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Analysis of Lateral Design Pressures, Vertical Frictional Forces and Bending Stresses on Horizontally Corrugated Steel Silo Wall Panels*

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Abstract: This study investigated the theoretical lateral design pressures, vertical frictional forces and bending stresses exerted by shelled corn on a wall configuration made of folded steel plates. Silo walls panels with trapezoidal corrugations were considered. Lateral design pressures and vertical frictional forces were calculated by using Janssen's equations for pressures in deep bins and bending stresses exerted by ensiled corn were calculated by using principle of engineering mechanics. Inclined sections of corrugations with 135º from the horizontal plane had the highest lateral design pressures and vertical frictional forces. Corrugations with tie bars had significantly lower bending stresses than the corrugations without tie bars. Using the results of this theoretical work, further studies can be performed for a complete silo model with a roof structure and a hopper bottom and wind forces and the shear stresses can be added to the model. This may give a better interpretation of the theoretical results on real models.
Key Words: Storage Bins, Silo Design, Wall Pressures, Steel Silos

Trapez Kesitli Çelik Silo Duvarlarında Yanal Basınç, Sürtünme Kuvveti ve Eğilme Gerilmesi Analizi

Özet: Bu çalışmada oluklu çelik silo duvarlarında mısır tarafından oluşturulan yanal tasarım basınçları, düşey sürtünme kuvvetleri ve eğilme gerilmeleri analiz edilmiştir. Silo duvarları trapez kesitli oluklara sahip duvar panellerinden oluşmaktadır. Yanal tasarım basınçları ve sürtünme kuvvetleri derin silolar için geliştirilmiş Janssen eşitlikleri kullanılarak ve eğilme gerilmeleri ise mühendislik mekaniği ilkeleri kullanılarak hesaplanmıştır. Trapez kesitlerin yatayla 135º derece açıya sahip olan kısımları en yüksek yanal basınca ve düşey sürtünme kuvvetine maruz kalmaktadır. Karşılıklı duvarları birbirine bağlayan bağlantı demirlerinin olduğu oluklar diğerlerine nazaran oldukça daha düşük eğilme gerilmesine maruz kalmaktadır. Bu teorik çalışmanın sonuçları kullanılarak çatı ve taban kısmı ile birlikte tam bir silo modeli dikkate alınarak rüzgar ve kesme kuvvetleri de eklenip daha ileri bir model çalışması yürütülebilir. Bu tür bir model, sonuçların daha iyi bir şekilde yorumlanabilmesine olanak sağlayacaktır.
Anahtar Kelimeler: Depolama Yapıları, Silo Tasarımı, Duvar Basınçları, Çelik Silolar

1. Introduction

Bins are designed to store materials which are more or less in granular forms. When the word 'silo' is mentioned, it usually brings to mind the storage facilities of large dimensions normally constructed in circular, square or rectangular plan-view configurations with vertical walls. Silos are often made of timber, steel or concrete. The size of a storage structure for granular materials can vary based on the type and maximum quantity of the material to be stored, storage duration, filling and discharge rates and the material handling system to be used. Filling and discharge rates have an effect on determination of size and flow characteristics of the solid.

The first bins and silos with large dimensions were built as early as the 1880s for storing large quantities of grain. In many industries, bulk solids are stored prior to processing. Early silo designs were based on the assumption that the bulk solids behave like liquids. A granular material or powder, unlike a liquid material, can resist static shear stresses. Furthermore, a cohesive material is capable of forming a pattern that obstructs the steady flow of the bulk solids.

*This paper is derived from M.S. Thesis

Unlike other structures, storage structures have an unusually high rate of structural failures (Molenda et al. 2009). This brings a need for more studies on silo design. Most of the failures occur primarily because the pressures on the walls of the structure cannot be exactly predicted when the grain is in motion. There are many reasons for silo failures and most occurrences were due to foundation failures, discharge over pressures, friction forces, pressure on silo bottom and abnormal outlets. The non-homogenous make up of grain makes the determination of the pressures exerted on bin walls difficult. Roberts (1882) studied the pressures exerted by wheat on the floor of square and hexagonal wood bins and concluded that in any silo cell which has parallel sides, the pressure of wheat on floor ceased when the grain was filled up to twice the diameter of the inscribed circle. Janssen (1895) made a significant contribution on the estimation of lateral and vertical pressures on silo walls and bottoms. Janssen studied deep bins made of wood. His study along with the corresponding pressure theories for lateral and vertical pressures on silo walls became the basis for calculating loads on silo walls for the next sixty years.

Wall friction is an important factor in determining the pressure exerted by grain on walls. Airy (1897) made some investigations to determine the coefficients of friction between the grain and the material of the bin. With the new materials used for bins in the beginning of 20th century, Jamieson (1904) conducted a series of grain pressure tests. Jamieson conducted his tests on round, square and rectangular model silos constructed with wood and steel. Results of these initial studies have been used for almost a century. Reimberts (1976) carried out tests on bins with sizes varying from small-scale models up to actual silos. He also developed his own formulas for lateral and vertical pressures exerted by the ensiled material on bin walls and floors.

The proper design of bulk storage structures requires knowledge of individual and bulk properties of the particulate material under static and dynamic loading conditions. Several researchers studied the bulk properties of various grains like bulk density, angle of repose, internal and external angles of friction and flow properties (Jenike (1964); Carr (1965); Reimberts (1987); Gaylord (1984), MWPS (1993), Ünal (2009), Thompson et al. (1998), Öztürk et al. (2008)).

In this study, lateral design pressures, vertical friction forces and bending stresses due to loads exerted by ensiled corn over the folded steel plates of silo walls were analyzed. Especially the difference in wall pressures over the different sections of corrugations and effects of tie bars on bending stresses were investigated.

2. Materials and Methods

The bin analyzed in this study has 500 x 500 cm square cross-section with a height of 725 cm. Bin walls were made of pre-fabricated folded steel plate wall panels. Corrugations were trapezoidal and height of each corrugation was 100 cm. There are seven corrugations along the height of the bin. Four tie bar sets were placed between opposite walls. A prefabricated wall panel made of folded steel plates with trapezoidal corrugations and bin configuration were shown in figure 1.

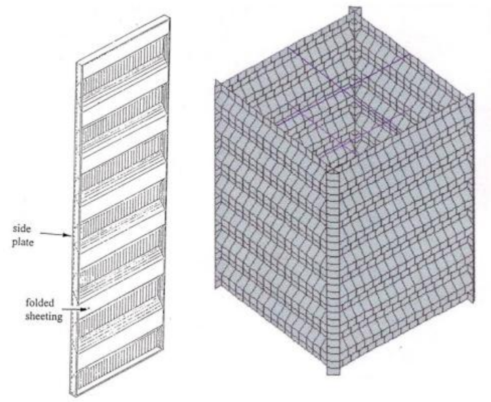

Figure 1. Pre-fabricated wall panel and bin configuration

The structure was composed of four walls made of horizontally corrugated steel wall panels. The wall panels are prefabricated and assembled at job site. The vertical sides of wall panels have side plates. The panels are welded to each other at corners and steel plate is used to form a triangular hollow core at corners. The bottom of structure is fixed to a flat bottom to prevent lateral movement and rotation. Because of the square cross-section of the bin, no corner rotations were considered. The top of the structure is open. There are no loads coming from the bottom and the top and the only loads considered are the design pressures and frictional forces due to ensiled grain inside the wall segments.

Grain Pressure Calculations

Grain pressures were calculated by using Janssen's equations for horizontal and vertical wall pressures. Loads acting on a bin were shown in Figure 2.

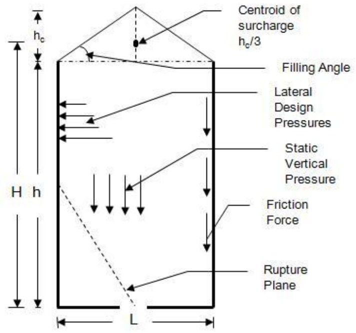

Figure 2. Loads acting in a silo

Janssen developed grain pressure equations for grain pressures on the walls of both shallow and deep bins. The first item to be determined to use Janssen's equations is to evaluate if the bin is a deep or shallow bin. Although there is no absolute or universally accepted definition of difference between shallow and deep bins, one reasonable distinction is based on whether or not the rupture plain within the grain mass intersects the bin wall. Applying these geometric criteria, bins can be classified as deep or shallow bins as shown in Table 1. The variables for height conditions were listed below in Table 1.

Table 1 . Silo classification

Height Condition	Bin Classification
H < (a / 2) tan (45 + ϕ / 2)	Shallow Bin
H > (a / 2) tan (45 + ϕ / 2)	Deep Bin

H= Height of the grain to centroid of surcharge (cm); a = Length of the side of square bin (cm); ϕ = Angle of internal friction, usually taken as the emptying angle of repose.

Static vertical pressures at any depth were calculated by using Eq. (1) (MWPS, 1983);

$$F = \frac{(R)(w)\left[1 - e^{-\frac{\mu' kY}{R}}\right]}{(\mu')(k)} \qquad (1)$$

Where;
F = Vertical pressure exerted by the stored granular material, (kg/cm^2)
R = Hydraulic radius (cm)
w = Weight of the stored material (g/cm^3)
μ'= Coefficient of friction between the stored material and the bin wall
k = Lateral-to-vertical pressure ratio
Y= Depth of grain mass from the level or sloping surface usually from the centroid of conical grain mass to the point where the bin loads are to be calculated, (cm)

Lateral static pressures were calculated by using Eq. (2);

$$L_s = (F)(k) \qquad (2)$$

Lateral-to-vertical pressure ratio is calculated by using Eq. (3) (MWPS, 1983);

$$k = \sin^2(\theta - \phi) \div \left\{\left[\sin^2(\theta)\sin(\theta + \phi')\right]T^2\right\} \qquad (3)$$

With

$$T = 1 + \left[\sin(\phi + \phi')\sin(\phi + \alpha) \div \sin(\theta - \alpha)\sin(\theta + \phi^2)\right]^{1/2} \qquad (4)$$

Where;
ϕ = Internal angle of friction (degrees)
θ = Angle of wall from the horizontal plane (degrees)
α = Filling angle of repose (degrees)
ϕ' = External angle of friction (degrees)

The grain pressures developed by Janssen apply for only static load conditions. Because the grain is in motion during the filling and unloading of silos, there is a significant increase in lateral pressure. To estimate the lateral grain pressures for dynamic load conditions, lateral grain pressures for static load conditions should be multiplied by appropriate over-pressure coefficient, Cd. The over-pressure coefficient converts static-load pressures to expected dynamic-load pressures. Over-pressure coefficients to be used were taken from Figure 3. Over-pressure factors do not apply to frictional forces. Then, the lateral design pressures are expressed as;

$$L_d = (L_s)(C_d) \qquad (5)$$

The grain pressures normal to sloping surfaces, F_n, is calculated by (MWPS, 1983);

$$F_n = F\cos^2\psi + L_d \sin^2\psi \qquad (6)$$

Where, ψ is the angle of sloping part from horizontal plane (degrees)

Vertical friction forces on the vertical sections of corrugations were calculated by using Eq.(7);

$$V_w = (L_d)(\mu') \qquad (7)$$

Friction forces on sloping sections of corrugations were calculated by using Eq.(8);

$$V_w = (F_n)(\mu') \qquad (8)$$

Shelled corn was considered as the ensiled grain to calculate the wall pressures and frictional forces exerted by the ensiled grain. Following grain properties were used in calculations (MWPS, 1983):

Bulk density of the shelled corn, w = 768,89 kg/m^3

Coefficient of friction between ensiled corn and steel, μ'= 0.20

Internal angle of friction, ϕ = 27º

Filling angle of repose, α = 16 º

External angle of friction, ϕ', equal to $\tan^{-1}\mu'$= 11.3 º

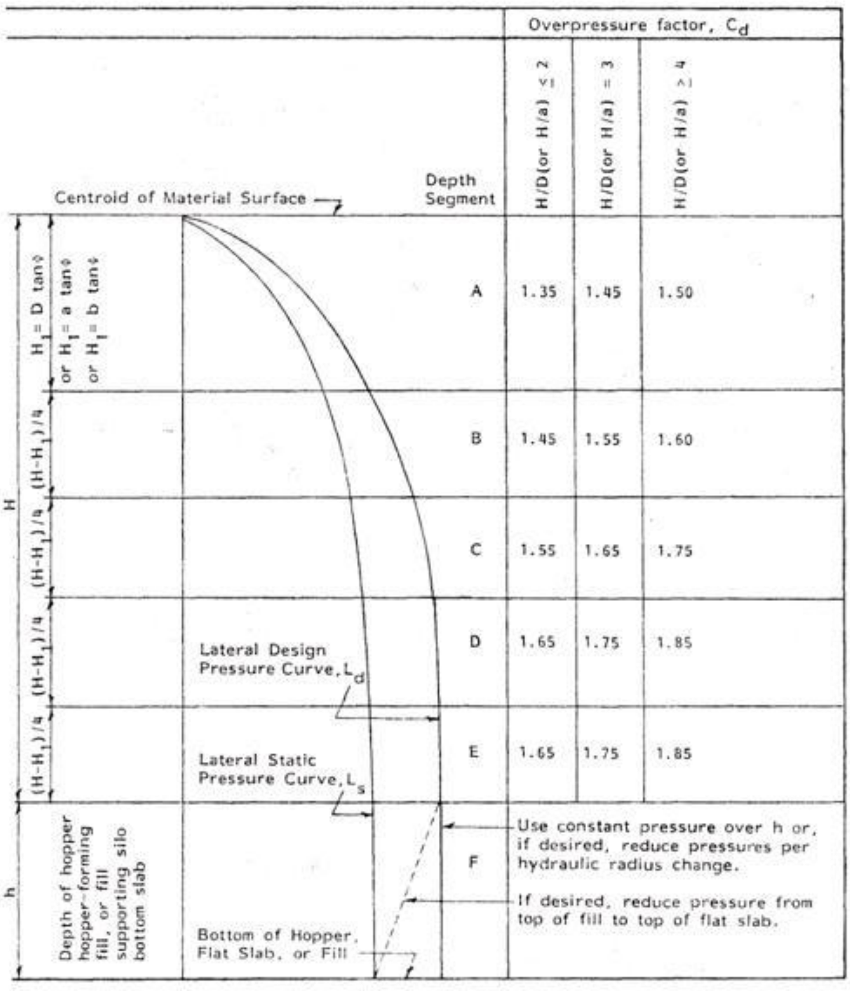

Figure 3. Minimum over-pressure factors, C_d, for deep bins (MWPS, 1983)

Bending stresses calculation

Cross-section of a corrugation was given in Figure 4. Sections of corrugations were numbered as shown in the figure. The moment of inertia of one corrugation along the Y-Y axis (Ravenet, 1984) was calculated by using Eq.(9);

$$I = \frac{2}{12} e\, L_1^{\,3} \cos^2(45) + 2\left[\frac{1}{12} L_2 e^3 + e L_2 \left(\frac{b}{2}\right)^2\right] \qquad (9)$$

Where,
e = thickness of the plate (cm)
L_1 = Length of the inclined straight profile (cm)
L_2 = Length of the vertical straight profile (cm)
b = Width of the corrugation (cm)

Section modules of the corrugation with respect to Y-Y axis was calculated by using Eq.(10);

$$S = \frac{2I}{b} \qquad (10)$$

Maximum bending moment at the ends of corrugations due to lateral design pressures and frictional forces were calculated by using Eq.(11);

$$M_{\max} = \frac{w_T L^2}{12} \qquad (11)$$

Where;
w_T = Total uniformly distributed load over a corrugation due to lateral design pressures and frictional forces (kg/cm^2)
L = length of the corrugation (cm)

The maximum bending moment at the ends of corrugations due to tension forces at the ends of tie bars was calculated by using Eq. (12);

$$M_{\max} = \frac{PL}{8} \qquad (12)$$

Where, P is the tension force (kg).

Bending stresses for each corrugation were calculated by using Eq. (13);

$$f_b = \frac{M_{\max}}{S} \qquad (13)$$

The corrugations were numbered from 1 to 7 starting from the bottom of the bin. The corrugation numbers and corresponding bin elevations were given in Figure 4.

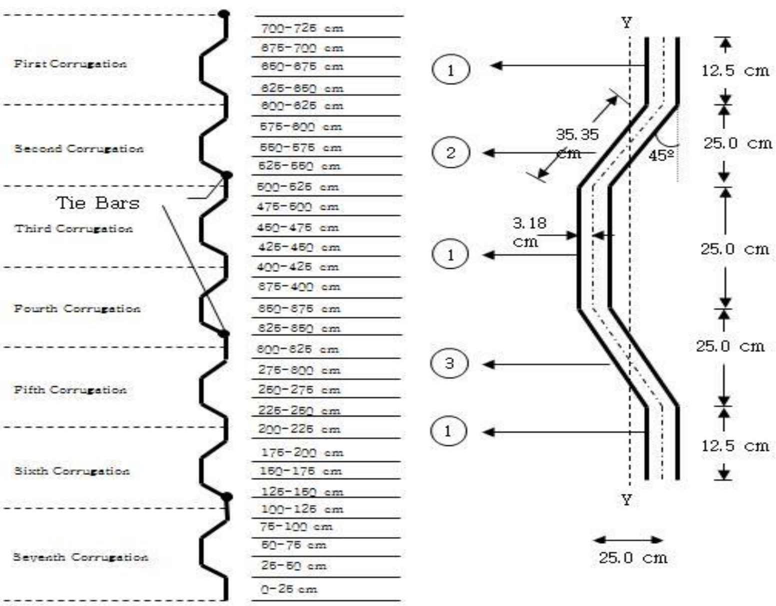

Figure 4. Corrugation numbers, elevations, dimensions and sections

3. Results and Discussion

Initially, calculations were performed to determine whether the bin is classified as shallow or deep bin and classification was made based on the criteria given in Table 1. The bin was classified as a deep bin.

Since horizontally corrugated steel silo wall panels and trapezoidal corrugations were taken into consideration, each segment of corrugations had different T and corresponding k values for pressure calculations. Values were calculated by using Equation 3 and 4. Following T and k values were used in calculation of lateral design pressures;

T =1.3474 and k = 0.446 for vertical sections of corrugations (section 1)

T =1.4506 and k = 0.109 for the sections with 45 º from the horizontal plane (section 2)

T =1.2739 and k = 2.010 for the sections with 135 º from the horizontal plane(section 3)

Over-pressure factors were taken from Figure 3 by using the criteria specified within the table. Over-pressure coefficients used to convert static pressures into dynamic pressures were given in Table 2.

Table 2. Over-pressure factors, C_d, for each depth segment of bin wall

Y (cm)	C_d
0 – 248	1.35
248 – 367	1.45
367 – 486	1.55
486 – 605	1.65
605 – 724	1.65

Lateral design pressures and vertical friction forces calculated by using Janssen's equations for each depth segment of wall panels were given in Table 3.

Change in lateral design pressures were presented in Figure 5. Lateral design pressures exerted by ensiled grain over different sections of the corrugations were presented as separate lines for each section. While section 3 of corrugations with 45 degrees from the horizontal plane had the highest lateral design pressures, section 2 with 135 degrees had the lowest pressures. Vertical sections of corrugations had pressure in between 2 and 3. Pressure calculations over section 2 were based on Eq. 6 developed for pressures over sloping surfaces. Therefore section 2 of corrugations had the highest lateral design pressures.

Change in vertical frictional forces was presented in Figure 6. Again, separate lines represent different sections of corrugations. Vertical frictional forces also yielded similar results with lateral design pressures. While section 3 of corrugations with 45 degrees from the horizontal plane had the highest vertical frictional forces, section 2 with 135 degrees had the lowest frictional forces. Vertical sections of corrugations had frictional forces in between 2 and 3. While grain-to-steel friction coefficients are effective for sections 2 and 3, grain-to-grain friction coefficient is effective for hallow sections of the corrugations.

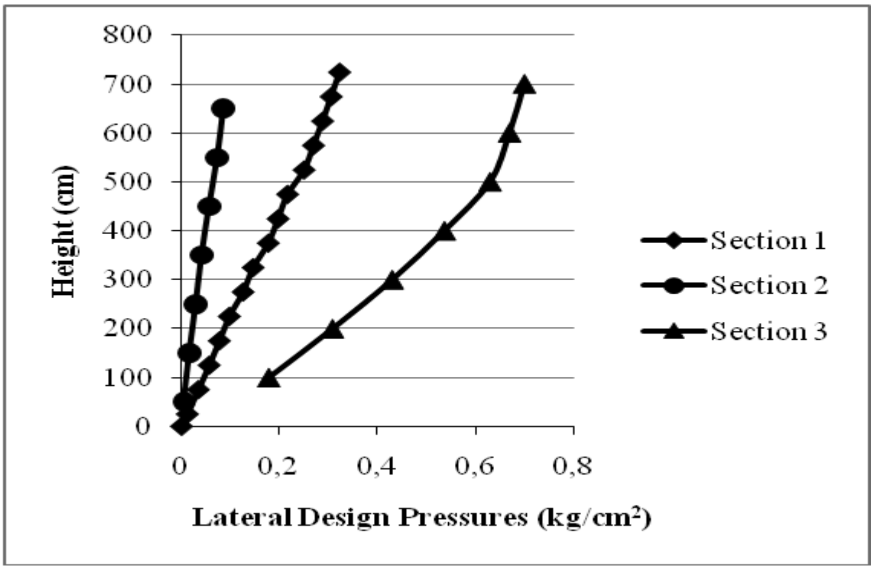

Figure 5. Change in lateral design pressures

Table 3. Lateral design pressures and vertical frictional forces

Y(cm)	F(kg/cm^2)	k	L_s(kg/cm^2)	C_d	L_d(kg/cm^2)	V_w(kg/cm^2)
0	0	0.446	0	1.35	0	0
25	0,019686	0.446	0,00878	1.35	0,011853	0,002371
50	0,038669	0.109	0,004215	1.35	0,00569	0,001138
75	0,056949	0.446	0,025399	1.35	0,034289	0,006858
100	0,066089	2.009	0,132772	1.35	0,179242*	0,035848
125	0,093508	0.446	0,041705	1.35	0,056301	0,01126
150	0,115303	0.109	0,012568	1.35	0,016967	0,003393
175	0,127959	0.446	0,05707	1.35	0,077044	0,015409
200	0,113897	2.009	0,22882	1.35	0,308906*	0,061781
225	0,161706	0.446	0,072121	1.35	0,097363	0,019473
250	0,191235	0.109	0,020845	1.45	0,030225	0,006045
275	0,194047	0.446	0,086545	1.45	0,12549	0,025098
300	0,147645	2.009	0,296618	1.45	0,430096*	0,086019
325	0,225685	0.446	0,100656	1.45	0,145951	0,02919
350	0,265057	0.109	0,028891	1.45	0,041892	0,008378
375	0,255917	0.446	0,114139	1.55	0,176916	0,035383
400	0,172252	2.009	0,346054	1.55	0,536384*	0,107277
425	0,284743	0.446	0,126995	1.55	0,196843	0,039369
450	0,337473	0.109	0,036785	1.55	0,057016	0,011403
475	0,312866	0.446	0,139538	1.55	0,216284	0,043257
500	0,189829	2.009	0,381366	1.65	0,629254*	0,125851
525	0,339583	0.446	0,151454	1.65	0,249899	0,04998
550	0,409186	0.109	0,044601	1.65	0,073592	0,014718
575	0,365596	0.446	0,163056	1.65	0,269042	0,053808
600	0,201781	2.009	0,405378	1.65	0,668874*	0,133775
625	0,390907	0.446	0,174344	1.65	0,287668	0,057534
650	0,47879	0.109	0,052188	1.65	0,08611	0,017222
675	0,414811	0.446	0,185006	1.65	0,305259	0,061052
700	0,210921	2.009	0,42374	1.65	0,699171*	0,139834
725	0,438012	0.446	0,195354	1.65	0,322333	0,064467

* Calculated by using $F_n = F\cos^2\psi + L_d \sin^2\psi$

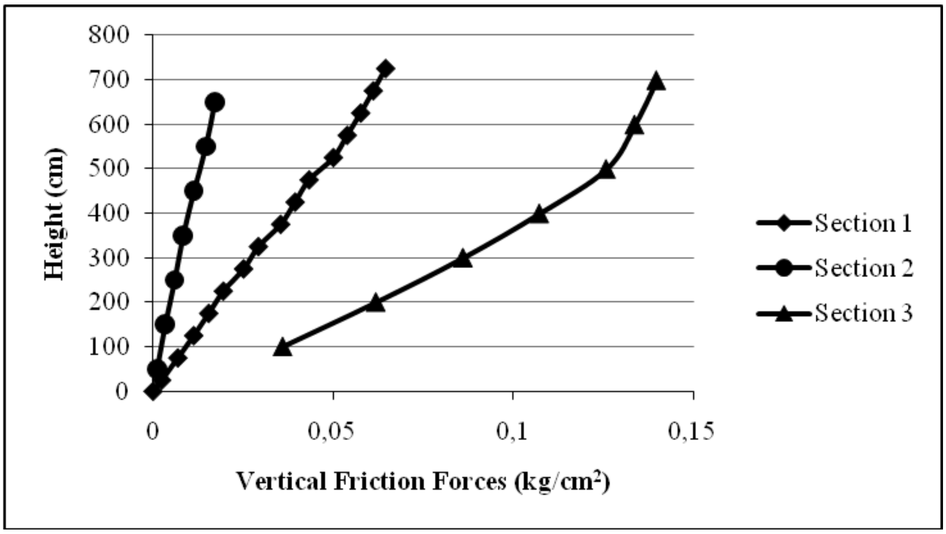

Figure 6. Change in vertical frictional forces

Change in bending stresses was presented in Figure 7. Bending stresses were presented for each corrugation. Corrugations 1, 2, 4 and 6 with tie bars had lower bending stresses than corrugations 3, 5 and 7. Results clearly indicate the effect of tie bar on bending stresses developed due to grain pressures over the walls.

Kibar et al (2006) developed a software for grain pressure calculations in grain bins and used Janssen equations for grain pressure and frictional force calculation. Researchers obtained increasing pressures with increasing height of grain ensiled into silo.

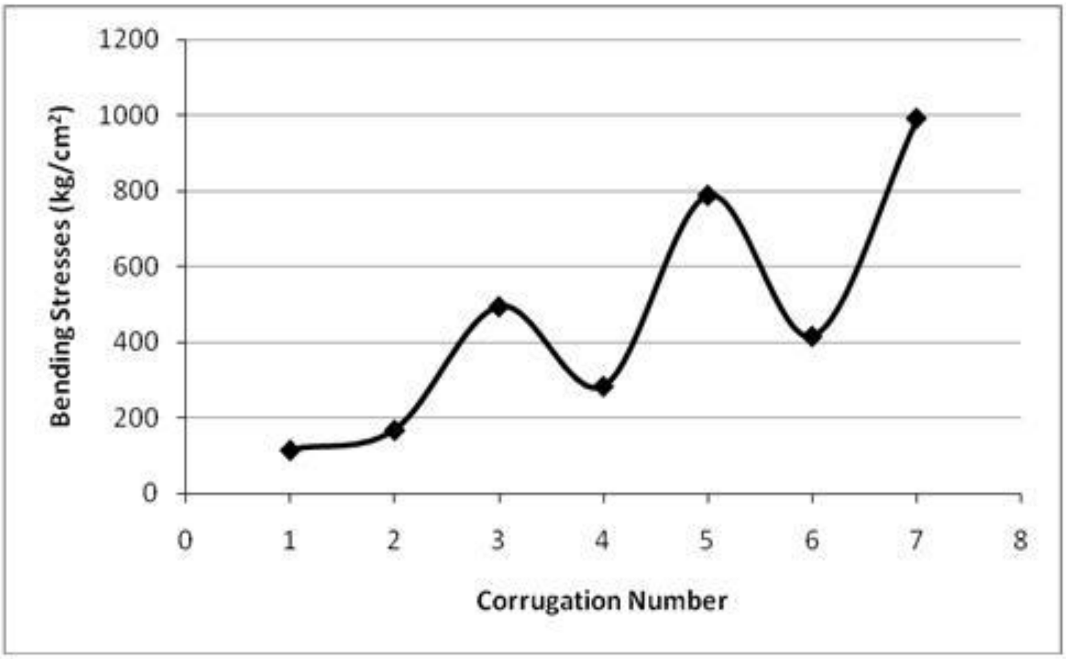

Figure 7. Change in bending stresses

4. Conclusions

This study investigated the theoretical lateral design pressures and vertical frictional forces and bending stresses exerted by shelled corn on a wall configuration made of folded steel plates. For the future studies, instead of analyzing only the walls, a complete silo model with a roof structure and a hopper bottom can be analyzed and wind forces and the shear stresses can be added to the model. Studies can be performed on a full scale model of the structure with the same characteristics mentioned above recommended study and a comparison can be made between a theoretical and a real model. This may give good interpretations of the theoretical results on real models.

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