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Regional Frequency Analysis of Maximum Daily Rainfalls Based on L-Moment Approach

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Abstract: The main goal of the study is to perform regional frequency analysis of maximum daily rainfalls selected for each year among daily rainfalls measured over Tokat region by using L-moment approach. Initially, using Runs and Mann-Whitney statistics to detect whether the conditions of randomness and homogeneity were implemented were applied to the maximum daily rainfalls. Thereafter, the most suitable distribution among the selected various statistical distributions whose parameters were predicted via L-moment approach for four hydrologic homogeneous regions of Tokat region, which was divided into four hydrologic homogeneous region as West-W, Central North-CN, Central South-CS and East-E, was determined according to the mean absolute deviation index (MADI) and mean square deviation index (MSDI) measures. The results of MADI and MSDI showed that the most suitable statistical distributions were, generalized logistic (GLO) for W and CN, generalized pareto (GPA) for CS, and generalized extreme value type I (GEV) for E.
Key Words: Maximum rainfall, hydrologic homogeneous region, L-moment

L-Moment Yaklaşımı ile Maksimum Günlük Yağmurların Bölgesel Frekans Analizi

Özet: Bu çalışmanın ana amacı, Tokat bölgesinde ölçülen günlük yağmurlar arasından her yıl için seçilen maksimum günlük yağmurların bölgesel frekans analizini yapmaktır. Öncelikle, rasgelelik ve homojenlik şartlarının yerine getirilip getirilmediğini saptamak için Runs ve Mann-Whitney istatistikleri maksimum günlük yağmurlara uygulandı. Daha sonra, Batı-W, Orta Kuzey-CN, Orta Güney-CS ve Doğu-E olarak dört hidrolojik homojen bölgeye ayrılan Tokat bölgesinin bu homojen yöreleri için parametreleri L-moment yöntemi ile tahmin edilen seçilmiş değişik dağılımlar arasından en uygun olanı, “the mean absolute deviation index (MADI) ve mean square deviation index (MSDI)” ölçütlerine göre belirlendi. MADI ve MSDI sonuçları, W ve CN için genelleşmiş logistic (GLO), CS için genelleşmiş pareto (GPA) ve E için genelleşmiş extreme value type I (GEV)’in en uygun olasılık dağılım olduklarını gösterdi.
Anahtar Kelimeler: Maksimum yağmur, hidrolojik homojen bölge, L-moment

1. Introduction

Knowledge related to distributions of extreme rainfall depths has a great important on flood estimation, the design of water-related structure, erosion, and agriculture. Therefore, the main goal should be to specify the most suitable probability distribution fit to the observations. But, a common problem encouraged in many aspects of water resources engineering is that of estimating the return period of rare events such as extreme flood and precipitation for a site or a group of sites. The selected quantile of under-or over design criterion concerning with hydraulic structures is exposed to risk as the return period is determined according to cost and economic-strategic significance of the structure. Selecting a reliable design quantile, which affect on design, operation, management and maintain of a hydraulic structure, considerably depends on statistical methods used in parameter estimation belonging to probability distribution (Hosking and Wallis, 1993).

Past observations is fit with a probability distribution used to predict the exceedance probability of future events. But, defining a true distribution for hydrological and meteorological observations continues to be major question facing researchers. However, many extreme event series are too short for a reliable estimation of extreme events. This condition complicates both the identification of appropriate statistical distribution for describing the observations and the estimation of the parameters of a selected distribution. But, the most popularized method to frequency analysis in recent time is that L-moment approach introduced by Hosking (1990). The advantages of this method over conventional moments are that they are relatively insensitive to outliers and do not have sample size related bounds.

Moreover, the parameter estimations are more reliable than the conventional method of moment estimates, particularly from small samples, and are usually computationally more tractable than maximum likelihood estimates. On the other hand, estimators of L-moments are virtually unbiased (Hosking, 1990; Park et al., 2001).

The main purpose of this article is to fulfill the identification of a suitable probability distribution, including normal (N), two-parameter lognormal (LN), three-parameters lognormal (LN3), logistic (LOG), generalized logistic (GLO), extreme value type I (EV), generalized extreme value type I (GEV), generalized pareto (GPA) by L-moment technique commonly used in recent time for maximum daily rainfalls selected for each year among daily rainfalls measured over Tokat region.

2. Material and Method

Tokat region, selected as the study region, is bounded by latitudes 39º 45' N and 40º 45' N, and longitudes 35º 30' E and 37º 45' E, covering 10,160.7 km^2. About 30% of the region is occupied by cropland. Wheat is the major food crop (the average sowing area is 68.5% of the total cropped area) not only in the district, but in all of Turkey. The major sources of irrigation are rainfall, canals and groundwater.

Rainfall amounts vary spatially within the region covered by a given storm. Therefore, this region should be divided into hydrologically homogeneous regions in which rainfall amounts recorded at the rain gauges are assumed to be identical to obtain reliable results in hydrologic studies related to rainfall (Okman, 1994). For this reason, the studied region was divided into four hydrologically homogeneous regions, West (W), Central North (CN), Central South (CS) and East (E), considering the mean, standard deviation and standard error of monthly rainfall recorded from the rain gauges and altitudes of the rain gauges in Tokat region. These four regions are separated from each other by Thiessen polygons. Average annual rainfall levels associated with these four regions are 415.8, 479.6, 413.3, and 557.2 for W, CN, CS, and E, respectively (Figure 1) (Yürekli, 1999). Considering the similarity principle of rainfall amounts from rain gauges in a hydrologically homogeneous region, a rain gauge with the longest observation period was selected for each hydrologically homogeneous region.

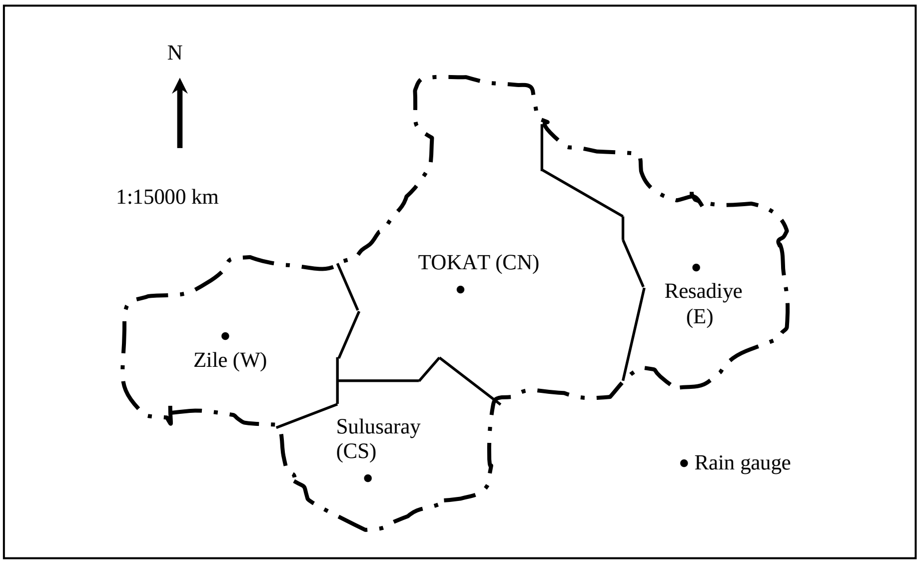

Figure 1. Hydrologically Homogeneous Regions of the Study Area.

The observations from hydrologic events should be come from the same population to be able to accurately perform frequency analysis related to the events. Therefore, the data used in frequency analysis should be random and homogeneous (Okman, 1975).

2.1 Testing of Randomness

The runs test can be used to decide if a data set is from a random process. A run is defined as a series of increasing values or a series of decreasing values. The number of increasing (or decreasing) values is the length of the run. In a random data set, the probability that the $(i + 1)^{th}$ value is larger or smaller than the i^{th} value follows a binomial distribution, which forms the basis of the runs test. The first step in the runs test is to compute the sequential differences (Y_i - Y_{i-1}). Positive values indicate an increasing value, whereas negative values indicate a decreasing value. In other terms, if $Y_i > Y_{i-1}$ a 1 (one) is assigned for an observation and a 0 (zero) otherwise. The series then has an associated series of 1s and 0s. To determine if the number of runs is the correct number for a series that is random, let T be the number of observations, T_A be the number above the mean, T_B be the number below the mean and R be the observed number of runs. Then, using combinatorial methods, the probability P(R) can be established and the mean and variance of R can be derived (Cromwell *et al.*, 1994; Gibbons, 1997). When T is relatively large (>20) the distribution of R is approximately normal.

$$E(R) = \frac{T + 2T_A T_B}{T} \qquad (1)$$

$$V(R) = \frac{2T_A T_B (2T_A T_B - T)}{T^2 (T-1)} \qquad (2)$$

$$Z_N = \frac{R - E(R)}{\sqrt{V(R)}} \approx N(0,1) \qquad (3)$$

The null hypothesis is rejected if the calculated Z_N value is greater than the selected critical value obtained from the standard normal distribution table at 0.05 significance level. In other words, the x(t) series is decided to be non-random.

2.2 Testing of Homogeneity

The occurrence of non-homogeneity in a hydrologic data set can result from gradual natural or man-induced changes in the hydrologic environment producing the data sequences. Changes in watershed conditions over period of several years and urbanization on a large scale may result in corresponding changes in streamflow characteristic, and changes in precipitation amounts, respectively (Huff and Changnon, 1973). Besides, natural events such as earthquakes, large forest fires and landslide that quickly and significantly alter hydrologic regime of an area cause jumps in the time series. Also, jumps in the time series results from man-made changes such as a new dam construction, and the beginning or cessation of pumping of ground water (Bayazit, 1981). Mann-Whitney U statistic is commonly used to decide whether observations of a hydrologic variable are comes from the same population. In order to apply the Mann-Whitney test, the raw data sequences with N elements should be divided into two samples P and Q groups, which have n_P and n_Q elements, respectively. The raw data set is then ranked from lowest to highest, including tied rank values where appropriate. The equations related to Mann-Whitney U statistic (|u|) are as follows (Bobee and Ashkar, 1991):

$$V = R - p(p + 1) / 2 \qquad (4)$$

$$W = pq - V \qquad (5)$$

$$\mu_U = pq / 2 \qquad (6)$$

$$S_U^2 = \left[\frac{pq}{N(N-1)}\right]\left[\frac{N^3 - N}{12} - \sum T\right] \qquad (7)$$

$$T = (J^3 - J) / 12 \qquad (8)$$

$$|u| = |(U - \mu_U) / [S_U^2]^{1/2}| \qquad (9)$$

The null hypothesis, which has homogeneity, for the observations is rejected if the calculated |u| statistic is greater than the selected critical value (1.96 at 0.05 significance level) obtained from the standard normal distribution table.

2.3 The Method of L-Moments

L-moments, as defined by Hosking (1990), are linear combinations of probability weighted moments (PWM). Greenwood et al. (1979) summarizes the theory of PWM and defined as

$$\beta_r = E\{X[F_X(x)]^r\} \qquad (10)$$

Where β_r is the r^{th} order PWM and $F_X(x)$ is the cumulative distribution function (cdf) of X. Hosking and Wallis (1997) defined unbiased sample estimators of PWMs as (b_i) and, obtained unbiased sample estimators of the first four L-moments by PWM sample estimators. Unbiased sample estimates of the PWM for any distribution can be computed from;

$$b_r = n^{-1} \sum_{j=1}^{n-r} \frac{\binom{n-j}{r}}{\binom{n-1}{r}} x_j \qquad (11)$$

Where x_j is an ordered set of observations $x_1 \le x_2 \le x_3 \le \ldots x_n$. For any distribution the first four L-moments are easily computed from PWM using;

$\lambda_1 = b_1$,

$\lambda_2 = 2b_2 - b_1$,

$\lambda_3 = 6b_3 - 6b_2 + b_1$,

$\lambda_4 = 20b_4 - 30b_3 + 12b_2 - b_1$ (12)

Hosking (1990) defines the L-moment ratios (L-coefficient of variation, L-skewness and L-kurtosis, respectively)

$\tau_2 = \lambda_2/\lambda_1$,

$\tau_3 = \lambda_3/\lambda_2$,

$\tau_4 = \lambda_4/\lambda_2$ (13)

Parameters belonging to statistical distributions used in the study, connection of these parameters to the L-moments and L-moment ratios are given in the Table 1 (Hosking, 1990; Stedinger, et al., 1993; Hosking and Wallis, 1997; Sankarasubramanian and Srinivasan,1999).

Table 1. L-Moments and L-Moment Ratios Associated with the Distributions

SDis*	Quantile Function	L-moments and L-moment Ratios
N	$X = \mu + \sigma\Phi^{-1}[F]$	$\lambda_1 = \mu$, $\lambda_2 = \sigma(\pi)^{-1/2}$ $\tau_3 = 0$, $\tau_4 = 0.1226$
LN3	$X = c + \exp[a + b\Phi^{-1}(F)]$	$K \cong s(0.999281 - 0.006118 s^2 + 0.000127 s^4$, $K = -b$, $\alpha = be^a$ $s = -(8/3)^{1/2}\Phi^{-1}[(1+\tau_3)/2]$, $\lambda_1 = \xi + \alpha K^{-1}(1 - e^{-K^2/2})$ $\lambda_2 = \alpha(Ke^{-K^2/2)} / [1 - 2\Phi(-K2^{-1/2}]$, $\xi = c + e^a$
LO	$X = \xi + \alpha \ln[F/(1-F)]$	$\lambda_1 = \xi$, $\lambda_2 = \alpha$, $\tau_3 = 0$, $\tau_4 = 6^{-1}$
GLO	$X = \xi + \alpha K^{-1}\{1 - [(1-F)/F]^K\}$	$\lambda_1 = \xi + \alpha K^{-1}\{1 - \Gamma(1+K)\Gamma(1-K)\}$, $\lambda_2 = \alpha\Gamma(1+K)\Gamma(1-K)$, $\tau_3 = -K$
EV	$X = \xi - \alpha \ln[-\ln F]$	$\lambda_1 = \xi + 0.5772\alpha$, $\lambda_2 = \alpha \ln 2$, $\tau_3 = 0.1699$, $\tau_4 = 0.1504$
GEV	$X = \xi + \alpha K^{-1}\{1 - (-\ln F)^K\}$	$\lambda_1 = \xi + \alpha K^{-1}\{1 - \Gamma(1+K)\}$, $\lambda_2 = \alpha K^{-1}(1 - 2^{-K})\Gamma(1+K)$, $\tau_3 = \frac{2(1-3^{-K})}{(1-2^{-K})} - 3$
GPA	$X = \xi + \alpha K^{-1}\{1 - [1-F]^K\}$	$\lambda_1 = \xi + \frac{\alpha}{(1+K)}$, $\lambda_2 = \frac{\alpha}{(1+K)(2+K)}$, $\tau_3 = \frac{(1-K)}{(3+K)}$,

*Statistical Distribution used in the study

2.4 Goodness of Fit Criteria for Comparison of Probability Distributions

For comparison of the probability distributions of fitting the data used in the study, two indices (mean absolute deviation index and mean square deviation index), which were proposed by Jain and Sing (1987), measured the relative goodness of fit were taken into account. The mean absolute deviation index (MADI) and mean square deviation index (MSDI) can be calculated by

$$\text{MADI} = N^{-1} \sum_{i=1}^{N} \left| \frac{x_i - z_i}{x_i} \right| \qquad (14),$$

$$\text{MSDI} = N^{-1} \sum_{i=1}^{N} \left(\frac{x_i - z_i}{x_i} \right)^2 \qquad (15)$$

Where x_i and z_i are observed and predicted low flows, respectively, for successive values of empirical probability of exceedence given by the Gringorten plotting position formula. Jain and Singh (1987) claimed that Gringorten formula ensures to maintain unbiasedness for different distributions. Therefore, they suggest the plotting position formula for comparison of the probability distributions of fitting the data.

3. Results and Discussion

The test results (runs and Mann-Whiney tests) related to randomness and homogeneity of the maximum daily rainfalls on four hydrologic homogeneous regions (W, CN, CS and E) of Tokat region are given in Table 2. These tests imply that the maximum daily rainfalls belonging to the hydrologic homogeneous regions are random and homogeneous.

Performance of the normal (N), 2-parameter lognormal (LN2), 3-parameter lognormal (LN3), logistic (LOG), generalized logistic (GLO), extreme value type I (EV), generalized extreme value type I (GEV) and generalized pareto (GPA) probability distributions used in the study for fitting the the maximum daily rainfall data of the hydrologic homogeneous region were tested by mean absolute deviation index (MADI) and mean square deviation index (MSDI). For this reason, estimations calculated by using the mentioned statistical distributions for the probabilities from Gringorten formula of each data point in the increasingly ordered data was made. Performance results based on MADI and MSDI were presented in Table 3 and 4. The MADI and MSDI results showed that the maximum daily rainfalls related to W, CN, CS and E hydrologic homogeneous regions is best fitted to GLO, GLO, GPA and GEV statistical distributions, respectively. The MADI and MSDI measures of GLO, GPA and GEV distributions were less than the measures of the other distributions used in the study. But, as can be seen in Table 3 and 4, the MADI and MSDI measures concern with the statistical distributions taken into consideration in the study is not significantly different from each other. However, although there is an insignificance difference among the estimates produced by the distributions up to 50 % probability level, this difference shows an increase for probability levels above 50 %. This may be seen in Table 5, which daily maximum rainfalls were computed for some selected probabilities by various distributions. Therefore, a conclusion related to performing frequency analysis of maximum daily rainfalls belonging to Tokat region could be given about being able to be used of the mentioned statistical distributions up to 50 % probability level. Figure 2 presents comparison of the maximum daily rainfalls from the selected distributions (GLO, GLO, GPA and GEV) for hydrologic homogeneous regions by using probabilities from Gringorten formula to the observed maximum daily rainfalls corresponding to the probabilities of the formula. The figure shows that maximum daily rainfalls from the selected statistical distributions are accurately predicted.

Table 2. Test Results Related to Randomness and Homogeneity of Maximum Daily Rainfalls

Region	Runs Test		Decision	Mann-Whitney Test		Decision
	Z_N	Z_{Table}		U_T	U_{Table}	
W	-0.543	± 1.96	R	-0.804	± 1.96	H
CN	-0.333	± 1.96	R	-0.942	± 1.96	H
CS	-0.779	± 1.96	R	-1.675	± 1.96	H
E	-1.461	± 1.96	R	-1.471	± 1.96	H
R, observations are random H, observations are homogeneous						

Table 3. The MADI Results of Statistical Distributions Used in the Study

Region	Statistical Distributions							
	N	LN2	LN3	LO	GLO	EV	GEV	GPA
W	0.05642	0.52228	0.16174	0.05091	0.02344*	0.02980	0.03093	0.05300
CN	0.06019	0.52606	0.17084	0.05913	0.02210*	0.02391	0.02330	0.04362
CS	0.02344	0.02771	0.11427	0.02696	0.02733	0.03767	0.02319	0.02313*
E	0.05780	0.03297	0.13421	0.05598	0.03122	0.03175	0.03071*	0.03951
* The best fitted distribution to the data								

Table 4. The MSDI Results of Statistical Distributions Used in the Study

Region	Statistical Distributions							
	N	LN2	LN3	LO	GLO	EV	GEV	GPA
W	0.00573	0.64306	0.05097	0.00535	0.00133*	0.00215	0.00212	0.00543
CN	0.00565	0.57283	0.05486	0.00570	0.00089*	0.00101	0.00099	0.00318
CS	0.00094	0.00120	0.02023	0.00132	0.00129	0.00194	0.00088	0.00078*
E	0.00498	0.00195	0.03072	0.00566	0.00179	0.00163	0.00161*	0.00229
* The best fitted distribution to the data								

Table 5. Computed Daily Maximum Rainfalls for Selected Probabilities by Various Distributions

Region	Distribution	Cumulative Probability, %							
		99	95	80	50	20	10	2	1
W	N	53.95	47.44	39.77	31.73	23.69	19.49	12.12	9.51
	LN2	60.82	49.57	38.96	30.27	23.51	20.61	16.34	15.06
	LN3	44.93	39.97	35.63	32.36	30.04	29.10	27.81	27.45
	LO	56.50	47.60	39.20	31.73	24.26	19.89	10.75	6.96
	GLO	67.65	50.16	38.08	30.03	23.88	20.99	16.23	14.67
	EV	63.03	50.35	38.91	30.09	23.54	20.75	16.63	15.36
	GEV	64.87	50.70	38.67	29.87	23.57	20.95	17.15	16.00
	GPA	58.03	50.68	40.04	29.56	22.74	20.84	19.42	19.25
CN	N	52.11	45.74	38.22	30.34	22.46	18.34	11.12	8.57
	LN2	58.43	47.63	37.43	29.08	22.59	19.80	15.70	14.47
	LN3	43.47	38.51	34.20	30.98	28.70	27.79	26.54	26.19
	LO	54.60	45.89	37.66	30.34	23.02	18.74	9.79	6.08
	GLO	65.76	48.42	36.53	28.64	22.66	19.85	15.26	13.76
	EV	61.00	48.58	37.38	28.74	22.32	19.59	15.56	14.31
	GEV	63.13	48.99	37.12	28.49	22.37	19.83	16.16	15.05
	GPA	56.96	49.32	38.56	28.21	21.56	19.72	18.35	18.19
CS	N	39.73	35.95	31.49	26.82	22.15	19.71	15.42	13.91
	LN2	42.89	37.17	31.40	26.31	22.05	20.10	17.09	16.14
	LN3	31.40	30.08	28.58	27.06	25.59	24.84	23.56	23.12
	LO	41.20	36.04	31.16	26.82	22.48	19.94	14.64	12.44
	GLO	41.81	36.22	31.11	26.71	22.43	19.99	15.01	13.00
	EV	45.00	37.64	30.99	25.87	22.06	20.44	18.04	17.31
	GEV	39.68	36.20	31.59	26.70	22.06	19.78	16.01	14.74
	GPA	36.48	35.54	32.42	26.67	21.22	19.44	18.03	17.86
E	N	46.63	41.68	35.84	29.72	23.60	20.40	14.79	12.81
	LN2	50.32	42.72	35.23	28.79	23.52	21.17	17.59	16.47
	LN3	39.91	36.05	32.71	30.21	28.45	27.74	26.78	26.51
	LO	48.56	41.79	35.40	29.72	24.04	20.71	13.76	10.88
	GLO	57.33	43.78	34.52	28.40	23.77	21.60	18.06	16.91
	EV	53.50	43.86	35.17	28.48	23.50	21.38	18.25	17.28
	GEV	55.24	44.22	34.98	28.29	23.54	21.58	18.74	17.88
	GPA	50.30	44.38	36.06	28.05	22.90	21.48	20.42	20.29

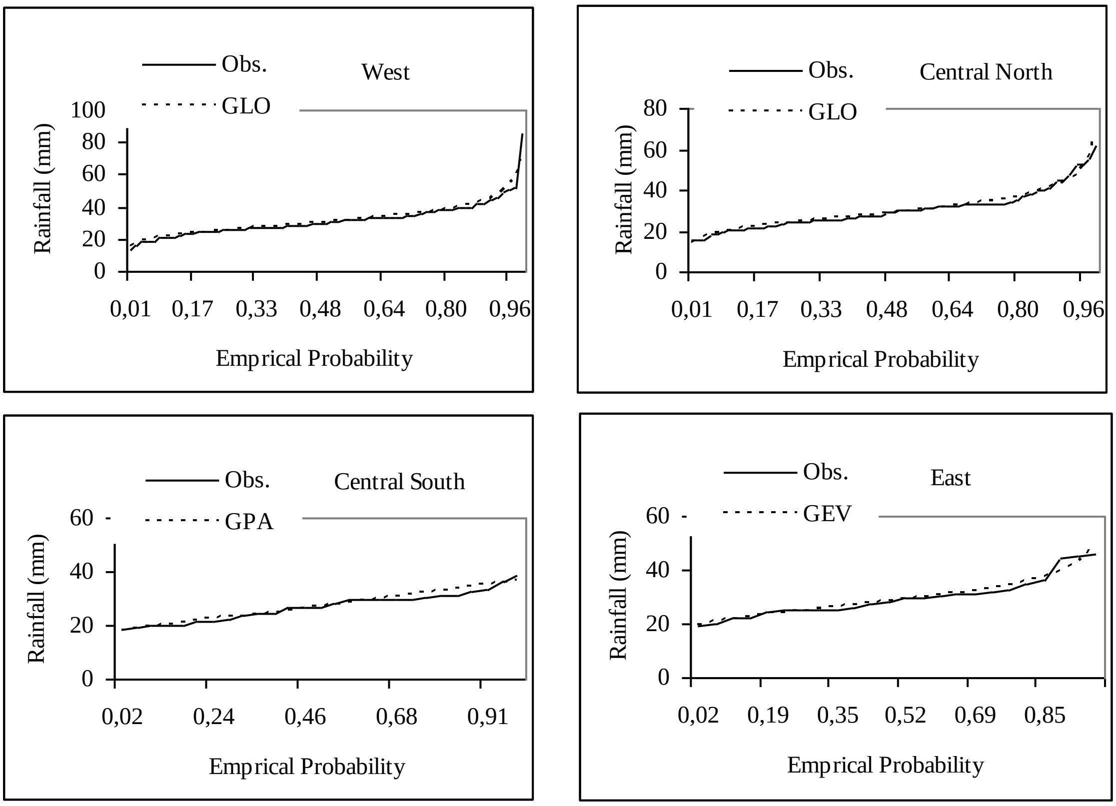

Figure 2. Comparison of the Maximum Daily Rainfalls Belonging to Hydrologic Homogeneous Regions with Rainfalls Predicted by Using the Selected Distributions

4. Conclusion

Knowledge related to distributions of rainfall amounts are of a great important on the design of water-related structure. Therefore, the main goal should be to specify the most suitable probability distribution fit to the observations. But, a reliable design quantile estimate is commonly impossible. In the study, the L-moment approach introduced by Hosking (1990), which is widely popularized to frequency analysis in recent time, was used for parameter estimation belonging to statistical distributions. Performances of the statistical distributions taken into consideration in the text for the maximum daily rainfall data of the hydrologic homogeneous region were evaluated by mean absolute deviation index (MADI) and mean square deviation index (MSDI). The MADI and MSDI results showed that the maximum daily rainfalls of W, CN, CS and E homogeneous regions are best fitted to GLO, GLO, GPA and GEV statistical distributions, respectively. Yürekli (1999) found LN2 as best fit distribution for the mentioned homogeneous regions by using moment method in parameter estimation.

5. Nomenclature

a	mean of the logarithmic values
b	standard deviation of the logarithmic values
c	location parameter
F	probability level
R	sum of ranks belonging to first group (P)
J	number of the tied observations in the raw data set

Greek Symbols

μ	mean of the x series
σ	standard deviation of the x series
$\Phi^{-1}(F)$	the inverse standard normal distribution function
ξ	location parameter
K	shape parameter
α	scale parameter

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