JOURNAL OF NEW RESULTS IN SCIENCE (JNRS)
ISSN: 1304-7981
http://dergipark.gov.tr/jnrs
Research Article

Volume : 8
Issue: 1
Year: 2019
Pages: 6-13

Received: 15.02.2018
Accepted: 25.09.2019

Editors-in-Chief: ***Çetin ÇEKİÇ***
Area Editor: ***Serkan DEMİRİZ/Bilge GÖZENER***

On Stereographic Semicircular Quasi Lindley Distribution

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ABSTRACT: In this paper, we made an attempt to construct a new Semicircular model, we call this as "**StereographicSemicircular Quasi Lindleydistribution**," by applying modified inverse Stereographic projection on Quasi Lindley distribution (Rama and Mishra (2013)) for modeling semicircular data. Probability density and cumulative distribution functions of said model are derived and their graphs are plotted for various values of parameters. The first two trigonometric moments are derived and the proposed model is extended for *l*-axial data also.
Keywords: Semicircular models, inverse stereographic projection, trigonometric moments.
AMS Subject Classification: 60E05, 62H11.

1. Introduction

Angular data/ circular data are very common in the areas of biology, geology, meteorology, earth science, political science, Economics, computer science, etc. Full circular models are prevalent at most of the text books (Fisher, 1993; Jammalamadaka and Sen Gupta, 2001; Mardia and Jupp, 2000). In some of the cases the angular data does not require full circular models for modeling; this fact is noted in Guardiola (2004), Jones (1968), Byoung et al (2008) and Phani et al (2013). For example, when sea turtles emerge from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is well sufficient for modeling such data. Similarly, when an aircraft is lost but its departure and its initial headings are known, a semicircular random variable is sufficient for such angular data. And few more examples of semicircular data are available in Ugai et al (1977).

Guardiola (2004) obtained the semicircular normal distribution by using a simple projection and Byoung et al (2008) developed a family of the semicircular Laplace distributions for modeling semicircular data by simple projection, Phani et al (2013) constructed some semicircular distributions by applying modified inverse Stereographic projection, Girija et al (2014) developed a family of semicircular Logistic distributions by applying simple projection. In this paper we develop new Semicircular model coined as Stereographic Semicircular Quasi Lindley distribution by applying modified inverse stereographic projection on Quasi Lindley distribution. The graphs of the density function and distribution function for various values of parameters are plotted. We derive the first two trigonometric moments of the proposed model to evaluate population characteristics.

2. Methodology of modified Inverse Stereographic Projection

Modified Inverse Stereographic Projection is defined by a one to one mapping given by $T(\theta) = x = v\tan\left(\frac{\theta}{2}\right)$, where $x \in (-\infty, \infty)$, $\theta \in (-\pi, \pi)$, $v \in \mathbb{R}^+$. Suppose x is randomly chosen on the interval $(-\infty, \infty)$.Let $F(x)$ and $f(x)$ denote the Cumulative distribution and probability density functions of the linear random variable X respectively. Then $T^{-1}(x) = \theta = 2\tan^{-1}\left\{\frac{x}{v}\right\}$ by Minh and Farnum (2003) is a random point on the unit circle. Let $G(\theta)$ and $g(\theta)$ denote the Cumulative distribution and probability density functions of this random point θ respectively. Then $G(\theta)$ and $g(\theta)$ can be written in terms of $F(x)$ and $f(x)$ using the following Theorem.

Theorem 2.1: For $v > 0$,

i) $G(\theta) = F\left(v\tan\left(\frac{\theta}{2}\right)\right)$

ii) $$g(\theta) = v\left(\frac{1+\tan^2\left(\frac{\theta}{2}\right)}{2}\right) f\left(v\tan\left(\frac{\theta}{2}\right)\right)$$

If a linear random variable X has a support on $\mathbb{R}$, then θ has a support on $(-\pi, \pi)$ and if X has a support on $\mathbb{R}^+$, then θ has a support on $(0, \pi)$. These means that, after the Inverse Stereographic Projection is applied, we can deal circular data if the support of X is on $\mathbb{R}$ and we can handle semicircular data if the support of X is on $\mathbb{R}^+$.

3. Stereographic Semicircular Quasi Lindley Distribution

Here a linear model, Quasi Lindley distribution is considered and by inducing modified inverse stereographic projection, a stereographic semicircular model is defined.

Definition: A random variable X on the real line is said to have Quasi Lindley Distribution with scale parameter $\alpha > 0$, shape parameter $\sigma > 0$ and location parameter α if the probability density and cumulative distribution functions of X are respectively given by

$$f(x;\alpha,\sigma) = \frac{\alpha}{(1+\sigma)}(\sigma + \alpha x)\exp(-\alpha x),\ \sigma > -1,\ 0 < \alpha \text{ and } 0 < x < \infty \tag{3.1}$$

and

$$F(x;\alpha,\sigma)=1-\exp(-\alpha x)\left[1+\frac{\alpha x}{(\sigma+1)}\right],\ \sigma>-1,\ 0<\alpha \text{ and } 0<x<\infty \tag{3.2}$$

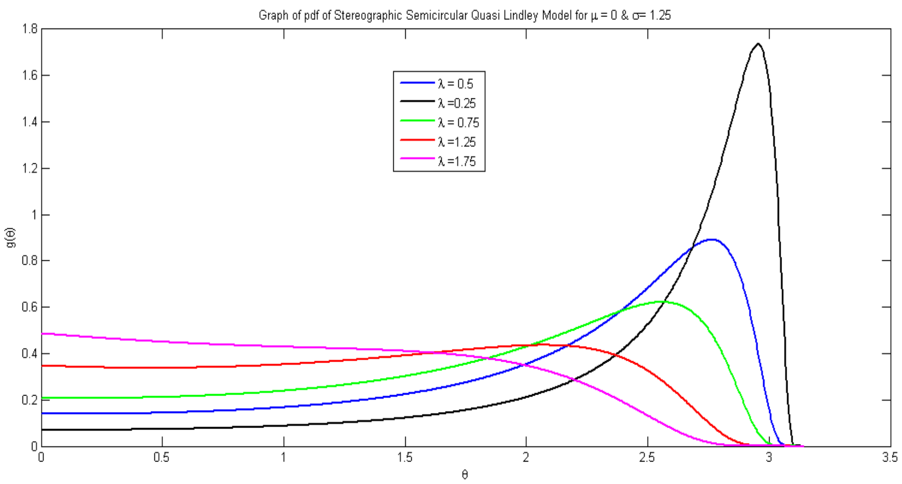
Then by applying modified inverse Stereographic projection defined by a one to one mapping $x = v\tan\left(\frac{\theta}{2}\right), v\in\mathbf{R}^{+}$, which leads to a Semicircular Model on unit semicircle. We call this model as Stereographic Semicircular Quasi Lindley Distribution.

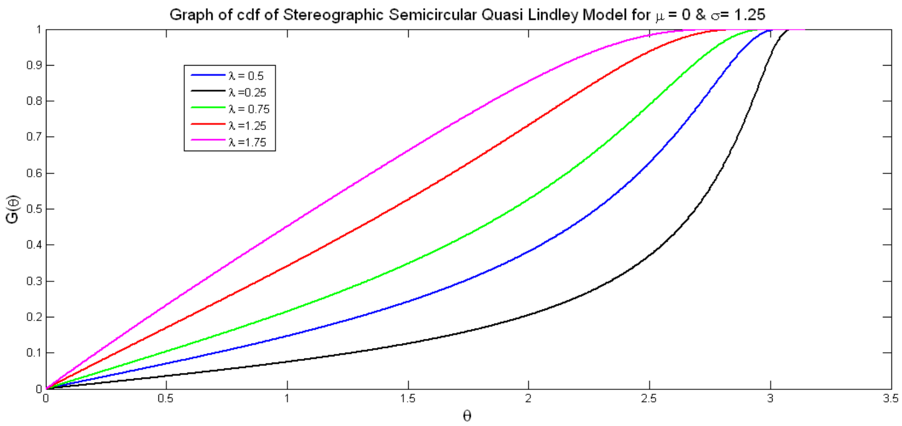
Definition: A random variable X_{SC} on the Semicircle is said to have the Stereographic Semicircular Quasi Lindley distribution with shape parameter $\sigma > -1$, location parameter μ and scale parameter $\sigma > 0$ denoted by **SSCQLD**(σ,λ,μ), if the probability density and the cumulative distribution functions are respectively given by

$$g(\theta)=\frac{\lambda\sec^2\left(\frac{\theta-\mu}{2}\right)}{2(\sigma+1)}\left(\sigma+\lambda\tan\left(\frac{\theta-\mu}{2}\right)\right)\exp\left(-\lambda\tan\left(\frac{\theta-\mu}{2}\right)\right),$$
where $0\le\mu,\theta<\pi,\ \lambda=v\alpha>0 \text{ and } \sigma>-1$ (3.3)

$$G(\theta)=1-\exp\left(-\lambda\tan\left(\frac{\theta-\mu}{2}\right)\right)\left[1+\frac{\lambda\tan\left(\frac{\theta-\mu}{2}\right)}{(\sigma+1)}\right]$$
where $0\le\mu,\theta<\pi,\ \lambda=v\alpha>0 \text{ and } \sigma>-1$ (3.4)

Graphs of pdf and cdf of Stereographic Semicircular Quasi Lindley Distribution for various values of parameters





3.Trigonometric moments of Stereographic Semicircular Quasi Lindley Distribution

It is customary to derive the trigonometric moments when a new distribution is proposed. Without loss of generality here we assume that $\mu = 0$, in (3.3) .The trigonometric moments of the distribution are given by $\{\varphi_{\mathrm{p}} : \mathrm{p} = 0, \pm 1, \pm 2, \pm 3, \ldots\}$, where $\varphi_{\mathrm{p}} = \alpha_{\mathrm{p}} + i\beta_{\mathrm{p}}$, with $\alpha_{\mathrm{p}} = E(\cos \mathrm{p}\theta)$ and $\beta_{\mathrm{p}} = E(\sin \mathrm{p}\theta)$ being the p^{th} order cosine and sine moments of the random angle θ, respectively.

Theorem 4.1 Under the pdf of Stereographic Semicircular Quasi Lindley Distribution with $\mu = 0$, the first four $\alpha_{\mathrm{p}} = E(\cos \mathrm{p}\theta)$ and $\beta_{\mathrm{p}} = E(\sin \mathrm{p}\theta)$, p=1, 2, 3, 4, are given as follows:

$$\alpha_1 = 1 - \frac{\lambda\sigma}{\sqrt{\pi}(\sigma+1)} G_{13}^{31}\left(\frac{\lambda^2}{4} \middle| \begin{matrix} -\frac{1}{2} \\ -\frac{1}{2}, 0, \frac{1}{2} \end{matrix} \right) - \frac{\lambda^2}{\sqrt{\pi}(\sigma+1)} G_{13}^{31}\left(\frac{\lambda^2}{4} \middle| \begin{matrix} -1 \\ -1, 0, \frac{1}{2} \end{matrix} \right),$$

$$\beta_1 = \frac{\lambda\sigma}{\sqrt{\pi}(\sigma+1)} G_{13}^{31}\left(\frac{\lambda^2}{4} \middle| \begin{matrix} 0 \\ 0, 0, \frac{1}{2} \end{matrix} \right) + \frac{\lambda^2}{\sqrt{\pi}(\sigma+1)} G_{13}^{31}\left(\frac{\lambda^2}{4} \middle| \begin{matrix} -\frac{1}{2} \\ -\frac{1}{2}, 0, \frac{1}{2} \end{matrix} \right),$$

$$\alpha_2 = 1 + \frac{4\lambda\sigma}{\sqrt{\pi}(\sigma+1)} \left[G_{13}^{31}\left(\frac{\lambda^2}{4} \middle| \begin{matrix} -\frac{3}{2} \\ -\frac{1}{2}, 0, \frac{1}{2} \end{matrix} \right) + G_{13}^{31}\left(\frac{\lambda^2}{4} \middle| \begin{matrix} -\frac{1}{2} \\ -\frac{1}{2}, 0, \frac{1}{2} \end{matrix} \right) \right]$$

$$- \frac{4\lambda^2}{\sqrt{\pi}(\sigma+1)} \left[G_{13}^{31}\left(\frac{\lambda^2}{4} \middle| \begin{matrix} -\frac{1}{2} \\ -\frac{1}{2}, 0, \frac{1}{2} \end{matrix} \right) + G_{13}^{31}\left(\frac{\lambda^2}{4} \middle| \begin{matrix} -1 \\ -1, 0, \frac{1}{2} \end{matrix} \right) \right]$$

$$\beta_2=\frac{2\lambda\sigma}{\sqrt{\pi}\left(\sigma+1\right)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)-2G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\0,0,\frac{1}{2}\end{matrix}\right)\right]$$

$$+\frac{2\lambda^2}{\sqrt{\pi}\left(\sigma+1\right)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)-2G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{3}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)\right]$$

where $\int_0^\infty x^{2\nu-1}\left(u^2+x^2\right)^{Q-1}e^{-\mu x}dx=\frac{u^{2\nu+2Q-2}}{2\sqrt{\pi}\Gamma(1-Q)}G_{13}^{31}\left(\frac{\mu^2u^2}{4}\middle|\begin{matrix}1-\nu\\1-Q-\nu,0,\frac{1}{2}\end{matrix}\right)$ (4.1)

for $\left|\arg u\pi\right|<\frac{\pi}{2}$, $\operatorname{Re}\mu>0$ and $\operatorname{Re}\nu>0$ and $G_{13}^{31}\left(\frac{\mu^2u^2}{4}\middle|\begin{matrix}1-\nu\\1-Q-\nu,0,\frac{1}{2}\end{matrix}\right)$ is called as

Meijer's **G-**function (Gradstein and Ryzhik, 2007, formula no. 3.389.2).

Proof:

$$\varphi_{\mathrm{p}}=\int_0^\pi e^{ip\theta}g\left(\theta\right)d\theta=\int_0^\pi\cos\left(\mathrm{p}\theta\right)g\left(\theta\right)d\theta+i\int_0^\pi\sin\left(\mathrm{p}\theta\right)g\left(\theta\right)d\theta$$

$$=E\left(\cos\left(\mathrm{p}\theta\right)\right)+i\,E\left(\sin\left(\mathrm{p}\theta\right)\right)=\alpha_{\mathrm{p}}+i\,\beta_{\mathrm{p}}\quad\text{for p}=0,\pm1,\pm2,\pm3,...,$$

For the first cosine and sine moments, use the transformations $x=\tan\left(\frac{\theta}{2}\right)$, $\cos\theta=1-\frac{2x^2}{1+x^2}$ and $\sin\theta=\frac{2x}{1+x^2}$,the results α_1 and β_1 follows by the integral formula **(4.1)**

Now $\mathrm{E}\left(\cos\left(\mathrm{p}\theta\right)\right)=\frac{\lambda}{2\left(\sigma+1\right)}\int_0^\pi\cos\left(\mathrm{p}\theta\right)\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$ and

$$\mathrm{E}\left(\sin\left(\mathrm{p}\theta\right)\right)=\frac{\lambda}{2\left(\sigma+1\right)}\int_0^\pi\sin\left(\mathrm{p}\theta\right)\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$\alpha_1=\frac{\lambda}{2\left(\sigma+1\right)}\int_0^\pi\cos\theta\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$=\frac{\lambda}{\left(\sigma+1\right)}\int_0^\infty\left[1\text{-}\frac{2x^2}{1+x^2}\right]\left(\sigma+\lambda x\right)e^{-\lambda x}dx$$

$$=1-\frac{2\lambda\sigma}{\left(\sigma+1\right)}\int_0^\infty\frac{x^2}{1+x^2}e^{-\lambda x}dx-\frac{2\lambda^2}{\left(\sigma+1\right)}\int_0^\infty\frac{x^3}{1+x^2}e^{-\lambda x}dx$$

$$=1-\frac{2\lambda\sigma}{\left(\sigma+1\right)}\int_0^\infty x^{2\left(\frac{3}{2}\right)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx-\frac{2\lambda^2}{\left(\sigma+1\right)}\int_0^\infty x^{2(2)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx$$

$$=1-\frac{2\lambda\sigma}{(\sigma+1)}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)\right]-\frac{2\lambda^2}{(\sigma+1)}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\ -1,0,\frac{1}{2}\end{matrix}\right)\right]$$

$$\alpha_1=1-\frac{\lambda\sigma}{\sqrt{\pi}(\sigma+1)}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\lambda^2}{\sqrt{\pi}(\sigma+1)}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\ -1,0,\frac{1}{2}\end{matrix}\right)$$

$$\beta_1=\frac{\lambda}{2(\sigma+1)}\int_0^{\pi}\sin\theta\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$=\frac{2\lambda}{(\sigma+1)}\int_0^{\infty}\left(\frac{x}{1+x^2}\right)(\sigma+\lambda x)e^{-\lambda x}dx$$

$$=\frac{2\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2(1)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx+\frac{2\lambda^2}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{3}{2}\right)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx$$

$$\beta_1=\frac{\lambda\sigma}{\sqrt{\pi}(\sigma+1)}G\left(\frac{\lambda^2}{4}\middle|\begin{matrix}0\\ 0,0,\frac{1}{2}\end{matrix}\right)+\frac{\lambda^2}{\sqrt{\pi}(\sigma+1)}G\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)$$

To obtain second cosine and sine moments α_2 and β_2, we use the transformations $x=\tan\left(\frac{\theta}{2}\right)$, $\cos 2\theta=1+\frac{8x^4}{\left(1+x^2\right)^2}-\frac{8x^2}{\left(1+x^2\right)}$ and $\sin 2\theta=\frac{4x}{\left(1+x^2\right)}-\frac{8x^3}{\left(1+x^2\right)^2}$, the results of α_2 and β_2 follows by the same integral formula of α_1.

$$\alpha_2=\frac{\lambda}{2(\sigma+1)}\int_0^{\pi}\cos 2\theta\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$=\frac{\lambda}{(\sigma+1)}\int_0^{\infty}\left[1+\frac{8x^4}{\left(1+x^2\right)^2}-\frac{8x^2}{\left(1+x^2\right)}\right](\sigma+\lambda x)e^{-\lambda x}dx$$

$$=1+\frac{8\lambda}{(\sigma+1)}\int_0^{\infty}\frac{x^4}{\left(1+x^2\right)^2}(\sigma+\lambda x)e^{-\lambda x}dx-\frac{8\lambda}{(\sigma+1)}\int_0^{\infty}\frac{x^2}{\left(1+x^2\right)}(\sigma+\lambda x)e^{-\lambda x}dx$$

$$=1+\frac{8\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{5}{2}\right)-1}\left(1+x^2\right)^{-1-1}e^{-\lambda x}dx+\frac{8\lambda^2}{(\sigma+1)}\int_0^{\infty}x^{2(3)-1}\left(1+x^2\right)^{-1-1}e^{-\lambda x}dx$$

$$-\frac{8\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{3}{2}\right)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx-\frac{8\lambda^2}{(\sigma+1)}\int_0^{\infty}x^{2(2)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx$$

$$\alpha_2=1+\frac{4\lambda\sigma}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{3}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)+G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)\right]$$

$$-\frac{4\lambda^2}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)+G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\ -1,0,\frac{1}{2}\end{matrix}\right)\right]$$

$$\beta_2=\frac{\lambda}{2(\sigma+1)}\int_0^{\pi}\sin 2\theta\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$=\frac{\lambda}{(\sigma+1)}\int_0^{\infty}\left[\frac{4x}{(1+x^2)}-\frac{8x^3}{(1+x^2)^2}\right](\sigma+\lambda x)e^{-\lambda x}dx$$

3. $$=\frac{\lambda}{(\sigma+1)}\int_0^{\infty}\frac{4x}{(1+x^2)}(\sigma+\lambda x)e^{-\lambda x}dx-\frac{\lambda}{(\sigma+1)}\int_0^{\infty}\frac{8x^3}{(1+x^2)^2}(\sigma+\lambda x)e^{-\lambda x}dx$$

~~4.~~ $$\frac{4\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2(1)-1}(1+x^2)^{0-1}e^{-\lambda x}dx+\frac{4\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{3}{2}\right)-1}(1+x^2)^{0-1}e^{-\lambda x}dx$$

$$-\frac{8\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2(2)-1}(1+x^2)^{-1-1}e^{-\lambda x}dx-\frac{8\lambda^2}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{5}{2}\right)-1}(1+x^2)^{-1-1}e^{-\lambda x}dx$$

$$\beta_2=\frac{2\lambda\sigma}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}0\\ 0,0,\frac{1}{2}\end{matrix}\right)-2G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\ 0,0,\frac{1}{2}\end{matrix}\right)\right]$$

$$+\frac{2\lambda^2}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)-2G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{3}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)\right]$$

Higher-order moments can be obtained similarly.

The first two trigonometric moments are sufficient for calculating population characteristics.

5. Stereographic -l-axial Quasi Lindley distribution

We extend the above Stereographic Semicircular model to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1,2,...,$. So it is desirable to extend the Stereographic Semicircular Quasi Lindley distribution to construct the Stereographic-l -axial Quasi Lindley distribution.We considers the density function of Stereographic Semicircular Quasi Lindley distribution and use the transformation $\theta=2\phi/l$, $l=1,2,...,$. The probability density function of Quasi Lindley distribution in ϕ is given by

$$g(\phi)=\frac{\lambda\sec^2\left(\frac{\phi}{2}\right)}{2(\sigma+1)}\left(\sigma+\lambda\tan\left(\frac{\phi}{2}\right)\right)\exp\left(-\lambda\tan\left(\frac{\phi}{2}\right)\right),$$

where $0 \le \mu, \phi < \pi,\ \lambda = v\alpha > 0$ and $\sigma > -1$

Then the corresponding Stereographic-l -axial Quasi Lindley distribution is

$$g(\theta) = \frac{\lambda l \sec^2\left(\frac{l\theta}{4}\right)}{4(\sigma+1)}\left(\sigma + \lambda \tan\left(\frac{l\theta}{4}\right)\right)\exp\left(-\lambda \tan\left(\frac{l\theta}{4}\right)\right),$$
$$0 < \theta < \frac{2\pi}{l},\ \sigma > -1, \lambda > 0 \text{ and } l = 1, 2, .. \quad (5.1)$$

We call it as **Stereographic -l -axial Quasi Lindley distribution.**

Case (1) When $l = 1$, in the probability density function (4.1), we get the density function

$$g(\theta) = \frac{\lambda \sec^2\left(\frac{\theta}{4}\right)}{4(\sigma+1)}\left(\sigma + \lambda \tan\left(\frac{\theta}{4}\right)\right)\exp\left(-\lambda \tan\left(\frac{\theta}{4}\right)\right),$$
$$0 < \theta < 2\pi,\ \sigma > -1, \lambda > 0 \quad (5.2)$$

We call it as **Stereographic CircularQuasi Lindley distribution**.

Case (2) When $l = 2$, the probability density function (5.1) is the same as that of **Stereographic Semicircular Quasi Lindley Distribution.**

6.Conclusion

In this paper, we discussed semicircular distribution induced by modified inverse stereographic projection on Quasi Lindley Distribution. The density and distribution function of Stereographic semicircular Quasi Lindley distribution admit explicit forms, as do trigonometric moments.As this distribution is asymmetric, suitable for modeling asymmetrical directional data.

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