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Real Time Performance Comparison of Buck Converter Circuit Controlled By Discrete Time PID, LQR and SMC Controllers in Continuous-Current Mode

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Abstract

In this study, a DC-DC buck converter circuit which is used to get direct voltage by decreasing the source voltage is researched. Firstly mathematical model for a DC-DC buck converter circuit is obtained and critical inductance value is calculated to make it work in continuous current mode. Then for controlling a DC-DC buck converter, three different controllers in discrete time (sliding mode control, proportional–integral–derivative controller and linear quadratic regulator) are designed. Finally, the performance of these controllers according to changing in the form of step function for input voltage, reference voltage and load conditions are examined and compared with real time studies. According to obtained results, a DC-DC buck converter circuit controlled by sliding mode controller has shown a better performance compared to the other controllers.

Keywords: DC-DC converter, Buck converter, PID, SMC, LQR

1. INTRODUCTION

Today, most of the energy consumed is unfortunately derived from fossil fuels. As the technology advances, worldwide energy consumption accelerates day by day [1]. Because the fossil fuels are nonrenewable and their bad effects to the environment, researchers are studying ways to get the maximum efficiency from the used energy [1,2].

Linear regulators are cheap and also have advantages like being robust and durable [3,4]. But also they are large, heavy and have low efficiency between (25% to 60%). Switch mode converters are relatively hard to control and expensive. They can produce electromagnetic interference (EMI). They are light, smaller and have high efficiency between (70% to 95%) [5]. With a right design and control, EMI can be reduced to reasonable values. Switch mode converters can have more than one output and some models have the option to adjust the output voltage poles as desired [6,7].

The DC-DC buck converters are power electronic circuits which get output voltage equal to or less than the input voltage. The DC-DC buck converters have at least one power switch to make a controlled switching. They have inductance and capacitor to filter the current and voltage fluctuations on the load [7,8].

In this study, firstly the DC-DC buck converter is presented and circuit analysis of the DC-DC converter is done. Then for controlling the DC-DC

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buck converter, in discrete time three different controllers (sliding mode control, proportional– integral–derivative controller, linear quadratic regulator) are designed. Finally, performances of these controllers according to changing input voltage, reference voltage and load conditions are examined and compared with real time studies.

2. DC-DC BUCK CONVERTERS

We have analyzed DC-DC buck converters circuit with the following assumptions.

-Power mosfet and diode are ideal switches.

-The inductor is in continuous current mode.

The structure of the DC-DC buck converters is as shown in Figure 1.



Figure 1. DC-DC buck converter circuit diagram

According to the opening and closing of the switching component which is shown in the Figure 1, as shown in Figure 2 can be analyzed in two modes.



Figure 2. (a) When the switching component is in off mode (b) When the switching component is in on mode.

The ratio of the on time (T_{on}) of switching component to the total switching period (T_s) gives the duty cycle (d) [9,10]

$$d = \frac{T_{on}}{T_{on} + T_{off}} \tag{1}$$

When the switching component is on (dT_s) , dynamic representations belonging to the DC-DC buck converters circuit shown in Figure 2.a are given below in equations 2, 3 and 4 [11].

$$v_g = L \frac{di_L}{d_t} + v_0 + r_L i_L \tag{2}$$

$$v_0 = r_C i_C + \frac{1}{C} \int i_C dt \tag{3}$$

$$i_C = i_L - \frac{v_0}{R_Y} \tag{4}$$

Similarly, when the switching component is off $((1-d)T_s)$, dynamic representations belonging to the DC-DC buck converters circuit shown in Figure 2.b are given below equations 5, 6 and 7 [11].

$$0 = L \frac{di_{L}}{d_{t}} + v_{0} + r_{L} i_{L}$$
(5)

$$v_0 = r_C i_C + \frac{1}{C} \int i_C dt \tag{6}$$

$$i_C = i_L - \frac{v_0}{R_\gamma} \tag{7}$$

If selected as inductance current and capacitor voltage state variables. Based on switching components on and off situations, the state space representation of the DC-DC buck converters are given below in equations 8 and 9 [12].

$$\dot{x} = \begin{bmatrix} -\frac{r_L}{L} & \frac{1}{L} \\ \frac{LR_Y - Cr_L r_C R_Y}{LCr_C + R_Y} & \frac{-L - Cr_C R_Y}{LC(r_C + R_Y)} \end{bmatrix} x + \begin{bmatrix} \frac{V_g}{L} \\ \frac{V_g r_C}{L} \end{bmatrix} u$$

$$\begin{bmatrix} 0 \end{bmatrix}$$

$$(8)$$

$$\dot{y} = \begin{bmatrix} 0\\1 \end{bmatrix} x \tag{9}$$

Derived from Figure 1, equation 8 and 9, the DC-DC buck converters transfer function between input and output voltages is given below in equation 10 and 11 [13].

$$\frac{V_0(s)}{V_g(s)} = \frac{d(RCr_c s + R)}{(R + r_c)LCs^2 + (m)s + (R + r_L)}$$
(10)

$$m = L + Cr_C R + Cr_L R + Cr_C r_L \tag{11}$$

The inductance value must be greater than the critical inductance value, for the buck converter to operate in continuous current mode. The value of critical inductance is given in equation 13 [11,12].

$$2I_{Y} = \frac{V_{0}(1-d)}{f_{s}L}$$
(12)

$$L_{K} = \frac{R(1-d)}{2f_{s}} = 600\,\mu H \tag{13}$$

Table 1. The system parameters used in real time studies

Parameter	Value		
İnductance	$L = 880 \ \mu H, r_L = 1.7 \Omega$		
Capacitor	$C = 390 \ \mu F, r_C = 14m\Omega$		
Load	$R = 15 \Omega$		
Input Voltage	$V_g = 13 V$		
Switching Frequency	$f_s = 10 \ kHz$		

3. CONTROL METHODS OF THE SYSTEM

3.1. Sliding Mode Control

Sliding mode control is a very robust control method under proper conditions indicating the desired dynamic behaviors despite the outer distortions, parasites and parameter changes [15,16]. SMC method is more advantageous than other methods because of its properties like convenience to be used on the system, indifference to parameter uncertainties, damping the noise and endurance [15-18]. Basically, the aim of SMC is reset the difference between the chosen control variables reference and measured value and the derivation of this difference to time and so leading the state variables, to the defined equilibrium point. SMC design is performed in two stages [15-19].

i-) Determining of a stable sliding surface.

ii-) Determining the control signal which will take the system from a starting point to the sliding surface and keep it in there.

In discrete time SMC systems, two types of orbital behavior can be defined as shown in Figure 3. These are [15,20];

- i-) Ideal orbit
- ii-) Realizable orbit



Figure 3. Phase plane for discrete-time SMC

In the SMC systems, for the cases when the system state orbit frequency in discrete time is not infinite, it cannot stay on the sliding surface as in continuous-time SMC systems. It shows a zigzag behavior around sliding surface in a limited band (Δ) which is also called "crackling". Thus,

discrete-time SMC systems are called Quasi-SMC [15,21].

$$2\Delta = 2\frac{\varepsilon T_s}{1 - qT_s} \tag{14}$$

For the system to be stable and robust, amplitude of the zigzags and system phase orbit should stay in a limited band as shown in equation 14. Ideal sliding happens when the limited band value is zero ($\Delta = 0$) [15].

If we rearrange the state variables as $(x_1 = V_0 - V_{ref})$ and $(x_2 = \dot{x}_1)$ to define the DC-DC buck converters SMC control signals, we get the equation 15.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ V_{in} \\ L \end{bmatrix} \begin{bmatrix} u \end{bmatrix} + \begin{bmatrix} 0 \\ V_{ref} \\ LC \end{bmatrix} (15)$$

If system parameter values and discrete time statespace model given in Table 1 is discretized by $(T_s = 1/f_s)$ frequency, we get these representations:

$$x(k+1) = Gx(k) + Hu(k) + d_{k}$$
(16)
$$G = \begin{bmatrix} 0.9855 & 0.0001 \\ -287.49 & 0.9687 \end{bmatrix}, H = \begin{bmatrix} 0.2 \\ 3737.5 \end{bmatrix}, d_{k} = \begin{bmatrix} 0 \\ -287.5 \\ -287.5 \end{bmatrix}$$
(17)

For a DC-DC buck converter circuit we can chose a sliding surface as in equation 18.

$$s(k) = C_g x(k) \tag{18}$$

With the help of Gao access rule, control signal u(k) [22];

$$s(k+1) - s(k) = -qT_s(s(k)) - \varepsilon T_s \operatorname{sgn}(s(k))$$
(19)

$$s(k+1) - s(k) = \begin{cases} C_g G(x(k)) + C_g H(u(k)) \\ -C_g(x(k)) \end{cases}$$
(20)

Derived from above given Gao access rule equations and using coefficients $C_g = \begin{bmatrix} 4 & 10^{-6} \end{bmatrix}$, e = 200 and q = 1500, for a DC-DC buck converter SMC control signal is as in equation 21.

$$u(k) = ([4\ 10^{-6}]H)^{-1} - [4\ 10^{-6}]Gx(k) - [4\ 10^{-6}]d_k + (1 - 1500T_s)(s(k)) - 200T_s\,\mathrm{sgn}(s(k)))$$
(21)

3.2. Linear Quadratic Regulator

Linear Quadratic Regulator is one of the most suitable control processes based on state feedback method [23,24]. Thus, except some special cases, LQR controlled system response is stable. LQR method is based on obtaining state feedback constants with the help of values which will minimize or maximize the performance index specified for the system. Schematic model controlled with LQR is given in Figure 4 [24].



Figure 4. Classical LQR schematic diagram.

As can be seen in the LQR block diagram, LQR method is a state variable feedback structured theory. What is different from LQR method and pole placement method is the way they determine the gain matrix K. In pole placement, designer implants the poles in determined spots to define the gain matrix. But in LQR technique for defining gain matrix K, performance index is used [15,25].

If system is affected from outer parasites and distortions, system deviates from the reference value and regulatory structure only would be insufficient to readjust the system to its reference value [26]. It is possible to eliminate this problem by adding an integral controller to the system. In this study, for making the DC-DC buck converter system immune to outer parasites and distortions and eliminating problems which may cause instability, system state vectors are augmented by appointing a new pole. Schematic model of increased poled LQR controller is given in Figure 5 [24,26].

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Figure 5. Servo system.

We can write equations augmented poled LQR controller schematic diagram, equation 22 and 23 from in Figure 5 [24].

$$v(k) = v(k-1) + r(k) - y(k)$$
(22)

$$v(k+1) = v(k) + r(k+1) - C(Gx(k) - Hu(k))$$
(23)

Using equations for increased poled discrete-time state space model x(k+1) and v(k+1) we can write;

$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} G & 0 \\ -CG & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} + \begin{bmatrix} H \\ -CH \end{bmatrix} u(k)$$
$$= + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(k+1)$$
(24)

$$y[k] = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x[k] \\ y[k] \end{bmatrix}$$
(25)

And augmented poled LQR controllers control equation would be given in equation 26 [24].

$$u(k) = -Kx(k) + K_i v(k)$$
(26)

In equation 27 performance index chosen for DC-DC buck converter is given [15].

$$J = \frac{1}{2} \sum_{k=0}^{\infty} \left(x^{T}[k] Q x[k] + u^{T}[k] R u[k] \right)$$
(27)

In the given performance index, x represents state variables, u represents control variables, Q and R represents positive defined weight matrixes. For increased poled system, gain matrix is given in equation 28 [26].

$$\begin{bmatrix} -K & k_i \end{bmatrix} = (R + \hat{H}^T P \hat{H})^{-1} \hat{H}^T P \hat{G}$$
(28)

To obtain gain matrix coefficients, positive defined symmetrical matrix (P) is needed. Using below given equation 29 for discrete-time Ricatti equation, positive defined symmetrical matrix (P) can be obtained [15,26].

$$P = \hat{G}^{T} P \hat{G} - (\hat{G}^{T} P \hat{H}) (R + \hat{H}^{T} P \hat{H})^{-1} (\hat{H}^{T} P \hat{G}) + Q_{(29)}$$

The choice of positive defined weight matrixes Qand R for LQR controller is the preference between density of the control signal and regulation speed. If Q > R is chosen control signals density would be low and system regulation speed would be high. Likewise, if R > Q is chosen, control signals density would be high and system regulation speed would be low. For this study, Q and R are chosen as [25]:

$$Q = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 \end{bmatrix}$$
(30)

In this study, gain matrix for augmented poled LQR controlled system is obtained as:

$$K = [0.7094 \quad 1.0248]$$
 ve $k_i = [0.1816]$ (31)

3.3. Proportional Integral Derivative Control

Proportional-integral-derivative (PID), made of a combination of proportional, integral and derivative methods has a relatively simple structure and it is easy to design, thus it is one of the most preferred controller types by the industry which is of classical feedback construction [26,27]. In simple terms, PID controller processes the error value which is the difference between desired reference voltage and system output to produce the control signal which will eliminate the error [28]. While producing the control signal, it processes the error value in three separate mathematical operations (proportional K_{P} , integral K_{I} and derivative K_{D}).Control block diagram for discrete time PID controller system is shown in Figure 6 [29].

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Figure 6. Classical discrete time PID schematic diagram

Mathematically algorithm of PID controller in discrete time is given in equation 32 [29].

$$G_{PID(Z)} = K_P + K_I \frac{z}{z-1} + K_D \frac{z-1}{z}$$
(32)

To determine the constants $(K_P, K_I \text{ and } K_D)$ in above given PID algorithm, there are various methods in literature for time domain and frequency domain. For this study Matlab/Sisotool program is used and K_P , K_I and K_D constants are obtained in equation 33 [24,27].

$$G_{PID(Z)} = 0.17 + +198.117 \frac{z+1}{z-1} + \frac{18.05}{10^6} \frac{z-1}{z}$$
(33)

4. REAL TIME STUDIES ON DC-DC BUCK CONVERTER

For real time studies of DC-DC buck converter working in continuous current mode, ARM based Texas Instrument LM4F120 processor is used. Firstly, while working in constant load and constant input voltage, the ability of the DC-DC buck converter controllers to follow reference step function voltage is examined. For this purpose reference voltage is increased to 8V from 7V and then reduced to 6V and again increased to 7V and controller performances are compared. The performances for reference value changes of SMC, PID and LQR are shown below respectively in Table 2 and Figures 7, 8 and 9.

Table 2. Real time performance comparison of the DC-DC buck converter, which is controlled with SMC, LQR and PID controllers for reference voltage change condition

	SMC	LQR	PID	
S. M.	0,8	2,9	3,3	7V → 8V
Settling Time (ms)	1,9	3,7	5,6	8V → 6V
	0,4	2,9	3,6	6V → 7V

9/ Marin	0	0	0,9	7V → 8V
%Maximum Overshoot	0	0	5,6	8V → 6V
	0	0	1,1	6V → 7V



Figure 7. Response curves of SMC examined for reference voltage change condition



Figure 8. Response curves of LQR controller examined for reference voltage change condition



Figure 9. Response curves of PID controller examined for reference voltage change condition

The system with the SMC has performed the fastest follow-up for all change moments and when overflow criteria is taken into account, SMC and LQR systems has the least overflow. However, in the system with the SMC a crackling problem has occurred as can be seen from the inductance and capacitor currents. There are various methods to overcome the crackling problem in literature. In responses of the three controllers, permanent state error wasn't observed.

Secondly, while working in constant load and constant reference voltage, the performances against the changing step functioned input voltage value are examined. For this purpose input voltage value is increased to 18.0 V from 13.0 V to compare the performances of the controllers. Performances belonging to SMC, PID and LQR controllers against the input voltage value change are given below respectively in Table 3 and Figures 10, 11 and 12.

Table 3. Real time performance comparison of the DC-DC buck converter, which is controlled with SMC, LQR and PID controllers for sudden changes of input voltage condition

	13,0V → 18,0V		
	SMC	LQR	PID
Settling Time (ms)	2,6	4,7	6,8
%Maximum Overshoot	11,2	24,5	28,7



Figure 10. Response curves of SMC examined for sudden changes of input voltage condition



Figure 11. Response curves of LQR controller examined for sudden changes of input voltage condition



Figure 12. Response curves of PID controller examined for sudden changes of input voltage condition

It is observed that the system with SMC has performed the fastest track and it is the one with the minimum overflow.

Lastly, for a DC-DC buck converter with controllers working in constant input voltage and constant reference voltage, the performances against the changing step functioned load values are examined. For this purpose load value is increased to 15.0 Ω from 7.5 Ω and then reduced to 7.5 Ω again to compare the performances of the controllers. Performances belonging to SMC, PID and LQR controllers against the load value change are given below respectively in Table 4. and Figures 13, 14 and 15.

Table 4. Real time performance comparison of the DC-DC buck converter, which is controlled with SMC, LQR and PID controllers for load change condition

	SMC	LQR	PID	
Settling	0	2,7	2,4	7,5Ω→15Ω
Time (ms)	0	3,1	2,1	15Ω → 7,5Ω
%Maximum	0	9,6	9,1	7 ,5Ω→ 15Ω
Overshoot	0	9,1	5,7	15Ω → 7,5Ω



Figure 13. Response curves of SMC examined for load change condition



Figure 14. Response curves of LQR controller examined for load change condition



Figure 15. Response curves of PID controller examined for load change condition

5. CONCLUSION

In this study for a DC-DC buck converter working in continuous current mode, SMC, LQR and PID controllers in discrete time are designed and with real time studies, their performances are examined comparatively. As the result of these studies, SMC controlled system is observed to be the least affected system in terms of reference voltage follow-up and distortion inputs like input voltage or load change.

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