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New Heuristics to Stochastic Dynamic Lot Sizing Problem

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ABSTRACT

Simultaneous consideration of both demand and price uncertainties is not studied extensively in the literature. This problem is mathematically intractable for cases where complex problem structure exists. This study proposes new heuristics that consider demand and purchasing price uncertainties simultaneously. When all the costs are constant over time, this is the classical dynamic lot sizing problem for which the optimal solution can be obtained by the Wagner-Whitin algorithm. Purchasing decisions are made on a rolling horizon basis rather than fixed planning horizon. Well-known Least Unit Cost and Silver-Meal algorithms are modified for both time varying purchasing price and rolling horizon. The proposed heuristic is basically based on a cost-benefit evaluation at decision points. A numerical example is explained for showing how heuristics are working in detail. The aim of study is to enlighten about problem that is taken into account.

Key Words: *Inventory; Stochastic lot sizing; Rolling horizon; Simulation.,*

1. INTRODUCTION

Inventory management is one of the important areas where management science has had a significant impact. Systems that need inventory can range from raw materials, spare parts, cash, and finished goods to hotel rooms and airline flight seats. The major decisions in inventory control concern when a replenishment order should be placed and what the quantity of such an order should be. The conventional inventory models can be placed into two categories: deterministic and stochastic. In deterministic models, all input data are assumed deterministic and given, and based upon the known data a model is applied to minimize the total inventory costs.

In stochastic models, a probabilistic distribution of the input data is specified, and a mathematical model is used to minimize the total expected inventory costs. Silver [1] suggested a heuristic procedure for the stochastic lot-sizing problem assuming that the forecast errors are normally distributed.

The rolling horizon approach is commonly applied because forecasts for periods further in the future are likely to be both of poorer quality and more expensive to make. In the rolling horizon procedure, one first solves a finite horizon forecast window (M) period problem but implements only the decision related to the first period. Next period the inventory status is revised, the multi-period problem is updated as better forecasts become available, and the approach continues until the end of the planning horizon [2].

The ''static-dynamic uncertainty'' strategy proposed by Bookbinder and Tan [3] is one approach to work out the parameters of non-stationary (R,S) policies. Bookbinder-Tan's solution heuristic is a two-stage process of firstly fixing the replenishment periods and then secondly determining what adjustments should be made to the planned orders as demand is realized. The total cost, composed of ordering and inventory holding costs, is minimized under a minimal service level constraint.

A formulation of Bookbinder and Tan problem that determines both the replenishment periods and the associated order-up-to-levels simultaneously, hence gives the optimal solution, is presented by Tarim and Kingsman [4]. In addition to the inventory holding and ordering costs of Bookbinder and Tan, Tarim and Kingsman take into account the direct item costs. The expected total cost during the planning horizon is minimized also under a minimal service level constraint.

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Chan et al. [5] consider the problem of determining order quantities of a single product to satisfy known demands over some finite set of future periods with varying cost parameters. When all the costs are constant over time, this is the classical dynamic lot sizing problem for which the optimal solution can be obtained by the Wagner-Whitin (WW) algorithm.

When the ordering, purchasing and holding costs vary over time, the problem becomes more complicated with the possibility of speculation and hedging on the timing of ordering and on-hand inventories. No heuristic algorithm has been developed and the existing heuristics are not readily applicable. Chan et al. develop new heuristic algorithms for the lot sizing problem with time varying cost parameters and without backlogging [5].

2. PROBLEM STATEMENT

This paper deals with the simulation modelling of the multi-period single-item lot sizing problem with stochastic non-stationary demand and price on a rolling horizon basis. This paper is similar to Bookbinder and Tan [3], Tarim and Kingsman [4] papers. Stochastic demand process in our simulation model is based on non-stationary stochastic demand that Tarim and Kingsman [4] proposed. We extend these works by new assumptions. We assumed that purchasing price has uniform distribution. The purchasing price uncertainty is not the only difference from these papers. The simulation model permits lost sale (outsourcing) and backordering. One of the new assumptions is minimum order quantity (R). A new order can not be less minimum order quantity. This paper proposes three new heuristics based on a rolling horizon to this problem. We revise the well-known Silver-Meal (SM) and Least-Unit Cost (LUC) heuristic for varying purchasing price and rolling horizon. So, we get Revised Silver-Meal heuristic (RSM) and Revised Least-Unit Cost (RLUC) heuristics. Last heuristic is a new proposed heuristic that evaluates the cost of giving a new order for the benefit of giving a new order from demand and price expectations at decision points. We call this new heuristic Cost-Benefit (CB) heuristic.

We explain our problem formulation starting with the simplest deterministic case and then extending the formulation with stochastic demand and price. We first briefly describe the deterministic version of problem.

Problem-1 (Deterministic Case)

In this problem, demand and price are known with certainty. Lead-time is zero. There is no permission to backorder. The problem is to determine lot sizing decisions that minimize the total relevant cost (TRC) subject to the demand constraint as the following [3].

$$\text{Minimize TRC} = \sum_{t=1}^{T} \left\{ S.\delta_t + hI_t + vX_t \right\}$$

Subject to, for $t = 1,2,...,T$:

$$I_t = I_0 + \sum_{t=1}^{T} \left(X_t - d_t \right) \tag{2.1}$$

$$X_t, I_t \geq 0 \tag{2.2}$$

$$\delta_t = \begin{Bmatrix} 1 & X_t > 0 \\ 0 & X_t = 0 \end{Bmatrix} \tag{2.3}$$

TRC is equal to the sum of set-up cost (S), holding cost (h) and purchasing cost (v). Equation 1 is the mathematical expression for inventory balance of all periods with no backorders. X_t, lot size in period t. $\delta(X_t)$ is a binary variable that takes the value of 1 if a replenishment order is placed in period t. It is inventory level at the end of period t.

Problem 2 (Stochastic Case)

Bookbinder and Tan [3] formulate the stochastic demand problem as a chance-constrained programming problem. The objective is minimization of the Expected Total Relevant Cost ($E[TRC]$) over a finite number of periods T, subject to inventory service-level constraints. Their original mathematical formulation is as follows:

$$\text{Minimize } E[TRC] = \int_{d_1}\int_{d_2}...\int_{d_T} \sum_{t=1}^{T}\left[vX_t + S.\delta(X_t) + hMax(I_t,0) \right] x g_1(d_1) g_2(d_2)....g_T(d_T) d(d_1) d(d_2)...d(d_T)$$

Subject to, for $t = 1,2,...,T$: $\Pr\{I_t \geq 0\} \geq \alpha,$ (2.4)

$$\delta_t = \begin{Bmatrix} 1 & X_t > 0 \\ 0 & X_t = 0 \end{Bmatrix} \tag{2.5}$$

$$I_t = I_0 + \sum_{t=1}^{T} \left(X_t - d_t \right), \tag{2.6}$$

$$X_t \geq 0 \tag{2.7}$$

Equation 4 states the service-level constraint. The static uncertainty decision rule requires that values of all decision variables be determined at the beginning of the time horizon. Since all X_t , in this strategy are decided at the beginning of period 1 and can be considered constants, the random variables I_t of Problem 2 can be obtained from (6) as

$$I_t = I_0 + \sum_{t=1}^{T} X_t - \sum_{t=1}^{T} d_t \tag{2.8}$$

Applying equation (8) to constraint (4), we have

$$\Pr\left\{ I_0 + \sum_{t=1}^{T} X_t \geq \sum_{t=1}^{T} d_t \right\} \geq \alpha$$

If $G_{d_1+d_2+....+d_t}(y)$ is the cumulative distribution function of $D(t) = d_1 + d_2 + ... + d_t$ then

$$\sum_{t=1}^{T} X_t \geq G^{-1}_{D(t)}(\alpha) - I_0 \qquad (2.9)$$

Problem 3 (Proposed Stochastic Formulation)

Bookbinder and Tan [3] showed the mathematical structure of static dynamic model was the same as the deterministic model in their work. They proposed heuristics to solve this problem like deterministic problem. They used a transformation procedure from stochastic problem to deterministic problem. They proposed a procedure for the probabilistic lot sizing problem in a rolling horizon environment. They ignored the unit variable cost in the determination of lot sizing.

Tarim and Kingsman [4] extend a single-item stochastic lot sizing problem with stochastic demands and service-level constraint with the purpose of determining replenishment quantity without considering the lead-time. Tarim and Kingsman expanded Bookbinder and Tan studies by taking deterministic unit variable cost into account. Tarim and Kingsman used fix planning instead of rolling horizon.

We use same analogy that was used by Bookbinder and Tan with some differences. We assumed that purchasing price (V_t) is non-stationary and stochastic as demand. We use demand ($E(D_t)$) and price expectations ($E(V_t)$) in our simulation decision process. The cost components which we taken purchasing cost, set-up cost (S), holding cost (h), lost sale cost (l) and backordering cost (e) into account. Total relevant cost (TRC) is equal to the sum of these cost components. We use simulation to model this non-stationary stochastic inventory model.

We made certain assumptions for modelling non-stationary stochastic inventory system. Our formulation is more realistic with the following characteristics:

Demand sizes are assumed to be normal random variable, and prices are uniform random variable.

Expected value for demand size and price are required before the lot sizing decisions.

Both stationary and non-stationary demand and price patterns are permitted.

Partial or complete backlogging (or lost sale) is permitted. Order lead time is zero.

We assumed that the demand in each period is normally distributed about the expected value with a constant coefficient variation (CV_d). It is a well accepted assumption in the non-stationary stochastic dynamic lot sizing literature [3, 4]. We assumed that the demand in each period is normally distributed about the forecast value with the same coefficient of variation in our work. The mean rate of demand and price may vary from period to period. Coefficient of variation is constant over time to use cumulative distribution of demand [3]. It is a well-accepted assumption in the literature [3, 4]. We assumed price is uniform random variable for the aim of easy understanding and coefficient of variation is constant over time to use cumulative distribution of price.

Overall, decision-making process undertaken in this study is illustrated in Figure 1. Figure-1 is a flow chart that summarizes associate steps and decisions for all heuristics through planning horizon length (T).

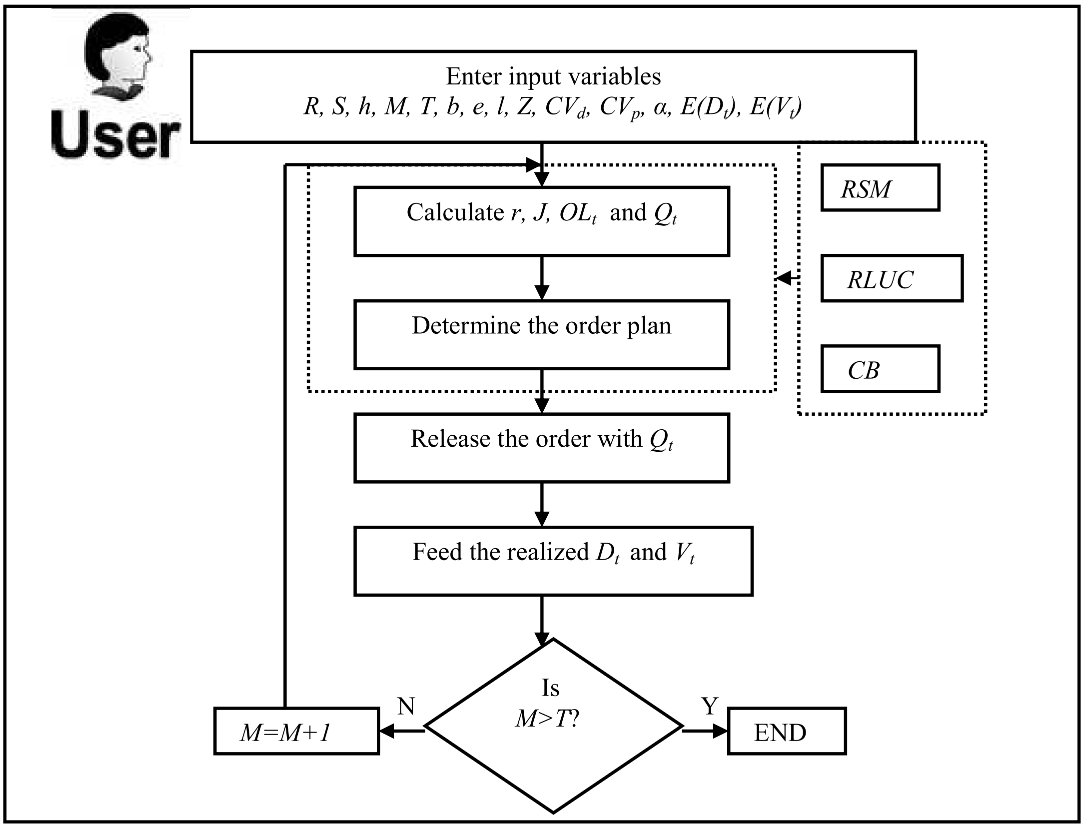

Figure 1. Illustration of decision-making process.

The initial inventory level (I_0) is taken as zero. In first step, user should enter R, S, h, M, T, b, e, l, Z, CV_d, CV_p, $\alpha, E(D_t)$, $E(V_t)$ values. Process aim is to make stock level upto target level, which called order-up level by orders. We calculated order-up level by equation (10). OL_t is order up level in period t. J is order number. Z is standard normal value corresponding to α service level. Backorder ratio (b) is determined by user at the beginning of the planning period. b value is a ratio of backordered demand to unsatisfied demand (Eq.11). Unsatisfied demand (UD_t) quantities are known at the end of the period. The backordered (BO_t) (Eq.12) and lost demand (LD_t) (Eq.13) quantities are calculated by b value.

$$OL_t = \sum_{t=r}^{r+J-1} E[d_t] + Z_\alpha . S_{D(r+J)} = \sum_{t=r}^{r+J-1} E[d_t] + Z_\alpha . CV_d \cdot \left(\sum_{t=r}^{r+J-1} E^2[d_t] \right)^{\frac{1}{2}} \quad (2.10)$$

$$b = \frac{BO_t}{UD_t} \quad (2.11)$$

$$BO_t = b.UD_t \quad (2.12)$$

$$LD_t = UD_t - BO_t \quad (2.13)$$

Calculation of order size Q_t depends on the following cases. The first two cases are common for all three heuristics. In case I, inventory level is enough to satisfy the total demand which covers j period. The order which is given is cancelled. In case II, inventory level is not enough and the system gives order with amount of Q_t. We assumed that the order quantity can not be less than minimum order quantity (R). We must satisfy the total backordered demand (TBO_{t-1}) up to period t. If there is an unsatisfied demand, Case III is valid for *RSM* and *RLUC* while Case IV is used for *CB*. The *CB* heuristic gives the amount of order in Case IV. The $\frac{S}{h}$ ratio, commonly used in lot sizing literature, shows whether total demand of covering periods are ordered as a single or multiple orders. We assume that $\frac{S}{h}$ is order up level in *CB* heuristic. Our intention is upgrade inventory level is up to $\frac{S}{h}$ ratio by new order. This is one of different character of *CB* heuristic from other heuristics. Q_t is always positive because of the inventory level of previous period in case 4 being negative.

Case I : if $I_{t-1} \geq OL_t$ then $Q_t = 0$

Case II : if $I_{t-1} < OL_t$ and $I_{t-1} \geq 0$ then $Q_t = Max((OL_t + TBO_{t-1} - I_{t-1}), R)$

Case III: if $I_{t-1} < OL_t$ and $I_{t-1} < 0$ then $Q_t = Max((OL_t + TBO_{t-1}), R)$

Case IV: if $I_{t-1} < OL_t$ and $I_{t-1} < 0$ then $Q_t = \left(\frac{S}{h} - I_{t-1}\right)$

These order quantities covering a forecast window (*M*) form an order plan. All heuristics may revise the order plan on a rolling horizon basis as actual demand size and prices are fed into the system until the end of planning horizon (*T*) is reached.

3. HEURISTICS

In this section of the study revisions which were carried out in order to use *SM*, *LUC* in the conditions where demand and price are uncertain. First, we revise the well-known *SM* heuristic for varying purchasing price and rolling horizon. *SM* heuristic seeks the minimum Total Relevant Cost per Unit Time *(TRCUT)* as shown in equation (14). We state different notations in order not to confuse formulation for stochastic case with formulations for deterministic case. We use *t* notation for stochastic case and *i* notation for deterministic case. Equation-14, 15, 16 and 17 are for deterministic cases. Equation 14 shows the calculation of total relevant cost per unit time that determines whether an order is given or not in *SM* heuristic. Similarly, equation 15 shows the calculation of total relevant cost per unit quantity. We also revise *LUC* heuristic in the same way as *SM*. In *LUC*, whether to include demand of time period *t* is determined by Total Relevant Cost per Unit Quantity (*TRCUQ*) as shown in equation (15). *N* shows the number of periods of order which X_i covers. The order size for deterministic case is X_i calculated by equation (16). Equation 17 shows the calculation of total cost for time varying price in deterministic case.

$$TRCUT_i = \frac{TRC_i}{i} = \frac{S + h\sum_{i=2}^{N}(i-1)D_i}{i} \tag{3.1}$$

$$TRCUQ_i = \frac{TRC_i}{\sum_{i=1}^{N} D_i} = \frac{S + h\sum_{i=2}^{N}(i-1)D_i}{\sum_{i=1}^{N} D_i} \tag{3.2}$$

$$X_i = \sum_{i=1}^{N} D_i \tag{3.3}$$

Extending TRC_i to include time-varying deterministic purchasing price and holding cost results in the following equation:

$$TRC_i = S + h\sum_{i=2}^{N}(i-1)D_i + V_i X_i \tag{3.4}$$

CB, *RLUC* and *RSM* heuristics take the simultaneous consideration of both demand and price uncertainties into account. Heuristics use expected demand and price of periods for calculating expected TRC_t. Expected TRC_t which includes time varying stochastic purchasing price and holding cost is presented by equation 18. Equation 18 states this case.

$$E(TRC_t) = S + h\sum_{t=2}^{N}(t-1)E(D_t) + E(V_t)Q_t \tag{3.5}$$

TRC_t of *CB* is the same as *RSM* and *RLUC*. *TB* is the benefit cost (no set-up cost and lower purchase cost) of combining ordering period and trial period demands instead of giving a new order in trial period. *TL* is the lost cost (higher holding cost and purchase cost) of combining ordering period and trial period demands instead of giving a new order in trial period. The rationale behind *CB* heuristic is evaluating cost with the benefit of combining trial period demand. We compare *TB* with *TL* for *CB* heuristic. *CB* heuristic determines order plan by these comparisons.

ACP is the value of total relevant cost per unit time in the ordering period, *($TRCUT_r$)*. *NCP* is the value of total relevant cost per unit time in the trial period *($TRCUT_u$)*. *ACD* is the value of total relevant cost per unit quantity in the ordering period, *($TRCUQ_r$)*. *NCP* is the value of total relevant cost per unit quantity in the trial period *($TRCUQ_u$)*. The rationale behind these heuristics is evaluating cost for the benefit of combining trial period demand. We compare *ACP* with *NCP* for *RSM* heuristic and *ACD* with *NCD* for *RLUC* heuristic. If first notation is greater than second notation then trial period demand is added to total demand and no order is given, else a new order is given with the amount of total demand until this period. Associate procedure of heuristics is given in Table 1.

Table 1. Associate procedure for heuristics.

Step 1:	Determine *R, S, h, M, b, e, l, Z*, CV_d, CV_p, $E(D_t)$, $E(V_t)$. Go to *Step 2* for *RSM*. Go to *Step 3* for *RLUC*. Go to *Step 4* for *CB*.
Step 2:	Calculate *ACP, NCP*. If $ACP > NCP$ then go to *Step 5* else go to Step *6*.
Step 3:	Calculate *ACD, NCD*. If $ACD > NCD$ then go to *Step 5 else* go to *Step 6*.
Step 4:	Calculate *TB, TL*. If $TB > TL$ then go to *Step 5* else go to Step *6*.
Step 5:	Assign $J = 0$ and $u = u + 1$. Go to *Step 7*.
Step 6:	Assign $t = r$. Determine $J = u - r$. Assign $r = u$. Go back Step *2* for RSM*,* Step *3* for *RLUC and Step 4* for *CB*.
Step 7:	If $J > 0$ then go to Step 8, *else Step 9*.
Step 8:	Calculate OL_t. If $I_{t-1} > OL_t$ then go to *Step 9* else go to *Step 12*.
Step 9:	Assign $Q_t = 0$. No order is given.
Step 10:	If $I_{t-1} < 0$ then go to *Step* 11 for *RSM* and *RLUC,* go to *Step* 12. Else go to *Step 13*.
Step 11:	Give the order which amount is $Q_t = Max\left(\left(OL_t + TBO_{t-1}\right), R\right)$ then go to *Step 14*.
Step 12:	Give the order which amount is $Q_t = \left(\dfrac{S}{h} - I_{t-1}\right)$ and go to *Step 14*.
Step 13:	Give the order which amount is $Q_t = Max\left(\left(OL_t + TBO_{t-1} - I_{t-1}\right), R\right)$.
Step 14:	Determine V_t, D_t and I_t. Calculate *TRC*.
Step 15:	If planning horizon is over then stop. Else go to *Step 1*6
Step 16:	If forecast window is over then determine new period's expectation. Then assign *r=r+1* and go to step *2* for *RSM* or *Step 3* for *RLUC*. Else assign *t=t+1* and go to s*tep 2* for *RSM*, *step 3* for *RLUC*, *step 4* for *CB*.

4. NUMERICAL EXAMPLE

Parameters considered in the example are shown in Table 2. The initial inventory level is taken as zero. It is assumed that the demand in each period is normally distributed about the expected value with $CV_d = 0.33$. The price in each period is uniformly distributed about the expected value with $CV_p = 0.33$. *b=0.5* means, *50 %* percent of unsatisfied demand are backordered and *50 %* percent of unsatisfied demand are lost. Expectations of demands and prices for periods are shown in Table 3. The rolling procedure and *RSM* heuristic solution of example are shown in Figure 2. Realized data of applications of heuristics are shown in Table 4, 5 and 6. *RSM* heuristic solved the given example by 4 orders and total cost of 20,096 currency unit. *RSM* heuristic is less sensitive than *CB*. For that reason *RSM* gives order in periods when prices are high. The rolling procedure and *RLUC* heuristic solution of example is shown in Figure 3. *RLUC* heuristic solved example as 5 orders and total cost of 15,180 currency unit. The rolling procedure and *CB* heuristic solution to the example is as shown in Figure 4.

Table 2. Parameters considered in the example.

CV_d	CV_p	T	M	S	h	b	R	α	e
0.33	0.33	10	5	400	1	0.5	100	90	20

Table 3. Expectations of demands and prices for periods.

Period	1	2	3	4	5	6	7	8	9	10
$E(d_t)$	150	134	129	148	125	158	155	165	137	177
$E(V_t)$	10	9	8	9	8	12	9	10	11	10

t=	1*	2	3	4*	5					
E(d_t)	150	134	129	148	125					
E(V_t)	10	9	8	9	8					

OL_1=515 Q_1=515

t=	1	2	3	4*	5	6*				
E(d_t)	150	134	129	148	125	158				
E(V_t)	10	9	8	9	8	12				

OL_4=370 Q_4=224

t=	1	2	3	4*	5	6*	7			
E(d_t)	150	134	129	148	125	158	155			
E(V_t)	10	9	8	9	8	12	9			

OL_6=407 Q_6=323

t=	1	2	3	4*	5	6*	7	8*		
E(d_t)	150	134	129	148	125	158	155	165		
E(V_t)	10	9	8	9	8	12	9	10		

t=	1	2	3	4	5	6*	7	8*	9	
E(d_t)	150	134	129	148	125	158	155	165	137	
E(V_t)	10	9	8	9	8	12	9	10	11	

t=	1*	2	3	4*	5	6*	7	8*	9	10
E(d_t)	150	134	129	148	125	158	155	165	137	177
E(V_t)	10	9	8	9	8	12	9	10	11	10

* ordering periods

OL_8=598 Q_8=558

Figure 2. The rolling procedure and *RSM* heuristic solution.

Table 4. Realized data of application *RSM* heuristic.

t=	1*	2	3	4*	5	6*	7	8*	9	10
d_t	109	91	169	161	125	170	197	210	26	212
v_t	6	7	7	8	9	9	4	15	8	5
I_t	406	315	146	209	84	237	40	388	362	150
TC_t	3896	4211	4357	6758	6842	10386	10426	19584	19946	20096

t=	1*	2	3*	4	5					
$E(d_t)$	150	134	129	148	125					
$E(V_t)$	10	9	8	9	8					

$OL_1=370$ $Q_1=370$

t=	1	2*	3	4	5*	6				
$E(d_t)$	150	134	129	148	125	158				
$E(V_t)$	10	9	8	9	8	12				

$OL_2=512$ $Q_2=251$

t=	1	2	3	4	5*	6	7*			
$E(d_t)$	150	134	129	148	125	158	155			
$E(V_t)$	10	9	8	9	8	12	9			

$OL_5=369$ $Q_5=278$

t=	1	2	3	4	5*	6	7*	8		
$E(d_t)$	150	134	129	148	125	158	155	165		
$E(V_t)$	10	9	8	9	8	12	9	10		

t=	1	2	3	4	5	6	7*	8	9*	
$E(d_t)$	150	134	129	148	125	158	155	165	137	
$E(V_t)$	10	9	8	9	8	12	9	10	11	

$OL_7=417$ $Q_7=343$

t=	1*	2*	3	4	5*	6	7*	8	9*	10
$E(d_t)$	150	134	129	148	125	158	155	165	137	177
$E(V_t)$	10	9	8	9	8	12	9	10	11	10

* ordering periods

$OL_9=410$ $Q_9=400$

Figure 3. The rolling procedure and *RLUC* heuristic solution.

Table 5. Realized data of application *RLUC* heuristic.

t=	1*	2*	3	4	5*	6	7*	8	9*	10
d_t	109	91	169	161	125	170	197	210	26	212
v_t	6	7	7	8	9	9	4	15	8	5
I_t	261	421	252	91	244	74	220	10	384	172
TC_t	2881	5459	5711	5802	8948	9022	11014	11024	15008	15180

Firstly, *M* period's data are taken into account. *CB* solves this problem. According to *CB* solution, *CB* plans to give order in the first and third periods. While solution is put into practice, the amount of the order is given as 370 units. After this, expected values of 6^{th} period are taken into account. *CB* avoids giving order in the third period dynamically because of the cost. *CB* order plan changes dynamically while the plan is rolling. Finally, *CB* gives four orders in order 1^{st}, 2^{nd}, 3^{rd} and 7^{th} periods. *CB* heuristic solved example as total cost of 13,320 currency unit. Randomly generated demands and prices are known, T-period realized-demand and price problem can be solved using *WW* to obtain a lower bound on the solution. We calculate the deviation from lower bound of heuristics by equation (4.1)

$$\text{Deviation\%} = \frac{TRC\left(Heuristic\right) - LB}{LB} \qquad (4.1)$$

t=	1*	2	3*	4	5					
$E(d_t)$	150	134	129	148	125					
$E(V_t)$	10	9	8	9	8					

$OL_1=370$ $Q_1=370$

t=	1	2*	3	4*	5	6				
$E(d_t)$	150	134	129	148	125	158				
$E(V_t)$	10	9	8	9	8	12				

$OL_2=342$ $Q_2=R=100$

t=	1	2	3*	4	5	6	7*			
$E(d_t)$	150	134	129	148	125	158	155			
$E(V_t)$	10	9	8	9	8	12	9			

$OL_3=680$ $Q_3=410$

t=	1	2	3	4	5	6	7*	8		
$E(d_t)$	150	134	129	148	125	158	155	165		
$E(V_t)$	10	9	8	9	8	12	9	10		

t=	1	2	3	4	5	6	7*	8	9	
$E(d_t)$	150	134	129	148	125	158	155	165	137	
$E(V_t)$	10	9	8	9	8	12	9	10	11	

* ordering periods $OL_7=770$ $Q_3=715$

t=	1*	2*	3*	4	5	6	7*	8	9	10
$E(d_t)$	150	134	129	148	125	158	155	165	137	177
$E(V_t)$	10	9	8	9	8	12	9	10	11	10

Figure 4. The rolling procedure and *CB* heuristic solution.

Table 6. Realized data of application *CB* heuristic.

t=	1*	2*	3*	4	5	6	7*	8	9	10
d_t	109	91	169	161	125	170	197	210	26	212
v_t	6	7	7	8	9	9	4	15	8	5
I_t	261	270	511	350	225	55	573	363	337	125
TC_t	2881	4251	8032	8382	8607	8662	12495	12858	13195	13320

Table 7. Lower Bound (*WW*) solution.

t=	1*	2	3*	4	5	6	7*	8	9	10*
d_t	109	91	169	161	125	170	197	210	26	212
v_t	6	7	7	8	9	9	4	15	8	5
I_t	91	0	456	295	170	0	236	26	0	0
TC_t	1691	1691	6922	7217	7387	7387	9755	9781	9781	11241
Q_t=	200		625				433			212

Realized data of applications of *WW* is shown in Table 7. The lower bound on the total cost based on *WW* solution for this example is 11,241 currency units. The *CB* heuristics has the best performance in terms of percent deviation from the lower bound (%18.49) while the *RSM* has the worst performance with %78.77 deviation.

5. CONCLUSIONS

This paper addresses lot-sizing problem in the system of the multi-period and single-item inventory with stochastic demand and price under rolling horizon. The demand does not have to be satisfied in all periods. It is assumed that the demand in a period can be backlogged. New heuristics for dynamically making lot sizing decisions on a rolling horizon basis under both demand and price uncertainties are proposed. We explain our problem formulation starting with the simplest case and then extending the formulation with stochastic demand and price. Proposed heuristics are explained in detail. A detailed numerical example is given.

There are some places that can be analyzed a number of directions for future research. Looking back at our model formulations, further research may be warranted concerning the calculation of lower bound, the probability distributions of demand and price. Lower bound would be calculated by stochastic linear programming. Different distributions can be tested for demand and price. In addition, it is possible to extend the current heuristics for non-zero replenishment lead-time.

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