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Experienced Elementary Teachers' Processes for Interpreting Artifacts of Student Thinking

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ABSTRACT

Consensus exists in mathematics education that classroom assessment is an essential component of effective practice; however, the importance of teacher interpretation of student thinking in the assessment process is often overlooked. The purpose of this interpretative qualitative research study was to examine teachers' interpretations of artifacts of student thinking. In particular, we sought to understand the personal resources teachers used to construct their interpretations. Nine experienced and professionally active teachers participated in two interviews. The first interview was semi-structured and focused on the participants' professional experiences, conceptions of assessment, and assessment practices. The second interview was task-based and involved participants in the interpretation of student artifacts collected from second grade students in the area of place value. The results indicate that teachers applied to the act of interpretation a complex, but personal awareness of student thinking that influenced interpretation, including: (a) conceptions of levels of student performance, (b) expectations for student performance, and (c) awareness of common student difficulties. These results provide support for the conclusion that professional development related to classroom assessment should address the interpretative process of examining student artifacts, with emphasis on developing personal resources used in this process.

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Consensus exists in mathematics education that formative assessment is an essential component of effective practice (National Council of Teachers of Mathematics [NCTM], 2000, 2014; National Research Council, 2000, 2001). Central to this claim is research that indicates effective formative assessment practices have a substantial, positive impact on student achievement (Black & William, 1998; Kingston & Nash, 2011). However, the potential benefit of formative assessment is dependent on teachers' interpretations of student thinking about mathematics (Jacobs, Lamb, & Philipp, 2010; Son, 2010).

The process of arriving at a reasonable interpretation of student thinking is complex (Ball, 1993, 1997, 2001; Schifter, 2001). Ball (1993, 1997) indicates that problems often arise in the interpretive process because children converse from a perspective that differs from that of an adult, and as a result, represent their conceptions in statements that are not understandable to an adult listener. Further, children cannot often represent their conceptions with detailed explanations, so teachers must reason about responses on which students cannot elaborate (Ball, 1993, 1997). Further, student responses that appear appropriate are often

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embedded with ideas that are underdeveloped or incorrect, and student responses that appear incorrect are often embedded with ideas that are sensible and useful to future instruction (Ball, 2001; Schifter, 2001).

Despite complexities embedded in the interpretive process, Even and Wallach (2003, 2005) note the interpretative process has been presented as an unproblematic aspect of formative assessment, and suggest the perceived ease of interpretation has resulted in teacher preparation and professional development programs that focus more on the collection of assessment data rather than the interpretation of and response to those data. However, recent recommendations have called for professional development that supports teachers in the interpretation of student thinking (Doerr, Goldsmith, & Lewis, 2010; NCTM, 2014). Toward that end, more needs to be understood about the interpretative process if we are to support teachers in arriving at useful interpretations of student thinking (Even & Wallach, 2004).

1. Relevant Literature

Two different components are found in the literature on teacher interpretation of student thinking: (a) one specific to the teacher and his or her *processes* in the act of interpretation, and (b) one specific to his or her *products* of the act of interpretation. Processes are related to *how* a teacher reasons about student artifacts, and *which* personal resources he or she draws on to arrive at an interpretation. Products describe *what* results from processes, such as inferences about what a student does or does not understand, and the instruction the teacher plans in response to those inferences. Whereas numerous studies have demonstrated that inservice (Jacobs et al., 2010; Kazemi & Franke, 2004; Whitenack, Knipping, Novinger, Coutts, & Standifer, 2000) and preservice (Crespo, 2000; Hines & McMahon, 2005; Otero, 2006) teachers often arrive at inaccurate or incomplete products of the act of interpretation, less is understood about the personal resources teachers use to arrive at their products (Even & Wallach, 2004).

In this paper, *personal resources* are defined as conceptions that are internal to the teacher, which include aspects of his or her knowledge and beliefs (Morgan & Watson, 2002). Morgan and Watson (2002) first used the term *resources* to describe the internal conceptions teachers used in the act of interpretation, and despite variation in terms used, consensus exists that teacher interpretation is influenced from that which is internal to the teacher. In this review, studies with a focus on the influence of that which is internal to the teacher in the interpretive process are considered, including those that refer to their *knowledge*, *beliefs*, and *conceptions*, and more recent studies related to their *conceptual frames* (Ebby, 2015, Ebby & Sam, 2015). Of those studies, two common influences on the interpretive process can be observed, including teachers' conceptions of the students and mathematical content.

The results of numerous studies suggest that teachers are influenced from their prior experiences with and conceptions of the student whose artifacts are considered (Wallach & Even, 2005; Morgan & Watson, 2002). For example, Morgan and Watson (2002) found that an experienced teacher who examined a student across multiple course sessions often arrived at conclusions about the student and her abilities that were "comparative to what she had done before" (p. 92). Further, his overall impressions of her mathematical abilities were slow to shift in consideration of evidence that was inconsistent with earlier impressions. These observations are consistent with Watson (1999), who noted that teachers are aware of their use of prior information about a student to interpret new evidence.

Numerous researchers have also noted the influence of the teacher and his or her conceptions of mathematics content on the interpretative process (Heid, Blume, Zbiek, & Edwards, 1999; Morgan & Watson, 2002; Son & Crespo, 2009; Van Dooren, Verschaffell, & Onghena, 2002; Wallach & Even, 2005). Wallach and Even (2005) noted that Ruth, an experienced classroom teacher studied, interpreted student responses to the problem in consideration of her solution to the problem, despite multiple alternative correct solutions. Further, when her students arrived at a correct solution that differed from her own, she was unable to determine whether or not the solution was correct. Much the same, studies of teachers of students in grades six through twelve reveals that teachers' limited understanding of the content contributed to challenges in determining the correctness of student responses (Morgan & Watson, 2002; Heid et al., 1999).

Descriptions of inservice teachers also point toward additional personal resources that are called on in the act of interpretation, including teacher awareness of children and their development and difficulties, and teacher beliefs and preferences for how mathematics should be approached, learned, and communicated (e.g., Ebby, 2015; Even & Wallach, 2003). Whereas the research is robust in its account of personal resources that limit the act of interpretation, far less is understood about the multifaceted scheme of personal resources that contribute to the interpretive process. From the research on the products of teacher interpretation, we are aware that at least some experienced teachers demonstrate elements of expertise (Jacobs et al., 2010); however, the personal resources these teachers call on in the act of interpretation is less clear.

The purpose of this research is to describe the personal resources that experienced teachers who are active in their professional development call on during the interpretative process, in order to arrive at inferences about the student and his or her conceptions of mathematics. We examined how first and second grade teachers interpret student artifacts collected from first and second grade students related to place value, using the following research question to guide the work:

> What personal resources and prior conceptions do experienced, professionally active first and second grade teachers' use to construct interpretations of student mathematical understanding from artifacts of student thinking?

2. Theoretical Perspective

The adopted social constructivist perspective supports the notion that an observer can construct a mental model to represent the conceptions of an observed (Even & Tirosh, 2008). Applied to the research, the mental models of interest are those the teacher constructs about the mathematical conceptions of the students whose responses are presented to them. These mental models take the form of teacher interpretations about what that student does or does not understand, and are referred to as *second-order models* (Steffe, von Glasersfeld, Richards, & Cobb, 1983; Steffe & Thompson, 2000). Within the broader research context, there are additional instances of model construction. These multiple levels of model construction are represented in Figure 1[1].

At the foundational level, learners construct their mathematical understanding. These constructions are called *first-order models*, and are described as "the knowledge the individual constructs to organize, comprehend, and control his or her experience" (Steffe & Thompson, 2000, p. 205). In the process of teaching and learning, first-order models are published through verbal and written language and student actions (Cobb, 1996; Ernest, 1991), and are the subject of interpretation in this research. Further, Wilson, Lee, and Hollebrands (2011) add a third level to account for the models a researcher constructs in his or her studies of a teaching and learning situation. Referred to as *third-order models*, Wilson et al. (2011) recognize the constructions of the researcher that represent the second-order models of teachers.

1 The model is adapted from Wilson et al. (2011). Most adaptations were minor and made in an effort to improve the fit of the description of model components with this research. As an example, instead of a label that named the first-order model as students' mathematical and technological conceptions, the term technological was removed because of the sole focus in this research on students' mathematical conceptions. The one instance when the adaptation was more substantial and an element was added to the model, is described in the related section below, titled researcher construction of second-order models.

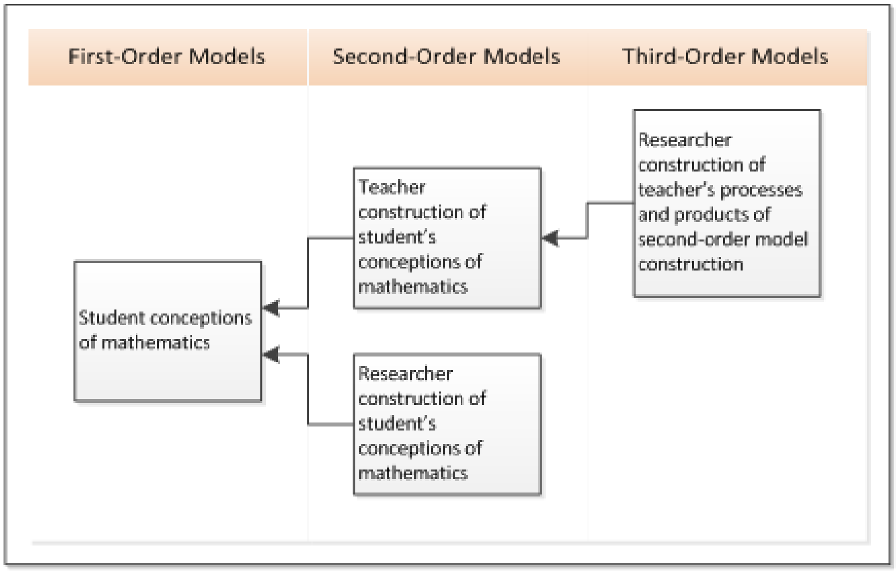

Figure 1. Occurrences of model construction. Modified from Wilson et al. (2011).

The adopted social constructivist perspective also assumes one constructs mental models in consideration of their immediate experiences and personal resources. For the teachers described in this research, their immediate experiences with the published conceptions of students are similar because each is presented with the same artifacts; however, the personal resources each applies to those experiences were different. The term *personal resources* refers to those conceptions that are internal to the teacher, and includes aspects of his or her *knowledge* and *beliefs* (Morgan & Watson, 2002). These personal resources are both the product of previous constructions, and the "building blocks" of future constructions (Ernest, 2010, p. 39).

This paper focuses on the personal resources that teachers use in the act of interpretation. The embedded assumption that underlies this consideration is that because personal resources are different for individuals, the models the teachers construct for the same student can be different—both from one another and from the first-order model of the student (Simon, 2000). To understand these differences, we must understand the nature of the personal resources used.

3. Method

3.1. Selection of Participants

Criterion sampling methods were used to locate first and second grade teachers for the research (Miles & Huberman, 1994). In particular, it was established that participants must meet two criteria: (a) have five or more years of teaching experience with first or second grade children, and (b) be professionally active, as identified by an administrator or instructional coach. The phrase *professionally active* is used to describe teachers who pursue opportunities for growth outside of the requirement of his or her contract. These opportunities might be formal, such as involvement in professional development inside or outside of the district, building- or district-level committees, or professional organizations. These opportunities might also be less formal, such as regular engagement with professional literature.

The selection of these criterion was made in consideration of research specific to teacher interpretation of assessment data, that indicates four or more years of classroom teaching experience contributes to statistically better interpretation abilities than are observed in preservice teachers (Jacobs et al., 2010). Further, the identification of teachers who are professionally active was added in response to research specific to teacher interpretation of assessment data (Jacobs et al., 2010) and general consensus that experience alone is not a sufficient predictor of teacher abilities (Berliner, 1986).

3.2. Description of Participants

Nine teachers from two different public school districts participated in the research. Of the nine participants, three were from District A and six were from District B. Both districts were located in the Midwestern United States. District A served students in a metropolitan area and a suburb of that area, and District B served students in a suburb of the same metropolitan area.

At the time of data collection, District A served about 14,000 students from kindergarten to twelfth grade, of which 73.7% of students were White, 16.6% were Black, and 5.1% were Hispanic. In District A, 68.5% of students were eligible for Free or Reduced-Price Lunch. In contrast, District B served about 11,000 students from kindergarten to twelfth grade, of which 83.9% of students were White and 5.7% were Black. In District B, 21.1% of students were eligible for Free or Reduced-Price Lunch.

Of the nine participants, all were female. At the time of data collection, five participants were teaching in first grade, and four were teaching in second grade. All nine participants held a master’s degree in the field of education. For more detailed information about participant teaching experience, see Table 1.

Table 1. Years of teaching experience per teacher participant

Participant	Current Grade	Years of Experience		
		First	Second	Total
A	1	5	0	12
B	2	0	5	7
C	2	2	7	10
D	2	0	5	11
E	1	13	0	13
F	1	11	0	15
G	2	0	10	27
H	1	23	0	25
I	1	11	0	24
Average	-	7.2	3.0	16.0

3.2. Data Collection

Two interviews were used to collect data. These included : (a) a semi-structured interview about the participant and his or her professional experiences, knowledge and beliefs about the teaching and learning of mathematics, conceptions of assessment, and assessment practices; and (b) a task-based interview that involved the participant in interpreting student mathematical thinking from student artifacts presented to the participant. The semi-structured interview was administered in one session that lasted about 45 minutes, and the task-based interview was administered in one session that lasted about 75 minutes.

In the creation and use of the instruments described above, *peer review* and *external audits* (Lincoln & Guba, 1985), two of several established validation strategies for qualitative research (Creswell, 2007), were used. To establish reliability, *intercoder agreement* was used in the data analysis process (Creswell, 2007), and the process for implementation is described in more detail in section *3.3. Data Analysis*.

3.2.1. Selection of student artifacts for the task-based interview

All student artifacts were first collected as data for the purposes of a separate research project that aimed to understand the development of student understanding related to various number and operations constructs for students in first and second grades (Lannin & van Garderen, 2009). For the purpose of the task-based interview, all student artifacts involved students in an assessment or instructional task that addressed mathematical ideas related to place value, in an effort to select a mathematical focus that was relevant to both first and second grades.

In addition, two factors guided the selection of artifacts from the data set, including: (a) the correctness of the student response, and (b) the nature of student approach or justification. Effort was made to select artifacts that differed in correctness, in part, because of research that indicates preservice teachers often attend more to the correctness of the response than other elements of the artifact (e.g., Crespo, 2000; Otero, 2006), such as the approach. Likewise, effort was made to select artifacts that differed in approach in response to research that suggests approach is more helpful in the act of interpretation than the correctness of student responses (Sowder, 2007), and research that indicates preservice (e.g., Schack et al., 2013) and inservice (e.g., Jacobs et al., 2010; Whitenack et al., 2000) teachers do not often notice subtleties in student approaches.

3.2.2. Design of the task-based interview

Two different approaches were used to develop the task-based interview. The first approach involved the design of prompts that focus on an individual student and his or her response to two or more related tasks (Figure 2), and the second approach involves the design of prompts that focus on multiple students and their individual responses to the same task (Figure 3). For all prompts, teachers were asked to consider and solve the mathematical task before the student artifact was observed, and then interpret student mathematical thinking and plan an instructional response after the student artifact was observed. Specific follow-up questions prompted the participant to consider: (a) strengths and weaknesses in the student response, (b) specific claims that could be made about the student, and (c) a planned response to the student.

Task 3.1

Interviewer: Make 56 with cubes.

Student: [Student touches five ten sticks, one touch per count, and then pull six ones. On table: | | | | | ......]

Interviewer: Using what you have out, make 66.

Student: [Student touches each of five tens.] I need another one of these [points to ten-sticks]. [On table: | | | | | | ......]

Interviewer: Using what you have out, make 46.

Student: [Student places left hand on four ten-sticks, and removes two ten-sticks with right hand. On table: | | | | ....]

Task 3.2

Interviewer: [Represents 24 with Unifix cubes | | ....] How many cubes do I have?

Student: 24, see 10, 20, 4. 24.

Interviewer: [Represents 34 with Unifix cubes | | .... |] How many cubes do I have?

Student: 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34. [Counts each block in last tens stick]

Interviewer: [Represents 54 with Unifix cubes | | .... | | |] How many cubes do I have?

Student: [On fingers] 24, 25, 26, 27... 54. 54. [Counts each block in last three tens sticks.]

Figure 2. Transcript of student artifacts presented for interpretation of an individual student response to two or more tasks, from Task 3.1 and 3.2

Interviewer Prompt
Josh has 4 bags with 10 toys in each bag. He also has 7 extra toys. How many toys does Josh have?

Task 2.4
Student: So, he has 4 bags.
[Student draws four boxes in one row with the number 10 written in each. Student draws fifth box in second row with the number 7 written in it. Student writes, "He has _ toys." Student fills in blank with 47.]

Task 2.5
[After interviewer reads, student writes. On paper: 10+7=17]

Student: I did this in first grade the tens plus a number would equal something. A teen number. So, 17.
[Student is asked to solve with another method. Student draws 4 circles. Student puts 10 dots in each circle. Student draws 7 balloon- like objects. Student writes 10, 20, 30, 40, 41, 42... 47 above objects on drawing. Student is asked which answer is correct. Student points to 17.]

Figure 3. Transcripts of two of five student artifacts presented for a *sort of student responses task*, from Task 2.4 and 2.5

3.3. Data Analysis

Inductive and comparative methods were used to make sense of these data (Boeije, 2002; Merriam, 2009). Each audio- or video-record was transcribed verbatim, and references to participant actions were made based on researcher discretion (Clement, 2000). Marginal remarks and memos from the transcription process were used to develop an initial list of descriptive codes. A process of *within-code comparison* was used to compare all instances where the same code was applied to establish consistent use of codes, *across-code comparison* was used to determine that differences across codes were "bold and clear" (Patton, 2002, p. 465). A second coder coded data collected from three participants, with 83% agreement. All disagreements were resolved through discussion, and data from the remaining participants were recoded in consideration of adjustments to the coding scheme.

Data matrices (i.e., retrieved data associated with particular codes for particular student artifacts) were used to reveal patterns in and variation across participants to inform the results presented below (Miles, Huberman & Saldaña, 2014; Patton, 2002), which are presented as claims. After claims were established, each claim was compared to the raw data in search of instances that disconfirmed the claims (Lincoln & Guba, 1985). Therefore, each claim characterizes what is representative of most teachers studied, and exceptions observed are described below in the presentation of evidence to support the claim.

4. Results

In considerations of those personal resources that influenced the act of interpretation, the teachers studied revealed a complex, but personal awareness of student thinking that impacted the act of interpretation, which are described within the context of the following three claims:

Claim One: Teachers applied their conceptions of levels of student performance to the act of interpretation.

Claim Two: Teachers applied their expectations for student performance to the act of interpretation.

Claim Three: Teachers applied their awareness of common student difficulties to the act of interpretation.

4.1. Conceptions of Levels of Student Performance

Before student artifacts were shared, teachers were prompted to describe possible student approaches and responses to the mathematical tasks represented in the student artifacts. These prompts revealed teacher awareness of multiple student approaches and responses for most mathematical tasks. In addition, these prompts revealed that teachers often associated the approaches and responses that were anticipated with various *levels of student performance,* or perceived differences in student understanding. That is, teachers often ordered the student approaches and responses that were anticipated from least to most sophisticated. I refer to these ordered approaches and responses as a *level scheme* for a particular mathematical task, and a common level scheme was shared across most teachers on 4 of the 5 mathematical tasks presented. Teachers often referenced their level scheme in the act of interpretation, and as such demonstrated its influence on the act of interpretation.

The approaches and responses that teachers anticipated for the mathematical task associated with Task 1.1 provide an example of a level scheme. In this mathematical task, a student was asked to determine whether one ten-stick and four blocks [| ....] was the equal to 14 blocks. In consideration of the mathematical task, 8 of 9 teachers anticipated two to three different correct student approaches that could be used to determine that one ten-stick and four blocks was the same as fourteen blocks, and teachers described these approaches as representative of different levels of student performance. For example, Ms. Pope (Teacher G) noted three levels,

> Beginning students are going to *count each one* and might even have to take it apart and see it spread and count it, and the more advanced students are going to *know that's a 10 and say 10 and count on*, and then the further along are just going to be able to *picture it and not have to count*.

All teachers who articulated a level scheme for this particular mathematical task included in their level scheme at least two of the three levels Ms. Pope described. That is, teachers anticipated that students would *count all*, *count on from ten*, or state *'fourteen blocks' but not count*, and ordered these approaches in terms of sophistication as can be observed from Ms. Pope.

In the act of interpretation, teachers often compared the observed student to their level scheme. For example, in response to the student artifact associated with Task 1.1, Ms. Pope stated,

> She didn't have to *count it out*. I noticed that she just seems to have a good grasp of place value. There wasn't a lot of thought. She just was able to *answer right away that this makes 14* [motions to one ten-stick and four ones].

In her interpretation, Ms. Pope noted an aspect of student performance that she did not observe; that is, she stated the student did not *count*. While it is unclear whether she intended to communicate that the student did not *count all*, *count on*, or both, she articulated that the student did not demonstrate the least advanced of the levels that she described. As demonstrated in her response, reference to levels that were not observed in the student artifact occurred across all teachers who articulated a level scheme for this particular mathematical task, and was common practice across other mathematical tasks for which a level scheme was articulated.

Teachers sometimes included in their level scheme incorrect approaches or responses that preceded a correct response. For the mathematical task described above, two teachers anticipated an error that involved the student identification of the ten-stick and the set of four blocks as *two sets of objects*, not fourteen blocks (Table 2). For these teachers, the scope of levels of student performance included all student approaches and responses the teachers anticipated as probable, both correct and incorrect.

Table 2. Anticipated incorrect student response for the mathematical task associated with Task 1.1

Teacher	Response
Ms. Allen (B)	I also have a student that would probably say that was *two because he sees two different items* [touches ten-stick and then touches ones].
Ms. Carson (C)	I could see them counting so they would consider this [touches a ten-stick] one block. So, they would be counting each individual one. A student that didn't get it right, I would see them like doing one and then two, so *two blocks since there's two separate ones*, would be another possibility, too.

Teacher awareness of incorrect approaches and responses is further discussed in **Claim Three**.

Level schemes sometimes differed across teachers for the same mathematical task. The mathematical task associated with Task 3.1 provides an example of such differences. In a portion of this mathematical task, the student was prompted to represent 56 with blocks, and then add to the 56 blocks to represent 66 blocks. Table 3 describes the level schemes of those teachers who articulated one for this portion of this particular mathematical task.

Table 3. Level scheme for the mathematical task associated with Task 3.1

Approach [from least to most sophisticated]	Teacher					
	B	C	D	E	F	G
Recount all						
Recount all [e.g., 1, 2... 64, 65, 66]					■	
Recount all [e.g., 10, 20... 60... 64, 65, 66]	■					■
Count on/back [e.g., 57, 58, ... 66]	■	■	■	■	■	
Add/remove ten-stick(s) without count	■	■	■	■	■	■

Notice that all teachers represented in Table 3 described a level that involved the addition of a ten-stick without a count, and all teachers indicated this was their preferred approach. That said, consensus on the student approach or response that teachers preferred was not observed for all mathematical tasks and is further discussed in **Claim Two**. Further, notice that the teachers represented in Table 3 were inconsistent in their description of those approaches that were less preferred, despite a pattern across teachers that includes three approaches that develop from least to most sophisticated.

Teachers drew on their level schemes in the act of interpretation, whether or not their levels were consistent with the level schemes of other teachers. For example, Ms. Spain (Teacher E) applied her level scheme to a student artifact that corresponded to the mathematical task in Task 3.1. In the student artifact, the student appeared to *recount* the five tens from the 56 blocks before he added a ten-stick.

> When he made sixty-six, he had to *go back and count by tens* [touches table six times], he wasn't able just to *add the ten*, but that's just kind of checking his work too, or remembering what it was. I don't know.

In her statement, she noted that the student did not demonstrate the more advanced of the two levels she described, but that he *recounted all*, a level she did not include in her level scheme before she observed the student. However, she compared the student response to the levels she referenced when she described her plans for an instructional response,

> I would probably, just probably throw out and have him do it one more time with different numbers to kind of see if he had to always *go back and count ten, twenty, thirty, forty* [touches table four times, one count per touch], or if he was able to *kind of just know fifty-six or count up ten more*.

As such, Ms. Spain planned to administer additional mathematical tasks to determine which level the student would demonstrate. In particular, she wanted to understand if he could demonstrate one of the two levels she referenced when she first considered the mathematical task.

In their semi-structured interviews that took place before teachers looked at student work, 7 of 8 teachers alluded to their awareness of various levels of student performance and its usefulness in the consideration of student artifacts. For example, when asked to describe what she considered when she looks at student artifacts, Ms. Graham (Teacher F) noted,

> Well, some of them have the facts memorized, some of them count on, some of them use their fingers, some of them need to draw a picture, some of them need cubes or some form of a manipulative. It's because they're all at different stages... Just to see how they arrived at their final response, was it a real primitive, you know they needed the concrete, or have they moved to abstract.

Notice that Ms. Graham explained that students are at different *stages* and indicates that she considers these as she is looking at student artifacts, providing additional support for the observation that teachers approach at least some student artifacts mindful of levels of student performance.

4.2. Expectations for Student Performance

A related aspect of teachers' personal resources that were called on in the act of interpretation involved their ideas about the student approaches or responses that were preferred from or expected of students at a particular point in time. In some instances, these approaches and responses were shared as the most sophisticated of the levels the teachers described, but at other times were shared separate from a level scheme. In either form, teacher *expectations for student performance* often differed across teachers for the same mathematical task, but informed their interpretation and planned instructional response nonetheless.

The responses teachers provided to the mathematical task associated with Task 3.2 (Figure 2) is an example of the differences observed in teacher expectations for the same mathematical task. In consideration of this mathematical task, most teachers described a level scheme; however, the level described as most preferred was inconsistent across teachers. For instance, when the student was prompted to name the total number of blocks after two ten-sticks were added to the right of two ten-sticks, four ones, and one ten-stick [| |.... | → | |.... | | |], Ms. Baehr (Teacher I) described three levels,

> A high student would look at this and say, '10, 20, 30, 40, 50' in their head for 54. The middle student would put all the ten-sticks together and separate the ones, 10, 20, 30, 40, 50, 1, 2, 3, 4. The low student would probably still have to count them one by one. But, you wouldn't have very many of those left, maybe one or two.

Notice in her response that Ms. Baehr expects that advanced students would *recount all* in order to provide a response. This expectation is one that others shared. In particular, four levels that involved a recount reoccurred across teachers. These included: (a) recount all in ones (1, 2, 3.... 54); (b) recount all in tens and then ones (10, 20, 30, 40, 50... 54); (c) recount all in tens, ones, and then tens (10, 20, 21, 22, 23, 24, 34, 44, 54); and (d) the movement of the tens to left of the ones before a recount occurs.

In contrast, other teachers stated that students should not need to recount all of the blocks after a ten-stick or ten-sticks were added, and did not include this approach as the most sophisticated in their level scheme.

Instead, these teachers preferred that the student *count on* from what was on the table. For example, Ms. Carson (Teacher C) noted,

> Hopefully they would say 34, and then 44, 54... Being able to remember what number they were on before, so how much they had before and then being able to add those tens on to it, I think would be the most difficult task, them not going all the way back to the beginning every time and being able to build upon where they were.

Similar to Ms. Carson, multiple teachers described a preferred approach in which the student *counted on* in tens. As such, their preferred approach was different than the preferred approach Ms. Baehr and others described.

The student artifact that was associated with Task 3.2 included a student who counted on by ones, not tens (24, 25… 54). In their interpretation of the student approach, teachers' descriptions of what the student did in response to the mathematical task were generally consistent for this particular mathematical task. However, their statements about what the student did not do reflected their preferred approach. For example, Ms. Spain (Teacher E), a teacher who preferred that students *recount*, stated,

> From a second grade standpoint, I think I might be a little concerned about his approach to that. To me, it showed me *he didn't really have that 10, 20, 30*... [touches table once per count].

In contrast, Ms. Pope (Teacher G), a teacher who preferred that students *count on* in tens, stated,

> He had to count on and I thought he would be able to just add on, I thought he had it… When given a number, he is able to count it out, but *he is not able to count on in the tens*, he has to count it out [points to each individual cube in a ten-stick], he's not to the point where he can just visually see that you change the number in the tens.

In both responses, we see that the teachers' preferred approach influenced their interpretations. That is, what teachers stated the student did not do was related to how she preferred the student approach the mathematical task.

The power of these differences in expectations for student performance came when these teachers were asked how they would respond to this child. That is, teachers often planned instructional responses that had a close connection to their interpretation and as such teachers differed in their approach to working with the child moving forward. For example, Ms. Spain planned an instructional response that involved guiding the student toward recounting all in tens and then ones, asking the student, 'Is there a faster method of counting this?" In contrast, Ms. Pope planned an instructional response that aimed to help the student count on in tens, through asking the student to consider what he or she notices about how the numeral that represents the number of base-ten materials changes after each tens-stick is counted by ones.

Across the interviews, teachers often described their awareness of what a student should be able to do and understand at a particular point in time, either within a grade level or across multiple grade levels, as the source of their expectations for student performance. For example, the mathematical task administered to students in Task 2 involved a contextual problem in which the student was prompted to determine the total number of objects one would have if he or she had four groups of ten objects and seven additional objects. In response, Beth, a second grader in the middle of second grade, wrote on her paper: *4x10=40*, *7-0=7*, and *47*. In consideration of this student response, most teachers shared a sense that the first part of Beth's response [*4x10=40*] demonstrated understanding of a mathematical idea that is outside of what is expected of a second grade student. For example, Ms. Allen (Teacher B) noted,

> She has the concept of *multiplication* [points to *4x10=40*] and not *repeated addition* or *skip counting*. You know, I would say that she definitely is very proficient, and for a second grader very

> advanced in… knowing multiplication, and in understanding very quickly and very easily what it was asking, which is what I would expect of a much older student, to be able to do that.

Notice that in her response Ms. Allen first alludes to less sophisticated levels of her level scheme (i.e., repeated addition, skip counting) before noting that the student response exceeds her expectation for second graders.

4.3. Awareness of Common Student Difficulties

Multiple aspects of teachers' *awareness of common student difficulties* influenced the act of interpretation. Teachers often communicated this awareness in their discussion of: (a) correct student approaches that were less preferred, and (b) incorrect student approaches and responses. The former involved references to common student difficulties that teachers associated with a lower level approach but a correct response, and the latter involved references to common student difficulties that teachers associated with an incorrect approach or response. Both means of communicating these ideas are presented in this section to describe this aspect of teachers' personal resources, and the connection between them and the act of interpretation.

4.3.1. Common student difficulties associated with correct, but less preferred approaches

Teachers described correct, but less preferred approaches in their discussion of their level schemes (see Claim One). In their discussion of these lower levels, teachers often noted that common student difficulties contributed to lower level approaches. Teachers' descriptions of these difficulties within their discussions of correct, but less preferred approaches are the focus of this section.

For example, recall that most teachers shared a common level scheme for the mathematical task associated with Task 3.1 (Table 3) that prompted students to build 56 and then add and remove tens to build other quantities (e.g., 66, 46). In their articulation of this level scheme, most teachers also articulated an awareness that the subtraction component of the mathematical task would be more difficult for children than the addition component, and as such teachers anticipated that a lower level approach would be more often observed for the subtraction component of the problem than for the addition component. For example, Ms. Carson (Teacher C) noted,

> I can see some kids being really puzzled and not knowing how to do that. And I could see some counting back – actually I could see a majority of my kids knowing what to do, but *counting back by ones*. So, 66, then 65, 64, all the way back to 46. And they might get off a little bit... But, I would want them just to be able to know to *take away two tens*... Just for my students every year, it seems like they can add, but... it always seems like subtraction is so much harder than addition every year.

As such, she maintained that common student difficulties around subtraction would influence the level of student performance. In particular, she indicated that students would be more apt to *count back* (Table 3) for the aspect of the mathematical task that involved subtraction than he or she would *count on* for an aspect of the mathematical task that involved addition.

For this mathematical task and others, teachers applied their ideas about common student difficulties to the act of interpretation. For example, in consideration of a student artifact in which the student responded to the mathematical task in Task 3.1 with the removal of a ten-stick *without a count*, Ms. Carson noted,

> Then, I thought it was interesting when he subtracted, he easily took those two away, so quickly, and I thought that was the hardest part... I think that's I good. I was impressed that he was able to do that so quickly.

Notice that Ms. Carson seems to have arrived at a positive interpretation of the student, in part, because she considered the student approach to be representative of an advanced level of performance for a component of the mathematical task that she considered difficult for students. A theme of positive appraisal when students overcame difficulties that a teacher had articulated as common across teachers and consistent across tasks.

4.3.2. Common student difficulties associated with incorrect responses

In addition to communicating their ideas about common student difficulties in their discussion of correct, but less preferred approaches, teachers also shared their awareness of common student difficulties in their discussion of incorrect approaches and responses. As noted above (Table 2), incorrect student approaches and responses were sometimes presented as a component of a level scheme. However, incorrect responses were also presented as separate from a level scheme when one was not described for a particular mathematical task, and some incorrect approaches and responses were not anticipated, but were referred to as common after the teachers considered a student artifact.

For example, various incorrect approaches were anticipated for the contextual situation in Task 2. Recall that this mathematical task prompted students to determine the total number of objects in four groups of 10 objects and seven extra objects. For this mathematical task, all but two teachers anticipated an incorrect approach that involved the addition of the three numbers presented in the contextual situation, and three teachers anticipated an incorrect approach that involved the addition of two numbers presented in the contextual situation. The reoccurrence of these two incorrect approaches across teachers indicated that teachers shared some awareness that these incorrect approaches were common. In fact, others described the *common* nature of such approaches (Table 4).

Table 4. Common student difficulties with the mathematical task associated with Task 2

Teacher	Response
Ms. Afton (A)	I think they would add the 4 plus the 10 plus the 7. They don't take the time to understand and to really draw it out –he has 4 actual bags and each one has 10. So, it's not just 4. I don't think they get it. They don't visualize it and I don't know if that's because they're still learning to read, or it's a higher level skill and they're not there yet, or if it's genetic –I really don't know, but I don't think they interpret the problem the right way.
Ms. Spain (E)	The thing that would be challenging, I think, is to understand that the four bags had ten toys in each bag, so that they had to count by tens. That they wouldn't throw that four in there. Sometimes they just see numbers; they see the four, they see the ten, and they see the seven. They don't really read and understand that this [points to the ten in the text of problem] is telling us how many are in each bag.

In contrast from the mathematical task above, other mathematical tasks produced few anticipated incorrect approaches and responses, and one produced no anticipated incorrect approaches or responses. For this particular mathematical task, two teachers noted that the student artifact demonstrated aspects of common student difficulties associated with the mathematical task after it was observed, but none anticipated or described them before it was observed. Of those teachers who noted the common student difficulties after consideration of the student artifact, one noted the student approach was incorrect and the other noted the student approach was correct. Both of these responses are described further below.

To provide context, the student artifact that teachers observed involved a mathematical task in which the student was asked to determine whether a ten-stick and four ones [| ....] were the same as fourteen ten-sticks. In response, the student pulled out four ten-sticks, and as each was pulled said, "10, 20, 30, 40." She then described the number of blocks on the table, but did not respond to the prompt about the number of ten-sticks. In response, Ms. Pope (Teacher G) noted,

> Yeah, she really doesn't have a good foundation in place value, because she can't, and I see this a lot, go back and forth from seeing this as a 10 or counting it 1, 2, 3, they have to count it as a 10 and they don't see that you can switch your counting depending on what the problem is.

As a part of her instructional response, Ms. Pope planned to demonstrate for the student the difference between the two units she described in her interpretation.

> I think I would have tried to teach her. I would say that is awesome that you know that this is a 10 and you can count by tens, but I asked you to do something different this time. I asked you to count the number of ten-sticks that you had out there. And I think I would, you know, grab some and start practicing, this is 1 ten-stick, 2 ten-sticks, and your friend thinks that you had 14 ten-sticks.

As such, Ms. Pope noted the approach was both incorrect and common, and planned an instructional response that addresses the incorrect approach. In particular, one that aimed to help the child consider the unit being counted (i.e., tens, ones).

Ms. Afton (Teacher A) seemed to recall that student difficulties with the mathematical idea related to the mathematical task were common for her students also. She stated,

> I totally forgot about that, but when we did teach them, it was hard for them to conceptualize that 14 of these [touches ten stick] would be more than the number 14, you would have over 100. So, I remember the first lesson we taught, we all kind of panicked. Then as we retaught it and got more and more manipulatives, the kids started to catch on. But, right away, they were all doing the same thing, and even my lower students would still—I don't know what it is. It's like a block in their brain, like they're not developmentally ready for that yet, it doesn't make sense to them to get that – that 14, they'd have to have 14 of these [touches ten-stick], but that would not mean the number 14, that would mean something different. So, I even had students where I'm like, 'No, you have to count like 10.' So, we would get our 14 out, and then we would have to count 10, 20, 30, 40, and some of them are still only able to count the 10, 20, 30, 40 and not conceptualize that this right here, right away I know if there's 14 it makes that number. So, flashbacks of first quarter.

Notice that her response differed from that of Ms. Pope because Ms. Afton did not communicate concern with the approach the student used, despite the fact that she stated that difficulties related to this mathematical idea are common. In fact, Ms. Afton described an instructional approach in which she modeled for her students the same count that the student used in the student artifact—a count that is not consistent with the unit identified in the mathematical task.

An implicit connection existed between teacher awareness of common student difficulties observed in incorrect approaches and responses and the act of interpretation. This connection is implicit because, unlike instances when teachers used their levels to support the act of interpretation, teachers did not often state that their awareness of common student difficulties helped them to see such difficulties in a response or approach. However, a notable pattern arose across the data that demonstrates that these teachers never failed to mention the presence of a student approach or response that reflected the common student difficulties that he or she had anticipated before consideration of the artifact, but these teachers often failed to mention aspects of approaches and responses that were representative of common student difficulties described in the literature when he or she had not anticipated or described them as common.

The notion that teachers' awareness of common student difficulties informed the act of interpretation was also supported in statements collected from 5 of 9 teachers in the semi-structured interview that took place before teachers examined student artifacts. While teachers were not asked whether or not their awareness of common student difficulties was useful in the act of interpretation, these teachers identified the usefulness of this awareness when asked to describe their confidence as it relates to looking at student work. For example, Ms. Carson noted,

> Over the years, not that all your kids are the same, but you kind of know what to look for as you're teaching certain things. Like, I know this is where kids can get caught up with this skill, and I know this is where kids can get caught up with that skill. So, I think that kind of helps me too, to kind of know what I'm looking for as well.

Notice that Ms. Carson seems to indicate that her awareness of common student difficulties, derived from their observations of students in her classrooms, supports her interpretations. In particular, this awareness informs what Ms. Carson looks for when considering a student artifact.

5. Discussion

The findings described above detail three aspects of teachers' personal resources that were drawn on in the act of interpretation. These included: (a) conceptions of levels of student performance, (b) expectations for student performance, and (c) awareness of common student difficulties. Each of these aspects of teachers' personal resources are interrelated and focus on what teachers understand about students—where *students* refers to all students, not just the particular student whose artifact is considered, as is sometimes described in the literature (e.g., Morgan & Watson, 2002; Wallach & Even, 2005).

The teachers in this study often approached student artifacts with *conceptions of levels of student performance* for mathematical tasks and used these level schemes to construct their interpretations of student artifacts. Their level schemes included multiple anticipated approaches that prompted both correct and incorrect responses, and as such served as an indicator that the experienced and active teachers accounted for far more than correctness of response in their consideration of student artifacts, an observation that is in direct contrast to studies of preservice and novice teachers who often focus on correctness of response (e.g., Crespo, 2000; Hines & McMahon, 2005; Otero, 2006). These level schemes are in contrast to the results of studies that describe teacher attention to one approach or response as an aspect of their personal resources that limit interpretations (Even, 2005; Even & Wallach, 2004; Wallach & Even, 2005).

Consistent with literature that emphasizes the importance of the approaches students use (e.g., Carpenter & Fennema, 1991, 1992; Romberg & Carpenter, 1986), these results point toward an element of these teachers' processes that are consistent with improved practice. In fact, the level schemes share similarities with research-based progressions in that both describe student development around a particular mathematical idea. Further, use of these level schemes is consistent with recommendations that teachers approach their practice mindful of student development (Daro, Mosher, & Corcoran, 2011; Sztajn et al., 2012). However, further research is needed to determine the extent to which these level schemes are consistent in terms of content with research-based progressions, and to consider the relationship between consistent level schemes and more useful interpretations.

The teachers studied also often approached student artifacts with *expectations for student performance* related to their preferences for a particular student response or approach, which is consistent with the results of other studies (e.g., Morris, 2006; Sleep & Boerst, 2012; Spitzer, Phelps, Johnson, & Sieminski, 2011; Van Dooren et al., 2002). However, these results build on the research, revealing that the experienced and active teachers studied often described their awareness of where a student should be at a particular point in time as the reason for their expectations, suggesting that their expectations were not static but fluid, which is consistent with recommendations that teachers be attentive to student development in the assessment process (Pellegrino, Chudowsky, & Glaser, 2001). However, it is notable that the teachers studied sometimes differed in their expectations for the same mathematical task, since the teachers studied were from the same state and therefore shared common content standards.

Teacher *awareness of common student difficulties* is referenced in the literature on interpretation of student artifacts, where it is described as a personal resource that limits useful interpretation (Even, 2005; Even & Wallach, 2004). In contrast to these studies, the results provide limited support for the conclusion that teacher awareness of common student difficulties does not just limit teachers' interpretations, but also supports interpretations of what occurred. This claim stems from an observation that teachers often described aspects of a student artifact that involved common student difficulties that were anticipated, but did not often describe aspects of the student artifact that involved common student difficulties that were not anticipated. As such, the potential positive impact of this personal resource could depend, in part, on teachers whose awareness of common student difficulties is robust and accurate.

6. Limitations and Implications

Multiple limitations should be considered as one reflects on these findings and their implications. The first of these limitations is related to sample size. Nine teachers from two different school districts participated in the research. Although the small sample size allowed for the collection of rich data from each participant, one must use caution in generalizing these claims to all experienced and active teachers. Second, the student artifacts that were selected were collected from students unfamiliar to the teachers studied, and as such claims made are the result of teacher participation in a *simulation* of actual practice. Because research indicates teachers' relationships with and conceptions of the student influences interpretation (Morgan & Watson, 2002; Wallach & Even, 2005), we must mindful of these results in consideration of this difference.

An additional limitation is reflective of the theoretical lens selected. That is, all claims presented as findings were the product of researcher constructed third-order models that represent the second-order models of teachers. The third-order models were constructed with methods that would optimize the fit of the claims with that which teachers published. However, published conceptions were dependent on teachers who were asked to share, but not filter their conceptions. Although the best intentions of participants are assumed, difficulties associated with the articulation of conceptions that often remain internal must be considered (Ericsson & Simon, 1993). As a result, it is probable these claims are consistent with some but not all that was internal of the teachers.

Taken as a whole, the findings point to the importance of professional development designed to assess and build on teacher awareness of how students develop and demonstrate understanding of a mathematical idea. This could involve professional development around research-based progressions that describe the development of student understanding of particular mathematical ideas and common difficulties that students demonstrate in developing that understanding. In conjunction, emphasis on content standards could help teachers become aware of the mathematical ideas children should understand at a particular point in time since these ideas inform interpretations. However, an essential element of this emphasis should be discussion around what constitutes understanding, and how understanding is represented in student responses.

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