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# High-temperature thermostatistical properties of deformed quantum gas in two dimensions

Mustafa Senay*

**Abstract**

In this study, we focus on the high-temperature thermostatistical properties of the $q$-deformed gas model in two spatial dimensions. Some important thermodynamical functions such as internal energy, entropy, specific heat are calculated depending on deformation parameter $q$. Moreover, the first five deformed virial coefficients in the equation of state of the model for two dimensions are derived. Also, the results obtained in this work are compared with the results of the undeformed gas model.

**Keywords:** Virial coefficient, $q$-deformed boson, $q$-deformed fermion, thermodynamics

## 1. INTRODUCTION

The investigation of the quantum groups and the quantum algebras has been become one of the interesting topics for physicsts and mathematicians over the last decades. In order to obtain the quantum groups and the quantum algebras, the usual Lie groups and Lie algebras can be deformed with some real or complex deformation parameters [1]. There are many studies in the literature to understand the physical interpretation of these deformation parameters [2-9]. For instance, in the Ref. [3,4], the $q$-deformed harmonic oscillators have investigated depending on the $q$-deformed creation and annihilation operators.

On the other hand, the high and low thermostatistical properties of the $q$-deformed bosons and fermions have been extensively examined for three dimensional space in the literature [10-17]. In these studies, the $q$-deformed theory has been found applications in several areas of physics due to its applications in a large variety areas such as Jaynes-Cummings model and the deformed oscillator algebra [10], generalized thermodynamics of $q$-deformed bosons and fermions [11], Bose-Einstein condensation of a relativistic $q$-deformed Bose gas [12], thermodynamic geometry of deformed bosons and fermions [13], high-temperature behavior of a deformed Fermi gas obeying interpolating statistics [14], thermal properties of a solid through $q$-deformed algebra [15], $q$-deformed Einstein equations [16], thermosize effects in a $q$-deformed fermion gas model [17]. However, so far, thermostatistics properties of $q$-deformed bosons and fermions have been less studied for two dimensional space in the literature.

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In the view of the above motivations, in this paper, we continue the work of [18] and study the high temperature thermostatistical properties of the $q$-deformed bosons and fermions for two dimensional space. In the work [18], some thermodynamical functions of such $q$-deformed bosons and fermions are investigated. We want to find other thermodynamical properties such as the equation of state as a virial expansion. The paper is organized as follows: In Sec. 2, we give a brief of the quantum algebraic properties concerning with the $q$-deformed algebra of bosons and fermions. In Sec. 3, we investigate some important thermodynamic quantities in the high temperature limit for two dimensional space. In the last Sec., we discuss the effects of fermionic and bosonic $q$-deformation on the thermodynamic functions and give our conclusions.

## 2. DEFORMED BOSON AND FERMION ALGEBRA

The symmetric $q$-deformed algebraic structure of the quantum oscillators is defined by $q$-deformed Heisenberg algebra in terms of creation and annihilation operators $c^*$ and $c$, respectively, and the total number operator $\widehat{N}$ as [6-9,11,18]

$$cc^* - \kappa q^{\kappa} c^* c = q^{-N}$$

$$\left[\widehat{N}, c^*\right] = c^*, \quad \left[\widehat{N}, c\right] = -c, \tag{2.1}$$

where $q$ is the real deformation parameter and the constants $\kappa = 1$ and $\kappa = -1$ are related to $q$-bosons and $q$-fermions, respectively. In addition, the operators obey the following relations [11]

$$c^* c = \left[\widehat{N}\right], \quad c^* c = \left[1 + \kappa \widehat{N}\right]. \tag{2.2}$$

The basic $q$-deformed quantum number is defined as

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}}. \tag{2.3}$$

Moreover, the Jackson derivative (JD) operator for the system is given as

$$D_x^{(q)} f(x) = \frac{1}{x}\left[\frac{f(qx) - f(q^{-1}x)}{q - q^{-1}}\right], \tag{2.4}$$

for any function $f(x)$. This JD operator reduces to the ordinary derivative operator in the limit $q \to 1$ [11].

## 3. THERMOSTATISTICS OF DEFORMED BOSON AND FERMION

In this section, we present the high temperature thermostatistical properties of deformed bosons and fermions in two dimensonal space. Now, we consider non-interacting $q$-deformed gas model constructed by Eqs. (2.1)-(2.3) confined in two dimensional space ($A$). Such non-interacting $q$-deformed gas model has Hamiltonian in the following [11]

$$H = \sum_i (\varepsilon_i - \mu) N_i, \tag{3.5}$$

where $\varepsilon_i$ is the kinetic energy of a particle in the $i$. state and $\mu$ is the chemical potential. In order to investigate the high-temperature properties of the $q$-deformed gas model in two dimensional space, the logarithm of the grand partition function of the model is given as

$$\ln Z = -\kappa \sum_i \ln(1 - \kappa z e^{-\beta \varepsilon_i}), \tag{3.6}$$

where $z = \exp(\mu / k_B T)$ is the fugacity and $\beta = 1/k_B T$. The total number of particles can be obtained by using JD operator in Eq. (2.4) instead of the standard derivative operator. Therefore, it can be found as

$$N^{(\kappa)} = z D_z^{(q)} \ln Z = \sum_i n_i^{(\kappa)}, \tag{3.7}$$

where $n_i^{(\kappa)}$ is the mean occupation number and expressed by the following form

$$n_i^{(\kappa)} = \frac{1}{q - q^{-1}} \ln\left(\frac{z^{-1} e^{\beta \varepsilon_i} - \kappa q^{-\kappa}}{z^{-1} e^{\beta \varepsilon_i} - \kappa q^{\kappa}}\right). \tag{3.8}$$

From the thermodynamic relation $P^{(\kappa)} A / k_B T = \ln Z$ the equation of state can be written as

$$\frac{P^{(\kappa)} A}{k_B T} = -\kappa \sum_i \ln(1 - \kappa z e^{-\beta \varepsilon_i}). \tag{3.9}$$

When the thermodynamic limit is taken into account, for a large area and a large number of particles, the sum of states can be replaced with the integral. Thus, for two dimensional space, the equation of state and the total number of particles can be, respectively, expressed as

$$\frac{P^{(\kappa)}}{k_B T} = \frac{1}{\lambda^2}\int_0^\infty dx \ln(1-\kappa z e^{-x}), \qquad (3.10)$$

$$\frac{N^{(\kappa)}}{A} = \frac{1}{\lambda}\int_0^\infty \frac{dx}{q-q^{-1}} \ln\left(\frac{1-\kappa q^{-\kappa} z e^{-x}}{1-\kappa q^{\kappa} z e^{-x}}\right), \qquad (3.11)$$

where $\lambda = h/(2\pi m k_B T)^{1/2}$ is the thermal wavelength, $x = \beta\varepsilon$ and $\varepsilon = p^2/2m$. Furthermore, these integrals can be expanded with Taylor series for the high temperature limit and defined as

$$\frac{P^{(\kappa)}}{k_B T} = \frac{1}{\lambda^2} h_2^{(\kappa)}(z,q), \qquad (3.12)$$

$$\frac{N^{(\kappa)}}{A} = \frac{1}{\lambda^1} h_1^{(\kappa)}(z,q), \qquad (3.13)$$

where $q$-deformed $h_n^{\kappa}(z,q)$ function is defined as [11]

$$h_n^{\kappa}(z,q) = \frac{1}{\Gamma(n)}\int_0^\infty \frac{x^{n-1}dx}{q-q^{-1}} \ln\left(\frac{1-\kappa q^{-\kappa} z e^{-x}}{1-\kappa q^{\kappa} z e^{-x}}\right)$$

$$= \frac{1}{q-q^{-1}}\left[\sum_{l=1}^{\infty}\frac{(\kappa z \;\; ^{\kappa})^l}{l^{n+1}} - \sum_{l=1}^{\infty}\frac{(\kappa z q^{-\kappa})^l}{l^{n+1}}\right], \qquad (3.14)$$

where $\Gamma(n) = \int_0^\infty x^{n-1}\exp(-x)$ is the Gamma function [19]. Now, we can find other thermodynamic quantities of the $q$-deformed gas model. For example, from the thermodynamic relation $U^{(\kappa)} = -(\partial \ln Z / \partial\beta)_{z,A}$, the internal energy of the model can be calculated for two dimensional space. Then, it can be obtained as

$$U^{(\kappa)} = \frac{k_B T A}{\lambda^2} h_2^{(\kappa)}(z,q). \qquad (3.15)$$

Moreover, the sepecific heat of the model can be found by the thermodynamic relation $C_A^{(\kappa)} = (\partial U^{(\kappa)}/\partial A)_{N,A}$ in the following form

$$\frac{C_A^{(\kappa)}\lambda^2}{k_B T} = 2zD_z^{(q)} h_3^{(\kappa)}(z,q) - \frac{(zD_z^{(q)} h_2^{(\kappa)}(z,q))^2}{zD_z^{(q)} h_1^{(\kappa)}(z,q)} (3.16)$$

By using the thermodynamic relation $F^{(\kappa)} = \mu N^{(\kappa)-} P^{(\kappa)} A$, the Helmholtz free energy can be determined from Eqs. (3.12) and (3.13) as

$$F^{(\kappa)} = N^{(\kappa)} k_B T\left[\ln z - \frac{h_2^{(\kappa)}(z,q)}{h_1^{(\kappa)}(z,q)}\right]. \qquad (3.17)$$

Then, the deformed entropy of the model in two dimensions can be derived from Eqs. (3.15) and (3.17) with the help of the thermodynamic relation $S^{(\kappa)} = (U^{(\kappa)-} F^{(\kappa)})/T$ as

$$\frac{S^{(\kappa)}}{N^{(\kappa)} k_B} = 2\frac{h_2^{(\kappa)}(z,q)}{h_1^{(\kappa)}(z,q)} - \ln z. \qquad (3.18)$$

In order to understand the effects of the bosonic and fermionic $q$-deformation on the $q$-deformed gas model, in Fig. 1-3, we plot the $q$-deformed bosonic and fermionic entropy functions and the undeformed entropy function as a function of $z$ for several values deformation parameter $q$. For both $q < 1$ and $q > 1$ and the same fugacity, the bosonic and fermionic entropy values of the deformed gas model in two dimensional space decrease with the values of the deformation parameter $q$. Also, at the same fugacity, they are lower than the values of the undeformed bosonic and fermionic entropy values in two dimensional space.

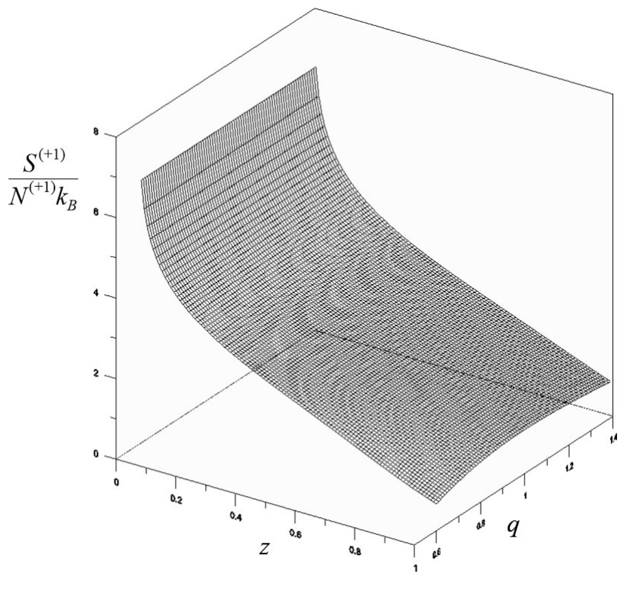


Figure 1. The bosonic entropy function $S^{(+1)}/N^{(+1)}k_B$ with respect to fugacity $z$ and several values of deformation parameter $q$ for boson.

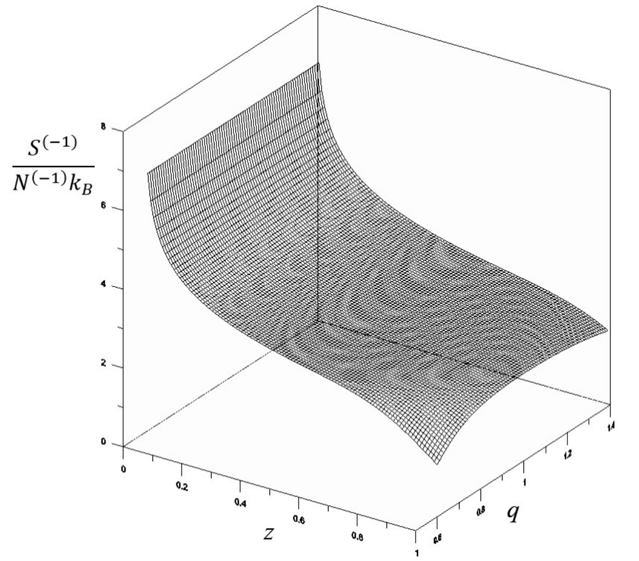


Figure 2. The fermionic entropy function $S^{(-1)}/N^{(-1)}k_B$ with respect to fugacity $z$ and several values of deformation parameter $q$ for fermion.

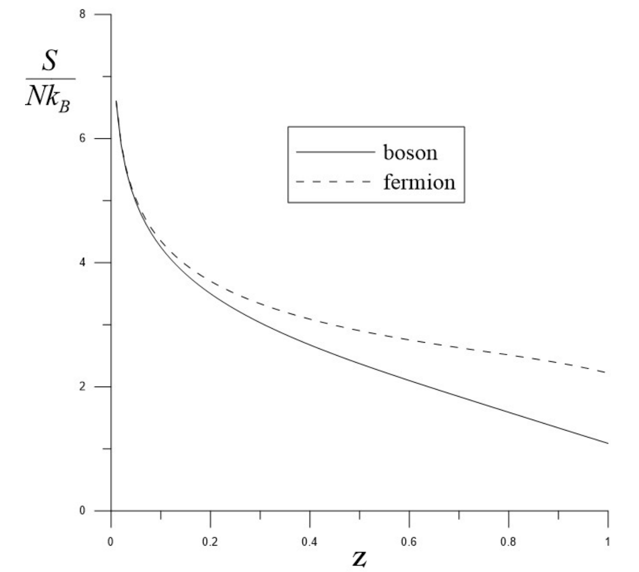


Figure 3. The undeformed bosonic and fermionic entropy function $S/Nk_B$ with respect to fugacity $z$.

On the other hand, the equation of state of the model can be derived from Eqs. (3.12) and (3.13) as a virial expansion for two dimensional space as

$$\frac{P^{(\kappa)}A}{N^{(\kappa)}k_BT} = a_1^{(\kappa,q)} + a_2^{(\kappa,q)}y + a_3^{(\kappa,q)}y^2 + a_4^{(\kappa,q)}y^3 + a_5^{(\kappa,q)}y^4 + \dots \quad (3.19)$$

where $y = (N^{(\kappa)}\lambda^2/A)$ and $a_n^{(\kappa,q)}$ are defined as deformed virial coefficients and the first five of them are found as below

$$a_1^{(\kappa,q)} = 1, \quad (3.20)$$

$$a_2^{(\kappa,q)} = -\kappa\left(\frac{1}{2^3}[2]\right), \quad (3.21)$$

$$a_3^{(\kappa,q)} = -\left(\frac{2}{3}[3] - \frac{1}{2^4}[2]^2\right), \quad (3.22)$$

$$a_4^{(\kappa,q)} = -\kappa\left(\frac{5}{2^7}[2]^3 - \frac{1}{12}[2][3] + \frac{3}{2^6}[4]\right), \quad (3.23)$$

$$a_5^{(\kappa,q)} = -\left(-\frac{7}{2^8}[2]^4 + \frac{1}{12}[2]^2[3] - \frac{1}{2^4}[2][4] - \frac{2}{3^4}[3]^2 + \frac{4}{5^3}[5]\right). \quad (3.24)$$

where $[x]$ $q$-deformed basic number is defined by Eq. (2.3). In Fig. 4-5, we plot these bosonic and fermionic virial coefficients as a function of deformation parameter $q$ in two dimensional space for the case $q < 1$ and $q > 1$. As it is shown in the Fig. 4, for all values of deformation parameter, the bosonic virial coefficients $a_2^{(+1,q)}$, $a_4^{(+1,q)}$ and $a_5^{(+1,q)}$ have negative values although the bosonic virial coefficient $a_3^{(+1,q)}$ has positive values. As it is shown in the Fig. 5, for all values of deformation parameter, the fermionic virial coefficients $a_2^{(-1,q)}$, $a_3^{(-1,q)}$ and $a_4^{(-1,q)}$ have positive values although the fermionic virial coefficient $a_5^{(-1,q)}$ has negative values.

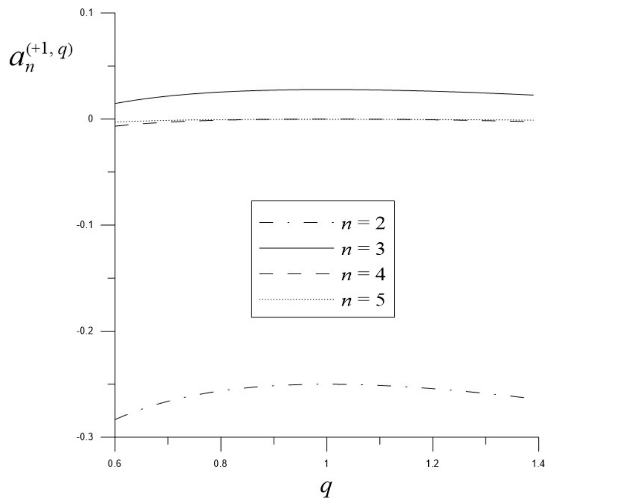


Figure 4. The virial coefficients $a_n^{(+1,q)}$ with respect to several values of deformation parameter $q$.

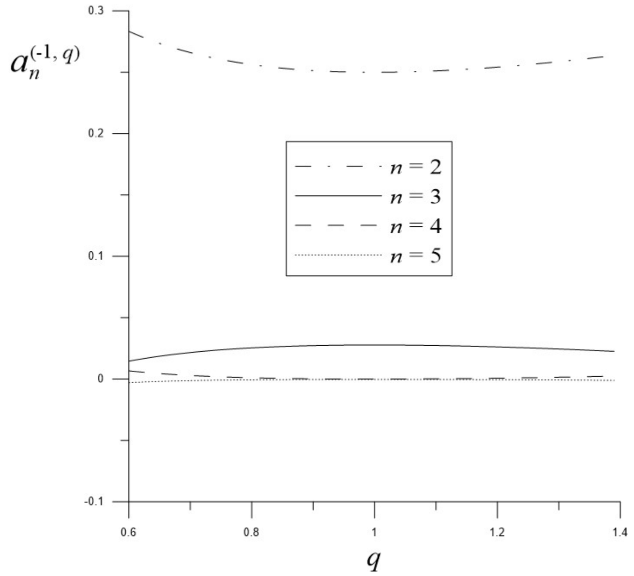


Figure 5. The virial coefficients $a_n^{(-1,q)}$ with respect to several values of deformation parameter $q$.

## 4. CONCLUSION

In this paper, we research the effects of the bosonic and fermionic $q$-deformation on some thermodynamic functions in two dimensional space. For instance, the $q$-deformed entropy function is obtained in Eq. (3.18). In order to compare, in Fig. 1-3, we demonstrate the plots of the $q$-deformed entropy functions and undeformed entropy function for boson and fermion. From these graphics, we can conclude that the values of bosonic and fermionic deformed entropy function decrease when the deformation parameter $q$ is increased. Moreover, all thermodyanamic functions found above reduce to undeformed functions in the limit $q \to 1$. The results obtained in this study may be used to investigate properties of some quantum systems in the microelectronic and material sciences.

On the other hand, in the work [20] Pazy and Argaman examined modified Newton theory by using standard Fermi Dirac statistics in two dimensional space. When used this statistics, interaction between particles don't take into account. Therefore, Modified Newton theory can be studied by using the properties of $q$-deformed fermion in two dimensional instead of standard one and the effects of $q$-deformation can be researched on this theory. Another important application area can be the investigation of the properties of $q$-deformation on thermosize effect on Seebeck-like thermosize effect [17,21]. For example, the Seebeck coefficients of the most of metals are negative but some of them are positive such as copper [22]. Free electron theoy is inadequate to explain the behavior of electoron and hole in metals and semicondutors. For this reason, when applied $q$-deformation on the Seebeck-like thermosize effect, positive and negative Seebeck coefficients of metals and semiconductors may be explained.

The studying of the low temperature thermostatistical properties of the present $q$-deformed model in two dimensions is one of the open problem.

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