Hacettepe Journal of Mathematics & Statistics

Hacet. J. Math. Stat.
Volume 48 (5) (2019), 1277 – 1285
DOI : 10.15672/HJMS.2018.557

Research Article

# Soft topology in ideal topological spaces

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## Abstract

In this paper, $(X, \tau, E)$ denotes a soft topological space and $\bar{\mathfrak{I}}$ a soft ideal over $X$ with the same set of parameters $E$. We define an operator $(F, E)^{\theta}(\bar{\mathfrak{I}}, \tau)$ called the $\theta$-local function of $(F, E)$ with respect to $\bar{\mathfrak{I}}$ and $\tau$. Also, we investigate some properties of this operator. Moreover, by using the operator $(F, E)^{\theta}(\bar{\mathfrak{I}}, \tau)$, we introduce another soft operator to obtain soft topology and show that $\tau_{\theta} \subseteq \sigma \subseteq \sigma_0$.

**Mathematics Subject Classification (2010).** 54A05, 54C10

**Keywords.** soft topological, ideal, $\theta$-local function, $\theta$-compatibility.

## 1. Introduction and preliminaries

In 1999, Molodtsov [5] introduced the concept of soft set theory and started to develop the basics of the corresponding theory as a new approach for modeling uncertainties. Shabir and Naz [6] gave the definition of soft topological spaces and studied soft neighborhoods of a point, soft separation axioms and their basic properties. At the same time, Aygünoğlu and Aygün [2] introduced soft topological spaces and soft continuity of soft mappings. Recently, in [3] it was introduced the concept of soft ideal theory and soft local function and a basis for this generated soft topologies were also studied. In this paper, We define an operator $(F, E)^{\theta}(\bar{\mathfrak{I}}, \tau)$ called the $\theta$-local function of $(F, E)$ with respect to $\bar{\mathfrak{I}}$ and $\tau$. Also, we investigate some properties of this operator. Moreover, by using the operator $(F, E)^{\theta}(\bar{\mathfrak{I}}, \tau)$, we introduce another soft operator to obtain soft topology and show that $\tau_{\theta} \subseteq \sigma \subseteq \sigma_0$.

**Definition 1.1.** [5] Let $X$ be an initial universe and $E$ be a set parameters. Let $P(X)$ denote the power set of $X$ and $A$ be a nonempty subset of $E$. A pair $(F, A)$ denoted by $F_A$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \to P(X)$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For a particular $e \in A$, $F(e)$ may be considered the set of $e$-approximate elements of the soft set $(F, A)$ and if $e \notin A$, then $F(e) = \phi$ i.e $F_A = \{F(e) : e \in A \subseteq E, F : A \to P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

**Definition 1.2.** [4] Let $F_A, G_B \in SS(X)_E$. Then $F_A$ is called a soft subset of $G_B$, denoted by $F_A \sqsubseteq G_B$ if

(1) $A \subseteq B$.

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Received: 28.03.2017; Accepted: 06.02.2018

(2) $F(e) \subseteq G(e)$, for all $e \in A$.

In this case $F_A$ is said to be a soft subset of $G_B$ and $G_B$ is said to be a soft superset of $F_A$, $F_A \sqsubseteq G_B$.

**Definition 1.3.** [1] A complement of a soft set $(F, E)$, denoted by $(F, E)^c$, is defined by $(F, E)^c = (F^c, E)$, $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$, for all $e \in E$ and $F^c$ is called a soft complement function of $F$.

Clearly $(F^c)^c$ is the same as $F$ and $((F, E)^c)^c = (F, E)$.

**Definition 1.4.** [6] A difference of two soft sets $(F, E)$ and $(G, E)$ over the common universe $X$, denoted by $(F, E) - (G, E)$, is the soft set $(H, E)$ where for all $e \in E$, $H(e) = F(e) - G(e)$.

**Definition 1.5.** [6] Let $(F, E)$ be a soft set over $X$ and $x \in X$. We say that $x \in (F, E)$ read as $x$ belongs to the soft set $(F, E)$ whenever $x \in F(e)$ for all $e \in E$.

**Definition 1.6.** [7] Let $\Delta$ be an arbitrary indexed set and $\Omega = \{(F_\alpha, E) : \alpha \in \Delta\}$ be a subfamily of $SS(X)_E$.

(1) The union of $\Delta$ is the soft set $(H, E)$, where $H(e) = \cup_{\alpha\in\Delta} F_\alpha(e)$ for each $e \in E$. We write $\sqcup_{\alpha\in\Delta}(F_\alpha, E) = (H, E)$.

(2) The intersection of $\Delta$ is the soft set $(M, E)$, where $M(e) = \cap_{\alpha\in\Delta} F_\alpha(e)$ for each $e \in E$. We write $\sqcap_{\alpha\in\Delta}(F_\alpha, E) = (M, E)$.

**Definition 1.7.** [7] A soft set $(F, E) \in SS(X)_E$ is called a soft point in $X_E$ if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e^c) = \phi$ for each $e^c \in E - \{e\}$. This soft point $(F, E)$ is denoted by $x_e$.

**Definition 1.8.** [6] Let $(X, \tau, E)$ be a soft topological space and $(F, E) \in SS(X)_E$. The soft closure of $(F, E)$, denoted by $cl(F, E)$ is the intersection of all closed soft super sets of $(F, E)$ i.e. $cl(F, E) = \{\sqcap(H, E) : (H, E)$ is closed soft and $(F, E) \sqsubseteq (H, E)\}$.

**Definition 1.9.** [7] A soft set $(G, E)$ in a soft topological space $(X, \tau, E)$ is called a soft neighborhood of the soft point $x_e \in X_E$ if there exists an open soft set $(H, E)$ such that $x_e \in (H, E) \sqsubseteq (G, E)$.

## 2. New type of soft local function

**Definition 2.1.** [3] Let $\tilde{\mathfrak{I}}$ be a non-null collection of soft sets over a universe $X$ with the same set of parameters $E$. Then $\tilde{\mathfrak{I}} \subseteq SS(X)_E$ is called a soft ideal on $X$ with the same set $E$ if

(1) $(F, E) \in \tilde{\mathfrak{I}}$ and $(G, E) \in \tilde{\mathfrak{I}}$, then $(F, E) \sqcup (G, E) \in \tilde{\mathfrak{I}}$.

(2) $(F, E) \in \tilde{\mathfrak{I}}$ and $(G, E) \sqsubseteq (F, E)$, then $(G, E) \in \tilde{\mathfrak{I}}$.

i.e. $\tilde{\mathfrak{I}}$ is closed under finite soft unions and soft subsets.

**Definition 2.2.** [3] Let $(X, \tau, E)$ be a soft topological space and $\tilde{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. Then $(F, E)^*(\tilde{\mathfrak{I}}, \tau)(\text{or} F^*_E) = \sqcup\{x_e \in (X, E) : O_{x_e} \sqcap (F, E) \notin \tilde{\mathfrak{I}}$ for every $O_{x_e} \in \tau\}$ is called the soft local function of $(F, E)$ with respect to $\tilde{\mathfrak{I}}$ and $\tau$, where $O_{x_e}$ is a $\tau$-open soft set containing $x_e$.

**Definition 2.3.** Let $(X, \tau, E)$ be a soft topological space and $\tilde{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. Then $(F, E)^\theta(\tilde{\mathfrak{I}}, \tau)(\text{or} F^\theta_E) = \sqcup\{x_e \in (X, E) : cl(O_{x_e}) \sqcap (F, E) \notin \tilde{\mathfrak{I}}$ for every $O_{x_e} \in \tau\}$ is called the soft $\theta$-local function of $(F, E)$ with respect to $\tilde{\mathfrak{I}}$ and $\tau$, where $O_{x_e}$ is a $\tau$-open soft set containing $x_e$.

**Lemma 2.4.** *Let $(X, \tau, E)$ be a soft topological space and $\tilde{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. Then $F^*_E \sqsubseteq F^\theta_E$ for ever subset $(F, E) \sqsubseteq (X, E)$.*

***Proof.*** Let $x_e \in F_E^*$. Then, $O_{x_e} \sqcap (F,E) \notin \bar{\mathfrak{I}}$ for every a $\tau$-open soft set $O_{x_e}$ containing $x_e$. Since $O_{x_e} \sqcap (F,E) \sqsubseteq cl(O_{x_e}) \sqcap (F,E)$, we have $cl(O_{x_e}) \sqcap (F,E) \notin \bar{\mathfrak{I}}$ and hence $x_e \in F_E^\theta$. □

**Lemma 2.5.** *Let $(X,\tau,E)$ be a soft topological space and $(F,E) \sqsubseteq (X,E)$. If $(F,E)$ is a soft open set, then $cl_\theta(F,E) = cl(F,E)$.*

**Theorem 2.6.** *Let $(X,\tau,E)$ be a soft topological space and $\bar{\mathfrak{I}}$ and $\bar{J}$ be two a soft ideals over $X$ with the same set of parameters $E$. Let $(F,E)$ and $(G,E)$ be subsets of $(X,E)$. Then the following properties hold:*

(1) *If $(F,E) \sqsubseteq (G,E)$, then $F_E^\theta \sqsubseteq G_E^\theta$.*
(2) *If $\bar{\mathfrak{I}} \sqsubseteq \bar{J}$, then $(F,E)^\theta(\bar{J},\tau) \sqsubseteq (F,E)^\theta(\bar{\mathfrak{I}},\tau)$.*
(3) *$F_E^\theta = cl(F_E^\theta) \sqsubseteq cl_\theta((F,E))$ and $F_E^\theta$ is $\tau$-closed soft.*
(4) *If $(F,E) \sqsubseteq F_E^\theta$ and $F_E^\theta$ is $\tau$-open soft, then $F_E^\theta = cl_\theta((F,E))$.*
(5) *If $(F,E) \in \bar{\mathfrak{I}}$, then $F_E^\theta = \phi$.*

***Proof.*** (1) Suppose that $x_e \notin G_E^\theta$. Then there exists $O_{x_e} \in \tau$ such that $cl(O_{x_e}) \sqcap (G,E) \in \bar{\mathfrak{I}}$. Since $cl(O_{x_e}) \sqcap (F,E) \sqsubseteq cl(O_{x_e}) \sqcap (G,E)$, $cl(O_{x_e}) \sqcap (F,E) \in \bar{\mathfrak{I}}$. Hence $x_e \notin F_E^\theta$. Thus $(X,E) \setminus G_E^\theta \sqsubseteq (X,E) \setminus F_E^\theta$ or $F_E^\theta \sqsubseteq G_E^\theta$.
(2) Suppose that $x_e \notin (F,E)^\theta(\bar{\mathfrak{I}},\tau)$. There exists $O_{x_e} \in \tau$ such that $cl(O_{x_e}) \sqcap (F,E) \in \bar{\mathfrak{I}}$. Since $\bar{\mathfrak{I}} \sqsubseteq \bar{J}$, $cl(O_{x_e}) \sqcap (F,E) \in \bar{J}$ and $x_e \notin (F,E)^\theta(\bar{J},\tau)$. Therefore, $(F,E)^\theta(\bar{J},\tau) \sqsubseteq (F,E)^\theta(\bar{\mathfrak{I}},\tau)$.
(3) We have $F_E^\theta \sqsubseteq cl(F_E^\theta)$ in general. Let $x_e \in cl(F_E^\theta)$. Then $O_{x_e} \sqcap F_E^\theta \neq \phi$ for every $O_{x_e} \in \tau$. Therefore, there exists some $y_e \in O_{x_e} \sqcap F_E^\theta$ and $O_{x_e}$ a $\tau$-open soft set containing $y_e$. Since $y_e \in F_E^\theta$, $cl(O_{x_e}) \sqcap (F,E) \notin \bar{\mathfrak{I}}$ and hence $x_e \in F_E^\theta$. Hence we have $cl(F_E^\theta) \sqsubseteq F_E^\theta$ and hence $F_E^\theta = cl(F_E^\theta)$. Again, let $x_e \in cl(F_E^\theta) = F_E^\theta$, then $cl(O_{x_e}) \sqcap (F,E) \notin \bar{\mathfrak{I}}$ for every a $\tau$-open soft set $O_{x_e}$ containing $x_e$. This implies $cl(O_{x_e}) \sqcap (F,E) \neq \phi$ for every a $\tau$-open soft set $O_{x_e}$ containing $x_e$. Therefore, $x_e \in cl_\theta((F,E))$. This show that $F_E^\theta = cl(F_E^\theta) \sqsubseteq cl_\theta((F,E))$.
(4) For any subset $(F,E) \sqsubseteq (X,E)$, by (3) we have $F_E^\theta = cl(F_E^\theta) \sqsubseteq cl_\theta((F,E))$. Since $(F,E) \sqsubseteq F_E^\theta$ and $F_E^\theta$ is a $\tau$-open soft, by Lemma 2.5 $cl_\theta((F,E)) \sqsubseteq cl_\theta(F_E^\theta) = cl(F_E^\theta) = F_E^\theta \sqsubseteq cl_\theta((F,E))$ and hence $F_E^\theta = cl_\theta((F,E))$.
(5) Suppose that $x_e \notin F_E^\theta$. Then for any $O_{x_e}$ a $\tau$-open soft set containing $x_e$, $cl(O_{x_e}) \sqcap (F,E) \notin \bar{\mathfrak{I}}$. But since $(F,E) \in \bar{\mathfrak{I}}$, $cl(O_{x_e}) \sqcap (F,E) \in \bar{\mathfrak{I}}$ for any $O_{x_e}$ a $\tau$-open soft set containing $x_e$. This is a contradiction. Hence $F_E^\theta = \phi$. □

**Lemma 2.7.** *Let $(X,\tau,E)$ be a soft topological space and $\bar{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. If $O$ is a $\tau_\theta$-open soft set, then $O \sqcap F_E^\theta = O \sqcap (O \sqcap F)_E^\theta \sqsubseteq (O \sqcap F)_E^\theta$ for any subset $(F,E)$ of $(X,E)$.*

***Proof.*** Suppose that $O$ is a $\tau_\theta$-open soft set and $x_e \in O \sqcap F_E^\theta$. Then $x_e \in O$ and $x_e \in F_E^\theta$. Since $O$ is a $\tau_\theta$-open soft set, then there exists a $\tau$-open soft set $W$ containing $x_e$ such that $W \sqsubseteq cl(W) \sqsubseteq O$. Let $V$ be any $\tau$-open soft set containing $x_e$. Then $V \sqcap W$ is a $\tau$-open soft set containing $x_e$ and $cl(V \sqcap W) \sqcap (F,E) \notin \bar{\mathfrak{I}}$ and hence $cl(V) \sqcap (O \sqcap (F,E)) \notin \bar{\mathfrak{I}}$. This shows that $x_e \in (O \sqcap F)_E^\theta$ and hence, we get $O \sqcap F_E^\theta \sqsubseteq (O \sqcap F)_E^\theta$. Moreover, $O \sqcap F_E^\theta \sqsubseteq O \sqcap (O \sqcap F)_E^\theta$ and by Theorem 2.6 $(O \sqcap F)_E^\theta \sqsubseteq F_E^\theta$ and $O \sqcap (O \sqcap F)_E^\theta \sqsubseteq O \sqcap F_E^\theta$. Therefore, $O \sqcap F_E^\theta = O \sqcap (O \sqcap F)_E^\theta$. □

**Theorem 2.8.** *Let $(X,\tau,E)$ be a soft topological space and $\bar{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$ and $(F,E)$, $(G,E)$ any subsets of $(X,E)$. Then the following properties hold:*

(1) *$\phi_E^\theta = \phi$.*
(2) *$(F \sqcup G)_E^\theta = F_E^\theta \sqcup G_E^\theta$.*

***Proof.*** (1) The proof is obvious.
(2) It follows from Theorem 2.6 that $(F \sqcup G)^{\theta}_{E} \sqsupseteq F^{\theta}_{E} \sqcup G^{\theta}_{E}$. To prove the reverse inclusion, let $x_e \notin F^{\theta}_{E} \sqcup G^{\theta}_{E}$. Then $x_e$ belongs neither to $F^{\theta}_{E}$ nor to $G^{\theta}_{E}$. Therefore, there exist a $\tau$-open soft sets $O_{x_e}$, $W_{x_e}$ containing $x_e$ such that $cl(O_{x_e}) \sqcap (F, E) \in \overline{\mathfrak{I}}$ and $cl(W_{x_e}) \sqcap (F, E) \in \overline{\mathfrak{I}}$. Since $\overline{\mathfrak{I}}$ is additive, $(cl(O_{x_e}) \sqcap (F, E)) \sqcup (cl(W_{x_e}) \sqcap (F, E)) \in \overline{\mathfrak{I}}$. Moreover, since $\overline{\mathfrak{I}}$ is hereditary and

$$\begin{aligned}
&(cl(O_{x_e}) \sqcap (F, E)) \sqcup (cl(W_{x_e}) \sqcap (G, E))\\
&= [(cl(O_{x_e}) \sqcap (F, E)) \sqcup cl(W_{x_e})] \sqcap [(cl(O_{x_e}) \sqcap (F, E)) \sqcup (G, E)] = (cl(O_{x_e}) \sqcup\\
&\quad cl(W_{x_e})) \sqcap (cl(W_{x_e}) \sqcup (F, E)) \sqcap (cl(O_{x_e}) \sqcup (G, E)) \sqcap ((F, E) \sqcup (G, E))\\
&\sqsupseteq cl(O_{x_e} \sqcap W_{x_e}) \sqcap ((F, E) \sqcup (G, E)).
\end{aligned}$$

Therefore, $cl(O_{x_e} \sqcap W_{x_e}) \sqcap ((F, E) \sqcup (G, E)) \in \overline{\mathfrak{I}}$. Since $O_{x_e} \sqcap W_{x_e}$ is a $\tau$-open soft set containing $x_e$, we have $x_e \notin (F \sqcup G)^{\theta}_{E}$ and $(F \sqcup G)^{\theta}_{E} \sqsubseteq F^{\theta}_{E} \sqcup G^{\theta}_{E}$. Hence we obtain $(F \sqcup G)^{\theta}_{E} = F^{\theta}_{E} \sqcup G^{\theta}_{E}$. □

**Lemma 2.9.** *Let $(X, \tau, E)$ be a soft topological space and $\overline{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$ and $(F, E)$, $(G, E)$ any subsets of $(X, E)$. Then $F^{\theta}_{E} - G^{\theta}_{E} = (F - G)^{\theta}_{E} - G^{\theta}_{E}$.*

***Proof.*** We have by Theorem 2.8 $F^{\theta}_{E} = [(F - G) \sqcup (F \sqcap G)]^{\theta}_{E} = (F - G)^{\theta}_{E} \sqcup (F \sqcap G)^{\theta}_{E} \sqsubseteq (F - G)^{\theta}_{E} \sqcup G^{\theta}_{E}$. Thus $F^{\theta}_{E} - G^{\theta}_{E} \sqsubseteq (F - G)^{\theta}_{E} - G^{\theta}_{E}$. By Theorem 2.6 $(F - G)^{\theta}_{E} \sqsubseteq F^{\theta}_{E}$ and hence $(F - G)^{\theta}_{E} - G^{\theta}_{E} \sqsubseteq F^{\theta}_{E} - G^{\theta}_{E}$. Hence $F^{\theta}_{E} - G^{\theta}_{E} = (F - G)^{\theta}_{E} - G^{\theta}_{E}$. □

**Corollary 2.10.** *Let $(X, \tau, E)$ be a soft topological space and $\overline{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$ and $(F, E)$, $(G, E)$ any subsets of $(X, E)$ with $(G, E) \in \overline{\mathfrak{I}}$. Then $(F \sqcup G)^{\theta}_{E} = F^{\theta}_{E} = (F - G)^{\theta}_{E}$.*

***Proof.*** Since $(G, E) \in \overline{\mathfrak{I}}$, by Theorem 2.6 $G^{\theta}_{E} = \phi$. By Lemma 2.9, $F^{\theta}_{E} = (F - G)^{\theta}_{E}$ and by Theorem 2.8 $(F \sqcup G)^{\theta}_{E} = F^{\theta}_{E} \sqcup G^{\theta}_{E} = F^{\theta}_{E}$. □

## 3. $\theta$-compatibility of soft topological spaces

**Definition 3.1.** [3] Let $(X, \tau, E)$ be a soft topological space and $\overline{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. We say that the soft topology $\tau$ is compatible with the soft ideal $\overline{\mathfrak{I}}$, denoted by $\tau \sim \overline{\mathfrak{I}}$. If the following holds for every $(F, E) \in SS(X)_E$, if for every soft point $x_e \in (F, E)$ there exists a $\tau$-open soft set $O_{x_e}$ containing $x_e$ such that $O_{x_e} \sqcap (F, E) \in \overline{\mathfrak{I}}$, then $(F, E) \in \overline{\mathfrak{I}}$.

**Definition 3.2.** Let $(X, \tau, E)$ be a soft topological space and $\overline{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. We say that the soft topology $\tau$ is $\theta$-compatible with the soft ideal $\overline{\mathfrak{I}}$, denoted by $\tau \sim_{\theta} \overline{\mathfrak{I}}$. If the following holds for every $(F, E) \in SS(X)_E$, if for every soft point $x_e \in (F, E)$ there exists a $\tau$-open soft set $O_{x_e}$ containing $x_e$ such that $cl(O_{x_e}) \sqcap (F, E) \in \overline{\mathfrak{I}}$, then $(F, E) \in \overline{\mathfrak{I}}$.

**Remark 3.3.** If $\tau$ is compatible with the soft ideal $\overline{\mathfrak{I}}$, then $\tau$ is $\theta$-compatible with the soft ideal $\overline{\mathfrak{I}}$.

**Theorem 3.4.** *Let $(X, \tau, E)$ be a soft topological space and $\overline{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. Then the following properties are equivalent:*

(1) $\tau \sim_{\theta} \overline{\mathfrak{I}}$;
(2) *If a soft subset $(F, E)$ of $(X, E)$ has a cover of $\tau$-open soft sets each of whose closure intersection with $(F, E)$ is in $\overline{\mathfrak{I}}$, then $(F, E) \in \overline{\mathfrak{I}}$;*
(3) *For every $(F, E) \sqsubseteq (X, E)$ with $(F, E) \sqcap F^{\theta}_{E} = \phi$ implies that $(F, E) \in \overline{\mathfrak{I}}$;*
(4) *For every $(F, E) \sqsubseteq (X, E)$, $(F, E) - F^{\theta}_{E} \in \overline{\mathfrak{I}}$;*

(5) *For every* $(F, E) \sqsubseteq (X, E)$, *if* $(F, E)$ *contains no nonempty subset* $(G, E)$ *with* $(G, E) \sqsubseteq G_E^\theta$, *then* $(F, E) \in \bar{\mathfrak{I}}$.

***Proof.*** (1) $\Rightarrow$ (2): The proof is obvious.
(2) $\Rightarrow$ (3): Let $(F, E) \sqsubseteq (X, E)$ and $x_e \in (F, E)$. Then $x_e \notin F_E^\theta$ and there exists $\tau$-open soft set $O_{x_e}$ containing $x_e$ such that $cl(O_{x_e}) \sqcap (F, E) \in \bar{\mathfrak{I}}$. Therefore, we have $(F, E) \sqsubseteq \sqcup\{O_{x_e} : x_e \in O_{x_e}\}$ and by (2) $(F, E) \in \bar{\mathfrak{I}}$.
(3) $\Rightarrow$ (4): For any $(F, E) \sqsubseteq (X, E)$, $(F, E) - F_E^\theta \sqsubseteq (F, E)$ and

$$\left[(F, E) - F_E^\theta\right] \sqcap \left[(F, E) - F_E^\theta\right]_E^\theta \sqsubseteq \left[(F, E) - F_E^\theta\right] \sqcap F_E^\theta = \phi.$$

By (3), $(F, E) - F_E^\theta \in \bar{\mathfrak{I}}$.
(4) $\Rightarrow$ (5): By (4), for every $(F, E) \sqsubseteq (X, E)$, $(F, E) - F_E^\theta \in \bar{\mathfrak{I}}$. Let $(F, E) - F_E^\theta = J \in \bar{\mathfrak{I}}$, then $(F, E) = J \sqcup [(F, E) \sqcap F_E^\theta]$ and by Theorem 2.6 (5) and Theorem 2.8 (2), $F_E^\theta = J_E^\theta \sqcup \left[(F, E) \sqcap F_E^\theta\right]_E^\theta = \left[(F, E) \sqcap F_E^\theta\right]_E^\theta$. Therefore, we have $(F, E) \sqcap F_E^\theta = (F, E) \sqcap \left[(F, E) \sqcap F_E^\theta\right]_E^\theta \sqsubseteq \left[(F, E) \sqcap F_E^\theta\right]_E^\theta$ and $(F, E) \sqcap F_E^\theta \sqsubseteq (F, E)$. By the assumption $(F, E) \sqcap F_E^\theta = \phi$ and hence $(F, E) = (F, E) - F_E^\theta \in \bar{\mathfrak{I}}$.
(5) $\Rightarrow$ (1): Let $(F, E) \sqsubseteq (X, E)$ and assume that for every $x_e \in (F, E)$, there exists $\tau$-open soft set $O_{x_e}$ containing $x_e$ such that $cl(O_{x_e}) \sqcap (F, E) \in \bar{\mathfrak{I}}$. Then $(F, E) \sqcap F_E^\theta = \phi$. Suppose that $(F, E)$ contains a subset $(G, E)$ with $(G, E) \sqsubseteq G_E^\theta$. Then $(G, E) = (G, E) \sqcap G_E^\theta \sqsubseteq (F, E) \sqcap F_E^\theta = \phi$. Therefore, $(F, E)$ contains no nonempty subset $(G, E)$ with $(G, E) \sqsubseteq G_E^\theta$. Hence $(F, E) \in \bar{\mathfrak{I}}$. □

**Theorem 3.5.** *Let* $(X, \tau, E)$ *be a soft topological space and* $\bar{\mathfrak{I}}$ *be a soft ideal over* $X$ *with the same set of parameters* $E$. *If* $\tau$ *is* $\theta$*-compatible with the soft ideal* $\bar{\mathfrak{I}}$. *Then the following equivalent properties hold:*

(1) *For every* $(F, E) \sqsubseteq (X, E)$, $(F, E) \sqcap F_E^\theta = \phi$ *implies that* $F_E^\theta = \phi$;
(2) *For every* $(F, E) \sqsubseteq (X, E)$, $\left[(F, E) - F_E^\theta\right]_E^\theta = \phi$;
(3) *For every* $(F, E) \sqsubseteq (X, E)$, $\left[(F, E) \sqcap F_E^\theta\right]_E^\theta = F_E^\theta$.

***Proof.*** First, we show that (1) holds if $\tau$ is $\theta$-compatible with the soft ideal $\bar{\mathfrak{I}}$. Let $(F, E) \sqsubseteq (X, E)$ and $(F, E) \sqcap F_E^\theta = \phi$. By Theorem 3.4, $(F, E) \in \bar{\mathfrak{I}}$ and by Theorem 2.6 (5) $F_E^\theta = \phi$.
(1) $\Rightarrow$ (2): Assume that for every $(F, E) \sqsubseteq (X, E)$, $(F, E) \sqcap F_E^\theta = \phi$ implies that $F_E^\theta = \phi$. Let $(G, E) = (F, E) - F_E^\theta$, then

$$\begin{aligned}
(G, E) \sqcap G_E^\theta &= \left[(F, E) - F_E^\theta\right] \sqcap \left[(F, E) - F_E^\theta\right]_E^\theta \\
&= \left[(F, E) \sqcap [(X, E) - F_E^\theta]\right] \sqcap \left[(F, E) \sqcap [(X, E) - F_E^\theta]\right]_E^\theta \\
&\sqsubseteq \left[(F, E) \sqcap \left[(X, E) - F_E^\theta\right]\right] \sqcap \left[F_E^\theta \sqcap \left[(X, E) - F_E^\theta\right]_E^\theta\right] = \phi.
\end{aligned}$$

By (1), we have $G_E^\theta = \phi$. Hence $\left[(F, E) - F_E^\theta\right]_E^\theta = \phi$.

(2) $\Rightarrow$ (3): Assume that for every $(F, E) \sqsubseteq (X, E)$, $\left[(F, E) - F_E^\theta\right]_E^\theta = \phi$.

$$\begin{aligned}
(F, E) &= \left[(F, E) - F_E^\theta\right] \sqcup \left[(F, E) \sqcap F_E^\theta\right] \\
F_E^\theta &= \left[\left[(F, E) - F_E^\theta\right] \sqcup \left[(F, E) \sqcap F_E^\theta\right]\right]_E^\theta \\
&= \left[(F, E) - F_E^\theta\right]_E^\theta \sqcup \left[(F, E) \sqcap F_E^\theta\right]_E^\theta
\end{aligned}$$

$$= \left[(F,E) \sqcap F_E^{\theta}\right]_E^{\theta}.$$

(3) $\Rightarrow$ (1): Assume that for every $(F,E) \sqsubseteq (X,E)$, $(F,E) \sqcap F_E^{\theta} = \phi$ and $\left[(F,E) \sqcap F_E^{\theta}\right]_E^{\theta} = F_E^{\theta}$. This implies that $\phi = \phi_E^{\theta} = F_E^{\theta}$. □

**Theorem 3.6.** *Let $(X,\tau,E)$ be a soft topological space and $\bar{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. Then the following properties are equivalent:*

(1) *$cl(\tau) \sqcap \bar{\mathfrak{I}} = \phi$, where $cl(\tau) = \{cl(U) : U$ is $\tau$-open soft set$\}$;*
(2) *If $S \in \bar{\mathfrak{I}}$, then $Int_{\theta}(S) = \phi$;*
(3) *For every clopen soft set $(F,E) \sqsubseteq (X,E)$, $(F,E) \sqsubseteq F_E^{\theta}$;*
(4) *$(X,E) = X_E^{\theta}$.*

***Proof.*** (1) $\Rightarrow$ (2): Let $cl(\tau) \sqcap \bar{\mathfrak{I}} = \phi$ and $S \in \bar{\mathfrak{I}}$. Suppose that $x_e \in Int_{\theta}(S)$. Then there exists $\tau$-open soft set $U$ such that $x_e \in U \sqsubseteq cl(U) \sqsubseteq S$. Since $S \in \bar{\mathfrak{I}}$ and hence $\phi \neq \{x_e\} \sqsubseteq cl(U) \in cl(\tau) \sqcap \bar{\mathfrak{I}}$. This is contrary to $cl(\tau) \sqcap \bar{\mathfrak{I}} = \phi$. Therefore, $Int_{\theta}(S) = \phi$.
(2) $\Rightarrow$ (3): Let $x_e \in (F,E)$. Assume $x_e \notin F_E^{\theta}$, then there exists $\tau$-open soft set $U_{x_e}$ containing $x_e$ such that $(F,E) \sqcap cl(U_{x_e}) \in \bar{\mathfrak{I}}$ and hence $(F,E) \sqcap U_{x_e} \in \bar{\mathfrak{I}}$. Since $(F,E)$ is clopen soft set, by (2) and Lemma 2.5 $x_e \in (F,E) \sqcap U_{x_e} = Int\left[(F,E) \sqcap U_{x_e}\right] \sqsubseteq Int\left[(F,E) \sqcap cl(U_{x_e})\right] = Int_{\theta}\left[(F,E) \sqcap cl(U_{x_e})\right] = \phi$. This is a contradiction. Hence $x_e \in F_E^{\theta}$ and $(F,E) \sqsubseteq F_E^{\theta}$.
(3) $\Rightarrow$ (4): Since $(X,E)$ is clopen soft set, then $(X,E) = X_E^{\theta}$.
(4) $\Rightarrow$ (1): $(X,E) = X_E^{\theta} = \{x_e \in (X,E) : cl(U) \sqcap (X,E) = cl(U) \notin \bar{\mathfrak{I}}$ for every $\tau$-open soft set $U$ containing $x_e\}$. Hence $cl(\tau) \sqcap \bar{\mathfrak{I}} = \phi$. □

**Theorem 3.7.** *Let $(X,\tau,E)$ be a soft topological space and $\bar{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. If $\tau$ is $\theta$-compatible with the soft ideal $\bar{\mathfrak{I}}$. Then for every $\tau_{\theta}$-open soft set $(G,E)$ and any subset $(F,E)$ of $(X,E)$, $cl\left([(G,E) \sqcap (F,E)]_E^{\theta}\right) = [(G,E) \sqcap (F,E)]_E^{\theta} \sqsubseteq \left[(G,E) \sqcap F_E^{\theta}\right]_E^{\theta} \sqsubseteq cl_{\theta}\left(\left[(G,E) \sqcap F_E^{\theta}\right]\right)$.*

***Proof.*** By Theorem 2.6 (1) and Theorem 3.5 (3) we have

$$[(G,E) \sqcap (F,E)]_E^{\theta} = \left[[(G,E) \sqcap (F,E)] \sqcap [(G,E) \sqcap (F,E)]_E^{\theta}\right]_E^{\theta} \sqsubseteq \left[(G,E) \sqcap F_E^{\theta}\right]_E^{\theta}.$$

Moreover, by Theorem 2.6 (3),

$$cl\left([(G,E) \sqcap (F,E)]_E^{\theta}\right) = [(G,E) \sqcap (F,E)]_E^{\theta} \sqsubseteq \left[(G,E) \sqcap F_E^{\theta}\right]_E^{\theta} \sqsubseteq cl_{\theta}\left(\left[(G,E) \sqcap F_E^{\theta}\right]\right).$$

□

## 4. $\mathcal{S}_E$-soft operator

**Definition 4.1.** Let $(X,\tau,E)$ be a soft topological space and $\bar{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. A soft operator $\mathcal{S}_E : SS(X)_E \to \tau$ is defined as follows: for every $(F,E) \sqsubseteq (X,E)$, $\mathcal{S}_E(F) = \{x_e \in (X,E)$ : there exists a $\tau$-open soft set $(G,E)$ containing $x_e$ such that $cl[(G,E)] - (F,E) \in \bar{\mathfrak{I}}\}$ and observe that $\mathcal{S}_E(F) = (X,E) - [(X,E) - (F,E)]_E^{\theta}$.

Several basic facts that are related to the behavior of the $\mathcal{S}_E$-soft operator are included in the following theorem.

**Theorem 4.2.** *Let $(X,\tau,E)$ be a soft topological space and $\bar{\mathfrak{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. Then the following properties are hold:*

(1) *If $(F,E) \sqsubseteq (X,E)$, then $\mathcal{S}_E(F)$ is a $\tau$-open soft.*
(2) *If $(F,E) \sqsubseteq (G,E)$, then $\mathcal{S}_E(F) \sqsubseteq \mathcal{S}_E(G)$.*

(3) *If* $(F, E), (G, E) \in SS(X)_E$, *then* $\mathcal{S}_E(F \sqcap G) = \mathcal{S}_E(F) \sqcap \mathcal{S}_E(G)$.
(4) *If* $(F, E) \sqsubseteq (X, E)$, *then* $\mathcal{S}_E(\mathcal{S}_E(F)) = \mathcal{S}_E(F)$ *if and only if*
$$[(X, E) - (F, E)]^\theta_E = \left([(X, E) - (F, E)]^\theta_E\right)^\theta_E.$$
(5) *If* $(A, E) \in \bar{\mathcal{I}}$, *then* $\mathcal{S}_E(A) = (X, E) - X^\theta_E$.
(6) *If* $(F, E) \sqsubseteq (X, E)$, $(A, E) \in \bar{\mathcal{I}}$, *then* $\mathcal{S}_E(F - A) = \mathcal{S}_E(F)$.
(7) *If* $(F, E) \sqsubseteq (X, E)$, $(A, E) \in \bar{\mathcal{I}}$, *then* $\mathcal{S}_E(F \sqcup A) = \mathcal{S}_E(F)$.
(8) *If* $(F, E), (G, E) \in SS(X)_E$ *and* $(F - G) \sqcup (G - F) \in \bar{\mathcal{I}}$, *then* $\mathcal{S}_E(F) = \mathcal{S}_E(G)$.

***Proof.*** (1) This follows from Theorem 2.6 (3).
(2) This follows from Theorem 2.6 (1).

$$\begin{aligned}
(3) \qquad \mathcal{S}_E(F \sqcap G) &= (X, E) - [(X, E) - ((F \sqcap G), E)]^\theta_E \\
&= (X, E) - [((X, E) - (F, E)) \sqcup ((X, E) - (G, E))]^\theta_E \\
&= (X, E) - \left[[(X, E) - (F, E)]^\theta_E \sqcup [(X, E) - (G, E)]^\theta_E\right] \\
&= \left[(X, E) - [(X, E) - (F, E)]^\theta_E\right] \sqcap \left[(X, E) - [(X, E) - (G, E)]^\theta_E\right] \\
&= \mathcal{S}_E(F) \sqcap \mathcal{S}_E(G).
\end{aligned}$$

(4) This follows from the facts:
(1) $\mathcal{S}_E(F) = (X, E) - [(X, E) - (F, E)]^\theta_E$.
(2) $\mathcal{S}_E(\mathcal{S}_E(F)) = (X, E) - \left([(X, E) - (F, E)]^\theta_E\right)^\theta_E$
$= (X, E) - \left((X, E) - [(X, E) - (F, E)]^\theta_E\right)^\theta_E$.

(5) By Corollary 2.10 we obtain that $[(X, E) - (F, E)]^\theta_E = X^\theta_E$ if $(F, E) \in \bar{\mathcal{I}}$ and $\mathcal{S}_E(A) = (X, E) - X^\theta_E$.
(6) This follows from Corollary 2.10 and
$\mathcal{S}_E(F - A) = (X, E) - [(X, E) - ((F, E) - (A, E))]^\theta_E =$
$(X, E) - [[(X, E) - (F, E)] \sqcup (A, E)]^\theta_E = (X, E) - [(X, E) - (F, E)]^\theta_E = \mathcal{S}_E(F)$.
(7) This follows from Corollary 2.10 and
$\mathcal{S}_E(F \sqcup A) = (X, E) - [(X, E) - ((F, E) \sqcup (A, E))]^\theta_E =$
$(X, E) - [[(X, E) - (F, E)] - (A, E))]^\theta_E = (X, E) - [(X, E) - (F, E)]^\theta_E = \mathcal{S}_E(F)$.
(8) Assume $[(F, E) - (G, E)] \sqcup [(G, E) - (F, E)] \in \bar{\mathcal{I}}$. Let $[(F, E) - (G, E)] = S_1$ and $[(G, E) - (F, E)] = S_2$. Observe that $S_1, S_2 \in \bar{\mathcal{I}}$ by heredity. Also observe that $(G, E) = [(F, E) - S_1] \sqcup S_2$. Thus $\mathcal{S}_E(F) = \mathcal{S}_E(F - S_1) = \mathcal{S}_E[(F - S_1) \sqcup S_2] = \mathcal{S}_E(G)$ by (6) and (7).

□

**Corollary 4.3.** *Let* $(X, \tau, E)$ *be a soft topological space and* $\bar{\mathcal{I}}$ *be a soft ideal over* $X$ *with the same set of parameters* $E$. *Then* $(F, E) \sqsubseteq \mathcal{S}_E(F)$ *for every* $\tau_\theta$*-open soft set* $(F, E)$ *of* $(X, E)$.

***Proof.*** We know that $\mathcal{S}_E(F) = (X, E) - [(X, E) - (F, E)]^\theta_E$. Now $[(X, E) - (F, E)]^\theta_E \sqsubseteq cl_\theta((X, E) - (F, E)) = (X, E) - (F, E)$, since $(X, E) - (F, E)$ is $\tau_\theta$-closed soft set. Therefore, $(F, E) = (X, E) - [(X, E) - (F, E)] \sqsubseteq (X, E) - [(X, E) - (F, E)]^\theta_E = \mathcal{S}_E(F)$. □

**Theorem 4.4.** *Let* $(X, \tau, E)$ *be a soft topological space and* $\bar{\mathcal{I}}$ *be a soft ideal over* $X$ *with the same set of parameters* $E$ *and* $(F, E) \sqsubseteq (X, E)$. *Then the following properties are hold:*
(1) $\mathcal{S}_E(F) = \sqcup\{(G, E) \in \tau : cl((G, E)) - (F, E) \in \bar{\mathcal{I}}\}$.
(2) $\mathcal{S}_E(F) \sqsupseteq \sqcup\{(G, E) \in \tau : [cl((G, E)) - (F, E)] \sqcup [(F, E) - cl((G, E))] \in \bar{\mathcal{I}}\}$.

***Proof.*** (1) This follows immediately from the definition of $\mathcal{S}_E$-soft operator.
(2) Since $\overline{\mathcal{I}}$ is heredity, it is obvious that

$$\mathcal{S}_E(F) = \sqcup\{(G,E) \in \tau : cl((G,E)) - (F,E) \in \overline{\mathcal{I}}\} \sqsupseteq$$
$$\sqcup\{(G,E) \in \tau : [cl((G,E)) - (F,E)] \sqcup [(F,E) - cl((G,E))] \in \overline{\mathcal{I}}\}$$

for every $(F,E) \sqsubseteq (X,E)$. □

**Theorem 4.5.** *Let $(X,\tau,E)$ be a soft topological space and $\overline{\mathcal{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. If $\sigma = \{(F,E) \sqsubseteq (X,E) : (F,E) \sqsubseteq \mathcal{S}_E(F)\}$. Then $\sigma$ is a soft topology for $(X,E)$.*

***Proof.*** Let $\sigma = \{(F,E) \sqsubseteq (X,E) : (F,E) \sqsubseteq \mathcal{S}_E(F)\}$. Since $(\phi,E) \in \overline{\mathcal{I}}$, by Theorem 2.6 (5) $\phi^\theta_E = \phi$ and $\mathcal{S}_E(X) = (X,E) - [X - X]^\theta_E = (X,E) - \phi^\theta_E = (X,E)$. Moreover, $\mathcal{S}_E(\phi) = (\phi,E) - [X - \phi]^\theta_E = (X,E) - (X,E) = (\phi,E)$. Therefore, we obtain that $(\phi,E) \sqsubseteq \mathcal{S}_E(\phi)$ and $(X,E) \sqsubseteq \mathcal{S}_E(X)$, and thus $(\phi,E)$ and $(X,E) \in \sigma$. Now if $(F,E)$, $(G,E) \in \sigma$, then by Theorem 4.2 $(F,E) \sqcap (G,E) \sqsubseteq \mathcal{S}_E(F) \sqcap \mathcal{S}_E(G) = \mathcal{S}_E(F \sqcap G)$ which implies that $(F,E) \sqcap (G.E) \in \sigma$. If $\{(A_\alpha,E) : \alpha \in \Delta\} \sqsubseteq \sigma$, then $A_\alpha \sqsubseteq \mathcal{S}_E(A_\alpha) \sqsubseteq \mathcal{S}_E(\sqcup A_\alpha)$ for every $\alpha$ and hence $\sqcup A_\alpha \sqsubseteq \mathcal{S}_E(\sqcup A_\alpha)$ This shows that $\sigma$ is a soft topology. □

**Lemma 4.6.** *If either $(F,E)$ or $(G,E)$ is a $\tau$-open soft sets, then $Int(cl((F,E) \sqcap (G,E))) = Int(cl((F,E))) \sqcap Int(cl((G,E)))$.*

**Theorem 4.7.** *Let $(X,\tau,E)$ be a soft topological space and $\overline{\mathcal{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. If $\sigma_0 = \{(F,E) \sqsubseteq (X,E) : (F,E) \sqsubseteq Int(cl(\mathcal{S}_E(F)))\}$. Then $\sigma_0$ is a soft topology for $(X,E)$.*

***Proof.*** By Theorem 4.2, for any subset $(F,E)$ of $(X,E)$, $\mathcal{S}_E(F)$ is $\tau$-open soft and $\sigma \sqsubseteq \sigma_0$. Therefore, $(\phi,E)$ and $(X,E) \in \sigma_0$. Let $(F,E)$, $(G,E) \in \sigma_0$. Then by Theorem 4.2 and Lemma 4.6, we have

$$\begin{aligned}(F,E) \sqcap (G,E) \sqsubseteq & Int(cl(\mathcal{S}_E(F))) \sqcap Int(cl(\mathcal{S}_E(G)))\\ =& Int(cl(\mathcal{S}_E(F) \sqcap \mathcal{S}_E(G)))\\ =& Int(cl(\mathcal{S}_E(F \sqcap G))).\end{aligned}$$

Therefore, $(F,E) \sqcap (G,E) \in \sigma_0$. Let $A_\alpha \in \sigma_0$ for each $\alpha \in \Delta$. By Theorem 4.2, for each $\alpha \in \Delta$, $(A_\alpha,E) \sqsubseteq Int(cl(\mathcal{S}_E(A_\alpha))) \sqsubseteq Int(cl(\mathcal{S}_E(\sqcup A_\alpha)))$ and hence $\sqcup(A_\alpha,E) \sqsubseteq Int(cl(\mathcal{S}_E(\sqcup A_\alpha)))$. Hence $\sqcup(A_\alpha,E) \in \sigma_0$. This shows that $\sigma_0$ is a soft topology. □

**Theorem 4.8.** *Let $(X,\tau,E)$ be a soft topological space and $\overline{\mathcal{I}}$ be a soft ideal over $X$ with the same set of parameters $E$. Then $\tau \sim_\theta \overline{\mathcal{I}}$ if and only if $\mathcal{S}_E(F) - (F,E) \in \overline{\mathcal{I}}$ for every subset $(F,E)$ of $(X,E)$.*

***Proof.*** *Necessity.* Assume $\tau \sim_\theta \overline{\mathcal{I}}$ and let $(F,E) \sqsubseteq (X,E)$. Observe that $x_e \in \mathcal{S}_E(F) - (F,E)$ if and only if $x_e \notin (F,E)$ and $x_e \notin [X - F]^\theta_E$ if and only if $x_e \notin (F,E)$ and there exists $(U_{x_e},E) \in \tau$ containing $x_e$ such that $cl((U_{x_e},E)) - (F,E) \in \overline{\mathcal{I}}$ if and only if there exists $(U_{x_e},E) \in \tau$ containing $x_e$ such that $x_e \in cl((U_{x_e},E)) - (F,E) \in \overline{\mathcal{I}}$. Now, for each $x_e \in \mathcal{S}_E(F) - (F,E) \in \overline{\mathcal{I}}$ and $(U_{x_e},E) \in \tau$ containing $x_e$, $cl[(U_{x_e},E)] \sqcap [\mathcal{S}_E(F) - (F,E)] \in \overline{\mathcal{I}}$ by heredity and hence $\mathcal{S}_E(F) - (F,E) \in \overline{\mathcal{I}}$ by assumption that $\tau \sim_\theta \overline{\mathcal{I}}$.
*Sufficiency.* Let $(F,E) \sqsubseteq (X,E)$ and assume that for each $x_e \in (F,E)$ there exists $(U_{x_e},E) \in \tau$ containing $x_e$ such that $cl((U_{x_e},E)) \sqcap (F,E) \in \overline{\mathcal{I}}$. Observe that $\mathcal{S}_E(X - F) - [(X,E) - (F,E)] = (F,E) - F^\theta_E = \{x_e :$ there exists $(U_{x_e},E) \in \tau$ containing $x_e$ such that $x_e \in cl((U_{x_e},E)) \sqcap (F,E) \in \overline{\mathcal{I}}\}$. Thus we have $(F,E) \sqsubseteq \mathcal{S}_E(X - F) - ((X,E) - (F,E)) \in \overline{\mathcal{I}}$ and hence $(F,E) \in \overline{\mathcal{I}}$ by heredity of $\overline{\mathcal{I}}$. □

**Proposition 4.9.** *Let $(X, \tau, E)$ be a soft topological space and $\bar{\mathcal{I}}$ be a soft ideal over $X$ with the same set of parameters $E$, $\tau \sim_\theta \bar{\mathcal{I}}$ and $(F, E) \sqsubseteq (X, E)$. If $(N, E)$ is nonempty $\tau$-open soft subset of $F_E^\theta \sqcap \mathcal{S}_E(F)$, then $(N, E) - (F, E) \in \bar{\mathcal{I}}$ and $cl(N, E) \sqcap (F, E) \notin \bar{\mathcal{I}}$.*

***Proof.*** If $(N, E) \sqsubseteq F_E^\theta \sqcap \mathcal{S}_E(F)$, then $(N, E) - (F, E) \sqsubseteq \mathcal{S}_E(F) - (F, E) \in \bar{\mathcal{I}}$ by Theorem 4.8 and hence $(N, E) - (F, E) \in \bar{\mathcal{I}}$ by heredity. Since $(N, E)$ is nonempty $\tau$-open soft and $(N, E) \sqsubseteq F_E^\theta$, we have $cl(N, E) \sqcap (F, E) \notin \bar{\mathcal{I}}$ by definition of $F_E^\theta$. □

**Theorem 4.10.** *Let $(X, \tau, E)$ be a soft topological space and $\bar{\mathcal{I}}$ be a soft ideal over $X$ with the same set of parameters $E$ and $\tau \sim_\theta \bar{\mathcal{I}}$, where $cl(\tau) \sqcap \bar{\mathcal{I}} = \phi$. Then for $(F, E) \sqsubseteq (X, E)$, $\mathcal{S}_E(F) \sqsubseteq F_E^\theta$.*

***Proof.*** Suppose $x_e \in \mathcal{S}_E(F)$ and $x_e \notin F_E^\theta$. Then there exists a nonempty soft neighborhood $(U_{x_e}, E) \in \tau(x_e)$ such that $cl((U_{x_e}, E)) \sqcap (F, E) \in \bar{\mathcal{I}}$. Since $x_e \in \mathcal{S}_E(F)$, by Theorem 4.4 $x_e \in \sqcup\{(G, E) \in \tau : cl((G, E)) - (F, E) \in \bar{\mathcal{I}}\}$ and there exists $(V, E) \in \tau$ containing $x_e$ and $cl((V, E)) - (F, E) \in \bar{\mathcal{I}}$. Now we have $(U_{x_e}, E) \sqcap (V, E) \in \tau$ and containing $x_e$, $cl((U_{x_e}, E) \sqcap (V, E)) \sqcap (F, E) \in \bar{\mathcal{I}}$ and $cl((U_{x_e}, E) \sqcap (V, E)) - (F, E) \in \bar{\mathcal{I}}$ by heredity. Hence by finite additivity we have $[cl((U_{x_e}, E) \sqcap (V, E)) \sqcap (F, E)] \sqcup [cl((U_{x_e}, E) \sqcap (V, E)) - (F, E)] = cl((U_{x_e}, E) \sqcap (V, E)) \in \bar{\mathcal{I}}$. Since $(U_{x_e}, E) \sqcap (V, E) \in \tau$, this is contrary to $cl(\tau) \sqcap \bar{\mathcal{I}} = \phi$. Therefore, $x_e \in F_E^\theta$. This implies that $\mathcal{S}_E(F) \sqsubseteq F_E^\theta$. □

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