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Research Article

Simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions

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Abstract

Novel approaches were proposed for constructing simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions, using the method of variance estimates recovery (MOVER) approach and the computational approach. They are then compared with the fiducial generalized confidence interval (FGCI) approach which was presented by (W. Thangjai, S. Niwitpong and S. Niwitpong, Simultaneous fiducial generalized confidence intervals for all differences of coefficients of variation of log-normal distributions, Lecture Notes in Artificial Intelligence, 2016). A Monte Carlo simulation was conducted to compare the performances of these simultaneous confidence intervals based on the coverage probability and average length. Simulation results show that the MOVER approach is satisfactory performances for all sample case (k) and sample size (n). Moreover, the computational approach performs as well as the MOVER approach when the sample size is large. Our approaches are applied to an analysis of a real data set from rainfall in regions of Thailand.

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1. Introduction

Log-normal distribution is a continuous probability distribution of a random variable whose logarithm follows a normal distribution. It is used as a model in real life applications. In particular, the log-normal distribution is used in analyzing medical data, pharmaceutical data, biological data, and hydrological data. For more information and applications, see Koch [14], Joulious and Debarnot [13], Shen et al. [23], Hanning et al. [10], Schaarschmidt [22], and Aghadoust et al. [2].

Standard deviation is a square root of a variance. The standard deviation and the variance are used to quantify the amount of dispersion of a set of data values in statistics

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and applications. However, the coefficient of variation, which is defined as the ratio of the standard deviation to the mean, has been used rather than the standard deviation and the variance. The coefficient of variation is important in many practical applications. For instance, in climatology, Ananthakrishnan and Soman [3] analyzed the rainfall data based on the coefficient of variation. In Business and Engineering applications, the problem of scheduling jobs to minimize the coefficient of variation was discussed by De et al. [5].

Simultaneous confidence intervals of log-normal parameters are useful in pharmaceutical statistics (Hanning et al. [10]). It is used to compare the equality of two or more drugs. This problem has been discussed in the literature. For example, Hanning et al. [10] introduced simultaneous confidence intervals based on fiducial generalized confidence interval (FGCI) approach for ratios of means of log-normal distributions. Sadooghi-Alvandi and Malekzadeh [20] used parametric bootstrap approach to construct the simultaneous confidence intervals for ratios of means of several log-normal distributions. Subsequently, Zhang and Falk [28] considered fiducial generalized pivotal quantity based simultaneous confidence interval for ratios of the means of several log-normal distributions when variances are heteroscedastic and group sizes are unequal. Recently, Thangjai et al. [24] proposed the simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions based on the FGCI approach.

In this paper, the research paper of Thangjai et al. [24] was extended to construct simultaneous confidence intervals of coefficients of variation of log-normal distributions based two new approaches: method of variance estimates recovery (MOVER) approach and computational approach. The concept of MOVER approach to construct confidence interval for the difference of parameters of two populations was extended from Zou and Donner [29], Zou et al. [30], and Donner and Zou [6]. The problem of constructing confidence interval based on the MOVER approach has received considerable attention in the literature. Sangnawakij et al. [19] proposed confidence interval for the ratio of coefficients of variation of the gamma distributions using the MOVER approach. Moreover, Sangnawakij and Niwitpong [21] presented the MOVER approach to construct confidence interval for coefficients of variation in two-parameter exponential distribution. The computational approach, was introduced by Pal et al. [17], uses the maximum likelihood estimates (MLEs) for simulation and numerical computations. The computational approach was used to test equality of several populations. For example, Gokpinar et al. [7] presented the computational approach to test equality of inverse Gaussian means under heterogeneity. Jafari and Abdollahnezhad [12] proposed a computational approach for comparing the means of two independent log-normal populations. Gokpinar and Gokpinar [8] proposed computational approach to test equality of coefficients of variation in k normal populations. To our knowledge, there is no research paper on simultaneous confidence intervals of coefficients of variation of log-normal distributions based on the MOVER approach and the computational approach. Therefore, to fill the gap, the MOVER approach and the computational approach were proposed to construct the simultaneous confidence intervals.

The rest of the paper is organized as follows. Our proposed approaches are presented in Section 2. The concept of FGCI approach is briefly presented in Section 2. Our simulation results are presented in Section 3. In our simulations, the MOVER and computational approaches are compared with the FGCI approach for the sample cases $k = 3$ and $k = 5$. Worked example is presented in Section 4. Some concluding remarks are presented in Section 5.

2. Simultaneous confidence intervals

Let $Y = (Y_1, Y_2, \ldots, Y_n)$ be a random sample of size n from the log-normal distribution $Y \sim LN(\mu, \sigma^2)$ where $LN(\mu, \sigma^2)$ refers to log-normal distribution with parameter μ and parameter σ^2, i.e. $X = \ln(Y) \sim N\left(\mu, \sigma^2\right)$. The mean and the variance of Y are

$$E(Y) = \exp\left(\mu + \frac{\sigma^2}{2}\right) \tag{2.1}$$

and

$$Var(Y) = \left(\exp\left(\sigma^2\right) - 1\right) \cdot \left(\exp\left(2\mu + \sigma^2\right)\right). \tag{2.2}$$

Then the coefficient of variation of Y is

$$\theta = \frac{\sqrt{Var(Y)}}{E(Y)} = \sqrt{\exp\left(\sigma^2\right) - 1}. \tag{2.3}$$

For $i = 1, 2, \ldots, k$, let $Y_i = (Y_{i1}, Y_{i2}, \ldots, Y_{in_i})$ be a random sample from k independent log-normal distributions $X_{ij} = \ln(Y_{ij}) \sim N\left(\mu_i, \sigma_i^2\right)$. The coefficient of variation of Y_i based on the i-th sample is defined as

$$\theta_i = \sqrt{\exp\left(\sigma_i^2\right) - 1}. \tag{2.4}$$

The interest of this paper is constructing simultaneous confidence intervals for all pairwise differences of the form

$$\theta_{il} = \theta_i - \theta_l = \sqrt{\exp\left(\sigma_i^2\right) - 1} - \sqrt{\exp\left(\sigma_l^2\right) - 1}, \tag{2.5}$$

where $i, l = 1, 2, \ldots, k$ and $i \neq l$.

The sample mean and sample variance for log-transformed data based on the i-th sample are

$$\bar{X}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} X_{ij} \quad \text{and} \quad S_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}\left(X_{ij} - \bar{X}_i\right)^2. \tag{2.6}$$

It is well known that $\bar{X}_i$ and S_i^2 are independent. Note that

$$\bar{X}_i \sim N\left(\mu_i, \frac{\sigma_i^2}{n_i}\right) \quad \text{and} \quad \frac{(n_i - 1)S_i^2}{\sigma_i^2} \sim \chi^2_{n_i - 1}, \tag{2.7}$$

where $\chi^2_{n_i-1}$ denotes chi-squared distribution with $n_i - 1$ degrees of freedom. The maximum likelihood estimator (MLE) of θ_i is

$$\hat{\theta}_i = \sqrt{\exp\left(S_i^2\right) - 1}. \tag{2.8}$$

Then

$$\hat{\theta}_{il} = \hat{\theta}_i - \hat{\theta}_l = \sqrt{\exp\left(S_i^2\right) - 1} - \sqrt{\exp\left(S_l^2\right) - 1}, \tag{2.9}$$

where $i, l = 1, 2, \ldots, k$ and $i \neq l$.

Following Thangjai et al. [24], the variance of $\hat{\theta}_i - \hat{\theta}_l$ is

$$Var\left(\hat{\theta}_i - \hat{\theta}_l\right) = \frac{\sigma_i^4 \cdot \left(\exp\left(2\sigma_i^2\right)\right)}{2(n_i - 1) \cdot \left(\exp\left(\sigma_i^2\right) - 1\right)} + \frac{\sigma_l^4 \cdot \left(\exp\left(2\sigma_l^2\right)\right)}{2(n_l - 1) \cdot \left(\exp\left(\sigma_l^2\right) - 1\right)}, \tag{2.10}$$

where $i, l = 1, 2, \ldots, k$ and $i \neq l$.

2.1. Method of variance estimates recovery approach

According to Niwitpong [16], the $100(1-\alpha)\%$ two-sided confidence interval for coefficient of variation of log-normal distribution is given by $[l_i, u_i]$ where $i = 1, 2, \ldots, k$. The l_i and u_i are defined by

$$l_i = \sqrt{\exp\left(\frac{(n_i - 1)S_i^2}{\chi^2_{(n_i-1),(1-\alpha/2)}}\right) - 1} \tag{2.11}$$

and

$$u_i = \sqrt{\exp\left(\frac{(n_i-1)S_i^2}{\chi^2_{(n_i-1),(\alpha/2)}}\right) - 1}, \tag{2.12}$$

where $\chi^2_{(n_i-1),(1-\alpha/2)}$ and $\chi^2_{(n_i-1),(\alpha/2)}$ denote the $(1-\alpha/2)$-th and $(\alpha/2)$-th quantiles of the chi-squared distribution with $n_i - 1$ degrees of freedom, respectively.

For $i = 1, 2$, Donner and Zou [6] introduced the MOVER approach to construct the $100\,(1-\alpha)\,\%$ two-sided confidence interval $[L_{12}, U_{12}]$ of $\theta_1 - \theta_2$ where θ_1 and θ_2 denote the parameters of interest and L_{12} and U_{12} denote the lower limit and upper limit of the confidence interval, respectively. The $[l_i, u_i]$ contains the parameter values for θ_i where $i = 1, 2$. The lower limit L_{12} is defined by

$$L_{12} = \hat{\theta}_1 - \hat{\theta}_2 - \sqrt{(\hat{\theta}_1 - l_1)^2 + (u_2 - \hat{\theta}_2)^2}. \tag{2.13}$$

The upper limit U_{12} is defined by

$$U_{12} = \hat{\theta}_1 - \hat{\theta}_2 + \sqrt{(u_1 - \hat{\theta}_1)^2 + (\hat{\theta}_2 - l_2)^2}. \tag{2.14}$$

For $i, l = 1, 2, \ldots, k$ and $i \neq l$, the lower limit L_{il} and the upper limit U_{il} are defined by

$$L_{il} = \hat{\theta}_i - \hat{\theta}_l - \sqrt{(\hat{\theta}_i - l_i)^2 + (u_l - \hat{\theta}_l)^2} \tag{2.15}$$

and

$$U_{il} = \hat{\theta}_i - \hat{\theta}_l + \sqrt{(u_i - \hat{\theta}_i)^2 + (\hat{\theta}_l - l_l)^2}, \tag{2.16}$$

where $\hat{\theta}_i$ and $\hat{\theta}_l$ are defined in equation (2.8), l_i and l_l are defined in equation (2.11), and u_i and u_l are defined in equation (2.12).

Then $100\,(1-\alpha)\,\%$ two-sided simultaneous confidence intervals for $\theta_i - \theta_l$ based on MOVER approach are

$$SCI_{il(MOVER)} = [L_{il}, U_{il}]\,, \tag{2.17}$$

where L_{il} and U_{il} are defined in equation (2.15) and equation (2.16), respectively.

Theorem 2.1. *Let $X_{ij} = ln(Y_{ij}) \sim N(\mu_i, \sigma_i^2)$ where $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n_i$. Let $\theta_i = \sqrt{\exp(\sigma_i^2) - 1}$ and $\theta_l = \sqrt{\exp(\sigma_l^2) - 1}$ be the coefficients of variation of log-normal distributions based on the i-th sample and l-th sample, respectively. Also, let $\hat{\theta}_i$ and $\hat{\theta}_l$ be the estimators of θ_i and θ_l, respectively. The lower limit and the upper limit of the confidence interval for $\theta_{il} = \theta_i - \theta_l$ are defined by $L_{il} = \hat{\theta}_i - \hat{\theta}_l - \sqrt{(\hat{\theta}_i - l_i)^2 + (u_l - \hat{\theta}_l)^2}$ and $U_{il} = \hat{\theta}_i - \hat{\theta}_l + \sqrt{(u_i - \hat{\theta}_i)^2 + (\hat{\theta}_l - l_l)^2}$ where $i, l = 1, 2, \ldots, k$ and $i \neq l$. Therefore*

$$P\{L_{il} \leq \theta_{il} \leq U_{il}, \forall i \neq l\} \to 1 - \alpha. \tag{2.18}$$

Proof. For $i, l = 1, 2, \ldots, k$ and $i \neq l$, the lower limit L_{il} and the upper limit U_{il} of the confidence interval for $\theta_{il} = \theta_i - \theta_l$ are

$$L_{il} = \hat{\theta}_i - \hat{\theta}_l - \sqrt{(\hat{\theta}_i - l_i)^2 + (u_l - \hat{\theta}_l)^2} = \hat{\theta}_{il} - \sqrt{(\hat{\theta}_i - l_i)^2 + (u_l - \hat{\theta}_l)^2}$$

and

$$U_{il} = \hat{\theta}_i - \hat{\theta}_l + \sqrt{(u_i - \hat{\theta}_i)^2 + (\hat{\theta}_l - l_l)^2} = \hat{\theta}_{il} + \sqrt{(u_i - \hat{\theta}_i)^2 + (\hat{\theta}_l - l_l)^2}.$$

Let $z_{\alpha/2}$ be the $(\alpha/2)$-th quantile of the standard normal distribution. The variance estimates for $\hat{\theta}_i$ at $\theta_i = l_i$ and $\hat{\theta}_l$ at $\theta_l = l_l$ are

$$\widehat{Var}(\hat{\theta}_i) = \frac{(\hat{\theta}_i - l_i)^2}{z^2_{\alpha/2}} \quad \text{and} \quad \widehat{Var}(\hat{\theta}_l) = \frac{(\hat{\theta}_l - l_l)^2}{z^2_{\alpha/2}}.$$

Also, the variance estimates for $\hat{\theta}_i$ at $\theta_i = u_i$ and $\hat{\theta}_l$ at $\theta_l = u_l$ are

$$\widehat{Var}(\hat{\theta}_i) = \frac{(u_i - \hat{\theta}_i)^2}{z^2_{\alpha/2}} \quad \text{and} \quad \widehat{Var}(\hat{\theta}_l) = \frac{(u_l - \hat{\theta}_l)^2}{z^2_{\alpha/2}}.$$

It can be written as

$$L_{il} = \hat{\theta}_{il} - z_{\alpha/2}\sqrt{\frac{(\hat{\theta}_i - l_i)^2}{z^2_{\alpha/2}} + \frac{(u_l - \hat{\theta}_l)^2}{z^2_{\alpha/2}}} = \hat{\theta}_{il} - z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\theta}_i) + \widehat{Var}(\hat{\theta}_l)}$$

and

$$U_{il} = \hat{\theta}_{il} + z_{\alpha/2}\sqrt{\frac{(u_i - \hat{\theta}_i)^2}{z^2_{\alpha/2}} + \frac{(\hat{\theta}_l - l_l)^2}{z^2_{\alpha/2}}} = \hat{\theta}_{il} + z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\theta}_i) + \widehat{Var}(\hat{\theta}_l)}.$$

Therefore

$$\begin{aligned} P(L_{il} \le \theta_{il} \le U_{il}) &= P\{\theta_{il} \in \left(\hat{\theta}_{il} \pm z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\theta}_i) + \widehat{Var}(\hat{\theta}_l)}\right), \forall i \ne l\} \\ &= P\{\max_{i \ne l}\left|\frac{\hat{\theta}_{il} - \theta_{il}}{\sqrt{\widehat{Var}(\hat{\theta}_i) + \widehat{Var}(\hat{\theta}_l)}}\right| \le z_{\alpha/2}\} \\ &= P\{Q_n \le z_{\alpha/2}\}. \end{aligned}$$

For $i = 1, 2, \ldots, k$, let $\frac{n_i}{N} \to r_i \in (0, 1)$ as $N \to \infty$ where $N = n_1 + n_2 + \ldots + n_k$. The central limit theorem implies that $N\left(\hat{\theta}_i - \theta_i\right) \to Z_i$ where $Z_1, Z_2, \ldots, Z_k$ are independent and identically distributed random variables (i.i.d.) $N\left(0, \sigma_i^2/r_i\right)$. It follows from Slutsky's theorem that as $n \to \infty$

$$Q_n \to Q,$$

where

$$Q = \max_{i \ne l}\left|\frac{Z_i - Z_l}{\sqrt{\frac{\sigma_i^2}{r_i} + \frac{\sigma_l^2}{r_l}}}\right|.$$

Using Skorohod's theorem, let Y_n and Y be the random variables on a common probability space are distributed as Q_n and Q, respectively. Then $Y_n \to Y$ and $Q_n \to Q$. Suppose that Z_i^* and Z_i are independent and identically distributed random variables. Therefore,

$$T(X, X^*, \mu, \sigma^2) \to Q^*,$$

where

$$Q^* = \max_{i \ne l}\left|\frac{Z_i^* - Z_l^*}{\sqrt{\frac{\sigma_i^2}{r_i} + \frac{\sigma_l^2}{r_l}}}\right|.$$

Since the limiting distribution of $T(X, X^*, \mu, \sigma^2)$ is continuous and the definition of convergence in distribution $z_{\alpha/2}(X) \to q_{\alpha/2}$ where $q_{\alpha/2}$ is the $(\alpha/2)$-th quantile of the distribution of Q^*. Therefore,

$$P(Q_n \le z_{\alpha/2}) \to P(Q \le q_{\alpha/2}) = P(Q^* \le q_{\alpha/2}) = 1 - \alpha, asN \to \infty.$$

Then

$$P\{\theta_{il} \in \left(\hat{\theta}_{il} \pm z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\theta}_i) + \widehat{Var}(\hat{\theta}_l)}\right), \forall i \ne l\} \to 1 - \alpha,$$

implies that

$$P\{L_{il} \le \theta_{il} \le U_{il}, \forall i \ne l\} \to 1 - \alpha.$$

□

2.2. Computational approach

The computational approach was introduced by Pal et al. [17]. It uses the maximum likelihood estimates for simulation and numerical computations. For $i = 1, 2, \dots, k$, let $\theta_i = \sqrt{\exp\left(\sigma_i^2\right) - 1}$ be the coefficient of variation of log-normal distribution.

It is well known that if $(\hat{\theta}_1, \hat{\theta}_2)$ is the MLE of (θ_1, θ_2), then $f(\hat{\theta}_1, \hat{\theta}_2)$ is the MLE of $f(\theta_1, \theta_2)$ provided f is a one-to-one function. Hence, the RMLs of μ_i and θ_i are obtained by

$$\hat{\mu}_{i(RML)} = \bar{X}_i \tag{2.19}$$

and

$$\hat{\theta}_{i(RML)} = \sqrt{\exp\left(S_i^2\right) - 1}. \tag{2.20}$$

Since the coefficient of variation depends on σ_i^2 only. Therefore, the RML of σ_i^2 is denoted by

$$\hat{\sigma}^2_{i(RML)} = \log\left(\hat{\theta}^2_{i(RML)} + 1\right). \tag{2.21}$$

Let artificial sample $X_{i(RML)} = (X_{i1(RML)}, X_{i2(RML)}, \dots, X_{in_i(RML)})$ be the normal distribution with the mean $\hat{\mu}_{i(RML)}$ in equation (2.19) and the variance $\hat{\sigma}^2_{i(RML)}$ in equation (2.21). Let $\bar{X}_{i(RML)}$ and $S^2_{i(RML)}$ be sample mean and sample variance for normal data for the i-th artificial sample and let $\bar{x}_{i(RML)}$ and $s^2_{i(RML)}$ be observed values of $\bar{X}_{i(RML)}$ and $S^2_{i(RML)}$, respectively. The difference of coefficients of variation estimator based on the artificial sample is defined by

$$\begin{aligned} \hat{\theta}_{il(RML)} &= \hat{\theta}_{i(RML)} - \hat{\theta}_{l(RML)} \\ &= \sqrt{\exp\left(S^2_{i(RML)}\right) - 1} - \sqrt{\exp\left(S^2_{l(RML)}\right) - 1}, \end{aligned} \tag{2.22}$$

where $i, l = 1, 2, \dots, k$ and $i \neq l$.

Then $100\,(1-\alpha)\,\%$ two-sided simultaneous confidence intervals for $\theta_i - \theta_l$ based on computational approach are

$$SCI_{il(CA)} = \left[\hat{\theta}_{il(RML),(\alpha/2)}, \hat{\theta}_{il(RML),(1-\alpha/2)}\right], \tag{2.23}$$

where $\hat{\theta}_{il(RML),(\alpha/2)}$ and $\hat{\theta}_{il(RML),(1-\alpha/2)}$ denote the $(\alpha/2)$-th and the $(1-\alpha/2)$-th quantiles of $\hat{\theta}_{il(RML)}$, respectively.

The computational approach is presented in Algorithm 1 and bellow:

Algorithm 1

Step 1 Obtain the MLE of the parameters as $\hat{\mu}_i = \bar{X}_i$ and $\hat{\sigma}_i = S_i$ where $i = 1, 2, \dots, k$. Then $\hat{\theta}_{il} = \sqrt{\exp\left(S_i^2\right) - 1} - \sqrt{\exp\left(S_l^2\right) - 1}$ where $i, l = 1, 2, \dots, k$ and $i \neq l$.

Step 2 Calculate the value of $\hat{\mu}_{i(RML)}$ as given by equation (2.19), calculate the value of $\hat{\theta}_{i(RML)}$ as given by equation (2.20), and calculate the value of $\hat{\sigma}^2_{i(RML)}$ as given by equation (2.21).

Step 3 Generate artificial sample $X_{i(RML)} = (X_{i1(RML)}, X_{i2(RML)}, \dots, X_{in_i(RML)})$ from $N\left(\hat{\mu}_{i(RML)}, \hat{\sigma}^2_{i(RML)}\right)$ a large number of times (say, m times). For each of these replicated samples, recalculate the MLE of $\theta_{il(RML)}$. Let these recalculated MLE values of $\theta_{il(RML)}$ be $\hat{\theta}_{il(RML),1}, \hat{\theta}_{il(RML),2}, \dots, \hat{\theta}_{il(RML),m}$.

Step 4 Let $\hat{\theta}_{il(RML),(1)} \leq \hat{\theta}_{il(RML),(2)} \leq \dots \leq \hat{\theta}_{il(RML),(m)}$ be the ordered values of $\hat{\theta}_{il(RML),g}$ where $g = 1, 2, \dots, m$, $i, l = 1, 2, \dots, k$ and $i \neq l$.

Step 5 Find the lower bound is defined by $\hat{\theta}_{il(RML),((\alpha/2)m)}$ and find the upper bound is defined by $\hat{\theta}_{il(RML),((1-\alpha/2)m)}$.

Theorem 2.2. *For $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n_i$, let $X_{ij} = ln(Y_{ij}) \sim N(\mu_i, \sigma_i^2)$. Let $\hat{\mu}_{i(RML)} = \bar{X}_i$, $\hat{\theta}_{i(RML)} = \sqrt{\exp\left(S_i^2\right) - 1}$, and $\hat{\sigma}^2_{i(RML)} = \log(\hat{\theta}^2_{i(RML)} + 1)$ be the restricted maximum likelihood estimators of μ_i, θ_i, and σ_i^2, respectively. Also, let $X_{ij(RML)} = ln(Y_{ij(RML)}) \sim N(\hat{\mu}_{i(RML)}, \hat{\sigma}^2_{i(RML)})$ be the i-th artificial sample. Let $\hat{\theta}_{il(RML),(\alpha/2)}$ and $\hat{\theta}_{il(RML),(1-\alpha/2)}$ be the lower limit and the upper limit of the confidence interval for $\hat{\theta}_{il(RML)}$ where $\hat{\theta}_{il(RML),(\alpha/2)}$ and $\hat{\theta}_{il(RML),(1-\alpha/2)}$ denote the $(\alpha/2)$-th quantile and the $(1-\alpha/2)$-th quantile of $\hat{\theta}_{il(RML)}$, respectively. Then*

$$P\{\hat{\theta}_{il(RML),(\alpha/2)} \leq \hat{\theta}_{il(RML)} \leq \hat{\theta}_{il(RML),(1-\alpha/2)}, \forall i \neq l\} \to 1 - \alpha. \tag{2.24}$$

Proof. It follows from Theorem 2.1. □

Finally, we briefly review the FGCI approach. Thangjai et al. [24] used the FGCI approach to construct the simultaneous confidence intervals.

Let $\bar{X}_i$ and $\bar{X}_i^*$ be independent and identically distributed and let S_i^2 and S_i^{2*} be independent and identically distributed. Hence, $\bar{X}_i^*$ and S_i^{2*} are independent. Since

$$\bar{X}_i^* \sim N\left(\mu_i, \frac{\sigma_i^2}{n_i}\right) \quad \text{and} \quad \frac{(n_i - 1)\, S_i^{2*}}{\sigma_i^2} \sim \chi^2_{n_i - 1}, \tag{2.25}$$

where $\chi^2_{n_i-1}$ denotes chi-squared distribution with $n_i - 1$ degrees of freedom.

According to Hanning et al. [10], the generalized pivotal quantities (GPQs) for μ_i and σ_i^2 are

$$R_{\mu_i} = \bar{X}_i - \frac{S_i}{S_i^*}\left(\bar{X}_i^* - \mu_i\right) \tag{2.26}$$

and

$$R_{\sigma_i^2} = \frac{S_i^2}{S_i^{2*}}\sigma_i^2. \tag{2.27}$$

The simultaneous fiducial generalized pivotal quantities (SFGPQs) for $\theta_i - \theta_l$ are

$$\begin{aligned} R_{\theta_{il}}\left(X, X^*, \mu, \sigma^2\right) &= R_{\theta_i}\left(X, X^*, \mu, \sigma^2\right) - R_{\theta_l}\left(X, X^*, \mu, \sigma^2\right) \\ &= \sqrt{\exp\left(R_{\sigma_i^2}\right) - 1} - \sqrt{\exp\left(R_{\sigma_l^2}\right) - 1}, \end{aligned} \tag{2.28}$$

where $i, l = 1, 2, \ldots, k$ and $i \neq l$.

According to Thangjai et al. [24], the variance of $\hat{\theta}_i - \hat{\theta}_l$ is obtained by

$$V_{il} = \frac{S_i^4 \cdot \left(\exp\left(2S_i^2\right)\right)}{2\left(n_i - 1\right) \cdot \left(\exp\left(S_i^2\right) - 1\right)} + \frac{S_l^4 \cdot \left(\exp\left(2S_l^2\right)\right)}{2\left(n_l - 1\right) \cdot \left(\exp\left(S_l^2\right) - 1\right)}; i \neq l. \tag{2.29}$$

According to Hannig et al. [10] and Hannig et al. [9], the simultaneous confidence intervals for $\theta_i - \theta_l$ are proposed on the random quantity T which is defined by

$$T = \max_{i \neq l}\left|\frac{\hat{\theta}_{il} - R_{\theta_{il}}\left(X, X^*, \mu, \sigma^2\right)}{\sqrt{V_{il}}}\right|, \tag{2.30}$$

where $\hat{\theta}_{il}$ is defined in equation (2.9), $R_{\theta_{il}}\left(X, X^*, \mu, \sigma^2\right)$ is defined in equation (2.28), and V_{il} is defined in equation (2.29).

Then $100\,(1-\alpha)\,\%$ two-sided simultaneous confidence intervals for $\theta_i - \theta_l$ based on FGCI approach are

$$SCI_{il(FGCI)} = \left[\hat{\theta}_{il} - d_{1-\alpha}\sqrt{V_{il}}, \hat{\theta}_{il} + d_{1-\alpha}\sqrt{V_{il}}\right], \tag{2.31}$$

where $d_{1-\alpha}$ denotes the $(1-\alpha)$-th quantile of the conditional distribution of T.

3. Simulation studies

A simulation study was performed to evaluate the coverage probabilities, average lengths, and standard errors of the simultaneous confidence intervals based on the MOVER approach (SCI_{MOVER}), the computational approach (SCI_{CA}), and compared with the FGCI approach (SCI_{FGCI}). Simultaneous confidence interval is satisfactory when the values of coverage probability are at least or close to the nominal confidence level $(1-\alpha)$ and also has the short average length.

The simulation study was performed with factors: (1) sample cases: $k = 3$ and $k = 5$; (2) population means: $\mu_1 = \mu_2 = \ldots = \mu_k = 1$; (3) population coefficients of variation: $\theta_1, \theta_2, \ldots, \theta_k$; (4) sample sizes: $n_1, n_2, \ldots, n_k$. The specific combinations are given in the following tables.

The following algorithm was used to estimate the coverage probabilities of three simultaneous confidence intervals:

Algorithm 2

Step 1 Generate X_i, a random sample of sample size n_i from normal population with parameters μ_i and σ_i^2 where $i = 1, 2, \ldots, k$. Calculate $\bar{x}_i$ and s_i (the observed values of $\bar{X}_i$ and S_i).

Step 2 Construct two-sided simultaneous confidence intervals based on MOVER approach ($SCI_{il(MOVER)}$) from equation (2.17) and record whether or not all the values of θ_{il} are in their corresponding SCI_{MOVER}.

Step 3 Using the computational procedure in Algorithm 1 with $m = 1000$, construct two-sided simultaneous confidence intervals based on computational approach ($SCI_{il(CA)}$) and record whether or not all the values of θ_{il} are in their corresponding SCI_{CA}.

Step 4 Construct two-sided simultaneous confidence intervals based on FGCI approach ($SCI_{il(FGCI)}$) from equation (2.31) and record whether or not all the values of θ_{il} are in their corresponding SCI_{FGCI}.

Step 5 Repeat step 1–step 4, a large number of times, $M = 5000$. Then, the fraction of times that all θ_{il} are in their corresponding SCIs provides an estimate of the coverage probability.

Table 1 and Table 2 presented the simulation results for $k = 3$ and $k = 5$, respectively. The results show that the coverage probabilities of the SCI_{FGCI} are close to 1.0000 for all cases. The SCI_{MOVER} has the coverage probabilities around the nominal confidence level 0.95 for all sample sizes (n) and all sample cases (k). The coverage probabilities of the SCI_{CA} are close to the nominal confidence level 0.95 for the same values of $\theta_1, \theta_2, \ldots, \theta_k$. For the different values of $\theta_1, \theta_2, \ldots, \theta_k$, the coverage probabilities of the SCI_{CA} provides underestimate the nominal confidence level 0.95 when the sample size is small, whereas the coverage probabilities of the SCI_{CA} are close to the nominal confidence level 0.95 when the sample size is large. Therefore, the MOVER approach can be used for estimating the simultaneous confidence intervals for all pairwise differences of coefficients of variation of several log-normal distributions. Moreover, the computational approach can be considered as an alternative to estimate the simultaneous confidence intervals for the same values of $\theta_1, \theta_2, \ldots, \theta_k$. Furthermore, the line graphs of these results are presented in Figure 1–Figure 4.

Table 1. The coverage probabilities (CP), average lengths (AL) and standard errors (s.e.) of 95% two-sided simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions: 3 sample cases.

n_1, n_2, n_3	$\theta_1, \theta_2, \theta_3$	SCI_{MOVER}		SCI_{CA}		SCI_{FGCI}	
		CP	AL (s.e.)	CP	AL (s.e.)	CP	AL (s.e.)
10,10,10	0.1,0.1,0.1	0.9590	0.1752 (0.0109)	0.9534	0.1280 (0.0081)	1.0000	0.2113 (0.0130)
	0.1,0.3,0.5	0.9501	0.6819 (0.1535)	0.9176	0.4700 (0.0942)	0.9855	0.9205 (0.1828)
20,20,20	0.1,0.1,0.1	0.9519	0.1038 (0.0044)	0.9479	0.0894 (0.0040)	0.9956	0.1239 (0.0053)
	0.1,0.3,0.5	0.9516	0.3830 (0.0773)	0.9341	0.3257 (0.0624)	0.9829	0.4899 (0.0932)
10,20,30	0.1,0.1,0.1	0.9534	0.1215 (0.0155)	0.9361	0.0985 (0.0097)	0.9901	0.1487 (0.0149)
	0.1,0.3,0.5	0.9562	0.3372 (0.0434)	0.9439	0.2896 (0.0407)	0.9904	0.4140 (0.0574)
30,30,30	0.1,0.1,0.1	0.9523	0.0801 (0.0027)	0.9504	0.0726 (0.0027)	0.9921	0.0955 (0.0032)
	0.1,0.3,0.5	0.9515	0.2922 (0.0572)	0.9414	0.2635 (0.0499)	0.9816	0.3639 (0.0686)
50,50,50	0.1,0.1,0.1	0.9527	0.0594 (0.0016)	0.9509	0.0561 (0.0017)	0.9891	0.0709 (0.0019)
	0.1,0.3,0.5	0.9462	0.2153 (0.0408)	0.9386	0.2028 (0.0378)	0.9801	0.2617 (0.0485)
30,50,100	0.1,0.1,0.1	0.9539	0.0612 (0.0059)	0.9468	0.0572 (0.0050)	0.9867	0.0736 (0.0064)
	0.1,0.3,0.5	0.9520	0.1736 (0.0199)	0.9464	0.1654 (0.0195)	0.9847	0.2081 (0.0242)
100,100,100	0.1,0.1,0.1	0.9457	0.0407 (0.0008)	0.9452	0.0395 (0.0009)	0.9814	0.0486 (0.0009)
	0.1,0.3,0.5	0.9515	0.1468 (0.0274)	0.9473	0.1427 (0.0264)	0.9806	0.1756 (0.0324)
200,200,200	0.1,0.1,0.1	0.9517	0.0283 (0.0004)	0.9521	0.0279 (0.0005)	0.9836	0.0339 (0.0004)
	0.1,0.3,0.5	0.9539	0.1019 (0.0188)	0.9496	0.1005 (0.0185)	0.9813	0.1210 (0.0222)
500,500,500	0.1,0.1,0.1	0.9500	0.0177 (0.0001)	0.9498	0.0176 (0.0003)	0.9814	0.0212 (0.0002)
	0.1,0.3,0.5	0.9482	0.0639 (0.0118)	0.9481	0.0636 (0.0117)	0.9791	0.0756 (0.0139)
1000,1000,1000	0.1,0.1,0.1	0.9515	0.0125 (0.0001)	0.9502	0.0125 (0.0002)	0.9811	0.0149 (0.0001)
	0.1,0.3,0.5	0.9480	0.0450 (0.0083)	0.9465	0.0449 (0.0083)	0.9803	0.0531 (0.0098)

Table 2. The coverage probabilities (CP), average lengths (AL) and standard errors (s.e.) of 95% two-sided simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions: 5 sample cases.

n_1, n_2, n_3, n_4, n_5	$\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$	SCI_{MOVER}		SCI_{CA}		SCI_{FGCI}	
		CP	AL (s.e.)	CP	AL (s.e.)	CP	AL (s.e.)
10,10,10,10,10	0.1,0.1,0.1,0.1,0.1	0.9573	0.1755 (0.0073)	0.9518	0.1282 (0.0054)	1.0000	0.2578 (0.0106)
	0.1,0.1,0.3,0.5,0.5	0.9532	0.7003 (0.1064)	0.9232	0.4765 (0.0655)	0.9954	1.1700 (0.1587)
20,20,20,20,20	0.1,0.1,0.1,0.1,0.1	0.9537	0.1037 (0.0029)	0.9512	0.0892 (0.0026)	0.9994	0.1468 (0.0041)
	0.1,0.1,0.3,0.5,0.5	0.9519	0.3858 (0.0510)	0.9354	0.3264 (0.0414)	0.9936	0.5898 (0.0744)
10,10,20,30,30	0.1,0.1,0.1,0.1,0.1	0.9530	0.1244 (0.0101)	0.9376	0.0997 (0.0064)	0.9976	0.1846 (0.0120)
	0.1,0.1,0.3,0.5,0.5	0.9556	0.3359 (0.0290)	0.9467	0.2880 (0.0275)	0.9970	0.5105 (0.0481)
30,30,30,30,30	0.1,0.1,0.1,0.1,0.1	0.9503	0.0802 (0.0019)	0.9486	0.0727 (0.0018)	0.9984	0.1126 (0.0026)
	0.1,0.1,0.3,0.5,0.5	0.9522	0.2939 (0.0373)	0.9409	0.2641 (0.0327)	0.9939	0.4330 (0.0533)
50,50,50,50,50	0.1,0.1,0.1,0.1,0.1	0.9524	0.0594 (0.0010)	0.9508	0.0560 (0.0011)	0.9972	0.0830 (0.0015)
	0.1,0.1,0.3,0.5,0.5	0.9491	0.2161 (0.0265)	0.9429	0.2031 (0.0246)	0.9913	0.3079 (0.0371)
30,30,50,100,100	0.1,0.1,0.1,0.1,0.1	0.9528	0.0613 (0.0038)	0.9470	0.0572 (0.0033)	0.9952	0.0868 (0.0050)
	0.1,0.1,0.3,0.5,0.5	0.9501	0.1692 (0.0138)	0.9460	0.1614 (0.0137)	0.9946	0.2386 (0.0201)
100,100,100,100,100	0.1,0.1,0.1,0.1,0.1	0.9497	0.0407 (0.0005)	0.9480	0.0395 (0.0006)	0.9951	0.0567 (0.0007)
	0.1,0.1,0.3,0.5,0.5	0.9518	0.1473 (0.0176)	0.9474	0.1430 (0.0170)	0.9925	0.2043 (0.0243)
200,200,200,200,200	0.1,0.1,0.1,0.1,0.1	0.9495	0.0283 (0.0002)	0.9485	0.0279 (0.0003)	0.9947	0.0394 (0.0003)
	0.1,0.1,0.3,0.5,0.5	0.9506	0.1025 (0.0121)	0.9487	0.1010 (0.0120)	0.9926	0.1403 (0.0166)
500,500,500,500,500	0.1,0.1,0.1,0.1,0.1	0.9486	0.0177 (0.0001)	0.9478	0.0176 (0.0002)	0.9938	0.0247 (0.0001)
	0.1,0.1,0.3,0.5,0.5	0.9483	0.0641 (0.0075)	0.9473	0.0638 (0.0075)	0.9907	0.0871 (0.0102)
1000,1000,1000,1000,1000	0.1,0.1,0.1,0.1,0.1	0.9486	0.0125 (0.0000)	0.9485	0.0125 (0.0001)	0.9938	0.0174 (0.0001)
	0.1,0.1,0.3,0.5,0.5	0.9512	0.0452 (0.0053)	0.9496	0.0451 (0.0053)	0.9922	0.0612 (0.0072)

4. An empirical verification

In the field of climate sciences and hydrology, precipitation and temperature are the most important variables frequently used to trace extent and magnitude of climate change and variability. The long-term climatic change related to changes in precipitation patterns,

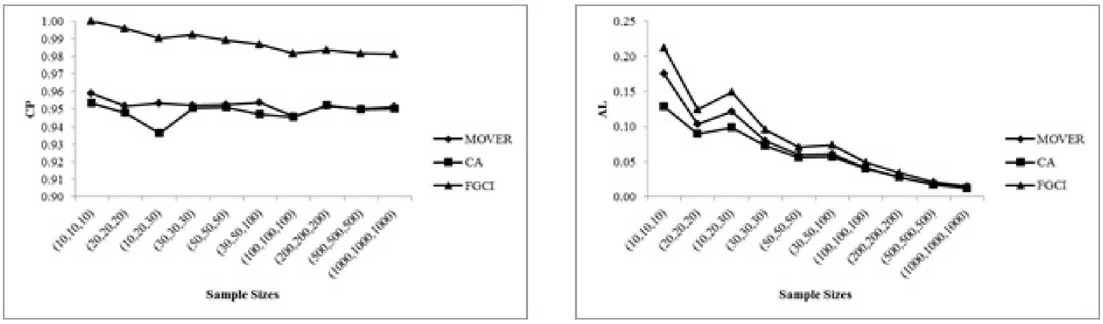

Figure 1. The coverage probabilities (CP) and average lengths (AL) of 95% two-sided simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions: 3 sample cases and $(\theta_1, \theta_2, \theta_3) = (0.1, 0.1, 0.1)$.

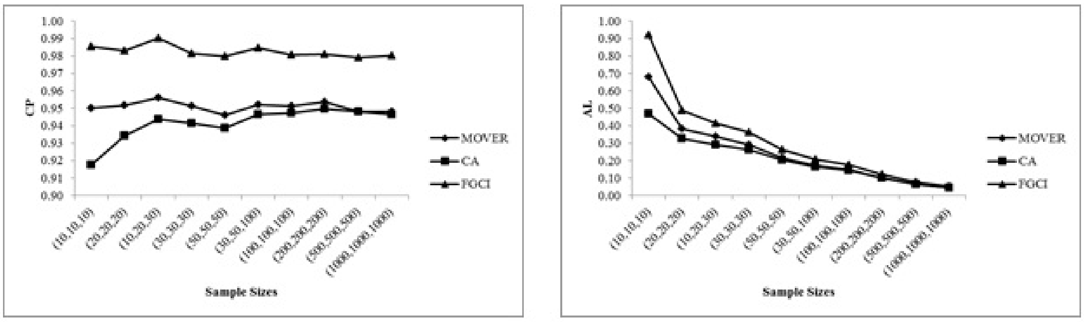

Figure 2. The coverage probabilities (CP) and average lengths (AL) of 95% two-sided simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions: 3 sample cases and $(\theta_1, \theta_2, \theta_3) = (0.1, 0.3, 0.5)$.

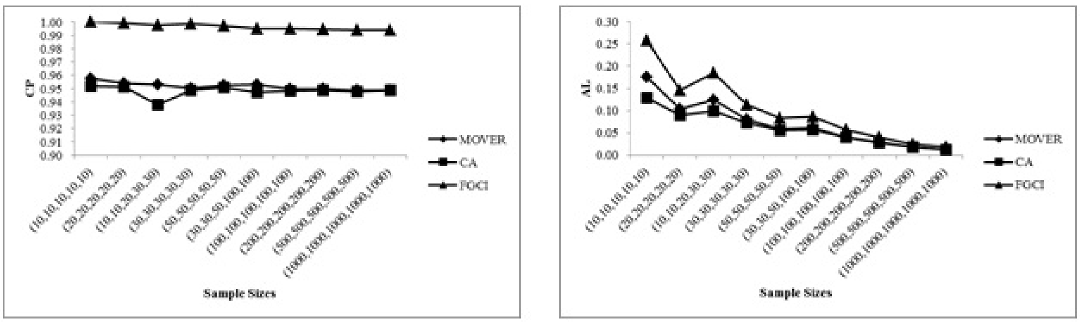

Figure 3. The coverage probabilities (CP) and average lengths (AL) of 95% two-sided simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions: 5 sample cases and $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) = (0.1, 0.1, 0.1, 0.1, 0.1)$.

rainfall variability and temperature. The climate will increase the risk of both droughts and floods. Rainfall and temperature are computed using the coefficient of variation. The coefficient of variation index for the climate moisture index is a statistical measure of variability in the ratio of plant water demand to precipitation. It is useful for identifying regions with highly variable climates as potentially vulnerable to periodic water stress and

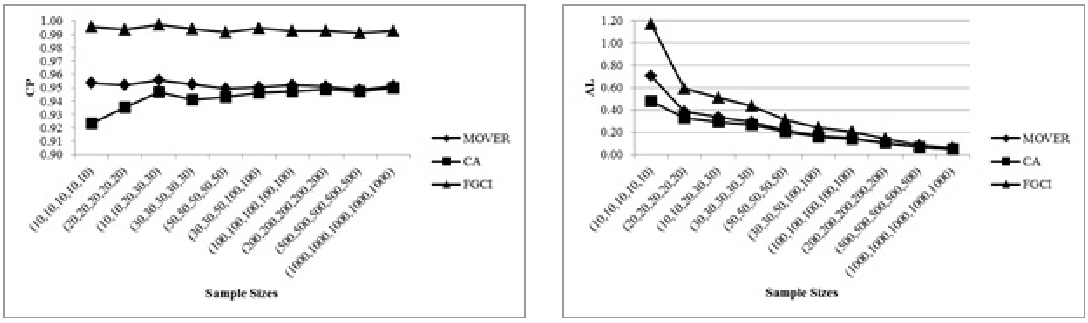

Figure 4. The coverage probabilities (CP) and average lengths (AL) of 95% two-sided simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions: 5 sample cases and $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) = (0.1, 0.1, 0.3, 0.5, 0.5)$.

scarcity. Thailand is divided into six geographical regions. The six regions include northern Thailand, northeastern Thailand, western Thailand, central Thailand, eastern Thailand, and southern Thailand. Rainfall varies widely both by location and season. As a result, the rainfall has changed in different regions of Thailand. The unit of precipitation is the millimeter (mm). The rainfall is often expressed in millimeters per day (mm/day). The probability models most commonly used to estimate rainfall frequency are the log-normal distribution. Cho et al. [4] compared gamma distribution and log-normal distribution for characterizing rain rates. Ritzema [18] analyzed extreme values of such variables as monthly and annual maximum values of daily rainfall and river discharge volumes using the log-normal distribution. Regarding confidence intervals for the coefficient of variation, Vangel [26] proposed confidence intervals for coefficient of variation of normal distribution. Wong and Wu [27] developed small sample asymptotic method to obtain approximate confidence intervals for the coefficient of variation for both normal and nonnormal models. Tian [25] presented the procedure for confidence interval estimation based on the concepts of generalized confidence interval. Furthermore, Hasan and Krishnamoorthy [11] and Nam and Kwon [15] developed approximate confidence interval estimation of the ratio of two coefficients of variation for log-normal distributions. In addition, the confidence interval for the coefficient of variation can be used in modeling and predicting the extreme rainfall events. The study of the characteristics of extreme rainfall events can help in the planning to reduce disaster losses. However, the statistical features of daily rainfall distribution at different regions are important aspects of rainfall climatology. Therefore, the coefficients of variation of rainfall in different regions are different values. Simultaneous confidence intervals for coefficients of variation of log-normal distributions are useful in climatology. In rainfall studies, comparing rainfall variability in different regions refers to compare the coefficient of variation of the different regions. Therefore, this paper considered simultaneous confidence intervals for estimating the differences of coefficients of variation of rainfall in different regions of Thailand. The MOVER approach, the computational approach, and the FGCI approach are applied to a real daily rainfall data in the section.

Thailand is divided into five regions based on the climate pattern and meteorological conditions. Five regions are Northern, Northeastern, Central, Eastern, and Southern regions. Daily rainfall data is given by Thai Meteorological Department. Daily rainfall data on 17 July 2018 are given to illustrate in this study. The data are showed in Table 3. The histograms of daily rainfall data are presented in Figure 5. These figures have shown that the daily rainfall data are right skewed distributions. Table 4 displays the sample sizes, the sample means, and the sample standard deviations of the five regions. Before applying our approaches to a set of real data, it is necessary to check the assumption that the log-data were drawn from a normal distribution. Traditionally, the Shapiro-Wilk normality test was used, with p-values 0.0113, 0.0003, 0.6367, 0.0554, and 0.9011 for Northern, Northeastern, Central, Eastern, and Southern regions, respectively. By now, it is known that the use of p-values in testing is not valid. Thus, alternatives for checking normality could be, either by graphical methods such as QQ-plot, or by Bayesian tests (model selection). From Table 5, the minimum Akaike Information Criterion (AIC) values by Bayesian test on the five regions indicate that Central, Eastern, and Southern regions are log-normal distributions. Furthermore, the normal QQ-plots of log-data in Figure 6 confirm the results of Bayesian test. Here we select only the data from log-normal distributions to illustrate our estimation approaches. The confidence intervals based on the MOVER approach, the computational approach, and the FGCI approach are given in Table 6.

From Table 6, the results show that the MOVER approach, the computational approach, and the FGCI approach contain true differences of coefficients of variation, but the computational approach is shorter interval than the other approaches. However, it is clear from this table that the MOVER approach and the computational approach are better than the FGCI approach in the sense of lengths. Moreover, this example indicates that the MOVER approach and the computational approach are satisfactory when the sample case is equal to three ($k = 3$).

Table 3. Daily rainfall data of five regions (mm).

Northern		**Northeastern**		**Central**		**Eastern**	**Southern**	
32.0	70.0	104.6	20.6	5.7	0.3	33.1	17.7	12.8
2.3	49.8	6.8	33.0	15.9	2.8	11.1	18.2	20.6
5.6	6.8	101.2	39.6	5.0	2.4	49.9	13.2	17.2
2.4	18.8	7.1	12.9	3.2	0.1	105.1	17.2	9.8
12.9	20.6	1.2	14.8	20.0	0.1	23.4	39.9	12.1
23.4	3.4	2.8	14.0	73.4	1.6	2.6	62.6	6.4
30.9	5.1	7.1	2.3	10.8	2.2	1.9	23.9	52.4
41.2	9.5	29.1	17.6	2.2	2.5	26.4	14.0	88.6
9.3	8.0	8.1	2.1	5.8	8.7	0.4	10.8	25.8
17.4	41.9	39.6	2.2	1.9	2.1	1.4	13.1	29.0
47.5	3.2	16.8	15.4	24.5	3.0	7.0	11.6	21.3
58.9	2.3	44.1	24.7			2.8	23.4	31.6
71.7	5.1	28.0	2.4			24.9	31.8	61.3
15.1	12.6	28.2	1.4			107.3	18.0	49.8
15.9	93.4					272.4		

Source: Thai Meteorological Department (www.tmd.go.th/climate/climate.php).

Table 4. Sample statistics.

Sample statistics	**Region**				
	Northern	Northeastern	Central	Eastern	Southern
n_i	30	28	22	15	28
$\bar{y}_i$	24.5667	22.4179	8.8273	44.6467	26.9321
s_{Y_i}	24.3103	26.1026	15.8404	71.9642	19.5774
$\bar{x}_i$	2.6727	2.4719	1.1642	2.5592	3.0854
s_{X_i}	1.1158	1.2552	1.6073	1.8549	0.6373

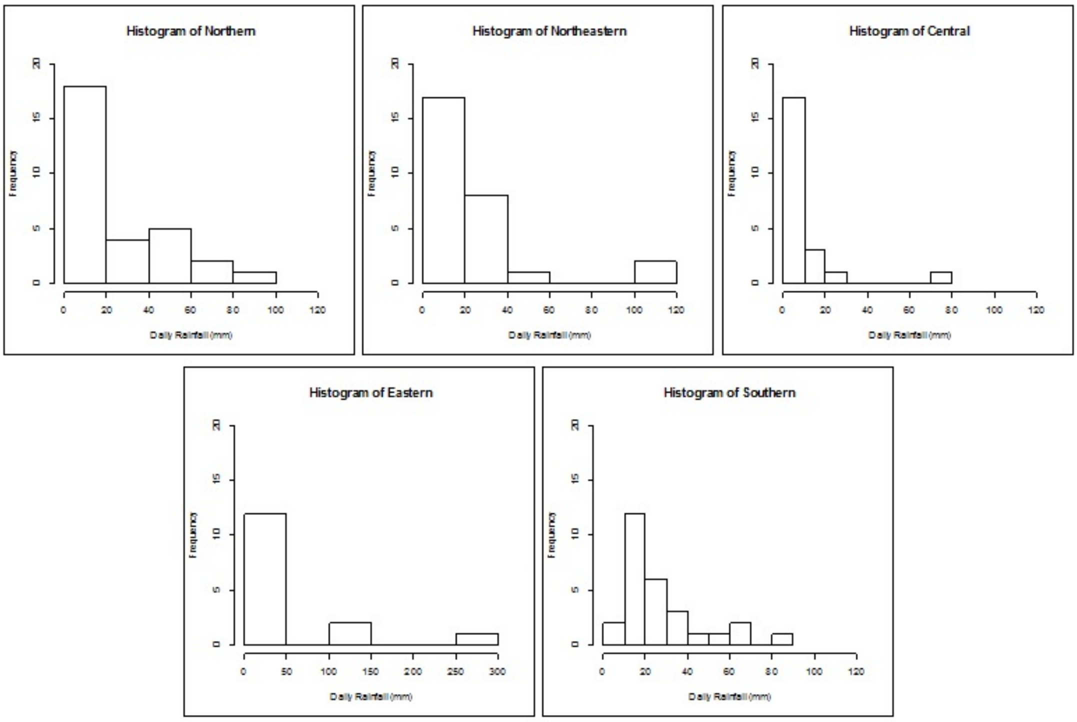

Figure 5. Histogram plots of daily rainfall data of five regions.

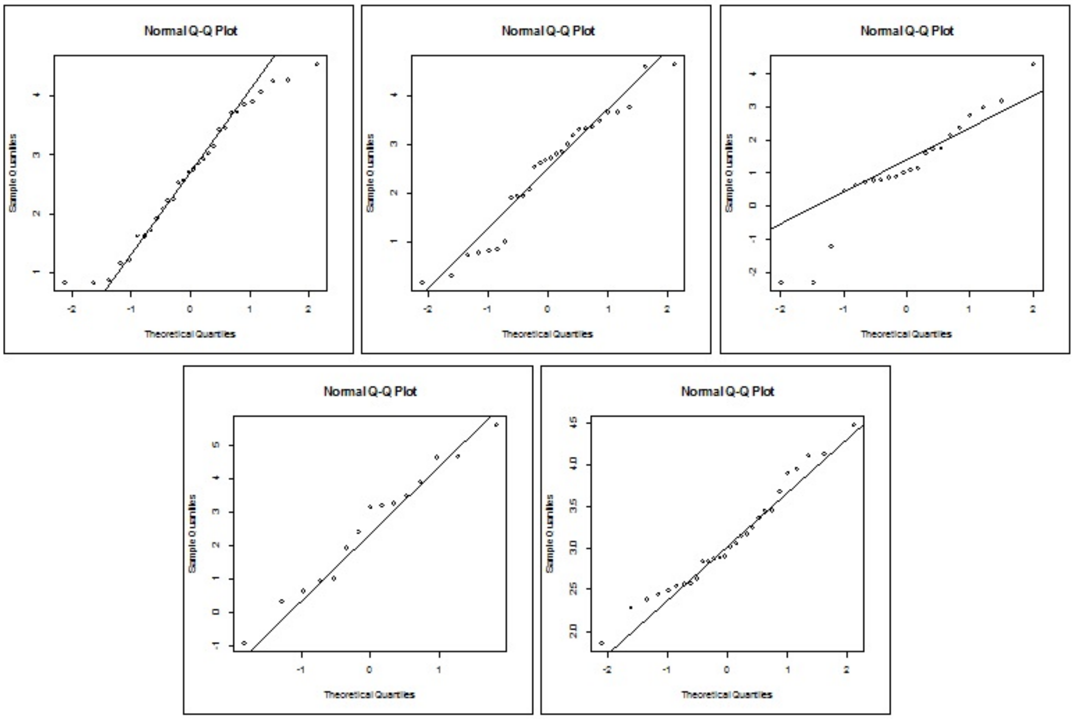

Figure 6. The normal QQ-plots of log-daily rainfall data of five regions.

Table 5. The minimum Akaike Information Criterion (AIC) values of five regions.

Distributions	**AIC values of region**				
	Northern	Northeastern	Central	Eastern	Southern
Normal	279.5734	265.1163	186.9627	173.8183	249.0074
Log-Normal	255.0532	233.5956	**137.5153**	**140.8451**	**229.9952**
Gamma	255.9677	234.0020	139.3899	141.9333	233.5485
Exponential	**253.0834**	**231.1520**	140.8252	144.9634	241.4259

Table 6. The 95% two-sided simultaneous confidence intervals for all pairwise differences of coefficients of variation of log-normal distributions.

Comparison	CI_{MOVER}		CI_{CA}		CI_{FGCI}	
	Lower	Upper	Lower	Upper	Lower	Upper
Central / Eastern	–8.9300	68.6753	–4.6153	22.9889	–49.8910	53.8852
Central / Southern	–13.2441	–1.1551	–8.4766	–0.8408	–22.4795	16.8970
Eastern / Southern	–71.4476	–1.5800	–27.4743	–0.9418	–52.8446	43.2679

5. Discussions and conclusions

Thangjai et al. [24] presented the FGCI approach for constructing the simultaneous confidence intervals for all differences of coefficients of variation of log-normal distributions. This paper provided the FGCI approach and proposed the MOVER approach and the computational approach to construct the simultaneous confidence intervals. Monte Carlo simulations were used to evaluate the coverage probabilities, average lengths, and standard errors of the simultaneous confidence intervals. The results indicated that the simultaneous confidence intervals based on FGCI approach are a conservative confidence interval which is similar to the result in research papers by Hannig et al. [10] and Abdel-Karim [1]. The coverage probabilities of the MOVER approach are close to the nominal confidence level 0.95. Therefore, the MOVER approach is recommended to construct the simultaneous confidence intervals for all pairwise differences of coefficients of variation from several log-normal distributions. Additionally, the computational approach can be used when the same values of k coefficients of variation. Furthermore, simulation results indicate that the MOVER approach and the computational approach are better than the FGCI approach.

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