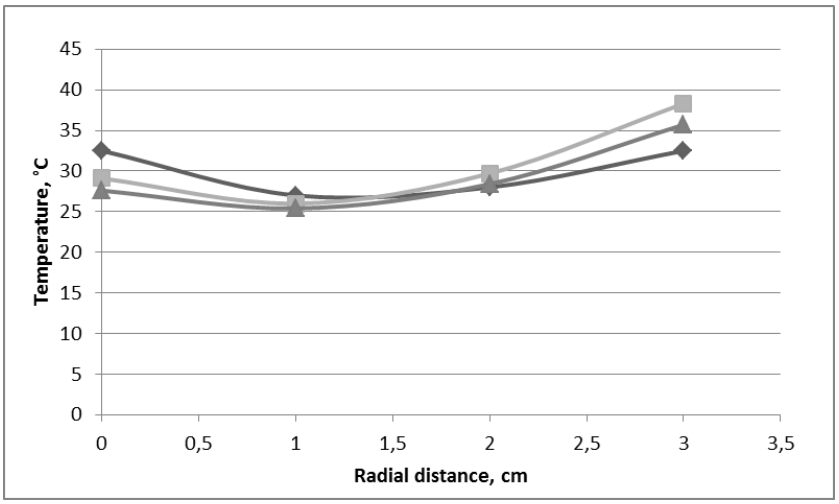
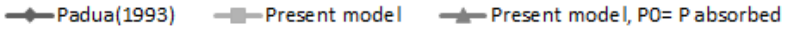


Fig.2. Comparison of the predictions of the present model with the experimental data of Padua (1993) for 40 % sucrose agar gel (R = 0.03 m, H= 0.086 m)



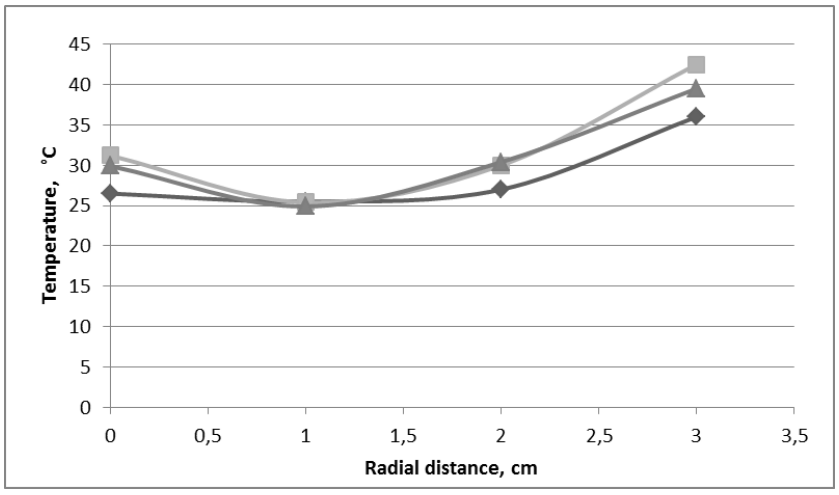
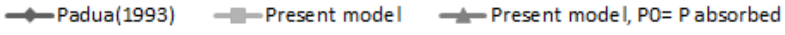


Fig.3. Comparison of the predictions of the present model with the experimental data of Padua (1993) for 60 % sucrose agar gel (R = 0.03 m, H= 0.086 m)



Numerical predictions where surface power P_0 was assumed to be equal to the average power absorbed (Eq.(10)) are also shown in Figs. (1) to 4. The predicted results were in better agreement with experimental temperatures of [5] for some of the data points. However, poor predictions were observed for the center temperatures.

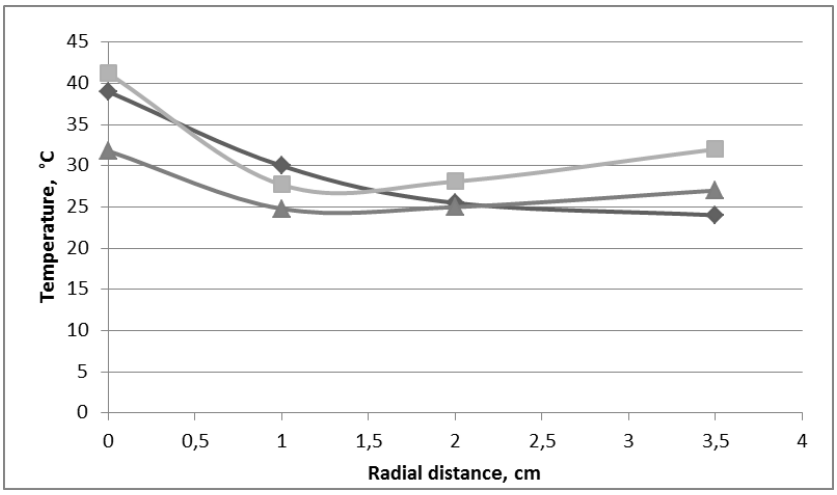


Fig.4. Comparison of the predictions of the present model with the experimental data of Padua (1993) for 2 % agar gel (R = 0.035 m, H= 0.086 m)

Padua(1993) Present model Present model, P0= P absorbed

The level of agreement between the experimental values and numerical predictions was estimated by calculating the percent mean error (% E_m), root mean square error (RMSE) and correlation coefficients which are shown in Equations (15), (16) and (17), respectively [8].

$$\%E_m = \frac{\sum_{i=1}^{N} \frac{V_i - V_e}{V_e} \times 100}{N} \qquad (15)$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(V_i - V_e)^2} \qquad (16)$$

$$r = \frac{N\sum_{i=1}^{N} V_i V_e - \sum_{i=1}^{N} V_i \sum_{i=1}^{N} V_e}{\sqrt{\left(N\sum_{i=1}^{N} V_i^2 - \left(\sum_{i=1}^{N} V_i\right)^2\right)\left(N\sum_{i=1}^{N} V_e^2 - \left(\sum_{i=1}^{N} V_e\right)^2\right)}} \quad (17)$$

In these equations, V_e is the experimental value at the observation, V_i is the predicted value for the observation, N is the number of observations and n is the number of parameters in the model.

The results of the statistical analysis are shown in Table (3) where the surface power P_0 was calculated from Eqn. (9) and in Table (4) where the surface power P_0 was calculated from the average power absorbed.

Table 3. Statistical analysis of the agreement between the predictions of the numerical model and experimental data (P_0 from Eqn. (9)) for 2 % agar gels containing different percentages of sucrose.

Run #	% sucrose	% E_m	RMSE, ^oC	r
1	0[a]	1.2	3.6	0.909
2	40[a]	2.4	3.5	0.674
3	60[a]	11.6	4.3	0.969
4	0[a]	10.4	4.5	0.793

Table 4. Statistical analysis of the agreement between the predictions of the numerical model and experimental data ($P_0 = P_{ab}$) for 2 % agar gels containing different percentages of sucrose.

Run #	% sucrose	% E_m	RMSE, ^oC	r
1	0[a]	-12.7	7.0	0.866
2	40[a]	-2.4	3.0	0.646
3	60[a]	8.2	3.0	0.956
4	0[a]	10.4	4.5	0.795

As may be observed from Tables (3) and (4) mean errors and root mean square errors are relatively small. However, the correlation coefficients are small for runs 2 and 4. Small correlation coefficients were attributed to the uncertainties in surface power, its propagation in the samples, uncertainties in the physical properties used and their temperature dependency.

The present model could not be compared with other literature data for the verification of the proposed expression for surface power since the experimental conditions under which the experimental data have been obtained and the parameters used in the model calculations were not fully reported.

4. Conclusions

The proposed expression for the evaluation of surface power for uniform, normal microwave energy may be used to evaluate the surface power from power absorbed. It may also be used to check the uniformity and normalcy of the incoming microwave. However, a thorough comparison with experimental data is required.

Nomenclature

c_p	specific heat capacity(J/kgK)
dm	mass of the differential element, (kg)
dr	differential distance in the r direction (m)
dV	differential volume element (m^3)
dz	differential distance between in the z direction (m)
% E	percent error
h	surface heat transfer coefficient (W/m^2K)
H	height of cylinder (m)
I	the i^{th} node
k	thermal conductivity (W/mK)
m	mass of the object, kg
M	number of intervals in r direction
MP1	M+1, symmetry axis of the cylinder
n	Number of parameters in the statistical model
N	number of observations
r	distance in radial direction (m)
R	the radius of the cylinder(m)
P	power (W)
P_0	normal surface power (W)
T	temperature (°C)

t time (s)
V volume, (m^3) or value used in statistical analysis
W sample weight, (g)
x normal distance from the surface, (m)
z distance in upward direction (m)

Subscripts

a ambient
ab absorbed
av average
e experimental
i ith observation
m mean
r, z the r and z directions

Greek symbols

A thermal diffusivity (m^2/s)
δ attenuation constant (1/m)
γ $\alpha\Delta t/\Delta r^2$ (dimensionless)
θ direction in cylindrical coordinates
Δr distance between two nodes in the radial direction (m)
Δt the time step (s)
ρ density (kg/m^3)

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