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Douglas-Square Metrics with Vanishing Mean Stretch Curvature

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ABSTRACT

In this paper, we consider the class of square metrics $F = \alpha + 2\beta + \beta^2/\alpha$ where $\alpha = \sqrt{a_{ij}y^iy^j}$ is a Riemannian metric and $\beta = b_i(x)y^i$ is a one-form on a manifold M. Let (M, F) be a Douglas-square manifold. We show that F is a Berwald metric if and only if it a weakly stretch metric. It results that, a Douglas-square metric is R-quadratic if and only if it is a Berwald metric.

Keywords: *Douglas metric; square metric; stretch metric; Berwald metric;*

AMS Subject Classification (2010): *Primary: 53B40; Secondary: 53C60.*

1. Introduction

In 1929, Berwald construct an interesting family of projectively flat Finsler metrics on the unit ball $\mathbb{B}^n$ which as follows

$$F = \frac{\left(\sqrt{(1-|x|^2)|y|^2 + \langle x, y\rangle^2} + \langle x, y\rangle\right)^2}{(1-|x|^2)^2\sqrt{(1-|x|^2)|y|^2 + \langle x, y\rangle^2}}. \tag{1.1}$$

He showed that this class of metrics has constant flag curvature [3]. Berwald's metric can be expressed as

$$F = \frac{(\alpha+\beta)^2}{\alpha}, \tag{1.2}$$

where

$$\alpha = \frac{\sqrt{(1-|x|^2)|y|^2 + \langle x, y\rangle^2}}{(1-|x|^2)^2}, \quad \beta = \frac{\langle x, y\rangle}{(1-|x|^2)^2}.$$

An Finsler metric in the form (1.2) is called a square metric.

Let (M, F) be a Finsler manifold. In local coordinates, a curve $c(t)$ is a geodesic if and only if its coordinates $(c^i(t))$ satisfy $\ddot{c}^i + 2G^i(\dot{c}) = 0$, where the local functions $G^i = G^i(x, y)$ are called the spray coefficients. F is called a Berwald metric, if G^i are quadratic in $y \in T_xM$ for any $x \in M$. In this case, there exists $\Gamma^i_{jk} = \Gamma^i_{jk}(x)$ such that

$$G^i = \frac{1}{2}\Gamma^i_{jk}(x)y^jy^k. \tag{1.3}$$

The Douglas metrics are extension of Berwald metrics, which introduced by Douglas as a projective invariant in Finsler geometry. A Finsler metric is called a Douglas metric if

$$G^i = \frac{1}{2}\Gamma^i_{jk}(x)y^jy^k + P(x, y)y^i, \tag{1.4}$$

where $\Gamma^i_{jk} = \Gamma^i_{jk}(x)$ is a scalar function on M and $P = P(x, y)$ is a homogeneous function of degree one with respect to y on TM_0. Equivalently, a Finsler metric is a Douglas metric if and only if $G^iy^j - G^jy^i$ are

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homogeneous polynomials in (y^i) of degree three [1]. If $P = 0$, then F reduces to a Berwald metric. If $\Gamma = 0$, then F is a projectively flat Finsler metric.

Other than Douglas metrics, there exist some extensions of Berwald metrics. Let (M, F) be a Finsler manifold. There are two basic tensors on Finsler manifolds: fundamental metric tensor $\mathbf{g}_y$ and the Cartan torsion $\mathbf{C}_y$, which are second and third order derivatives of $\frac{1}{2}F_x^2$ at $y \in T_xM_0$, respectively. The rate of change of the Cartan torsion along geodesics, $\mathbf{L}_y$ is said to be Landsberg curvature. Finsler metrics with vanishing Landsberg curvatures are called Landsberg metrics. Every Berwald metric is a Landsberg metric. Taking trace with respect to $\mathbf{g}_y$ in first and second variables of $\mathbf{C}_y$ and $\mathbf{L}_y$ gives rise mean Cartan torsion $\mathbf{I}_y$ and mean Landsberg curvature $\mathbf{J}_y$, respectively. The mean Landsberg curvature is the rate of change of the mean Cartan torsion along geodesics. As a generalization of Landsberg curvature, Berwald introduced the notion of stretch curvature and denoted it by $\mathbf{\Sigma}_y$ [4]. He showed that $\mathbf{\Sigma} = 0$ if and only if the length of a vector remains unchanged under the parallel displacement along an infinitesimal parallelogram. Then, this curvature investigated by Matsumoto in [7]. Taking trace with respect to $\mathbf{g}_y$ in first and second variables of $\mathbf{\Sigma}_y$ gives rise mean stretch curvature $\bar{\mathbf{\Sigma}}_y$ [9][14][18]. A Finsler metric is said to be weakly stretch metric if $\bar{\mathbf{\Sigma}} = 0$. By definition, every weakly Landsberg metric is a weakly stretch metric.

In [19], the first author with Tabatabeifar proved that every Douglas-Randers metric with vanishing stretch curvature is a Berwald metric. In this paper, we consider Douglas-square metrics with vanishing mean stretch curvature and obtain the following.

Theorem 1.1. *Let (M, F) be a Douglas-square manifold of dimension $n \geq 3$. Then F is a Berwald metric if and only if it is a weakly stretch metric.*

A Finsler spaces is said to be R-quadratic if its Riemann curvature $\mathbf{R}_y$ is quadratic in $y \in T_xM$ [2]. By definition, every Berwald metric is R-quadratic. It is proved that every R-quadratic metric is a stretch metric (see [8][10]). Since every stretch metric is a weakly stretch metric, then by Theorem 1.1 we conclude the following.

Corollary 1.1. *Let $F = \alpha + 2\beta + \beta^2/\alpha$ be a Douglas metric on a manifold M of dimension $n \geq 3$. Then F is R-quadratic if and only if it is a Berwald metric.*

There are many connections in Finsler geometry [5][11][12]. Throughout this paper, we use the Cartan connection on Finsler manifolds. The h- and v- covariant derivatives of a Finsler tensor field are denoted by "|" and ", ", respectively.

2. Proof of Theorem 1.1

Let $F = \alpha + 2\beta + \beta^2/\alpha$ be a square metric on an n-dimensional manifold M, where $\alpha = \sqrt{a_{ij}(x)y^i y^j}$ is a Riemannian metric and $\beta(y) = b_i(x)y^i$ is a 1-form on M. Define $b_{i;j}$ by $b_{i;j}\theta^j := db_i - b_j\theta_i^{\ j}$, where $\theta^i := dx^i$ and $\theta_i^{\ j} := \gamma^j_{ik}dx^k$ denote the Levi-Civita connection forms of α. Let

$$r_{ij} := \frac{1}{2}\left(b_{i;j} + b_{j;i}\right), \quad s_{ij} := \frac{1}{2}\left(b_{i;j} - b_{j;i}\right), \quad s^i_{\ j} := a^{ih}s_{hj}, \quad s_j := b^i s_{ij}, \quad r_j := b^i r_{ij},$$
$$s_0 := s_i y^i, \quad r_0 := r_i y^i, \quad r_{j0} := r_{ji}y^i, \quad r_{00} := r_{ij}y^i y^j, \quad s^i_{\ 0} := s^i_{\ j}y^j,$$

where $b^i := a^{ij}b_j$ and $(a^{ij}) = (a_{ij})^{-1}$.

To finding the relation between the Levi-Civita and Cartan connections of α and F, we put the difference tensor $D^i_{jk} := \Gamma^i_{jk} - \gamma^i_{jk}$, where Γ^i_{jk} and γ^i_{jk} denoted the Cheristoffel symbols of the Cartan connection of F and the Levi-Civita connection of α, respectively. The tensor D^j_{ik} is called the difference tensor which is computed by Matsumoto in [7]. Now, let ∇_k be the covariant differentiation by x^k with respect to associated Riemannian connection. Let us put

$$b_{i;j} := \nabla_j b_i := \frac{\partial b_i}{\partial x^j} - b_r\gamma^r_{ij}, \quad r_{ijk} := \nabla_k r_{ij}, \quad s_{ijk} := \nabla_k s_{ij}. \tag{2.1}$$

We recall that the index 0 mean contraction by y^i. For example $r_{0jk} = r_{ijk}y^i$.

For a square metric $F = \alpha + 2\beta + \beta^2/\alpha$, the following holds

$$G^i = G^i_\alpha + \frac{2\alpha}{1-s}s^i_0 + \frac{\theta}{1-s}\Big\{(1-s)r_{00} - 4\alpha s_0\Big\}\Big\{\frac{y^i}{\alpha} + \chi b^i\Big\}, \tag{2.2}$$

where G^i and G^i_α are the spray coefficients of F and α, respectively, and

$$\theta := \frac{1-2s}{-3s^2+2b^2+1}, \qquad \chi := \frac{1}{1-2s}, \quad s := \frac{\beta}{\alpha}$$

and $b^2 := ||\beta||_\alpha = \sqrt{a_{ij}b^ib^j}$ (see [13], [15], [16] and [20]).

In [6], Li-Shen-Shen characterized (α,β)-metrics with vanishing Douglas curvature. In particular, we will use the following.

Lemma 2.1. *Let $F = \alpha + 2\beta + \beta^2/\alpha$ be a square metric on an open subset $U \subset \mathbb{R}^n$ $(n \geq 3)$. Then F is a Douglas metric if and only if*

$$b_{i;j} = 2\tau\Big\{(1+2b^2)a_{ij} - 3b_ib_j\Big\}, \tag{2.3}$$

where $\tau = \tau(x)$ is a scalar function on U.

Put

$$D^i := 2(G^i - G^i_\alpha),$$

By assumption, the square metric $F = \alpha + 2\beta + \beta^2/\alpha$ has vanishing Douglas curvature. Then, by Lemma 2.1, β is a close 1-form, i.e., $s_{ij} = 0$. In this case, by (2.2) we get

$$D^i = 2\theta\left(\frac{y^i}{\alpha} + \chi b^i\right) r_{00}. \tag{2.4}$$

Taking a vertical derivative of (2.4) yields

$$D^i{}_j = By^ib_j + Cy^iy_j - D\delta^i_j - Ey^ir_{0j} + Gb^ib_j - Hb^iy_j + Kb^ir_{0j}, \tag{2.5}$$

where

$$\begin{aligned}
B &:= \frac{-4(2\alpha^2b^2+3\beta^2-3\alpha\beta+\alpha^2)r_{00}}{(2\alpha^2b^2-3\beta^2+\alpha^2)^2},\\
C &:= \frac{2\Big[2\beta(2\alpha^2b^2+3\beta^2-3\alpha\beta+\alpha^2)+(2\beta-\alpha)(2\alpha^2b^2-3\beta^2+\alpha^2)\Big]r_{00}}{\alpha^2(2\alpha^2b^2-3\beta^2+\alpha^2)^2},\\
D &:= \frac{2(2\beta-\alpha)r_{00}}{(2\alpha^2b^2-3\beta^2+\alpha^2)},\\
E &:= \frac{4(2\beta-\alpha)}{(2\alpha^2b^2-3\beta^2+\alpha^2)},\\
G &:= \frac{12\alpha^2\beta r_{00}}{(2\alpha^2b^2-3\beta^2+\alpha^2)^2},\\
H &:= \frac{12\beta^2 r_{00}}{(2\alpha^2b^2-3\beta^2+\alpha^2)^2},\\
K &:= \frac{4\alpha^2}{(2\alpha^2b^2-3\beta^2+\alpha^2)}.
\end{aligned}$$

By contracting (2.5) with b_i, we have

$$b_iD^i{}_j = (B\beta - D + Gb^2)b_j + (C\beta - Hb^2)y_j + (Kb^2 - E\beta)r_{0j}. \tag{2.6}$$

Putting $b_{i|j} = b_{i;j} - b_rD^r_{ij}$ in (2.6) yields

$$b_{i|0} = -(B\beta - D + Gb^2)b_i - (C\beta - Hb^2)y_i + (1 + E\beta - Kb^2)r_{0i}. \tag{2.7}$$

Multiplying (2.7) with y^i implies that

$$b_{0|0} = -(B\beta - D + Gb^2)\beta - (C\beta - Hb^2)\alpha^2 + (1 + E\beta - Kb^2)r_{00}. \tag{2.8}$$

On the other hand, the mean Cartan torsion of a square metric $F = \alpha + 2\beta + \beta^2/\alpha$ is given by

$$I_i := -\frac{1}{2\alpha} A \left(b_i - \alpha^{-2} \beta y_i \right), \tag{2.9}$$

where

$$A := \frac{2(n-2)s}{1-s^2} - \frac{2(n+1)}{1+s} + \frac{6s}{1-3s^2+2b^2}.$$

By (2.9) we have

$$J_i = \frac{1}{2\alpha^4} \left(A\alpha_{|0} - A_{|0}\alpha \right) \left(\alpha^2 b_i - \beta y_i \right) - \frac{A}{2\alpha} b_{i|0} + \frac{A}{2\alpha^4} (\alpha b_{0|0} - 2\beta\alpha_{|0}) y_i + \frac{A\beta}{2\alpha^3} y_{i|0}. \tag{2.10}$$

By putting (2.7) and (2.8) in (2.10), one can get the following

$$J_i = X b_i + Y y_i - W r_{0i}, \tag{2.11}$$

where X, Y and W are listed in Appendix. By taking a horizontal derivation of (2.11) along a geodesic and contracting the result with b^i, we get

$$b^i J_{i|0} = \gamma \Big(b_1 r_{00|0} + b_2 r_0 r_{00} + b_3 r_{00}^2 + b_4 b^i r_{0i|0} + b_5 r_0 + b_6 r_0^2 + b_7 r_{0|0} + b_8 r_{00} \Big), \tag{2.12}$$

where b_i $(1 \le i \le 8)$ are listed in Appendix, and

$$\gamma := \frac{1}{\alpha^2 \left(\alpha^2 - \beta^2 \right)^2 \left(2\alpha^2 b^2 + \alpha^2 - 3\beta^2 \right)^5}.$$

By some computations, we get

$$r_{0|0} = b^i r_{i0|0}, \tag{2.13}$$

$$r_{i0|0} = r_{i00} - r_{im} D^m_{00} - r_{m0} D^m_{i0}, \tag{2.14}$$

$$\begin{aligned} r_{00|0} \;=\; & \mathfrak{A} \Big(4\alpha^5 b^2 r_{00}^2 + 2\alpha^5 r_{00}^2 + 4\alpha^4 b^4 r_{000} - 8\alpha^4 b^2 \beta r_{00} - 8\alpha^4 b^2 r_0 r_{00} - 6 r_{00}^2 \alpha^3 \\ & - 18\alpha^3 \beta^2 r_{00}^2 + 24\alpha^2 \beta^3 r_{00}^2 + 4\alpha^4 b^2 r_{000} - 4\alpha^4 \beta r_{00} - 4\alpha^4 r_0 r_{00} - 12\alpha^3 b^2 r_{00}^2 \\ & - 12\, \alpha^3\, \beta^2 r_{00} - 6\alpha^2 \beta^2 r_{000} - 12\alpha^2 b^2 \beta^2 r_{000} + 20\alpha^2 b^2 \beta r_{00}^2 + 12\alpha^2 \beta^3 r_{00} \\ & + \alpha^4 r_{000} + 12\alpha^2 \beta^2 r_0 r_{00} + 9\beta^4 r_{000} + 10\alpha^2 \beta r_{00}^2 \\ & + 18\alpha\beta^2 r_{00}^2 - 30\beta^3 r_{00}^2 \Big), \end{aligned} \tag{2.15}$$

where

$$\mathfrak{A} := \frac{1}{(2b^2\alpha^2 + \alpha^2 - 3\beta^2)^2}$$

Thus, we have

$$\begin{aligned} b^i r_{i0|0} = b^i r_{i00} + \mathfrak{A} \Big(& 8\alpha^4 b^4 r_{00}^2 - 12\alpha^4 b^2 \beta r_0 r_{00} + 4\alpha^4 b^4 b^i r_{i00} - 4\alpha^4\, b^2 r_0^2 - 4\alpha^4 b^2 r_0 r_{00} \\ & + 4\alpha^4 b^2 r_{00}^2 - 8\alpha^3 b^2 \beta r_{00}^2 + 4\alpha^2 b^2 \beta^2 r_{00}^2 + 12\beta^3 r_{00} \alpha^2 r_0 - 12\alpha^2 b^2 \beta^2 b^i r_{i00} \\ & - 4\alpha^2 \beta^2\, r_{00}^2 - 6\alpha\beta^3 r_{00}^2 + 4\alpha^4\, b^2 b^i r_{i00} - 2\alpha^4\, r_0^2 - 2\alpha^4\, r_0 r_{00} - 4\alpha^3 b^2 r_0 r_{00} \\ & - 2\alpha^3 r_0 r_{00} - 8\alpha^2 b^2 \beta^2 r_{00} + 4\alpha^2 b^2 \beta r_0 r_{00} + 6\alpha^2 \beta^2 r_0^2 - 4\alpha^2 \beta^2 r_{00} \\ & + 6\alpha\beta^2 r_0 r_{00} + 9\beta^4 b^i r_{i00} - 6\alpha^2 \beta^2 b^i r_{i00} + 6\alpha^2 \beta^2 r_0 r_{00} + 2\alpha^2 \beta r_0 r_{00} \\ & + \alpha^4 b^i r_{i00} - 12\alpha\beta^3 r_{00} + 12\beta^4 r_{00}^2 + 2\alpha^3 \beta\, r_{00}^2 + 12\beta^4 r_{00} - 6\beta^3 r_0 r_{00} \Big). \end{aligned} \tag{2.16}$$

By definition of the Cartan connection, we have $g_{ij|s} = 0$ and then $g^{ij}{}_{|s} = 0$. Therefore, we have

$$\bar{\Sigma}_{kl} = g^{ij} \Sigma_{ijkl} = g^{ij} (L_{ijk|l} - L_{ijl|k}) = J_{k|l} - J_{l|k}.$$

By assumption, F is a weakly stretch metric $\bar{\Sigma} = 0$. Thus $J_{i|k} = J_{k|i}$. Contracting it with y^k yields $J_{i|k}y^k = 0$ or $J_{i|0} = 0$. This equation is equivalent to that for any linearly parallel vector field u along a geodesic c, the following holds:

$$\frac{d}{dt}\Big[\mathbf{J}_{\dot{c}}(u)\Big] = 0.$$

The geometric meaning of this is that the rate of change of the mean Landsberg curvature is constant along any Finslerian geodesic.

Since $J_{i|0} = 0$, then $b^i J_{i|0} = 0$. Then by putting (2.13), (2.14), (2.15) and (2.16) in (2.12), we get

$$\gamma\Big(A_{16}\alpha^{16} + \cdots + A_1\alpha + A_0\Big) = 0. \tag{2.17}$$

where A_i $(0 \leq i \leq 16)$ are given in the Appendix. By (2.17) we get

$$A_{16}\alpha^{16} + A_{14}\alpha^{14} + \cdots + A_2\alpha^2 + A_0 = 0, \tag{2.18}$$
$$A_{15}\alpha^{14} + A_{13}\alpha^{12} + \cdots + A_3\alpha^2 + A_1 = 0. \tag{2.19}$$

By (2.18) and (2.19) we get

$$3(3n+10)\beta r_{000} + (97n+350)r_{00}^2 = f\alpha^2, \tag{2.20}$$
$$3(4n-10)\beta r_{000} - (353n+759)r_{00}^2 = g\alpha^2, \tag{2.21}$$

where $f = f(x,y)$ and $g = g(x,y)$ are homogeneous functions of degree 2 with respect to y on TM. (2.21) implies that

$$r_{000} = h\,\alpha^2 + k\,r_{00}^2, \tag{2.22}$$

where $h = h(x,y)$ and $k = k(x,y)$ are homogeneous functions of degree 1 with respect to y on TM. Putting (2.22) in (2.20) yields

$$r_{00} = t\alpha^2, \tag{2.23}$$

where $t = t(x,y)$ is a homogeneous function of degree 0 with respect to y on TM.

By Lemma 2.1, the following holds

$$b_{i;j} = 2\tau\Big\{(1+2b^2)a_{ij} - 3b_ib_j\Big\}, \tag{2.24}$$

where $\tau = \tau(x)$ is a scalar function. We claim that $\tau = 0$. On contrary, suppose that $\tau \neq 0$. Thus by (2.24), we get

$$r_{ij} = 2\tau\Big\{(1+2b^2)a_{ij} - 3b_ib_j\Big\}. \tag{2.25}$$

which yields

$$r_{00} = 2\tau\Big\{(1+2b^2)\alpha^2 - 3\beta^2\Big\}. \tag{2.26}$$

By (2.23) and (2.26), we get

$$\alpha^2 = \left[\frac{6\tau}{2\tau(1+2b^2) - t}\right]\beta^2.$$

This contradicts with the positive-definiteness of α. Then $\tau = 0$. By considering (2.24), it follows that β is parallel with respect to α. Therefore, F reduces to a Berwald metric. $\Box$

3. Appendix

$$\begin{aligned}
X \;:=\; & \tilde{X}\big(4n\,b^4\beta\,r_{00}\alpha^9 - 8nb^4\,\beta^2 r_{00}\alpha^8 - 4nb^4\beta^3 r_{00}\alpha^7 + 4n\,b^4\beta^4 r_{00}\alpha^6 + r_{00}\alpha^8 + 4b^4\beta\,r_{00}\alpha^9 - 2nb^2\beta r_{00}\alpha^9 + 4b^4\beta^2 r_{00}\alpha^8 \\
& -4b^4\beta^3 r_{00}\alpha^7 - 40nb^2\beta^3 r_{00}\alpha^7 + 4b^4\beta^4 r_{00}\alpha^6 + 22nb^2\beta^4 r_{00}\alpha^6 + 30nb^2\beta^5 r_{00}\alpha^5 - 18nb^2\beta^6 r_{00}\alpha^4 - 2b^2\beta r_{00}\alpha^9 - 2n\beta r_{00}\alpha^9 \\
& +4b^2\beta^2 r_{00}\alpha^8 + 10n\beta^2 r_{00}\alpha^8 - 8nb^4\beta r_{00}\alpha^7 - 4b^2\beta^3 r_{00}\alpha^7 - 10n\beta^3 r_{00}\alpha^7 + 8nb^4\beta^2 r_{00}\alpha^6 - 20b^2\beta^4 r_{00}\alpha^6 - 26n\beta^4 r_{00}\alpha^6 \\
& +30b^2\beta^5 r_{00}\alpha^5 + 60n\beta^5 r_{00}\alpha^5 - 8nb^4\beta^4 r_{00}\alpha^4 - 24b^2\beta^6 r_{00}\alpha^4 - 18n\beta^6 r_{00}\alpha^4 - 36n\beta^7 r_{00}\alpha^3 + 18n\beta^8 r_{00}\alpha^2 - 2\beta r_{00}\alpha^9 \\
& -12b^2\beta r_0\alpha^8 + 4nb^2 r_{00}\alpha^8 + 10\beta^2 r_{00}\alpha^8 - 8b^4\beta r_{00}\alpha^7 - 8nb^2\beta r_{00}\alpha^7 - 10\beta^3 r_{00}\alpha^7 - 4b^4\beta^2 r_{00}\alpha^6 + 24b^2\beta^3 r_0\alpha^6 \\
& -20\beta^4 r_{00}\alpha^6 + 8b^4\beta^3 r_{00}\alpha^5 - 9\beta^7 r_{00}\alpha + 14nb^2\beta^3 r_{00}\alpha^5 + 42\beta^5 r_{00}\alpha^5 - 2b^4\beta^4 r_{00}\alpha^4 - 12b^2\beta^5 r_0\alpha^4 - 26nb^2\beta^4 r_{00}\alpha^4 \\
& -12nb^2\beta^5 r_{00}\alpha^3 - 54\beta^7 r_{00}\alpha^3 + 18nb^2\beta^6 r_{00}\alpha^2 + 36\beta^8 r_{00}\alpha^2 + b^2 r_{00}\alpha^8 - 6\beta r_0\alpha^8 + n r_{00}\alpha^8 - 8b^2\beta r_{00}\alpha^7 - 2n\beta r_{00}\alpha^7 \\
& +30\beta^3 r_0\alpha^6 - n\beta^2 r_{00}\alpha^6 + 32b^2\beta^3 r_{00}\alpha^5 + 5n\beta^3 r_{00}\alpha^5 - 5b^2\beta^4 r_{00}\alpha^4 - 42\beta^5 r_0\alpha^4 - 11n\beta^4 r_{00}\alpha^4 - 12b^2\beta^5 r_{00}\alpha^3 \\
& +3b^2\beta^6 r_{00}\alpha^2 + 18\beta^7 r_0\alpha^2 + 18n\beta^6 r_{00}\alpha^2 - 9n\beta^8 r_{00} - 2\beta r_{00}\alpha^7 + 8\beta^2 r_{00}\alpha^6 + 5\beta^3 r_{00}\alpha^5 - 29\beta^4 r_{00}\alpha^4 - 6\beta^5 r_{00}\alpha^3 \\
& +24\beta^6 r_{00}\alpha^2 + 3n\beta^5 r_{00}\alpha^3 + 5b^2\beta^2 r_{00}\alpha^6 + 6\beta^6 r_{00}\alpha^4 + 2nb^2\beta^2 r_{00}\alpha^6 - 2b^4 r_{00}\alpha^8 + 8nb^4\beta^3 r_{00}\alpha^5 + 4nb^4 r_{00}\alpha^8 + 16nb^2\beta^2 r_{00}\big),
\end{aligned}$$

$$
\begin{aligned}
Y \;:=\;& \tilde{Y}\Big(8b^4\beta n r_{00}\alpha^{10} - 8nb^4\beta^2 r_{00}\alpha^9 - 16b^4\beta^5\alpha^6 + 8nb^4\beta^5 r_{00}\alpha^6 + 8b^4\beta\, r_{00}\alpha^{10} + 8nb^2\beta\, r_{00}\alpha^8 - 16n\, b^4\beta^2\alpha^5 + 16b^4\beta^2 r_{00}\alpha^9 \\
& -8nb^4\beta r_{00}\alpha^9 - 20nb^2\beta^2 r_{00}\alpha^9 + 24nb^4\beta^2 r_{00}\alpha^8 - \alpha^9 r_{00} - 24n\alpha^8 b^2 r_{00}\beta^3 + 16nb^4\beta^4\alpha^7 - 8nb^4\beta^3 r_{00}\alpha^7 + 72nb^2\beta^4 r_{00}\alpha^7 \\
& +8b^4\beta^5 r_{00}\alpha^6 - 16nb^4\beta^4 r_{00}\alpha^6 - 4nb^2\beta^5 r_{00}\alpha^6 - 36nb^2\beta^6 r_{00}\alpha^5 + 12nb^2\beta^7 r_{00}\alpha^4 + 8b^2\beta r_{00}\alpha^{10} + 2n\beta r_{00}\alpha^{10} - 16b^4\beta^2\alpha^9 \\
& +8nb^4\beta^5 r_{00}\alpha^5 - 4nb^4 r_{00}\alpha^9 - 16nb^2\beta^2\alpha^9 - 8b^2 r_{00}\beta^2\alpha^9 - 8nb^2\beta r_{00}\alpha^9 - 8n\beta^2 r_{00}\alpha^9 - 16b^4\beta^3\alpha^8 + 8nb^2\beta^3\alpha^8 \\
& +24nb^2\beta^2 r_{00}\alpha^8 - 6n\beta^3 r_{00}\alpha^9 + 16b^4\beta^4\alpha^7 + 16nb^4\beta^3 r_{00}\alpha^7 + 48nb^4\beta^2 r_{00}\alpha^7 + 112nb^2\beta^4\alpha^7 + 24b^2\beta^4 r_{00}\alpha^7 + 28nb^2\beta^3 r_{00}\alpha^7 \\
& +54n\beta^4 r_{00}\alpha^7 - 16b^4\beta^4 r_{00}\alpha^6 - 28nb^4\beta^3 r_{00}\alpha^6 - 88nb^2\beta^5\alpha^6 + 56b^2\beta^5 r_{00}\alpha^6 - 124nb^2\beta^4 r_{00}\alpha^6 - 22n\beta^5 r_{00}\alpha^6 + 8b^4\beta^5 r_{00}\alpha^5 \\
& -28nb^4\beta^4 r_{00}\alpha^5 - 48b^2\beta^6 r_{00}\alpha^5 + 44nb^2\beta^5 r_{00}\alpha^5 - 18n\beta^6 r_{00}\alpha^5 + b^4\beta^5 r_{00}\alpha^4 + 48nb^2\beta^7\alpha^4 + 72nb^2\beta^6 r_{00}\alpha^4 + 78n\beta^7 r_{00}\alpha^4 \\
& +2\beta r_{00}\alpha^{10} - 36nb^2\beta^7 r_{00}\alpha^3 + 54n\beta^8 r_{00}\alpha^3 - 36n\beta^9 r_{00}\alpha^2 - 4b^4 r_{00}\alpha^9 - 16b^2\beta^2\alpha^9 - 8b^2\beta r_{00}\alpha^9 - 4nb^2 r_{00}\alpha^9 - 4n\beta^2\alpha^9 \\
& -8\beta^2 r_{00}\alpha^9 - 2n\beta r_{00}\alpha^9 - 24b^4\beta r_{00}\alpha^8 - 16b^2\beta^3\alpha^8 - 24b^2\beta^2 r_0\alpha^8 - 6\beta^3 r_{00}\alpha^8 + 6n\beta^2 r_{00}\alpha^8 + 40b^2\beta^4\alpha^7 + 40b^2\beta^3 r_{00}\alpha^7 \\
& +52n\beta^4\alpha^7 + 48\beta^4 r_{00}\alpha^7 + 16n\beta^3 r_{00}\alpha^7 + 32b^4\beta^3 r_{00}\alpha^6 + 8b^2\beta^5\alpha^6 + 48b^2\beta^4 r_0\alpha^6 - 40b^2\beta^4 r_{00}\alpha^6 - 28nb^2\beta^3 r_{00}\alpha^6 - 4n\beta^5\alpha^6 \\
& -58n\,\beta^4 r_{00}\alpha^6 - 28\,b^4 r_{00}\beta^4\alpha^5 - 72b^2\beta^6\alpha^5 - 40b^4\beta^5 r_{00}\alpha^5 - 172nb^2\,\beta^4 r_{00}\alpha^5 - 144n\,\beta^6\alpha^5 - 96\beta^6 r_{00}\alpha^5 + 24b^4\beta^5 r_{00}\alpha^4 \\
& -24b^2\beta^6 r_0\alpha^4 + 84b^2\beta^6 r_{00}\alpha^4 + 96nb^2\beta^5 r_{00}\alpha^4 + 54\beta^7 r_{00}\alpha^4 + 144n\beta^6 r_{00}\alpha^4 - 48b^2\beta^7 r_{00}\alpha^3 + 84nb^2\beta^6 r_{00}\alpha^3 + 72n\beta^8\alpha^3 \\
& -36n\beta^9\alpha^2 - 72\beta^9 r_{00}\alpha^2 - 72n\beta^8 r_{00}\alpha^2 + 36\beta^9 r_{00}\alpha - 4\alpha^9 b^2 r_{00} - 4\beta^2\alpha^9 - 2\beta r_{00}\alpha^9 - nr_{00}\alpha^9 - 12b2\beta r_{00}\alpha^8 - 4\beta^3\alpha^8 \\
& -36nb^2\beta^7 r_{00}\alpha^2 + 6\beta^2 r_{00}\alpha^8 + 36b^2\beta^2 r_{00}\alpha^7 + 52\beta^4\alpha^7 + 16\beta^3 r_{00}\alpha^7 + 18n\beta^2 r_{00}\alpha^7 + 44b^2\beta^3 r_{00}\alpha^6 + 8\beta^5\alpha^6 + 60\beta^4 r_0\alpha^6 \\
& -7n\beta^3 r_{00}\alpha^6 - 76b^2 r_{00}\beta^4\alpha^5 - 108\beta^6\alpha^5 - 22\beta^5 r_{00}\alpha^5 - 88n\beta^4 r_{00}\alpha^5 + 12b^2\beta^5 r_{00}\alpha^4 + 36\beta^7\alpha^4 - 84\beta^6 r_0\alpha^4 + 114\beta^6 r_{00}\alpha^4 \\
& +45n\,\beta^5 r_{00}\alpha^4 + 108\,b^2\beta^6 r_{00}\alpha^3 - 24\,\beta^7 r_{00}\alpha^3 + 150n\,\beta^6 r_{00}\alpha^3 - 108b^2\beta^7 r_{00}\alpha^2 - 72\beta^9\alpha^2 + 36\beta^8 r_0\alpha^2 - 108\beta^8\, r_{00}\alpha^2 \\
& +72\beta^9 r_{00}\alpha - 63n\beta^8 r_{00}\alpha + 27n\beta^9 r_{00} + 18\beta^2 r_{00}\alpha^7 + 14\beta^3 r_{00}\alpha^6 - 76\beta^4 r_{00}\alpha^5 + 72\beta^8 r_{00}\alpha^3 - 18\beta^5 r_{00}\alpha^4 + 126\beta^6 r_{00}\alpha^3 \\
& -99\beta^8 r_{00}\alpha + 90\beta^9 r_{00} - 16n\beta^5 r_{00}\alpha^5 + 60n\beta^7\alpha^4 - 48b^2\beta^3 r_{00}\alpha^8 - 8b^4\beta r_{00}\alpha^9 - 16nb^4\beta^5\alpha^6 + 32nb^4\beta^3\alpha^8 + 60nb^2\beta^2 r_{00}\alpha^7 \\
& +12b_2\beta^2 r_{00}\alpha^8 - 4n\beta^3\alpha^8 - 72nb^2\beta^6\alpha^5 - 54n\beta^7 r_{00}\alpha^3 + 72b^2\beta^7\alpha^4 - 10\beta^5 r_{00}\alpha^6 + 108\beta^8\alpha^3 - 52\beta^4 r_{00}\alpha^6 - 12\beta^2 r_0\alpha^8 \\
& -81n\,\beta^7 r_{00}\alpha^2 - 54\beta^7 r_{00}\alpha^2\Big),
\end{aligned}
$$

$$
\begin{aligned}
W \;:=\;& \tilde{W}\Big(2nb^2\alpha^5 - 4nb^2\beta\alpha^4 - 2nb^2\beta^2\alpha^3 - 6\beta^5 + 2nb^2\beta^3\alpha^2 + 2b^2\alpha^5 + n\alpha^5 + 2b^2\beta\alpha^4 - 2n\beta\alpha^4 - 2b^2\beta^2\alpha^3 - 4n\beta^2\alpha^3 \\
& -4\beta^2\alpha^3 + 2b^2\beta^3\alpha^2 + 7n\beta^3\alpha^2 + 3n\beta^4\alpha - 3n\beta^5 + \alpha^5 - 2\beta\alpha^4 + 4\beta^3\alpha^2 + 3\beta^4\alpha\Big),
\end{aligned}
$$

$$
\begin{aligned}
b_1 \;:=\;& 32nb^{10}\beta\alpha^{15} - 64nb^{10}\beta^2\alpha^{14} - 32nb^{10}\beta^3\alpha^{13} + 32b^{10}\beta^4\alpha^{12} + 32b^{10}\beta\alpha^{15} + \beta\alpha^{13} + 16nb^8\beta\alpha^{15} + 32b^{10}\beta^2\alpha^{14} + 32nb^8\beta^2\alpha^{14} \\
& -416nb^8\beta^3\alpha^{13} + 192b^6\beta^3\alpha^{13} + 32b^{10}\beta^4\alpha^{12} + 496nb^8\beta^4\alpha^{12} + 336nb^8\beta^5\alpha^{11} - 64nb^{10}\beta\alpha^{13} + 32nb^{10}\alpha^{14} - 224b^8\beta^3\alpha^{13} \\
& +32b^8\beta^2\alpha^{14} - 272nb^8\beta^6\alpha^{10} - 44\beta^6\alpha^{10} - 176b^2\beta^4\alpha^{11} - 134nb^2\beta^3\alpha^{11} - 96\beta^5\alpha^{11} - 40b^4\beta^3\alpha^{11} - 28n\beta^4\alpha^{11} + 64nb^2\beta^4\alpha^{10} \\
& -1028b^4\beta^4\alpha^{10} + 392b^2\beta^5\alpha^{10} + 94n\beta^5\alpha^{10} + 18\beta^4\alpha^{12} - 6n\beta^3\alpha^{12} + 632b^2\beta^6\alpha^9 + 1046b^2\beta^5\alpha^9 + 384b^4\beta^5\alpha^9 + \beta^7\alpha^8 \\
& -1224b^4\beta^7\alpha^7 - 720b^2\beta^8\alpha^7 - 3258nb^2\beta^7\alpha^7 - 60\beta^8\alpha^8 + 32nb^8\beta^2\alpha^{13} + b^2\beta^4\alpha^{12} + 128nb^6\beta^2\alpha^{14} - 24nb^6\beta\alpha^{15} + 64nb^{10}\beta^3\alpha^{11} \\
& +336b^8\beta^5\alpha^{11} + 32nb^8\beta^4\alpha^{11} + 1416nb^6\beta^5\alpha^{11} - 64nb^{10}\beta^4\alpha^{10} - 320b^8\beta^6\alpha^{10} - 128nb^8\beta^2\alpha^{13} - 24nb^6\beta^4\alpha^{12} - 96nb^8\beta^3\alpha^{12} \\
& -224b^8\beta^4\alpha^{12} - 20nb^4\beta\alpha^{15} - 112nb^8\beta\alpha^{13} + 64nb^6\beta^2\alpha^{13} - 32nb^8\beta^4\alpha^{10} - 16b^{10}\beta^4\alpha^{10} + b^8\beta^5\alpha^{10} - 168b^6\beta^4\alpha^{12} + 888b^6\beta^5\alpha^{11} \\
& -308nb^4\beta^4\,\alpha^{12} - 176nb^6\beta^4\alpha^{11} - 64b^8\beta^4\alpha^{11} + 432nb^4\beta^5\alpha^{11} + 176\,nb^8\beta^3\alpha^{11} + 64b^{10}\beta^3\alpha^{11} - 192nb^6\beta^3\alpha^{12} + 32b^8\beta^2\alpha^{13} \\
& -64nb^6\beta\,\alpha^{13} + 32nb^4\beta^3\,\alpha^{13} - 14n\,b^4\beta^3\alpha^{12} - 24b^8\beta^6\alpha^8 + 360b^4\beta^5\alpha^{11} - 300nb^2\,\beta^5\alpha^{11} + 512b^8\beta^3\alpha^{11} - 320nb^6\beta^4\alpha^{11} \\
& -312n\,b^4\beta^4\,\alpha^{11} - 88\,b^8\beta^4\alpha^{10} + 224b^6\beta^5\,\alpha^{10} - 160nb^6\beta^4\alpha^{10} - 48b^4\,\beta^6\alpha^{10} - 1152b^4\beta^7\alpha^9 + 240nb^4\beta^6\alpha^9 + 756nb^2\,\beta^7\alpha^9 \\
& +384nb^6\beta^6\alpha^8 + 1176nb^4\beta^5\alpha^{10} + 260nb^2\beta^6\alpha^{10} - 176b^8\beta^5\alpha^9 + 320b^6\beta^6\alpha^9 + 728n\,b^6\beta^5\alpha^9 - 2664nb^4\beta^7\alpha^8 + 576b^4\beta^8\alpha^8 \\
& -192b^2\beta^5\alpha^{11} - 164nb^2\beta^4\alpha^{11} + 1296nb^4\beta^9\alpha^6 + 648nb^2\beta^{10}\alpha^6 - 648nb^4\beta^{10}\alpha^5 - 864nb^2\beta^9\alpha^7 - 360nb^6\beta^8\alpha^6 - 26nb^2\beta^{12}\alpha^4 \\
& -4b^4\beta\alpha^{15} + 12b^6\alpha^{14} + 16nb^4\alpha^{14} + 4b^2\beta^2\alpha^{14} - 2n\beta^2\alpha^{14} - 64b^6\beta\alpha^{13} + 48b^4\beta^2\alpha^{13} - 8nb^4\beta\alpha^{13} + 44b^2\beta^3\alpha^{13} + 16nb^2\beta^2\alpha^{13} \\
& -1080b^4\beta^{10}\alpha^6 - 102n\beta^5\,\alpha^{11} - 976b2^3\beta^4\alpha^{10} + 528b^4\beta^5\alpha^{10} + 100nb^4\beta^4\alpha^{10} + 572nb^2\beta^5\alpha^{10} - 32n\,\beta^6\alpha^{10} - 736\,b^6\beta^5\alpha^9 \\
& +452nb^2\beta^6\alpha^9 - 288nb^4\beta^6\alpha^8 - 2304nb^2\beta^7\alpha^8 + 72b^2\beta^8\alpha^8 - 156n\beta^8\alpha^8 - 8b^8\alpha^{14} + 88b^4\beta^6\alpha^9 - 16b^2\beta^6\alpha^{10} + 1680nb^4\beta^5\alpha^9 \\
& +528b^6\beta^6\alpha^8 - 1368b^4\beta^7\alpha^8 - 14nb^2\beta^2\alpha^{12} - 408n\,b^6\beta^7\alpha^7 - 288b^4\beta^8\alpha^7 - 540b^2\beta^9\alpha^7 + 72nb^2\beta^8\alpha^7 - 1026b^2\beta^8\alpha^6 \\
& +1296b^2\beta^{12}\alpha^3 - 1512nb2\beta^{11}\alpha^3 - 648\beta^{13}\alpha^3 + 702n\beta^{12}\alpha^3 + n\beta\alpha^{13} + 648\beta^{14}\alpha^2 + 486nb^2\beta^{12}\alpha^2 + 648n\beta^{13}\alpha^2 - 324n\beta^{14}\alpha \\
& +4b^2\beta\alpha^{13} + 2\beta^2\,\alpha^{13} + 16b^2\beta^2\alpha^{12} - 6\beta^3\alpha^{12} - 10b^2\beta^3\alpha^{11} + 1260\beta^9\alpha^6 - 28\beta^4\alpha^{11} - 24\beta^3\alpha^{11} - 308b^2\beta^4\alpha^{10} + 88\,\beta^5\alpha^{10} \\
& +205n\beta^5\alpha^9 + 7n\beta^4\alpha^{10} + 740b^2\beta^5\alpha^9 + 110b^2\beta^6\alpha^8 - 252\beta^8\alpha^7 - 840n\beta^7\alpha^7 + 48nb^6\beta^5\alpha^{10} + 60nb^4\beta^6\alpha^{10} - 32b^8\beta^6\alpha^9 \\
& -102\alpha^9 b^6\beta^7\alpha^9 - 304nb^6\beta^6\alpha^9 + 1032b^6\beta^8\alpha^8 - 1296nb^4\beta^7\alpha^9 + 2nb^8\beta^6\alpha^8 - 480nb^6\beta^7\alpha^8 + 104nb^4\beta^8\alpha^8 + 240nb^6\beta^8\alpha^7 \\
& -540nb^4\beta^{10}\alpha^6 - 20b^4\beta\alpha^{15} - 4nb^2\beta\alpha^{15} + 48nb^6\alpha^{14} + 40b^4\beta^2\alpha^{14} + 4nb^2\beta^2\alpha^{14} - 112b^8\beta\alpha^{13} + 414n\beta^8\alpha^6 + 3240b^2\beta^9\alpha^5 \\
& -18\beta^{10}\alpha^5 + 1755n\,beta^9\alpha^5 - 1332\,b^2\beta^{10}\alpha^4 - 1674\beta^{11}\alpha^4 - 2322\,b^2\beta^{11}\,\alpha^3 + 648\beta^{12}\,\alpha^3 - 1728n\,\beta^{11}\alpha^3 + 2052b^2\,\beta^{12}\alpha^2 \\
& -324\beta^{13}\alpha^2 + 891n\,\beta^{12}\alpha^2 - 648\beta^{14}\alpha + 567n\,\beta^{13}\alpha - 24\beta^3\alpha^{11} - 14\beta^4\alpha^{10} + 193\beta^5\,\alpha^9 - 744\beta^7\alpha^7 + 102\,\beta^6\alpha^8 - 180\beta^8\alpha^6 \\
& +891\beta^{13}\alpha - 252\beta^{10}\alpha^4 - 810\beta^{14} - 1728\beta^{11}\alpha^3 + 1026\beta^{12}\alpha^2, +4nb^8\alpha^{14} + 64nb^{10}\beta^2\alpha^{12} - 64b^{10}\beta\alpha^{13} + 1539\,\beta^9\alpha^5 \\
& +1980b^4\beta^6\alpha^8 + 16b^8\beta\alpha^{15} - 296nb^6\beta^3\alpha^{13} - 32b^{10}\beta^3\alpha^{13} + 108b^4\beta^8\alpha^7 + 34nb^2\beta^{11}\alpha^5 - 612n\,b^2\beta^8\alpha^8 - 528\,b^6\beta^7\alpha^8 \\
& -24\,nb^6\beta^3\alpha^{11} + 64\,b^6\beta^2\alpha^{13} + 2b^2\alpha^{14} - 3276n\,b^4\beta^7\alpha^7 + 504\beta^7\alpha^9 + 432b^2\beta^7\alpha^9 + 972\beta^{13}\alpha^2 - 918\beta^{10}\alpha^4 + 76nb^4\beta^9\alpha^7 \\
& -176nb^8\beta^5\alpha^9 + 136\beta^6\alpha^9
\end{aligned}
$$

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\begin{aligned}
b_2 \ :=\ & 16nb^8\alpha^{15} - 384nb^6\beta^2\alpha^{14} - 22b^2\beta^2\alpha^{12} + 48\beta^3\alpha^{12} - 96n\beta^2\alpha^{12} + 1920b^2\beta^3\alpha^{11} + 324b^2\beta^4\alpha^{10} - 616\beta^5\alpha^{10} + 88\beta^4\alpha^{11} \\
& -12528nb^2\beta^5\alpha^9 - 136\beta^6\alpha^9 - 812n\beta^5\alpha^9 + 4836b^2\beta^6\alpha^8 + 570n\beta^4\alpha^{10} + 2376\beta^7\alpha^8 - 1352n\beta^6\alpha^8 + 2150b^2\beta^7\alpha^7 - 600\beta^8\alpha^7 \\
& -3672\beta^9\alpha^6 - 2700b^2\beta^9\alpha^6 + 264b2\beta^7\alpha^8 - 11454b^2\beta^8\alpha^6 + 128nb^6\beta^4\alpha^{12} - 192nb^6\beta^5\alpha^{11} + 128b^6\beta\alpha^{15} - 192nb^4\beta^2\alpha^{14} \\
& +96nb^4\beta\alpha^{15} + 64nb^6\beta^6\alpha^{10} + 128b^6\beta^4\alpha^{12} + 192nb^6\beta^3\alpha^{12} + 1056nb^4\beta^4\alpha^{12} - 64nb^6\beta^4\alpha^{11} + 1056nb^4\beta^5\alpha^{11} - 12b^6\beta^5\alpha^{11} \\
& -576nb^4\beta^3\alpha^{13} + 416b^6\beta\alpha^{13} + 960n\beta^5\alpha^{11} + 4b^4\beta^4\alpha^{11} + 1752nb^4\beta^3\alpha^{11} + 1440b^2\beta^5\alpha^{11} + 432nb^2\beta^4\alpha^{11} - 192b^2\beta^4\alpha^{12} \\
& -34\beta^4\alpha^{12} - 64nb^6\beta^2\alpha^{13} - 6b^6\beta^3\alpha^{11} - 96b^4\beta^2\alpha^{13} + 24nb^4\beta\alpha^{13} - 192b^2\beta^3\alpha^{13} + 32nb^6\alpha^{15} - 48nb^2\beta^2\alpha^{13} - 456b^6\beta^2\alpha^{12} \\
& -48n\beta^3\alpha^{13} + 192b^4\beta^3\alpha^{12} - 1200nb^4\beta^2\alpha^{12} + 160b^6\beta^6\alpha^{10} - 576nb^4\beta^7\alpha^9 - 128nb^6\beta^5\alpha^{10} + 576nb^4\beta^8\alpha^8 - 1632nb^4\beta^6\alpha^{10} \\
& +64nb^6\beta^6\alpha^9 + 96nb^2\beta^2\alpha^{14} + 48nb^6\alpha^{14} - 144b^4\beta^2\alpha^{14} + 288nb^4\beta^3\alpha^{12} - 96\alpha^{14}b^4 - 12nb^2\beta^4\alpha^{12} - 58b^4\beta^4\alpha^{12} + 12b^6\beta^4\alpha^{11} \\
& +288b^4\beta^5\alpha^{11} + 38nb^4\beta^4\alpha^{11} + 2928nb^2\beta^5\alpha^{11} + 1568nb^6\beta^3\alpha^{11} + 32nb^6\beta\alpha^{13} - 64b^6\beta^2\alpha^{13} - 192b^4\beta^3\alpha^{13} - 816nb^6\beta^2\alpha^{12} \\
& +1104nb^4\beta^4\alpha^{10} + 768b^2\beta^6\alpha^{10} - 1536nb^2\beta^5\alpha^{10} + 1656b^6\beta^4\alpha^{10} + 440n\beta^6\alpha^{10} - 832b^6\beta^5\alpha^9 - 480b^4\beta^6\alpha^9 - 724nb^4\beta^5\alpha^9 \\
& -38n\beta^7\alpha^9 + 16n\beta^8\alpha^8 + 312nb^4\beta^7\alpha^7 - 148nb^2\beta^8\alpha^7 + 128\beta^6\alpha^{10} - 192nb^4\beta^8\alpha^6 + 5832n\beta^9\alpha^7 - 672b^4\beta^8\alpha^7 - 2880b^2\beta^{10}\alpha^6 \\
& -2592n\beta^{11}\alpha^5 - 4104n\beta^{10}\alpha^6 + 1728n\beta^{12}\alpha^4 + 1008nb^2\beta^{10}\alpha^5 + 36nb^2\alpha^{14} + 48\beta^2\alpha^{14} + 408b^4\beta\alpha^{13} - 48b^2\beta^2\alpha^{13} + 1056\beta^5\alpha^{11} \\
& +600nb^2\beta^5\alpha^{10} - 1488b^2\beta^5\alpha^{10} + 258nb^2\beta^4\alpha^{10} - 376n\beta^5\alpha^{10} - 528b^2\beta^6\alpha^9 - 4872nb^2\beta^5\alpha^9 - 39\beta^7\alpha^9 - 376\beta^6\alpha^9 - 72b^4\beta^6\alpha^8 \\
& -432nb^2\beta^3\alpha^{13} - 576b^4\beta^5\alpha^{10} + 2688nb^4\beta^4\alpha^{10} - 368b^2\beta^7\alpha^9 - 480nb^2\beta^6\alpha^9 + 1944b^4\beta^4\alpha^{10} - 2508nb^2\beta^6\alpha^8 + 1824\beta^8\alpha^8 \\
& +8n\beta^3\alpha^{13} + 240b^6\beta^2\alpha^{12} - 72b^4\beta^3\alpha^{12} - 76nb^4\beta^2\alpha^{12} - 16b^2\beta^4\alpha^{12} - 48nb^2\beta^3\alpha^{12} + 18n\beta^4 - 384\alpha^{11}b^4\beta^4\alpha^{12} - 22nb^4\beta^3\alpha^{11} \\
& -72\beta^3\alpha^{13} - 8n\beta^2\alpha^{13} + 192b^2\beta^3\alpha^{12} - 588nb^2\beta^2\alpha^{12} + 24n\beta^3\alpha^{12} - 272\beta^4\alpha^{12} + 228n\beta^3\alpha^{12} + 384b^2\beta^4\alpha^{11} + 648nb^2\beta^3\alpha^{11} \\
& -1368b^2\beta^9\alpha^5 + 1872\beta^{10}\alpha^5 - 4626n\beta^9\alpha^5 + 1800n\beta^{10}\alpha^4 + 95b^2\beta^{10}\alpha^4 + 2376\beta^{11}\alpha^4 - 1728\beta^{12}\alpha^3 + 1890n\beta^{11}\alpha^3 + 18\beta^2\alpha^{12} \\
& -576\beta^4\alpha^{10} - 4772\beta^5\alpha^9 + 3748\beta^6\alpha^8 + 14196\beta^7\alpha^7 + 224b^3\beta^5\alpha^{10} - 384nb^4\beta^5\alpha^{10} + 720nb^4\beta^6\alpha^9 - 14nb^4\beta^7\alpha^8 - 128b^3\beta^5\alpha^{10} \\
& -192b^4\beta^7\alpha^9 + 56nb^4\beta^6\alpha^9 - 594nb^2\beta^7\alpha^9 - 48b^4\beta^8\alpha^8 + 960nb^4\beta^7\alpha^8 + 486n\beta^8\alpha^8 - 480nb^4\beta^8\alpha^7 + 3024nb^2\beta^9\alpha^7 - 20nb^2\beta^{10}\alpha^6 \\
& +6744b^4\beta^7\alpha^7 - 672b^2\beta^8\alpha^7 - 17370\beta^9\alpha^5 + 13194\beta^{10}\alpha^4 + 8154\beta^{11}\alpha^3 - 6588\beta^{12}\alpha^2 - 126n\beta^{12}\alpha^2 + 6\alpha^{14} - 2\beta\alpha^{13} - 104nb^6\beta^5\alpha^9 \\
& +512nb^6\beta^6\alpha^8 - 8n\beta\alpha^{15} - 120b^6\alpha^{14} + 72nb^4\alpha^{14} + 48n\beta^2\alpha^{14} - 10698\beta^8\alpha^6 + 96\alpha^{15}b^4\beta + 304n\beta^7\alpha^7 + 82n\beta^3\alpha^{11} - 192b^6\beta^3\alpha^{13} \\
& -12nb^6\beta\alpha^{14} + 144nb^2\beta^3\alpha^{12} - 8\alpha^{15}\beta + 56\beta^8\alpha^6 - 96b^4\beta^2\alpha^{13} + 368b^2\beta^9\alpha^7 - 206nb^2\beta^9\alpha^6 + 12n\beta^4\alpha^{11} + 368b^2\beta^7\alpha^8 + 152\beta^7\alpha^8 \\
& +48b^6\beta^3\alpha^{11} - 74b^4\beta^2\alpha^{12} + 562\beta^3\alpha^{11} + 64b^6\beta^6\alpha^9
\end{aligned}
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\begin{aligned}
b_3 \ :=\ & 128nb^6\beta\alpha^{15} - 64nb^8\beta\alpha^{14} + 48nb^8\beta^3\alpha^{12} - 16nb^8\beta^4\alpha^{11} - 16b^8\beta^4\alpha^{11} + 16nb^8\beta^5\alpha^{10} + 64nb^6\beta\alpha^{14} + 256nb^6\beta^4\alpha^{11} \\
& -36nb^6\beta^2\alpha^{13} - 32b^8\beta^5\alpha^{10} - 4320nb^6\beta^5\alpha^{10} + 80nb^6\beta^6\alpha^9 - 80b^6\beta^7\alpha^8 + 16b^2\beta\alpha^{14} - 120nb^4\alpha^{15} + 16b^6\beta\alpha^{14} + 96nb^4\beta\alpha^{14} \\
& +640nb^8\beta^2\alpha^{11} - 1760b^6\beta^3\alpha^{12} - 16nb^6\beta^2\alpha^{12} + 64b^6\beta^4\alpha^{11} - 416b^6\beta^3\alpha^{11} - 4n\beta\alpha^{14} + 240nb^4\beta^4\alpha^{11} - 112nb^8\beta^3\alpha^{10} \\
& +32nb^6\beta\alpha^{13} + 56nb^6\alpha^{13} + 4nb^4\beta\alpha^{13} - 240b^4\beta^2\alpha^{13} + 14nb^4\beta^2\alpha^{13} + 48nb^4\beta^2\alpha^{13} + 6nb^2\beta^3\alpha^{13} - 24b^2\beta^2\alpha^{13} + 128b^6\beta^2\alpha^{12} \\
& -86b^4\beta^3\alpha^{11} - 16nb^2\beta^4\alpha^{11} + 68b^8\beta^3\alpha^{10} + 24b^4\beta^3\alpha^{12} - 24b^4\beta^2\alpha^{12} + 152b^6\alpha^{13} - 320b^4\beta^2\alpha^{11} - 24b^6\beta^3\alpha^{11} + 128b^6\beta^2\alpha^{11} \\
& -448b^8\beta^4\alpha^9 + 52b^6\beta^5\alpha^9 + 4952nb^6\beta^4\alpha^9 + 48b^4\beta^6\alpha^9 + 242nb^4\beta^5\alpha^9 - 4n\beta^3\alpha^{12} + 1464nb^2\beta^6\alpha^9 - 448b^6\beta^6\alpha^8 + 24nb^6\beta^5\alpha^7 \\
& -240nb^6\beta^7\alpha^7 + 192b^6\beta^7\alpha^7 - 2800nb^6\beta^6\alpha^7 + 1152b^4\beta^7\alpha^8 + 2160nb^4\beta^6\alpha^8 + 384b^2\beta^8\alpha^8 + 3984nb^2\beta^7\alpha^8 - 192b^6\beta^4\alpha^{10} \\
& +1128b^4\beta^7\alpha^8 - 4632nb^4\beta^6\alpha^8 - 660nb^2\beta^7\alpha^8 + 256b^8\beta^5\alpha^8 - 396b^4\beta^8\alpha^7 + 120b^4\beta^2\alpha^{12} + 3120nb^4\beta^7\alpha^7 - 2124nb^2\beta^8\alpha^7 \\
& +142nb^2\beta^9\alpha^6 - 576nb^2\beta^{11}\alpha^4 + 86nb^2\beta^{10}\alpha^5 - 864nb^4\beta^9\alpha^5 - 60b^2\beta^2\alpha^{13} + 24nb^2\beta\alpha^{13} + 8b^4\beta\alpha^{13} + 36nb^4\alpha^{13} + 24n\beta^2\alpha^{13} \\
& -672nb^4\beta^2\alpha^{11} - 176b^2\beta^4\alpha^{11} + 1496b^6\beta^2\alpha^{11} + 264b^2\beta^3\alpha^{12} - 12nb^2\beta^2\alpha^{12} + 384nb^4\beta\alpha^{12} - 504nb^2\beta^3\alpha^{11} - 196n\beta^4\alpha^{11} \\
& -7104nb^4\beta^3\alpha^{10} - 520b^2\beta^5\alpha^{10} + 600nb^2\beta^4\alpha^{10} + 228n\beta^5\alpha^{10} - 3256b^6\beta^4\alpha^9 + 1392b^2\beta^6\alpha^9 + 2700nb^2\beta^5\alpha^9 + 532n\beta^6\alpha^9 \\
& +18240nb^4\beta^5\alpha^8 - 216b^2\beta^7\alpha^8 - 4080b^2\beta^6\alpha^8 - 2304b^4\beta^7\alpha^7 - 34608nb^4\beta^6\alpha^7 + 2016b^4\beta^5\alpha^9 + 11160nb^4\beta^4\alpha^9 - 880\beta^7\alpha^8 \\
& +4480nb^6\beta^3\alpha^{12} - 480b^6\beta^2\alpha^{13} - 312nb^4\beta^2\alpha^{13} - 320nb^8\beta\alpha^{12} + 396nb^4\beta^3\alpha^{12} - 32nb^6\beta\alpha^{12} + 60b^4\beta^4\alpha^{11} - 9360b^4\beta^5\alpha^{10} \\
& -6240b^4\beta^3\alpha^{11} + 2400nb^4\beta^8\alpha^6 - 316nb^2\beta^5\alpha^{10} + 24nb^2\beta^2\alpha^{12} - 5004nb^4\beta^6\alpha^8 - 456b^4\beta^4\alpha^{10} + 2480nb^2\beta\alpha^{12} - 4\beta^3\alpha^{12} \\
& -2n\beta^{2\alpha^{12}} + 1308b^4\beta^2\alpha^{11} - 456b^2\beta^3\alpha^{11} - 448nb^2\beta^2\alpha^{11} - 148n\beta^4\alpha^{11} - 600b^2\beta^4\alpha^{10} - 44840nb^2\beta^3\alpha^{10} - 100n\beta^3\alpha^{11} \\
& +108\beta^4\alpha^{10} - 122b^2\beta^6\alpha^{10} + 204nb^2\beta^5\alpha^8 - 76b^4\beta^4\alpha^9 + 2352b^2\beta^5\alpha^9 + 72nb^2\beta^4\alpha^9 + 34\beta^6\alpha^9 + 70n\beta^5\alpha^9 + 14688b^4\beta^5\alpha^8 \\
& -4320b^2\beta^7\alpha^7 - 3578nb^2\beta^6\alpha^7 - 216\beta^8\alpha^7 - 2298n\beta^7\alpha^7 - 1459b^4\beta^7\alpha^6 + 4908b^2\beta^8\alpha^6 - 2658nb^2\beta^7\alpha^6 + 564\beta^9\alpha^6 + 3480n\beta^8\alpha^6 \\
& +1296\beta^{11}\alpha^5 - 26n\beta^{10}\alpha^5 - 18b^4\beta^{10}\alpha^4 - 2052b^2\beta^{11}\alpha^4 - 1728nb^2\beta^{10}\alpha^4 - 98\beta^{12}\alpha^4 - 1728n\beta^{11}\alpha^4 - 54\beta^{11}\alpha^4 - 5562n\beta^{10}\alpha^4 \\
& +612nb^2\beta^8\alpha^5 + 2646n\beta^9\alpha^5 - 123b^4\beta^9\alpha^4 - 72\beta^{10}\alpha^5 - 6732b^2\beta^{10}\alpha^4 - 5382nb^2\beta^9\alpha^4 + 108\beta^{12}\alpha^3 + 41\beta^{11}\alpha^3 + 7182nb^2\beta^{11}\alpha^2 \\
& +4\beta\alpha^{13} + 308b^2\beta - 2\beta^2\alpha^{12} + 4n\beta\alpha^{12} + 10b^2\alpha^{13} + 80b^2\beta^2\alpha^{11} + 60\beta^4\alpha^{10} - 893\beta^3\alpha^{10} - 620b^2\beta^4\alpha^9 - 87n\beta^6\alpha^8 - 2160b^2\beta^7\alpha^7 \\
& -428\beta^6\alpha^8 + 5671n\beta^5\alpha^8 - 9752n\beta^6\alpha^7 + 18608\alpha^7b2\beta^5\alpha^7 - 2424\beta^7\alpha^7 - 30672b^2\beta^7\alpha^6 - 100\beta^3\alpha^{11} - 80n\alpha^{11}\beta^2 - 4148b^2\beta^3\alpha^{10} \\
& -352nb^8\beta^4\alpha^{10} - 16b^6\beta^6\alpha^9 - 400nb^6\beta^6\alpha^8 - 480\beta^7\alpha^8 - 16nb^6\beta^5\alpha^9 + 840nb^4\beta^6\alpha^9 + 112nb^8\beta^5\alpha^8 + 304b^6\beta^7\alpha^8 - 108nb^4\beta^7\alpha^8 \\
& -4nb^2\alpha^{15} - 12b^4\alpha^{15} + 48b^4\beta\alpha^{14} + 16nb^2\beta\alpha^{14} + 128b^8\alpha^{13} - 4770\beta^{10}\alpha^4 + 121n\beta^9\alpha^4 + 2208\beta^8\alpha^6 - 14498n\beta^7\alpha^6 + 37458b^2\beta^8\alpha^5 \\
& -45648b^2\beta^{10}\alpha^3 + 4212\beta^{12}\alpha^2 + 3807n\beta^{11}\alpha^2 - 1944\beta^{13}\alpha + 11583n\beta^{12}\alpha - 3429n\beta^{13} + 44\beta\alpha^{12} - 80\beta^2\alpha^{11} - 986\beta^3\alpha^{10} + 457\beta^4\alpha^9 \\
& +15507\beta^8\alpha^5 + 2016\beta^9\alpha^4 - 31896\beta^{10}\alpha^3 - 1170\beta^{11}\alpha^2 + 2467\beta^{12}\alpha - 756\beta^{11}\alpha^3 - 34200n\beta^{10}\alpha^3 + 22932b^2\beta^{11}\alpha^2 - 504b^2\beta^{10}\alpha^5 \\
& +4158nb^2\beta^9\alpha^5 - 932n\beta^6\alpha^8 - 5196b^4\beta^6 - 3224\beta^6\alpha^7 - 17972\beta^7\alpha^6 + 2815n\beta^8\alpha^5 + 10188b^2\beta^9\alpha^4 - 756nb^4\beta^8\alpha^7 + 672nb^6\beta^4\alpha^{10} \\
& -90nb^4\beta^5\alpha^{10} - 756n\beta^{13}\alpha + 216\beta^{13}\alpha^2 + n\alpha^{13} + 736\beta^5\alpha^9 + 1477n\beta^4\alpha^9 - 236nb^2\beta^{10}\alpha^3 + 32b^2\beta^9\alpha^5 + 516nb^4\beta^9\alpha^6 \\
& +6442\beta^5\alpha^8 + 3204\beta^9\alpha^5 - 144b^6\beta^5\alpha^{10} + 14nb^6\beta^7\alpha^7 - 8496b^2\beta^6\alpha^7 + 68n\beta^{12}\alpha^2 + 32b^8\beta\alpha^{14} - 4b^2\alpha^{15} + 32b^8\alpha^{13} \\
& +32nb^6\beta\alpha^{13} - 24b^8\beta\alpha^{12} + 8nb^2\beta^3\alpha^{12} + 288b^6\beta^8\alpha^7 + 4448b^6\beta^5\alpha^8 + 972b^4\beta^9\alpha^7 - 5184b^4\beta^3\alpha^{10} - 352\beta^7\alpha^8\alpha^7 \\
& +250b^4\beta^8\alpha^5 + 3240b^2\beta^{11}\alpha^3 + 1200\beta^5\alpha^{10} - 3064b^6\beta^6\alpha^7 - 928b^6\beta^3\alpha^{10} + 16b^6\beta\alpha^{12} - 264b^4\beta^9\alpha^6 - 3472nb^6\beta^3\alpha^{10}
\end{aligned}
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\begin{aligned}
b_4 \;:=\;& 16b^8\alpha^{15} - 16nb^8\beta^5\alpha^{10} + 80nb^8\beta^4\alpha^{11} - 48nb^8\beta^3\alpha^{12} + 96nb^8\beta\alpha^{14} - 48nb^4\beta^2\alpha^{13} - 24b^4\alpha^{15} + 520b^6\beta^5\alpha^{10} - 936nb^4\beta^3\alpha^{12}\\
&-32nb^6\alpha^{15} + 48b^8\beta\alpha^{14} + 192nb^6\beta\alpha^{14} + 64b^8\beta^2\alpha^{13} - 160nb^6\beta^2\alpha^{13} - 96b^8\beta^3\alpha^{12} - 672nb^6\beta^3\alpha^{12} + 80b^8\beta^4\alpha^{11} + 928b^6\beta^4\alpha^{11}\\
&+1704nb^4\beta^5\alpha^{10} - 576b2^3\beta^6\alpha^9 + 48b^4\beta^2\alpha^{13} + 48nb^2\beta^2\alpha^{13} - 4b^4\beta^3\alpha^{12} - 24b^4\alpha^{15} + 936b^2\beta^3\alpha^{12} - 432b^4\beta^4\alpha^{11} - 106nb^2\beta^4\alpha^{11}\\
&+120b^6\beta^7\alpha^8 - 54nb^4\beta^7\alpha^8 + 1080nb^4\beta^8\alpha^7 - 2448nb^4\beta^6\alpha^9 + 4b^4\beta^4\alpha^{11} - 32b^6\alpha^{15} + 400nb^2\beta^4\alpha^{11} - 720b^4\beta^3\alpha^{12} - 456nb^2\beta^3\alpha^{12}\\
&-864b^4\beta^6\alpha^9 + 1512b^4\beta^8\alpha^7 + 2808nb^2\beta^8\alpha^7 - 1872b^2\beta^6\alpha^9 - 1008b^4\beta^7\alpha^8 + 81\beta^9\alpha^6 - 1872nb^2\beta^7\alpha^8 + 48nb^2\beta\alpha^{14} + 96b^4\beta^2\alpha^{13}\\
&-216nb^4\beta^9\alpha^6 - 75n\beta^3\alpha^{12} - 198b^2\beta^7\alpha^8 - 798n\beta^7\alpha^8 + 1944b^2\beta^8\alpha^7 - 384n\beta^6\alpha^9 - 84b^2\beta^6\alpha^9 + 3n\beta^5\alpha^{10} + 112b^2\beta^4\alpha^{11} + 47n\beta^4\alpha^{11}\\
&+756n\beta^9\alpha^6 + 1324b^2\beta^5\alpha^{10} - 1728b^2\beta^{10}\alpha^5 + 378b^2\beta^{11}\alpha^4 - 1188n\beta^{10}\alpha^5 - 72\beta^3\alpha^{12} - 135n\beta^{11}\alpha^4 + 4\beta^2\alpha^{13} - 288\beta^6\alpha^9 + 35\beta^4\alpha^{11}\\
&+837\beta^8\alpha^7 - 480nb^6\beta^6\alpha^9 + 96nb^6\beta^7\alpha^8 + 120b^6\beta\alpha^{14} + 144nb^4\beta\alpha^{14} + 128b^6\beta^2\alpha^{13} - 1188\beta^{10}\alpha^5 - 216\beta^{11}\alpha^4 + 729\beta^{12}\alpha^3 - 162\beta^{13}\alpha^2\\
&-1704nb^2\beta^5\alpha^{10} + 256nb^6\beta^5\alpha^{10} + 35\beta^4\alpha^{11} - 32b^6\beta^4\alpha^{11} + 1056b^4\beta^4\alpha^{11} + 108b^4\beta\alpha^{14} + 1504nb^2\beta^5\alpha^{10} + 1053n\beta^8\alpha^7 + 405n\beta^{12}\alpha^3\\
&-768\beta^7\alpha^8 - 50b^6\beta^3\alpha^{12} - 16b^8\beta^5\alpha^{10} + 720b^4\beta^5\alpha^{10} + 150b^4\beta^5\alpha^{10} + 80n\,b^2\beta^2\alpha^{13} + 8b^2\beta^9\alpha^6 + 338\beta^5\alpha^{10} - 8nb^2\alpha^{15}
\end{aligned}
$$

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\begin{aligned}
b_5 \;:=\;& 32nb^8\beta\alpha^{14} + 32nb^8\beta^3\alpha^{12} + 25nb^6\beta^4\alpha^{12} + 18nb^6\beta^5\alpha^{11} - 6nb^8\beta^2\alpha^{13} + 6nb^8\beta^4\alpha^{11} - 32nb^8\beta^5\alpha^{10} - 128nb^6\beta^3\alpha^{13}\\
&-32b^8\beta^5\alpha^{10} - 12nb^6\beta^3\alpha^{12} + 16nb^4\beta^5\alpha^{11} - 128b^3\beta^6\alpha^{10} + 2b^8\beta\alpha^{11}\alpha^{14} + 64nb^6\beta\alpha^{14} - 128b^6\beta^3\alpha^{13} - 12nb^6\beta^2\alpha^{13} - 12nb^4\beta^3\alpha^{13}\\
&-64b^8\beta^3\alpha^{12} + 74nb^6\beta^4\alpha^{11} - 48b^6\beta^2\alpha^{13} - 192b^4\beta^3\alpha^{13} - 144nb^4\beta^2\alpha^{13} - 272b^6\beta^3\alpha^{12} - 19b^4\beta^4\alpha^{12} - 24b^4\beta^3\alpha^{12} - 192nb^2\beta^4\alpha^{12}\\
&+38b^4\beta^5\alpha^{11} - 96nb^2\beta^3\alpha^{13} + 90nb^4\beta^4\alpha^{11} + 1440nb^2\beta^5\alpha^{11} + 22b^6\beta^5\alpha^{10} - 432b^6\beta^6\alpha^9 - 960b^4\beta^7\alpha^9 - 1872nb^4\beta^6\alpha^9 + 720nb^4\beta^7\alpha^8\\
&+288nb^2\beta^6\alpha^{10} - 418nb^2\beta^7\alpha^9 + 40b^6\beta^7\alpha^8 + 960b^4\beta^8\alpha^8 + 1152nb^2\beta^8\alpha^8 + 84nb^4\beta^8\alpha^7 - 42nb^4\beta^9\alpha^6 + 216nb^2\beta^9\alpha^7 - 864nb^2\beta^{10}\alpha^6\\
&-96b^2\beta^3\alpha^{13} - 48nb^2\beta^2\alpha^{13} - 72b^4\beta^2\alpha^{13} + 16nb^2\beta\alpha^{14} + 144b^4\beta^2\alpha^{13} + 352\,\beta^5\alpha^{11} + 600\,b^4\beta^5\alpha^{10} + 96\,b^2\beta^6\alpha^{10} + 272nb^2\beta^5\alpha^{10}\\
&+144nb^2\beta^7\alpha^8 - 16n\beta^3\alpha^{13} + 1152b^2\beta^5\alpha^{11} + 46nb^2\beta^4\alpha^{11} + 456b^2\beta^4\alpha^{11} - 220\beta^3\alpha^{12} - 312\,b^4\beta^3\alpha^{12} - 128nb^2\beta^3\alpha^{12} - 96b^2\beta^4\alpha^{12}\\
&-1584nb^2\beta^6\alpha^9 - 1824\,n\beta^7\alpha^9 - 72b^4\beta^7\alpha^8 + 1248b^2\beta^8\alpha^8 - 432n\beta^8\alpha^8 + 1080\,b^4\beta^8\alpha^7 + 2160n\,b^2\beta^8\alpha^7 - 16\beta^4\alpha^{12} - 220n\beta^3\alpha^{12}\\
&-256nb^6\beta^5\alpha^{10} - 960nb^4\beta^6\alpha^{10} + 320b^2\beta^4\alpha^{11} + 416b^2\beta^5\alpha^{10} + 32\beta^6\alpha^{10} + 82n\beta^5\alpha^{10} - 1080b^2\beta^6\alpha^9 - 2256\beta^7\alpha^9 - 360b^2\beta^7\alpha^8\\
&+360\beta^9\alpha^7 + 864n\beta^8\alpha^7 - 42b^2\beta^9\alpha^6 - 1440\beta^{10}\alpha^6 - 162n\beta^9\alpha^6 - 118b^2\beta^{10}\alpha^5 - 22\beta^{11}\alpha^5 - 918n\beta^{10}\alpha^5 + 76b^2\beta^{11}\alpha^4 + 126\beta^{12}\alpha^4\\
&+70\beta^4\alpha^{11} - 162n\beta^{13}\alpha^2 - 132\beta^7\alpha^8 - 54\beta^9\alpha^6 + 88\beta^5\alpha^{10} - 324\beta^6\alpha^9 - 34nb^6\beta^4\alpha^{10} + 756\beta^8\alpha^7 + 1104b^4\beta^6\alpha^{10} - 162nb^4\beta^5\alpha^{10}\\
&+576nb^4\beta^8\alpha^8 - 960nb^4\beta^7\alpha^9 + 192nb^6\beta^7\alpha^8 + 64b^6\beta\alpha^{14} + 48nb^4\beta\alpha^{14} - 918\beta^{10}\alpha^5 + 378\beta^{11}\alpha^4 + 486\beta^{12}\alpha^3 - 324\beta^{13}\alpha^2\\
&-2400b^2\beta^7\alpha^9 - 6\beta^2\alpha^{13} - 108b^4\beta^6\alpha^9 - 84n\beta^7\alpha^8 + 128b^2\beta^8\alpha^7 + 496\beta^5\alpha^{11} + 76n\beta^4\alpha^{11} - 384nb^6\beta^6\alpha^9 - 128nb^6\beta^6\alpha^{10}\\
&+384\beta^8\alpha^8 + 18b^6\beta^5\alpha^{11} + 2880b^2\beta^9\alpha^7 + 2\beta\alpha^{14} + 324n\beta^{12}\alpha^3 - 1680nb^2\beta^6\alpha^{10} + 38n\beta^{11}\alpha^4 - 640n\,\beta^4\alpha^{12} + 368n\,\beta^6\alpha^{10}\\
&+48b^4\beta\alpha^{14} + 9nb^4\beta^5\alpha^{10} + 224b^6\beta^4\alpha^{11} - 28b^6\beta^4\alpha^{12}
\end{aligned}
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\begin{aligned}
b_6 \;:=\;& 256nb^6\beta^2\alpha^{13} + 96nb^6\beta^3\alpha^{12} - 160nb^6\beta^4\alpha^{11} + 64nb^8\beta^5\alpha^{10} + 104b^3\beta^6\alpha^{10} - 131nb^6\beta^6\alpha^{10} - 32nb^8\beta^6\alpha^9 - 936nb^6\beta^7\alpha^9\\
&+32b^6\beta^5\alpha^{10} + 48nb^4\alpha^{15} + 96b^6\beta\alpha^{14} - 288nb^4\beta\alpha^{14} - 128b^6\beta^2\alpha^{13} + 108nb^4\beta^3\alpha^{12} + 240b^4\beta^2\alpha^{13} - 192b^6\beta^3\alpha^{12} - 160b^6\beta^4\alpha^{11}\\
&+504nb^2\beta^7\alpha^8 + 2448nb^2\beta^6\,\alpha^9 + 1104b^4\beta^6\alpha^9 + 960b^2\beta^3\alpha^{12} + 228n\beta^3\alpha^{12} + 4n\alpha^{15} - 1248b^2\beta^4\alpha^{11} - 200n\,\beta^4\alpha^{11} - 86b^2\beta^5\alpha^{10}\\
&-768b^2\beta^7\alpha^8 + 936n\beta^7\alpha^8 - 2232b^2\beta^8\alpha^7 + 216nb^2\beta^9\alpha^6 - 216b^2\beta\alpha^{14} - 1080nb^2\beta^8\alpha^7 - 24n\beta\alpha^{14} + 144b^2\beta^2\alpha^{13} - 4n\beta^2\alpha^{13}\\
&-1080\beta^9\alpha^7 + 1944b^4\beta^9\alpha^5 + 24b^2\beta^9\alpha^6 + 486\beta^{10}\alpha^6 + 1296nb^2\beta^8\alpha^6 + 1458n\beta^9\alpha^6 - 242\beta^8\alpha^7 + 10\beta^9\alpha^6 - 1208\beta^5\alpha^{10}\\
&+10\beta^7\alpha^8 + 14\beta^{10}\alpha^5 + 24b^2\alpha^{15} - 50\beta^{11}\alpha^4 - 14b^4\beta\alpha^{14} + 48nb^4\alpha^{15} + 32b^6\alpha^{15} - 92\beta^7\alpha^8 + 24b^2\alpha^{15} - 75\beta^5\alpha^{10}\\
&+468\beta^3\alpha^{12} - 56\beta^4\alpha^{11} - 18\beta^8\alpha^7 - 54b^4\beta^8\alpha^6 + 25b^2\beta^6\alpha^9 + 66nb^6\beta^8\alpha^8 - 24b^6\beta\alpha^{15} + 96n\beta^6\alpha^9 - 132nb^4\beta^4\alpha^{11}\\
&+1752\beta^6\alpha^9 - 14n\alpha^{15}b^2\beta\alpha^{15}
\end{aligned}
$$

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\begin{aligned}
b_7 \;:=\;& 48b^6\beta^3\alpha^{12} + 152b^8\beta^2\alpha^{12} - 120nb^6\beta^2\alpha^{12} - 128b^4\beta^4\alpha^{12} - 52n + 816b^6\beta^3\alpha^{12} + 528b^6\beta^7\alpha^8 - 1200b^6\beta^5\alpha^{10} - 96b^8\beta\alpha^{14}\\
&+72b^4\beta^3\alpha^{12} - 24b^4\beta^5\alpha^{10} + 26b^4\beta^7\alpha^8 - 12\beta^5\alpha^{10} - 18b^2\beta^5\alpha^{10} + 52\beta^7\alpha^8 + 12\beta^3\alpha^{12} + 972b^2\beta^{11}\alpha^4 - 10\beta^9\alpha^6 - 72b^4\beta\alpha^{14}\\
&+192b^8\beta^3\alpha^{12} - 96b^8\beta^5\alpha^{10} - 4b^6\beta\alpha^{14} + 972\beta^{11}\alpha^4,
\end{aligned}
$$

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\begin{aligned}
b_8 \;:=\;& 96b^6\beta^2\alpha^{13} + 128b^6\beta^3\alpha^{12} - 192nb^4\beta^3\alpha^{12} - 128b^6\beta^4\alpha^{11} - 1008nb^4\beta^4\alpha^{11} + 1008nb^4\beta^6\alpha^9 + 32b^6\beta^6\alpha^9 - 128b^6\beta^7\alpha^8\\
&+1104n\,b^4\beta^5\alpha^{10} - 256n\,b^6\beta^3\alpha^{12} - 128n\,b^6\beta^4\alpha^{11} + 96n\,b^6\beta^2\alpha^{13} + 48\,nb^4\beta^9\,\alpha^6 + 72n\,b^2\beta^2\alpha^{13} - 288b^4\beta^4\alpha^{11}\\
&-1800nb^2\beta^7\alpha^8 - 288b^4\beta^8\,\alpha^7 - 1296n\,b^2\beta^8\alpha^7 - 5304b^4\beta^5\alpha^9 + 432b^4\beta^6\alpha^9 - 48\,\beta^5\alpha^{10} + 2208b^2\beta^6\alpha^9 - 672nb^4\beta^7\alpha^8\\
&+60n\beta^5\alpha^{10} + 1416\alpha^9b^2\,\beta^6\alpha^9 + 856n\beta^6\alpha^9 - 576b^2\beta^7\alpha^8 - 664n\beta^7\alpha^8 - 1656b^2\,\beta^8\alpha^7 - 936nn\beta^8\alpha^7 + 1536b^2\beta^9\alpha^6\\
&-504n\beta^{11}\alpha^4 + 324n\beta^{12}\alpha^3 - 108n\,\beta^{13}\alpha^2 - 220\beta^4\alpha^{11} + 496\beta^6\alpha^9 - 432\beta^8\alpha^7 - 136\beta^7\alpha^8 + 600\,\beta^9\alpha^6 + 288nb^6\beta^5\alpha^{10}\\
&-648\beta^{11}\alpha^4 + 36\beta^{10}\alpha^5 + 108\,\beta^{12}\alpha^3 - 192b^2\beta^5\alpha^{10} - 220n\beta^4\alpha^{11} - 192b^4\,\beta^5\alpha^{10} + 16\beta^3\,\alpha^{12} + 12\beta^2\alpha^{13} - 36n\beta^{10}\alpha^5\\
&+16n\beta^3\alpha^{12} - 552b^2\beta^4\alpha^{11} - 960nb^4\beta^7\alpha^8 - 144nb^4\beta^8\alpha^7 - 912nb^2\beta^4\alpha^{11} + 192\,b^4\beta^3\alpha^{12} + 1200n\beta^9\alpha^6 + 720\,b^2\beta^{10}\alpha^5\\
&-864b^2\beta^{11}\alpha^4 + 216\beta^{13}\alpha^2 + 32nb^6\beta^6\alpha^9 - 32\,b^6\beta^7\alpha^8,
\end{aligned}
$$

$$
A_{16} \;:=\; 4(n+1)(1+2b^2)^2(6b^2\beta + 2b^2 + 1)r_{00}^2,
$$

$$
\begin{aligned}
A_{15} \;:=\;& -(1+2b^2)\Big(16nb^8\beta r_{000} - 32b^6\beta^2 r_{00} - 32nb^6\beta^2 r_{00}^2 - 32nb^6\beta^2 r_{00} + 16b^6\beta^2 r_{00}^2 - 2nb^6\beta r_0 r_{00} + 4nb^4\beta^2 r_{00}^2 + 16b^8\beta r_{000}\\
&+16nb^4\beta^2 r_{00} + 104nb^4\beta r_0 r_{00} + 24\beta^2 r_{00}^2 - 8nb^6 b^i r_{i00} - 16r_{00}^2 b^6 + 16b^4\beta^2 r_{00} - 12nb^4\beta r_{000} - 4nb^2\beta r_{000} + 16b^2\beta^2 r_{00}\\
&+24b^2\beta^2 r_{00}^2 + 44b^2\beta r_0 r_{00} - 4n\beta^2 r_{00}^2 - 8b^6 b^i r_{i00} - 12b^2\beta r_{000} - 12nb^4 b^i r_{i00} + 24r_0^2 b^4 + 8r_{00}r_0 b^4 - 16r_{00}^2 b^4 + 12r_0^2\\
&+2nr_0^2 b^2 + 8nr_{00}r_0 b^2 - 4nb^2 r_{00}^2 - 4\beta^2 r_{00}^2 - 8n\beta r_0 r_{00} - 2b^4 b^i r_{i00} - 4b^2\beta r_{000} - 6nb^2 b^i r_{i00} + 4r_0^2 b^2 + 8r_{00}r_0 b^2 - 4r_{00}^2 b^2\\
&+8nr_{00}r_0 b^4 - 4nr_{00}^2 b^4 - 32b^6\beta r_0 r_{00} + 8nr_{00}^2 b^6 - 6b^2 b^i r_{i00} - nb^i r_{i00} + 2r_{00}r_0 + 16nb^2\beta^2 r_{00} + 44b^2\beta r_0 r_{00} - 8n\beta r_0 r_{00}\\
&+14b^4\beta r_0 r_{00} + 4nr_0^2 b^4 + 2nr_{00}r_0 - b^i r_{i00}r\Big),
\end{aligned}
$$

$$
\begin{aligned}
A_{14} \;:=\;& 2(1+2b^2)\Big(8b^6\beta^3r_{00}^2+16nb^8\beta^2r_{000}-32nb^6\beta^3r_{00}+8b^6\beta^3r_{00}^2-32nb^6\beta^2r_0r_{00}+108nb^4\beta^3r_{00}^2-8b^8\beta^2r_{000}+16b^6\beta^3r_{00}\\
&+72nb^4\beta^3r_{00}+6b^4\beta^3r_{00}^2+216nb^4\beta^2r_0r_{00}-8nb^4\beta^2r_{00}^2-36nb^2\beta^3r_{00}^2-4b^6\beta^2r_{000}-24nb^6\beta b^ir_{i00}-8nb^3\beta r_0+16b^6\beta r_{00}\\
&-24nb^4\beta^2r_{000}-6b^4\beta^2r_0r_{00}-8b^4\beta^2r_{00}^2+72nb^4\beta r_0^2+24nb^4\beta r_0r_{00}-8b^4\beta r_{00}^2-24b^2\beta^3r_{00}^2+60nb^2\beta^2r_0r_{00}-8nb^2\beta^2r_{00}^2\\
&-8b^6\beta r_{00}-12nb^6r_{000}-8b^6r_0r_{00}-b^4\beta^2r_{000}-36nb^4\beta b^ir_{i00}-12nb^4\beta r_0-36b^4\beta r_0^2-12b^4\beta r_0r_{00}-b^4\beta r_{00}^2-4b^2\beta^2r_{000}\\
&+24nb^2\beta r_0r_{00}-38nb^2\beta r_{00}^2-4n\beta^3r_{00}-8\beta^3r_{00}^2-8n\beta^2r_0r_{00}-2n\beta^2r_{00}^4+4b^4\beta r_{00}-6nb^4r_{000}+30b^2r_0r_{00}-4b^2\beta^2r_{000}\\
&+6b^2\beta r_0r_{00}-8b^2\beta r_{00}^2-2b^2r_0r_{00}-4\beta^3r_{00}+n\beta^2r_{00}-8\beta^2r_0r_{00}-2\beta^2r_{00}^2+6\beta r_0r_{00}+2n\beta r_{00}^2-3b^4r_{000}-9b^2\beta b^ir_{i00}\\
&-\beta r_0+72nb^2\beta r_0^2+\beta^2r_{000}-3n\beta b^ir_{i00}+36\beta r_0^2+6\beta r_0r_{00}+2\beta r_{00}^2-4nr_0r_{00}-18nb^2\beta r_0b^ir_{i00}-3\beta b^ir_{00}+24b^4\beta^3r_{00}\\
&+9b^2r_0-12b^4\beta r_0+8nb^6\beta r_{00}^2-8nb^8r_{000}+16nb^6r_0r_{00}+16b^6\beta^2r_0r_{00}-8b^2\beta^2r_{00}^2+12b^6\beta b^ir_{i00}-nb^2r_{000}-b^2r_{000}r_{00}\\
&-6b^2\beta r_0+b^6\beta r_{00}^2-16nb^6\beta^2r_{000}+4b^8r_{000}-4r_0r_{00}-n\beta r_0+16nb^4\beta r_{00}+18b^2\beta^2r_0r_{00}+54b^2\beta r_0^2+4b^2\beta r_{00}-8n\beta^3r_{00}^2\\
&+4nb^2\beta r_{00}-8b^6\beta r_0\Big),
\end{aligned}
$$

$$
\begin{aligned}
A_{13} \;:=\;& -32n\,b^8\beta^4r_{00}^2+320nb^{10}\beta^3r_{000}-640\,nb^8\beta^4r_{00}-32\,b^8\beta^4r_{00}^2-64nb^8\beta^3r_0r_{00}-768\,nb^6\beta^4r_{00}^2+32\,b^{10}\beta^3r_{000}-64\,b^8\beta^4r_{00}\\
&-160nb^8\beta^2\,r_{00}^2-896nb^6\beta^4r_{00}+96\,b^6\beta^4r_{00}^2-1280n\,b^6\beta^3r_0r_{00}+96n\,b^6\beta^3r_{00}^2+504n\,b^4\beta^4r_{00}^2+64nb^{10}\beta r_{000}+224\,b^8\beta^3r_{000}\\
&-128nb^4\beta^2\,r_{00}-32nb^8\beta^2\,r_{000}-160nb^8\beta^2\,r_{00}^2-128n\,b^8\beta r_0r_{00}-224b^6\,\beta^4r_{00}+128nb^6\beta^3r_0+64n\,b^6\beta^3r_{00}+64b^6\beta^3r_0r_{00}\\
&+1120nb^6\beta^2\,r_{00}^2+48nb^2\beta^4\,r_{00}+144b^4\beta^4\,r_{00}^2+144nb^4\beta^3r_0\,r_{00}+144n\,b^4\beta^3r_{00}^2+64b^{10}\beta r_{000}-64b^8\beta^2\,b^ir_{i00}-128b^8\beta^2r_{00}\\
&+112nb^8\beta r_{000}-128b^8\beta r_0r_{00}+64nr_{00}^2b^8+128b^6\beta^3r_0+64b^6\beta^3r00+\beta^3r_{00}^2+152b^6\beta^3r_{000}+160nb^6\beta^2b^ir_{i00}+192nb^6\beta^2r_0\\
&+192b^6\beta^2\,r_0^2+192b^6\,\beta^2r_0\,r_{00}-272\,b^6\beta^2r_{00}^2-80n\,b^6\beta r_0r_{00}-32nb^6\beta r_{00}^2-144b^4\,\beta^4r_{00}+192nb^4\beta^3\,r_0+96nb^4\beta^3\,r_{00}\\
&+144b^4\beta^3r_0r_{00}+416nb^8\beta^3r_{000}-64nb^6\beta^2r_{000}+34nb^2\beta^4r_{00}^2+26nb^6\beta^3r_{000}+12b^8\beta^2b^ir_{i00}+48b^4\beta^3r_{00}^2-360nb^4\beta^2r_0^2\\
&+114nb^4\beta^2r_{00}^2+26nb^2\beta^4r_{00}+22b^2\beta^4r_{00}^2+58nb^2\beta^3r_0r_{00}+72nb^2\beta^3r_{00}^2-4n\beta^4r_{00}^2+112b^8\beta r_{000}-176b^8r_{00}^2-128b^6\beta^2b^ir_{i00}\\
&-36b^6\beta^2r_{00}-6b^6\beta^2r_{000}-48\beta^4r_{00}^2+6nb^6\beta r_{000}-68b^6\beta r_0r_{00}-32b^6\beta r_{00}^2+8nr_{00}^2b^6+192b^4\beta^3r_0+48b^2\beta^3r_{00}^2+96b^4\beta^3r_{00}\\
&+48b^4\beta^2b^ir_{i00}+144nb^4\beta^2r_0-144nb^4\beta^2r_{00}+6\beta^2r_0-48nb^4\beta^2r_{000}+24b^4\beta^2r_0^2+264b^4\beta^2r_0r_{00}+432b^4\beta^2r_{00}^2+96nb^4\beta r_0r_{00}\\
&-48nb^4\beta r_{00}^2+28b^2\beta^4r_{00}+96nb^2\beta^3r_0+48nb^2\beta^3r_{00}-56nb^2\beta^3r_{000}+16\beta^3r_0+28b^2\beta^3r_0r_{00}-72nb^2\beta^2r_0^2+72nb^2\beta^2r_0r_{00}\\
&+92n\beta^3r_0r_{00}+12n\,\beta^3r_{00}^2+64b^6\beta r_{000}-152\,r_{00}^2b^6-96b^4\beta^2b^ir_{i00}+72b^4\beta^2r_0-216b^4\beta^2r_{00}-48b^4\beta^2r_{000}+8nb^4\beta r_{000}\\
&+96b^2\beta^3r_0+48b^2\beta^3r_{00}-4b^2\beta^3r_{000}-8nb^2\beta^2b^ir_{i00}+48nb^2\beta^2r_0-64nb^2\beta^2r_{00}-16nb^2\beta^2r_{000}-120b^2\beta^2r_0^2+12b^2\beta^2r_0r_{00}\\
&-48b^4\beta r_{00}^2+36nr_{00}^2b^4+92n\,b^2\beta r_0r_{00}-24nb^2\beta r_{00}^2+8\beta^4r_{00}+16n\beta^3r_0+8n\beta^3r_{00}-8n\,\beta^3r_{000}+116\beta^3r_0r_{00}+18n\beta^2r_0r_{00}\\
&+8b^4\beta r_{000}-r_{00}^2-24r_{00}^2b^4-2b^2\beta^2b^ir_{i00}+36b^2\beta^2r_0-6b^2\beta^2r_{00}-16b^2\beta^2r_{000}-4nb^2\beta r_{000}-4b^2\beta r_0r_{00}-4b^2\beta r_{00}^2+nb^2r_{00}^2\\
&-8\beta^3r_{000}-4n\beta^2b^ir_{i00}+6n\beta^2r_0-12n\beta^2r_{00}-2n\beta^2r_{000}-42\beta^2r_0^2+18\beta^2r_0r_{00}-52\beta^2r_{00}^2+20n\beta r_0r_{00}-4\beta r_{00}^2-4b^2\beta r_{000}\\
&-2\beta^2r_{000}-n\beta r_{000}+20\beta r_0r_{00}-4\beta r_{00}^2-nr00^2-\beta r_{000}+8\beta^3r_{00}+6n\beta^2r_0^2-4\beta^2b^ir_{i00}-32b^8\beta^2r_{000}-64b^8\beta^3r_0r_{00}\\
&+16b^2\beta^2r_{00}^2+14nb^2\beta^2r_{00}^2+96nb^8\beta^2r_0+2\,r_{00}^2b^2-12\beta^2r_{00}-384n\,b^6\beta^2r_0^2-192nb^6\beta^2r_{00}-52n\beta^2r_{00}^2-32n\,b^4\beta^3\,r_{000}\\
&+72nb^4\beta^2r_0r_{00}+48b^6\beta^2r_0-432\,b^4\beta r_0r_{00}-32b^4\beta^3r_{000}+8n\beta^4r_{00},
\end{aligned}
$$

$$
\begin{aligned}
A_{12} \;:=\;& -\beta\,\Big(32nb^{10}\beta^3r_{000}-64nb^8\beta^4r_{00}-64nb^8\beta^3r_0r_{00}+32b^{10}\beta^3r_{000}-64b^8\beta^4r_{00}+496nb^8\beta^3r_{000}-64b^8\beta^3r_0r_{00}-28nb^8\beta^2r_{00}^2\\
&-49nb^6\beta^3r_0r_{00}+32nb^6\beta^3r_{00}^2+2184nb^4\beta^4r_{00}^2+64nb^{10}\beta r_{000}-224b^8\beta^3r_{000}-48nb^8\beta^2b^ir_{i00}+32nb^8\beta^2r_0-128nb^8\beta^2r_{00}\\
&-128nb^8\beta\,r_0r_{00}+480b^6\beta^4r_{00}+256nb^6\beta^3r_0+192nb^6\beta^3r_{00}-24nb^6\beta^3r_{000}+224\,b^6\beta^3r_0r_{00}-64b^6\beta^3r_{00}^2+144nb^6\beta^2r_0^2\\
&+1104nb^4\beta^4r_{00}+792b^4\beta^4r_{00}^2+2592nb^4\beta^3r_0r_{00}-240nb^4\beta^3r_{00}^2+168nb^2\beta^4r_{00}^2-320b^{10}\beta r_{000}+96b^8\beta^2b^ir_{i00}-604\,b^8\beta^2r_0\\
&+32nr_{00}^2b^8-128b^6\beta^3\,r_0-168b^6\beta^3r_{000}-672n\,b^6\beta^2b^ir_{i00}-128nb^6\beta^2r_0-256n\,b^6\beta^2\,r_{00}-192nb^6\beta^2r_{000}-288b^6\beta^2\,r_0^2\\
&-592nb^3\beta r_0r_{00}-112nb^6\beta r_{00}^2+384b^4\beta^4r_{00}+192nb^4\beta^3r_{00}-308nb^4\beta^3r_{000}-72b^4\beta^3r_0r_{00}-336b^4\beta^3r_{00}^2+112nb^4\beta^2r_0^2\\
&+208nb^2\beta^4r_{00}+42b^2\beta^4r_{00}^2+72nb^2\beta^3r_0r_{00}-24nb^2\beta^3r_{00}^2-30n\beta^4r_{00}^2+152b^8\beta r_{000}-12r_{00}^2b^8+312b^6\beta^2b^ir_{i00}-272b^6\beta^2r_0\\
&-12nb^6\beta r_{000}-856b^6\beta r_0r_{00}+32b^6\beta r_{00}^2+48nr_{00}^2b^6-12b^4\beta^3r_0-14n\beta^4r_{00}-96nb^4\beta^2b^ir_{i00}-20nb^4\beta^2r_0-12nb^4\beta^2r_{00}\\
&+96\,b^4\beta^2r_0r_{00}+744b^4\,\beta^2r_{00}^2-840nb^4\beta r_0r_{00}-168nb^4\beta r_{00}^2+208\,b^2\beta^4r_{00}-192n\,b^2\beta^3r_0+48nb^2\beta^3r_{00}-52nb^2\beta^3r_{000}\\
&-28b^2\beta^3r_{00}^2-240b^4\beta^2r_0^2+86nb^4\beta^2r_{00}^2+64b^8\beta r_0r_{00}+56b^6\beta^4r_{00}^2+1888nb^6\beta^2r_{00}^2+52b^8\beta^2r_{00}^2-16nb^8\beta r_{000}+92b^6\beta^2r_{00}^2\\
&+1404nb^2\beta^2r_0^2+756nb^2\beta^2r_0r_{00}-36nb^2\beta^2r_{00}^2-14n\beta^4r_{00}-288\beta^4r_{00}^2-320n\beta^3r_0r_{00}-68n\beta^3r_{00}^2+240b^6\beta r_{000}+256r_{00}^2b^6\\
&-72b^4\beta^2r_{000}-76nb^4\beta r_{000}-12b^4\beta r_0r_{00}-24b^4\beta r_{00}^2+56nr_{00}^2b^4-96b^2\beta^3r_0-16b^2\beta^3r_{000}-46nb^2\beta^2b^ir_{i00}-12nb^2\beta^2r_0\\
&+1056b^2\beta^2\,r_0^2+384b^2\beta^2\,r_0r_{00}+124b^2\beta^2\,r_{00}^2-460nb^2\beta r_0r_{00}-84nb^2\beta r_{00}^2+38r_{00}^2-64n\beta^3r_0+18n\beta^3r_{000}-248\beta^3r_0r_{00}\\
&-148n\beta^2r_{00}^2+110b^4\beta r_{000}+636r_{00}^2b^4-186b^2\beta^2b^ir_{i00}-140b^2\beta^2r_0-16b^2\beta^2r_{00}-36b^2\beta^2r_{000}-14nb^2\beta r_{000}-298b^2\beta r_0r_{00}\\
&+18\beta^3\,r_{000}-75n\beta^2\,b^ir_{i00}-22n\,\beta^2r_0-8n\beta^2r_{00}-6n\,\beta^2r_{000}+552\beta^2\,r_0^2-8\beta^2r_{00}-148\beta^2r_{00}^2-86n\beta r_0r_{00}-14n\beta\,r_{00}^2\\
&-60\beta^2b^ir_{i00}-14\beta r_{00}^2-6\beta^2r_{000}+28\beta r_0r_{00}+38nr_{00}^2-48b^6\beta^2r_{000}-128b^4\beta^3r_{000}+156\beta^2r_0r_{00}+162n\beta^2r_0r_{00}-22\beta^2r_0\\
&+232b^2\beta^3r_0r_{00}-62nb^6\beta^4r_{00}-96nb^8\beta^2r_{000}+432n\,b^6\beta^2r_0r_{00}+316b^2r_{00}^2-48nb^2\beta^2r_{000}+256nb^2r_{00}^2-68\beta^3r_{00}^2\\
&-48b^2\beta r_{00}^2+48b^4\beta^2r_{00}+640b^8\beta^2r_{00}-96\,b^6\,\beta^2r_0r_{00}+108nb^4\beta^2r_0r_{00}+12b^6\beta^2r_{00}-14nb^4\beta^2r_{000}+16\,b^2\beta r_{000}\\
&-16\beta^3r_0-312b^4\beta^2r_0-64nb^2\beta^2r_{00}+342n\,\beta^2r_0^2\Big),
\end{aligned}
$$

$$
\begin{aligned}
A_{11} \;:=\; & -\beta^2\big(336nb^8\beta^3 r_{000} - 512nb^6\beta^4 r_{00}^2 - 176nb^8\beta^2 r_{00}^2 - 672nb^6\beta^4 r_{00} - 560b^6\beta^4 r_{00}^2 - 1008n\,b^6\beta^3 r_0 r_{00} + 64nb^6\beta^3 r_{00}^2 \\
& +336b^8\beta^3 r_{000} + 80\,b^8\beta^2 b^i r_{i00} + 64n\,b^8\beta^2 r_0 - 128nb^8\beta^2 r_{00} + 32nb^8\beta^2 r_{000} - 128nb^8\beta r_0 r_{00} - 672b^6\beta^4 r_{00} + 128nb^6\beta^3 r_0 \\
& -12b^8\beta^2 r_{00}^2 - 108b^6\beta^3 r_0 r_{00} + 64b^6\beta^3 r_{00}^2 - 240nb^6\beta^2 r_0^2 - 24\beta r_{000} - 70r_{00}^2 - 20nb^6\beta^2 r_0 r_{00} - 20nb^6\beta^2 r_{00}^2 - 24nb^4\beta^4 r_{00} \\
& -2208nb^4\beta^3 r_0 r_{00} + 1152nb^4\beta^3 r_{00}^2 + 528nb^2\beta^4 r_{00}^2 + 64b^{10}\beta r_{000} + 128nb^8 r_{00}^2 + 80b^8\beta^2 b^i r_{i00} + 64b^8\beta^2 r_0 - 128\,b^8\beta^2 r_{00} \\
& +176nb^8\beta r_{000} - 128b^8\beta r_0 r_{00} + 128b^6\beta^3 r_{00} + 888b^6\beta^3 r_{000} + 704nb^6\beta^2 r_0 - 160nb^6\beta^2 r_{00} - 176nb^6\beta^2 r_{000} - 240b^6\beta^2 r_0^2 \\
& +1392nb^6\beta r_0 r_{00} + 32nb^6\beta r_{00}^2 - 528b^4\beta^4 r_{00} + 1536nb^4\beta^3 r_0 + 576nb^4\beta^3 r_{00} + 42nb^4\beta^3 r_{000} - 768b^4\beta^3 r_0 r_{00} + 384b^4\beta^3 r_{00}^2 \\
& +936nb^4\beta^2 r_{00}^2 + 576nb^2\beta^4 r_{00} - 480b^2\beta^4 r_{00}^2 + 2400nb^2\beta^3 r_0 r_{00} + 1104nb^2\beta^3 r_{00}^2 + 128n\beta^4 r_{00}^2 + 512b^8\beta\, r000 - 544b^8 r_{00}^2 \\
& -928b^6\beta^2 r_{00} - 320b^6\beta^2 r_{000} - 24nb^6\beta\, r_{000} - 1536b^6\beta\, r_0 r_{00} + 1056nb^4\beta^2 b^i r_{i00} - 64b^6\beta r_{00}^2 - 272nb^6 r_{00}^2 + 384\,b^4\beta^3 r_0 \\
& +960nb^4\beta^2 r_0 - 768nb^4\beta^2 r_{00} - 312nb^4\beta^2 r_{000} - 456b^4\beta^2 r_0^2 + 936b^4\beta^2 r_0 r_{00} - 792b^4\beta^2 r_{00}^2 + 2472nb^4\beta r_0 r_{00} - 144nb^4\beta r_{00}^2 \\
& +62nb^2\beta^3 r_{00} - 300nb^2\beta^3 r_{000} + 1104b^2\beta^3 r_0 r_{00} + 696b^2\beta^3 r_{00}^2 - 1584nb^2\beta^2 r_0^2 + 24nb^2\beta^2 r_0 r_{00} + 62nb^2\beta^2 r_{00}^2 + 240n\beta^4 r_{00} \\
& +64b^6 r_{00}^2 + 48b^6\beta^2 b^i r_{i00} + 46b^4\beta^2 r_0 - 888b^4\beta^2 r_{00} - 384b^4\beta^2 r_{000} - 220nb^4\beta r_{000} + 912b^4\beta r_0 r_{00} - 20b^4\beta r_{00}^2 - 64nb^4 r_{00}^2 \\
& +272n\beta^3 r_{00}^2 + 408\,b^6\beta r_{000} + 1248n\beta^3 r_0 r_{00} + 1440n\,b^2\beta^3 r_0 + 360\,b^4\beta^3 r_{00} + 160n\beta^3 r_{00} + 70\,\beta^2 r_0 - 24nb^4\beta^2 r_0 r_{00} \\
& +128b^6\beta^3 r_0 + 1416nb^6\beta^3 r_{000} + 64nb^{10}\beta r_{000} - 192b^2\beta^3 r_{000} + 400nb^2\beta^2 b^i r_{i00} + 464nb^2\beta^2 r_0 - 102n\beta^3 r_{000} - 44n\beta r_{00}^2 \\
& -784nb^2\beta^2 r_{00} - 164nb^2\beta^2 r_{000} - 1392b^2\beta^2 r_0^2 + 624b^2\beta^2 r_0 r_{00} - 436b^2\beta^2 r_{00}^2 + 1436nb^2\beta r_0 r_{00} - 168nb^2\beta r_{00}^2 + 216\beta^4 r_{00} \\
& +1320\beta^3 r_0 r_{00} + 26\beta^3 r_{00}^2 - 30n\beta^2 r_0^2 + 10n\beta^2 r_0 r_{00} - 28n\beta^2 r_{00}^2 - 40b^4\beta r_{000} + 684b^4 r_{00}^2 + 112b^2\beta^2 b^i r_{i00} + 320b^2\beta^2 r_0 \\
& +2288b^2\beta r_0 r_{00} - 192b^2\beta r_{00}^2 - 380nb^2 r_{00}^2 + 496\beta^3 r_0 + 160\beta^3 r_{00} - 96\beta^3 r_{000} + 47n\beta^2 b^i r_{i00} + 76n\beta^2 r_0 - 212n\beta^2 r_{00} \\
& -23\beta^2 r_{00}^2 + 28n\beta r_0 r_{00} - 110b^2\beta r_{000} - 32b^2 r_{00}^2 + 35\beta^2 b^i r_{i00} - 212n\beta^2 r_{00} - 28\beta^2 r_{000} - 24n\beta r_{000} + 72\beta r_0 r_{00} - 44n\beta r_{00}^2 \\
& +352n\beta^3 r_0 - 176b^2\beta^2 r_{000} - 760b^2\beta^2 r_{00} + 224b^6\beta^2 r_0 - 70nr_{00}^2 - 134nb^2\beta r_{000} - 28n\beta^2 r_{000} + 928nb^6\beta^2 b^i r_{i00} - 612\beta^2 r_0^2 \\
& +100\beta^2 r_0 r_{00} - 464b^6\beta^2 r_{00}^2 + 528b^2\beta^3 r_{00} + 12b^2\beta^3 r_0 - 96b^2\beta^4 r_{00} + 104\beta^4 r_{00}^2 + 480\,b^4\beta^3 r_{00} - 32b^6\beta^2 b^i r_{i00} \\
& -64b^8\beta^2 r_{000} - 144b^4\beta^4 r_{00}^2 - 64nb^6\beta^3 r_{00} - 3168nb^4\beta^4 r_{00}^2 + 176b^6\beta^2 r_0 r_{00} - 208nb^4\beta^2 r_0^2\big),
\end{aligned}
$$

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\begin{aligned}
A_{10} \;:=\; & \beta^3\big(-192nb^6\beta^4 r_{00}^2 + 272nb^8\beta^3 r_{000} - 144nb^8\beta^2 r_{00}^2 - 448nb^6\beta^4 r_{00} - 560n\,b^6\beta^3 r_0 r_{00} + 32nb^6\beta^3 r_{00}^2 - 408nb^4\beta^4 r_{00}^2 \\
& +16nb^8\beta^2 b^i r_{i00} + 32nb^8\beta^2 r_0 - 12nb^8\beta^2 r_{00} - 64nb^8\beta^2 r_{000} + 96b^8\beta^2 r_{00}^2 - 128nb^8\beta r_0 r_{00} - 54b^6\beta^4 r_{00} + 128nb^6\beta^3 r_0 \\
& -104b^6\beta^3 r_0 r_{00} + 32b^6\beta^3 r_{00}^2 - 48nb^6\beta^2 r_0^2 + 20nb^6\beta^2 r_0 r_{00} - 768nb^6\beta^2 r_{00}^2 - 52nb^4\beta^4 r_{00} + 228b^4\beta^4 r_{00}^2 + 92nb^4\beta^3 r_0 r_{00} \\
& +16b^{10}\beta r_{000} + 12b^8\beta^2 b^i r_{i00} + 32b^8\beta^2 r_0 - 32b^8\beta^2 r_{00} - 64b^8\beta^2 r_{000} - 32b^8\beta r_0 r_{00} - 16nb^8 r_{00}^2 + 18b^6\beta^3 r_0 + 128b^6\beta^3 r_{00} \\
& -26nb^6\beta^2 b^i r_{i00} + 26nb^6\beta^2 r_0 - 64nb^6\beta^2 r_{00} - 88nb^6\beta^2 r_{000} + 144b^6\beta^2 r_0 r_{00} + 3168b^6\beta^2 r_{00}^2 + 64nb^6\beta r_0 r_{00} - 96nb^6\beta r_{00}^2 \\
& +1440nb^4\beta^3 r_{00} - 336b^6\beta^2 r_0^2 - 60nb^4\beta^3 r_{000} - 28b^4\beta^3 r_0 r_{00} - 432b^4\beta^3 r_{00}^2 + 56nb^4\beta^2 r_0^2 + 316nb^4\beta^2 r_0 r_{00} + 256nb^4\beta^2 r_{00}^2 \\
& +3924n\,b^2\beta^3 r_0 r_{00} + 96n\,b^2\beta^3 r_{00}^2 - 1332\,n\beta^4 r_{00}^2 + 88b^8\beta r_{000} - 256b^8 r_{00}^2 + 680b^6\beta^2 b^i r_{i00} - 224\,b^6\beta^2 r_0 + 128b^6\beta^2 r_{00} \\
& -1928b^6\beta r_0 r_{00} + 366nb^6 r_{00}^2 + 12b^4\beta^3 r_{00} + 48b^4\beta^3 r_{000} - 1704nb^4\beta^2 b^i r_{i00} - 96nb^4\beta^2 r_0 - 282nb^4\beta^2 r_{00} - 116nb^4\beta^2 r_{000} \\
& +330b^4\beta^2 r_{00}^2 - 154nb^4\beta r_0 r_{00} - 816nb^4\beta r_{00}^2 + 52b^2\beta^4 r_{00} - 288nb^2\beta^3 r_0 + 912nb^2\beta^3 r_{00} - 20nb^2\beta^3 r_{000} - 1080b^2\beta^3 r_0 r_{00} \\
& +3636nb^2\beta^2 r_0 r_{00} - 1836nb^2\beta^2 r_{00}^2 - 632n\,\beta^4 r_{00} - 1200\beta^4 r_{00}^2 + 44n\beta^3 r_0 r_{00} - 176n\beta^3 r_{00}^2 + 976b^6\beta r_{000} + 1360r_{00}^2 b^6 \\
& -528b^4\beta^2 r_{000} - 100nb^4\beta\, r_{000} - 3504b^2\beta r_0 r_{00} - 24b^4\beta r_{00}^2 + 6840nr_{00}^2 b^4 - 96\,b^2\beta^3 r_0 + 240b^2\beta^6 r_{00} + 16b^2\beta^3 r_{000} \\
& -2272nb^2\beta^2 r_{00} - 572nb^2\beta^2 r_{000} + 600b^2\beta^2 r_0^2 + 1992b^2\beta^2 r_0 r_{00} - 588b^2\beta^2 r_{00}^2 - 1480nb^2\beta r_0 r_{00} - 744nb^2\beta r_{00}^2 - 608\beta^4 r_{00} \\
& +1176nb^2\beta^4 r_{00}^2 + 1312nb^6\beta^3 r_{000} + 320b^8\beta^3 r_{000} - 60b^4\beta^2 r_0 + 624b^2\beta^4 r_{00}^2 + 960nb^4\beta^3 r_0 - 336b^6\beta^2 r_0^2 + 160nb^6\beta r_{000} \\
& +2556nb^2\beta^2 r_0^2 + 40b^4\beta^2 r_0 r_{00} + 32n\beta^3 r_{000} + 112n\beta^3 r_{00} + 128\beta^3 r_0 r_{00} - 188\beta^3 r_{00}^2 + 128n\beta^2 r_0^2 + 106n\beta^2 r_0 r_{00} - 164n\beta^2 r_{00}^2 \\
& +1028b^4\beta r_{000} + 4632r_{00}^2\, b^4 - 196b^2\beta^2 b^i\, r_{i00} - 416\,b^2\beta^2 r_0 - 1120\,b^2\beta^2 r_{00} + 14\beta r_{000} - 392\,b^2\beta^2 r_{000} - 64nb^2\beta r_{000} \\
& +88\beta^3 r_{00} + 44\beta^3 r_{000} - 359n\beta^2 b^i r_{i00} - 82n\beta^2 r_0 - 500n\beta^2 r_{00} - 94n\beta^2 r_{000} + 1056\beta^2 r_0 r_{00} - 1392\beta^2 r_{00}^2 - 328n\beta r_0 r_{00} \\
& -206\beta^2 b^i r_{i00} - 88\beta^2 r_0 - 46\beta^2 r_{00} - 88\beta^2 r_{000} - 7n\beta r_{000} + 680\beta r_0 r_{00} - 132n\beta r_{00}^2 - 192b^6\beta^4 r_{00}^2 - 276b^2\beta r_{00}^2 + 404nr_{00}^2 b^2 \\
& -32\beta^3 r_0 + 352nb^8\beta r_{000} + 144b^4\beta^2 r_{00} + 80r_{00}^2 + 18\beta^2 r_0^2 + 308b^2\beta r_{000} + 356r_{00}^2 b^2 - 180n\beta r_{00}^2 - 940b^2\beta r_0 r_{00} + 92nb^4\beta^3 r_{00}^2 \\
& +128nb^6\beta^3 r_{00} + 64nb^{10}\beta r_{000} - 1200b^4\beta^2 r_0^2 - 60b^2\beta^3 r_{00}^2 + 74\,b^4\beta^2 b^i r_{i00} - 1504nb^2\beta^2 b^i r_{i00} - 14b^6\beta^3 r_{000} + 624b^4\beta^4 r_{00} \\
& +96nb^2\beta^4 r_{00} - 224\,b^6\beta^2 r_{000} - 368n\beta^3 r_0 - 272n\,b^2\beta^2 r_0 + 767r_{00}^2\big),
\end{aligned}
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\begin{aligned}
A_9 \; := \; & \beta^4\Big(-2328nb^4\beta^4r_{00}^2 + 32nb^8\beta^2r_{000} - 64nb^6\beta^3r_{00} + 936nb^6\beta^3r_{000} - 128nb^6\beta^2r_0r_{00} - 2592nb^6\beta^2r_{00}^2 - 1536nb^4\beta^4r_{00} \\
& -12nb^4\beta^3r_0r_{00} + 38nb^4\beta^3r_{00}^2 - 144nb^2\beta^4r_{00}^2 + 32b^8\beta^2r_{000} + 16nb^8\beta r_{000} - 160nb^8r_{00}^2 - 64b^6\beta^3r_{00} + 102b^6\beta^3r_{000} \\
& -224nb^6\beta^2r_{00} + 34nb^6\beta^2r_{000} - 128b^6\beta^2r_0r_{00} - 2256b^6\beta^2r_{00}^2 + 78nb^6\beta r_0r_{00} + 48nb^6\beta r_{00}^2 - 182b^4\beta^4r_{00} + 960nb^4\beta^3r_0 \\
& -256b^4\beta^3r_0r_{00} + 108b^4\beta^3r_{00}^2 - 1080nb^4\beta^2r_0^2 - 1416nb^4\beta^2r_0r_{00} - 4944nb^4\beta^2r_{00}^2 - 240nb^2\beta^4r_{00} - 2160b^2\beta^4r_{00}^2 \\
& +176b^8\beta r_{000} + 80b^8r_{00}^2 + 576b^6\beta^2b^ir_{i00} + 42b^6\beta^2r_0 - 80b^6\beta^2r_{00} - 320b^6\beta^2r_{000} - 78nb^6\beta r_{000} + 40b^6\beta r_0r_{00} - 584nb^6r_{00}^2 \\
& +152b^4\beta^3r_{000} + 2448n\,b^4\beta^2r_{00} + 1872n\,b^4\beta^2r_0 + 1632n\,b^4\beta^2r_{00} - 240nb^4\beta^2r_{000} - 1560\,b^4\beta^2r_0^2 + 504b^4\beta^2r_0r_{00} \\
& -720b^2\beta^4r_{00} + 4128n\,b^2\beta^3r_0 + 1776nb^2\beta^3r_{00} - 756nb^2\beta^3r_{000} + 2280b^2\beta^3r_0r_{00} + 2304b^2\beta^3r_{00}^2 - 3672nb^2\beta^2r_0^2 \\
& +1440\beta^4r_{00}^2 + 4440n\,\beta^3r_0r_{00} + 171n\,\beta^3r_{00}^2 + 736\,b^6\beta r_{000} + 952b^6r_{00}^2 + 1080\beta^4r_{00} + 864b^4\beta^2b^ir_{i00} + 1080b^4\beta^2r_0 \\
& +450b^4\beta r_0r_{00} - 240b^4\beta r_{00}^2 - 104nb^4r_{00}^2 + 2400b^2\beta^3r_0 + 1296b^2\beta^3r_{00} - 432b^2\beta^3r_{000} + 182nb^2\beta^2b^ir_{i00} + 154nb^2\beta^2r_0 \\
& -3144b^2\beta^2r_0^2 + 744b^2\beta^2r_0r_{00} - 2508b^2\beta^2r_{00}^2 + 7212nb^2\beta r_0r_{00} + 12nb^2\beta r_{00}^2 + 1824n\beta^3r_0 + 952n\beta^3r_{00} - 504n\beta^3r_{000} \\
& +1644n\beta^2r_0r_{00} + 744n\beta^2r_{00}^2 - 384b^4\beta r_{000} + 1908b^4r_{00}^2 + 864b^2\beta^2b^ir_{i00} + 1080b^2\beta^2r_0 - 960b^2\beta^2r_{00} - 632b^2\beta^2r_{000} \\
& -312b^2\beta r_{00}^2 - 6236ntb^2tr_{00}^2 + 256\beta^3r_0 + 904\beta^3r_{00} - 456\beta^3r_{000} + 384n\beta^2b^ir_{i00} + 372n\beta^2r_0 - 472n\beta^2r_{00} \\
& +166n\beta r_0r_{00} + 50\beta^2r_{00}^2 - 54n\beta r_{00}^2 - 740b^2\beta r_{000} - 44b^2r_{00}^2 + 288\beta^2r_{00} + 3244\beta^2r_0 + 13344b^2\beta r_0r_{00} \\
& +68\beta^2r_0\,r_{00} + 158\beta^3r_{00}^2 - 104n\beta^2r_0^2 + 1848n\beta^4r_{00}^2 + 672b^4\beta^3r_{00} + 264nb^4\beta r_{00}^2 + 1344n\,\beta^4r_{00} + 1296n\,b^4\beta^3r_{000} \\
& -888b^4\beta^2r_{000} + 2256nb^2\beta^2r_{00}^2 + 8400nb^4\beta r_0r_{00} + 3624nb^2\beta^3r_{00}^2 - 280\beta^2r_{00} - 136\beta^2r_{000} - 205n\beta r_{000} + 5376\beta r_0r_{00} \\
& -12nr_{00}^2 - 193\beta r_{000} - 33r_{00}^2b^2\beta^2b^ir_{i00} + 100b^2\beta^2r_0 - 960b^2\beta^2r_{00} - 632b^2\beta^2r_{000} - 472n\beta^2r_{00} + 372n\beta^2r_0 - 130n\beta^2r_{000} \\
& -1046nb^2\beta r_{000} + 13344b^2\beta r_0r_{00} - 312b^2\beta r_{00}^2 - 62ntb^2tr_{00}^2 + 226\beta^3r_0 + 94\beta^3r_{00} - 46\beta^3r_{000} + 384n\beta^2b^ir_{i00} - 136\beta^2r_{000} \\
& -2100\beta^2r_0^2 + 68\beta^2r_0r_{00} + 504\beta^2r_{00}^2 + 1656n\beta r_0r_{00} - 54n\beta r_{00}^2 - 740b^2\beta r_{000} - 44b^2r_{00}^2 + 288\beta^2r_{00} + 324\beta^2r_0 - 280\beta^2r_{00} \\
& -96\beta r_{00}^2 - 1225nr_{00}^2 - 193\beta r_{000} + 5376\beta r_0r_{00} - 205n\beta r_{000} - 331r_{00}^2 + 3432n\,b^2\beta^3r_0r_{00} - 28nb^4\beta^3r_{00} + 40nb^6\beta^2b^ir_{i00} \\
& -2736b^4\beta^4r_{00}^2 - 840\,b^4\beta^2r_{00} - 456nb^2\beta^2r_0r_{00} - 42nb^2\beta^2r_{000} - 130n\beta^2r_{000} - 4704\,b^4\beta^2r_{00}^2 + 90b^4\beta^3r_0\Big),
\end{aligned}
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\begin{aligned}
A_8 \; := \; & -\beta^5\Big(-1056nb^4\beta^4r_{00}^2 + 696nb^6\beta^3r_{000} - 1504nb^6\beta^2r_{00}^2 - 720nb^4\beta^4r_{00} - 1344b^4\beta^4r_{00}^2 - 336nb^4\beta^3r_0r_{00} + 672nb^4\beta^3r_{00}^2 \\
& -208nb^8r_{00}^2 + 1032b^6\beta^3r_{000} + 96nb^6\beta^2b^ir_{i00} + 192nb^6\beta^2r_0 - 384nb^6\beta^2r_{00} - 480nb^6\beta^2r_{000} - 640b^6\beta^2r_{00}^2 + 112nb^6\beta r_0r_{00} \\
& +576nb^4\beta^3r_0 + 86nb^4\beta^3r_{00} + 144nb^4\beta^3r_{000} - 68b^4\beta^3r_0r_{00} + 720b^4\beta^3r_{00}^2 - 216nb^4\beta^2r_0^2 + 166nb^4\beta^2r_0r_{00} - 240nb^4\beta^2r_{00}^2 \\
& +5676nb^2\beta^3r_0r_{00} + 4056n\,b^2\beta^3r_{00}^2 - 1524n\beta^4r_{00}^2 - 24b^8\beta r_{000} + 176b^8r_{00}^2 + 648b^6\beta^2b^ir_{i00} + 240b^6\beta^2r_0 + 192b^6\beta^2r_{00} \\
& +184b^6\beta\,r_0r0_0 - 32b^6\beta r_{00}^2 - 632n\,b^6r_{00}^2 + 960b^4\beta^3r_0 + 96b^4\beta^3r_{00} + 576b^4\beta^3r_{000} - 504nb^4\beta^2b^ir_{i00} + 720nb^4\beta^2r_0 \\
& +1536b^4\beta^2r_0r_{00} + 2472\,b^4\beta^2r_{00}^2 + 2808\,nb^2\beta r_0r_{00} - 552\,nb^4\beta\,r_{00}^2 + 336b^2\beta^4r_{00} + 1152n\,b^2\beta^3r_0 + 3456n\,b^2\beta^3r_{00} \\
& +756nb^2\beta^2r_0^2 + 7164n\,b^2\beta^2r_0\,r_{00} - 4680nb^2\beta^2r_{00}^2 - 720n\beta^4r_{00} - 1632\beta^4r_{00}^2 + 3084n\beta^3r_0r_{00} + 1212n\beta^3r_{00}^2 + 528b^6\beta r_{000} \\
& -72b^4\beta^2r_0 - 720b^4\beta^2r_{00} - 1368b^4\beta^2r_{000} - 288nb^4\beta r_{000} - 156b^4\beta r_0r_{00} + 52r_{00}^2 - 264b^4\beta r_{00}^2 + 154nb^4r_{00}^2 + 1248b^2\beta^3r_0 \\
& -1872nb^2\beta^2b^ir_{i00} + 144nb^2\beta^2r_0 - 630n\,b^2\beta^2r_{00} - 2304n\,b^2\beta^2r_{000} - 1536\,b^2\beta^2r_0^2 + 4128b^2\beta^2r_0r_{00} - 3708b^2\beta^2r_{00}^2 \\
& +1080n\beta^3r_{00} - 156n\beta^3r_{000} + 226\beta^3r_0r_{00} + 900\beta^3r_{00}^2 + 1404n\beta^2r_0^2 + 10\beta r_{000} + 3492n\beta^2r_0r_{00} - 5396n\beta^2r_{00}^2 + 180b^4\beta r_{000} \\
& -360b^2\beta^2r_0 - 264b^2\beta^2r_{00} - 1512b^2\beta^2r_{000} - 378nb^2\beta r_{000} + 308b^2\beta r_0r_{00} - 636b^2\beta r_{00}^2 + 1750nb^2r_{00}^2 + 384\beta^3r_0 + 816\beta^3r_{00} \\
& -84n\beta^2r_0 - 2736n\beta^2r_{00} - 546n\beta^2r_{000} + 984\beta^2r_0^2 + 3408\beta^2r_0r_{00} - 4496\beta^2r_{00}^2 - 244n\beta r_0r_{00} - 556n\beta r_{00}^2 + 1140b^2\beta r_{000} \\
& -1992\beta^2r_{00} - 480\beta^2r_{000} - 87n\,\beta r_{000} + 4016\beta r_0r_{00} - 148\beta r_{00}^2 + 4889nr_{00}^2 - 432n\beta^3r_0 - 816\beta^4r_{00} + 1728\,b^4\beta^2b^ir_{i00} \\
& -798n\beta^2b^ir_{i00} + 756b^2\beta^2b^ir_{i00} + 384nb^6\beta r_{000} - 132\beta^2r_0 - 264b^2\beta^4r_{00}^2 - 1248b^4\beta^4r_{00} + 288nb^8\beta r_{000} + 72b^2\beta^3r_{000} \\
& -216\beta^2b^ir_{i00} - 1392b^4\beta^2r_0^2 - 1392nb^2\beta r_{00}^2 - 612nb^2\beta^3r_{000} - 528\,b^6\beta^2r_{000} + 56nb^2\beta^4r_{00} + 16nb^6\beta r_{00}^2 - 336nb^2\beta^4r_{00}^2 \\
& +240nb^2\beta r_0r_{00} + 1248b^2\beta^3r_{00} + 404b^6r_{00}^2 - 1416b^2\beta^3r_0r_{00} - 1536nb^4\beta^2r_{00} - 2664nb^4\beta^2r_{000} + 432b^2\beta^3r_{00}^2 + 14704b^2r_{00}^2 \\
& -60\beta^3r_{000} + 1168b^4r_{00}^2 - 96\beta r_{00}^2 + 384n\,b^6\beta^2r_0 - 160nb^4\beta r_{000} - 2100\beta^2r_0^2 - 1046nb^2\beta r_{000} + 4512\beta^3r_0r_{00} - 72nb^2\beta^2r_{00}\Big),
\end{aligned}
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\begin{aligned}
A_7 \;:=\; & -\beta^6\Big(-192nb^4\beta^3r_{00}^2 - 1152nb^2\beta^4r_{00}^2 + 240n\,b^6\beta^2\,r_{000} - 16nb^6\beta\,r_{00}^2 - 384nb^4\beta^3r_{00} + 756nb^4\beta^3r_{000} - 192b^4\beta^3r_{00}^2\\
& -288nb^2\beta^4r_{00} - 2448b^2\beta^4r_{00}^2 + 1764nb^2\beta^3r_0r_{00} + 528nb^2\beta^3r_{00}^2 + 4068n\beta^4r_{00}^2 + 28b^6\beta^2r_{000} - 240nb^6\beta r_{000} + 32b^6\beta r_{00}^2\\
& +1080nb^4\beta^2b^ir_{i00} + 864nb^4\beta^2r_0 + 1056nb^4\beta^2r_{00} + 1008nb^4\beta^2r_{000} + 1044b^2\beta^3r_0r_{00} - 108b^2\beta^4r_{00} + 2904b^2\beta^3r_{00}^2\\
& -1152b^4\beta^2r_0r_{00} - 4320b^4\beta^2r_{00}^2 + 4200n\,b^4\beta r_0r_{00} + 264nb^4\beta r_{00}^2 + 2016nb^2\beta^3r_0 - 240nb^2\beta^3r_{00} - 864n\,b^2\beta^3r_{000}\\
& -3132nb^2\beta^2r_0r_{00} + 2448n\beta^4r_{00} + 3420\beta^4r_{00}^2 + 6120n\beta^3r_0r_{00} + 3120n\beta^3r_{00}^2 - 408b^6\beta r_{000} - 3320b^6r_{00}^2 + 1512b^4\beta^2b^ir_{i00}\\
& +2232b^4\beta^2r_{00} - 288b^4\beta^2r_{000} - 3276nb^4\beta r_{000} + 6864b^4\beta r_0r_{00} + 192b^4\beta r_{00}^2 - 30264nb^4r_{00}^2 + 2880b^2\beta^3r_0 + 1104b^2\beta^3r_{00}\\
& +2160nb^2\beta^2r_0 + 5160nb^2\beta^2r_{00} + 72nb^2\beta^2r_{000} - 3060b^2\beta^2r_0^2 - 1404b^2\beta^2r_0r_{00} - 1368b^2\beta^2r_{00}^2 + 636nb^2\beta r_{00}^2 + 2088\beta^4r_{00}\\
& +1704n\beta^3r_{00} - 1188n\beta^3r_{000} + 6624\beta^3r_0r_{00} + 2796\beta^3r_{00}^2 - 2106n\beta^2r_0^2 - 702n\beta^2r_0r_{00} + 6648n\beta^2r_{00}^2 - 1224\,b^4\beta r_{000}\\
& +1728b^2\beta^2r_0 + 2232b^2\beta^2r_{00} - 720\,b^2\beta^2r_{000} - 3258n\,b^2\beta r_{000} + 22416b^2\beta r_0r_{00} + 312b^2\beta r_{00}^2 - 29736nr_{00}^2b^2 + 3600\beta^3r_0\\
& +1053n\beta^2b^ir_{i00} + 864n\beta^2r_0 + 1344n\beta^2r_{00} - 186n\beta^2r_{000} - 302\beta^2r_0^2 - 104\beta r_{00}^2 - 1062\beta^2r_0r_{00} + 5040\beta^2r_{00}^2 + 4740n\beta r_0r_{00}\\
& -7824r_{00}^2b^2 + 837\beta^2b^ir_{i00} + 756\beta^2r_0 + 1368\beta^2r_{00} - 840n\beta r_{000} + 15216\beta r_0r_{00} - 7936nr_{00}^2 - 2284r_{00}^2 - 480b^4\beta^3r_{00}\\
& -2160b^2\beta r_{000} - 252\beta^2r_{000} - 1080\beta^3r_{000} - 744\beta r_{000} + 1944b^2\beta^2b^ir_{i00} - 4332nb^4\beta^2r_{00}^2 + 13092nb^2\beta r_0r_{00}\\
& -1620nb^2\beta^2r_0^2 - 376nr_{00}^2b^6 - 864nb^4\beta^2r_0r_{00} + 2808nb^2\beta^2b^ir_{i00} + 97b^4\beta^3r_{000} + 3168n\beta^3r_0 - 6876r_{00}^2b^4\\
& +1536\beta^3r_{00} + 250n\beta r_{00}^2 + 1080b^4\beta^2r_0 + 5568nb^2\beta^2r_{00}^2 - 540b^2\beta^3r_{000}\Big),
\end{aligned}
$$

$$
\begin{aligned}
A_6 \;:=\; & 2\beta^7\Big(-288nb^2\beta^4r_{00}^2 + 270nb^4\beta^3r_{000} - 1410nb^4\beta^2r_{00}^2 - 864b^2\beta^4r_{00}^2 + 180n\,b^6\beta r_{000} + 954n\,b^2\beta^3r_0r_{00} + 1260nb^2\beta^3r_{00}^2\\
& -940nr_{00}^2b^6 + 540b^4\beta^3r_{000} + 108nb^4\beta^2b^ir_{i00} + 216nb^4\beta^2r_0 + 888nb^4\beta r_0r_{00} + 48nb^4\beta r_{00}^2 - 24nb^2\beta^2r_{00} - 68nb^2\beta^2r_{000}\\
& -162nb^2\beta^2r_0^2 + 1674nb^2\beta^2r_0r_{00} - 144b^2\beta^4r_{00} + 432nb^2\beta^3r_0 + 936nb^2\beta^3r_{00} - 324nb^2\beta^3r_{000} + 504b^2\beta^3r_0r_{00} + 154b^2\beta^3r_{00}^2\\
& +702b^4\beta^2b^ir_{i00} + 32b^4\beta^2r_0 + 684\,b^4\beta^2r_{00} - 816nb^2\beta^2r_{00}^2 + 216n\beta^4r_{00} - 792\,b^4\beta^2r_{000} + 2718n\,\beta^3r_0r_{00} + 2250n\beta^3r_{00}^2\\
& -132b^4\beta r_{00}^2 - 378nb^4\beta r_{000} + 2184b^4\beta r_0r_{00} - 1332nb^4r_{00}^2 + 1008b^2\beta^3r_0 + 1224b^2\beta^3r_{00} - 54b^2\beta^3r_{000} - 216nb^2\beta^2b^ir_{i00}\\
& +2106b^2\beta^2r_0r_{00} - 235b^2\beta^2r_{00}^2 + 2904nb^2\beta r_0r_{00} - 132nb^2\beta r_{00}^2 + 36\beta^4r_{00} + 216n\beta^3r_0 + 132n\beta^3r_{00} - 351n\beta^3r_{000}\\
& +2412\beta^3r_0r_{00} + 166\beta^3r_{00}^2 + 12n\beta^2r_0^2 + 2538n\beta^2r_0r_{00} - 2688n\beta^2r_{00}^2 + 27b^4\beta r_{000} + 720\beta^3r_0 + 504b^4r_{00}^2 + 945b^4\beta^2b^ir_{i00}\\
& -1044b^2\beta^2r_{00} - 1332b^2\beta^2r_{000} - 648nb^2\beta r_{000} + 5973b^2\beta r_0r_{00} - 366b^2\beta r_{00}^2 + 11379nb^2r_{00}^2 + 1044\beta^3r_{00} - 243\beta^3r_{000}\\
& +81n\beta^2r_0 - 2148n\beta^2r_{00} - 729n\beta^2r_{000} - 288\beta^2r_0^2 + 2538\beta^2r_0r_{00} - 2658\beta^2r_{00}^2 + 74n\beta r_0r_{00} - 132n\beta r_{00}^2 + 513\,b^2\beta r_{000}\\
& -1476\beta^2r_{00} - 630\beta^2r_{000} - 207n\beta r_{000} + 5640\beta r_0r_{00} + 120\beta r_{00}^2 + 6337nr_{00}^2 + 90\beta r_{000} + 7624r_{00}^2 - 1782n\,b^2\beta^2r_{000}\\
& +135\beta^2b^ir_{i00} + 27\beta^2r_0 - 954b^2\beta^2r_0^2 + 1736\,b^2r_{00}^2 - 378n\beta^2b^ir_{i00} + 216b^2\beta^2r_0 + 432nb^2\beta^2r_0 - 960n\,b^2\beta^2r_{00}\\
& -306b^6\beta r_{000} - 48b^6r_{00}^2 - 984b^4\beta^2r_{00}^2 + 450n\beta^4r_{00}^2\Big),
\end{aligned}
$$

$$
\begin{aligned}
A_5 \;:=\; & 3\beta^8\Big(68n\beta^4r_{00}^2 - 288nb^2\beta^3r_{00}^2 + 26nb^4\beta^2r_{000} - 32nb^4\beta r_{00}^2 + 360n\beta^4r_{00} - 240nb^2\beta^3r_{00} - 18nb^2\beta^3r_{000} - 384b^2\beta^3r_{00}^2\\
& +708\,b^2\beta^2r_{00}^2 + 80b^4\beta r_{00}^2 - 4684nb^4r_{00}^2 - 384\,b^2\beta^3r_{00} + 1008\beta^4r_{00}^2 + 900n\,\beta^3r_0r_{00} + 312b^4\beta^2r_{000} - 432nb^4\beta r_{000}\\
& -10b^2\beta^3r_{000} + 30nb^2\beta^2b^ir_{i00} + 28nb^2\beta^2r_0 + 108nb^2\beta^2r_{00} + 468n\,b^2\beta^2r_{000} - 1056b^2\beta^2r_0r_{00} - 20n\beta^2r_0^2 + 132b^2\beta^2r_{00}^2\\
& +576\beta^4r_{00} + 432n\beta^3r_0 + 24n\beta^3r_{00} - 432n\beta^3r_{000} + 1188\beta^3r_0r_{00} + 456\beta^3r_{00}^2 - 738n\beta^2r_0r_{00} + 397n\beta^2r_{00}^2 - 648b^4\beta r_{000}\\
& +576b^2\beta^2b^ir_{i00} + 396b^2\beta^2r_0 + 1824b^2\beta^2r_{00} + 168b^2\beta^2r_{000} - 1386nb^2\beta r_{000} + 4824b^2\beta r_0r_{00} + 272\,b^2\beta r_{00}^2 - 16642nb^2r_{00}^2\\
& +1404n\beta^2r_{00} + 72n\beta^2r_{000} - 630\beta^2r_0^2 - 978\beta^2r_0r_{00} + 3636\beta^2r_{00}^2 + 2028n\beta r_0r_{00} + 106n\beta r_{00}^2 - 1080b^2\beta r_{000} - 10370b^2r_{00}^2\\
& +396\beta^2b^ir_{i00} + 306\beta^2r_0 + 1452\beta^2r_{00} + 6\beta^2r_{000} - 585n\beta r_{000} + 6084\beta r_0r_{00} - 7573nr_{00}^2 - 513\beta r_{000} - 3989r_{00}^2\\
& -28\beta r_{00}^2 + 864\beta^3r_0 - 6980\,b^4r_{00}^2 + 396n\beta^2b^ir_{i00} - 432\beta^3r_{000} + 240\beta^3r_{00} + 306n\beta^2r_0 - 56nb^2\beta^2r_0r_{00}\\
& +52nb^2\beta r_{00}^2 + 1896nb^2\beta r_0r_{00}\Big),
\end{aligned}
$$

$$
\begin{aligned}
A_4 \;:=\; & 9\beta^9\Big(+24nb^2\beta^3r_{000-9n6\beta^4r_{00}^2} + 4nb^2\beta^2r_{00}^2 - 48n\beta^4r_{00} - 192\beta^4r_{00}^2 - 192r_{00}^2 - 228n\beta^3r_0r_{00} - 264n\beta^3r_{00}^2 + 12nb^4\beta r_{000}\\
& +584nb^4r_{00}^2 + 24b^2\beta^3r_{000} - 24nb^2\beta^2b^ir_{i00} - 48nb^2\beta^2r_0 - 80nb^2\beta^2r_{00} + 168nb^2\beta^2r_{000} - 144b^2\beta^2r_{00}^2 - 316nb^2\beta r_0r_{00}\\
& -52nb^2\beta r_{00}^2 - 96\beta^4r_{00} - 48n\beta^3r_0 - 144n\,\beta^3r_{00} + 102n\beta^3r_{000} - 456\beta^3r_0r_{00} - 432\beta^3r_{00}^2 + 18n\beta^2r_0^2 - 234n\,\beta^2r_0r_{00}\\
& +1068b^4r_{00}^2 - 150b^2\beta^2b^ir_{i00} - 84b^2\beta^2r_0 - 280b^2\beta^2r_{00} + 28b^2\beta^2r_{000} - 240\beta^3r_{00} + 192nb^2\beta r_{000} - 124b^2\beta r_0r_{00} + 36b^2\beta r_{00}^2\\
& +102\beta^3r_{000} + 15n\beta^2b^ir_{i00} - 42n\beta^2r_0 + 104n\beta^2r_{00} + 12n\beta^2r_{000} + 96\beta^2r_0^2 - 372\beta^2r_0r_{00} - 36\beta^2r_{00}^2 - 32n\beta r_0r_{00} - 124nr_{00}^2\\
& -74n\beta r_{00}^2 + 148b^2\beta r_{000} - 868b^2r_{00}^2 - 84\beta^2r_{00} - 1588\beta r_0r_{00} + 112\beta^2r_{00} + 186\beta^2r_{000} + 102n\beta r_{000} - 66n\beta r_{00}^2\\
& -42\beta^2r_0 + 28\beta r_{000} + 450b^2r_{00}^2 - 184n\beta^2r_{00}^2 + 202\,b^4\beta r_{000} - 144\beta^3r_0\Big),
\end{aligned}
$$

$$
\begin{aligned}
A_3 &:= -9\beta^{10}\Big(84nb^2\beta^2 r_{000} - 28\beta r_{00}^2 - 96n\beta^3 r_{00}^2 - 36nb^2\beta r_{00}^2 - 48n\beta^3 r_{00} - 54n\beta^3 r_{000} - 12\beta^3 r_{00}^2 - 120n\beta^2 r_0 r_{00} + 54n\beta^2 r_{00}^2 \\
&\quad -168nb^2\beta\, r_{000} + 40b^2\beta\, r_{00}^2 - 2140nb^2 r_{00}^2 - 96\beta^3 r_{00} - 72\beta^3 r_{000} + 45n\,\beta^2 b^i r_{i00} + 36n\beta^2 r_0 + 228n\beta^2 r_{00} + 78n\,\beta^2\, r_{000} \\
&\quad +984\beta^2 r_{00}^2 + 258n\beta r_0 r_{00} - 30n\beta r_{00}^2 - 258b^2\beta r_{000} - 4096b^2 r_{00}^2 + 81\beta^2 b^i r_{i00} + 54\beta^2 r_0 + 444\beta^2 r_{00} + 72\beta^2 r_{000} - 12n\beta r_{000} \\
&\quad -3050nr_{00}^2 - 192\beta r_{000} + 48\beta r_0 r_{00} - 288\beta^2\, r_0 r_{00} - 2846r_{00}^2 + 14b^2\beta^2 r_{000}\Big),
\end{aligned}
$$

$$
\begin{aligned}
A_2 &:= -9\beta^{11}\Big(36n\beta^3 r_{000} - 36\beta^2 r_0 - 26n\beta^2 r_{00}^2 + 54nb^2\beta r_{000} + 678b^2 r_{00}^2 + 72\beta^3 r_{000} - r_{00}^2 - 9n\beta^2 b^i r_{i00} - 18n\beta^2 r_0 - 60n\beta^2 r_{00} \\
&\quad -528\beta^2 r_{00}^2 - 144n\beta r_0 r_{00} - 60n\beta r_{00}^2 + 228b^2\beta r_{000} + 1988b^2 r_{00}^2 - 54\beta^2 b^i r_{i00} - 168\beta^2 r_{00} - 36\beta r_{00}^2 + 108\beta^2 r_{000} + 99n\beta r_{000} \\
&\quad +249nr_{00}^2 + 114\beta r_{000} - 81\beta r_0 r_{00} + 72n\beta^2 r_{000}\Big),
\end{aligned}
$$

$$
A_1 := 27\beta^{13}\Big(3[4n - 10]\beta r_{000} - [353n + 759]r_{00}^2\Big),
$$

$$
A_0 := 27\beta^{13}\Big(3[3n + 10]\beta r_{000} + [97n + 350]r_{00}^2\Big),
$$

and

$$
\begin{aligned}
\tilde{X} &:= \frac{2}{(\alpha^2 - \beta^2)(2\alpha^2 b^2 + \alpha^2 - 3\beta^2)^3}, \\
\tilde{Y} &:= \frac{2}{\alpha^2(\alpha^2 - \beta^2)^2(2b^2\alpha^2 + \alpha^2 - 3\beta^2)^3}, \\
\tilde{W} &:= -\frac{\alpha^2 - 4\beta\alpha + \beta^2}{(2b^2\alpha^2 + \alpha^2 - 3\beta^2)^2(\alpha^2 - \beta^2)^2}.
\end{aligned}
$$

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