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An Analysis for Losses and Confinement Factors for the Regions of a Semiconductor Single Asymmetric Step-Index Laser in Terms of Normalized Propagation Constants for Even and Odd Fields

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ABSTRACT

A requested quantity of the laser can be obtained in terms of normalized propagation constant, which is represented by alpha, belonging to active region. As a new computation procedure this alpha method based on structural property of material contains the computing of the requested quantity for the semiconductor laser theoretically when width of the active region, refractive indices of the regions and wave length are given. In this work, the loss and absorption constants or confinement factors have been analyzed in terms of normalized propagation constants for even and odd fields in a semiconductor single asymmetric step-index laser. Some important parameters, such as propagation constants, effective index of active region, phase constant, phase velocity, absorption constants or confinement factors of the regions, percent of device loss and percent of active region loss, coordinate variables η, ζ for energy eigenvalues for charged carriers in the perpendicular coordinate system $\zeta-\eta$ for single asymmetric step-index laser have been obtained. The validities of found formulas have been tested, numerically. Since effective refractive index belong to active region of the laser is constant, the phase constant and the phase velocity are also constant for each of even and odd fields

Key Words: *Absorption coefficient, Confinement factor*

1. INTRODUCTION

In this work the analysis for a semiconductor single step-index laser (SCSSIL) in terms of normalized propagation constant (NPC) α is presented [1]. The SCSSIL, as shown in Figure 1, consists of three regions. The region II, which is called active region (AR), has thickness 2*a*, refractive index n_{II}, and the regions I and III, which are called cladding layers (CLs), have refractive indices n_I and n_{III}, respectively. Generally, if it is taken as $n_{II} \rangle n_I \rangle n_{III}$ and $n_{II} \rangle n_I$, $n_{III} = n_I = n_{I,III}$, then we have a semiconductor single asymmetric step-index laser (SCSASIL) and semiconductor single symmetric step-index laser (SCSSSIL), respectively [2]. Referring to Figure 1, the SCSASIL is a representative of many single semiconductor laser structures.

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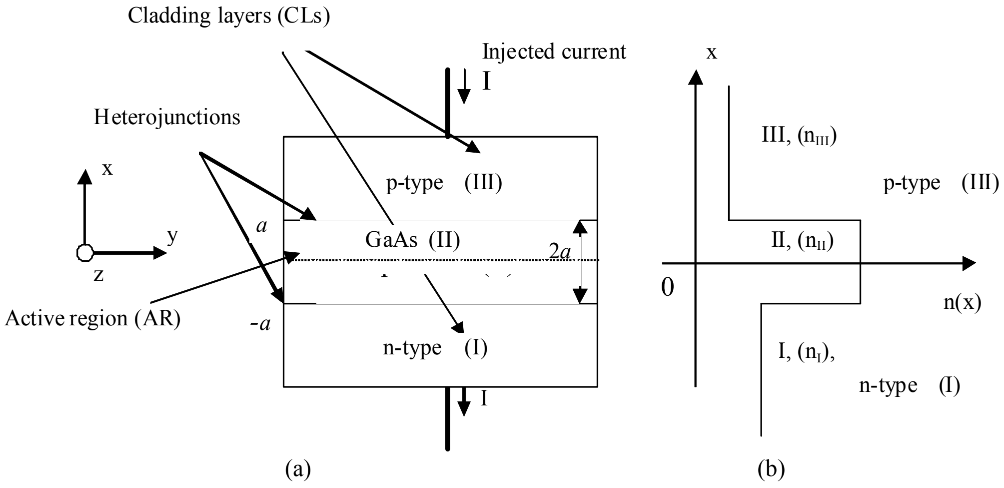

Figure 1 (a) The structure of a SCSASIL (b) Refractive index profile of a SCSASIL.

The SCSASIL is formed by two junctions between two dissimilar materials, such as GaAs (Gallium Arsenide) and p-type or n-type $Al_xGa_{1-x}As$ Aluminium Gallium Arsenide), with x being the fraction of aluminium atoms being replaced by gallium atoms. GaAs and AlAs semiconductors have almost identical lattice constant [3,4].

The operational functions of the SCSASIL are strongly affected by materials used. The CLs of the SCSASIL act as confining the light closely around the AR through action of the total internal reflection with the reflection constant $r_t = e^{j2\zeta}$, as shown in Figure 2 [3]. The requirement is that all upward propagating plane waves in the SCSASIL must be precisely in phase so that the same is true for all downward propagating waves. This condition will be satisfied if the net transverse phase shift of the wave over this path is an integer multiple of 2π. Consider a starting electric field wave with amplitude value E_u, and its amplitude value E_d at the end of a complete round trip. That is, the amplitude E_u of field wave to upward is equal to the amplitude E_d of field wave to down according to transverse resonant condition*. With the well controlled waveguiding to confine the light to the AR, the electrons and holes recombine most strongly. The AR provides a stable platform for the electronic interaction with the changes in optical power [5].

In this work, it has been emphasized the important of normalized propagation constant (NPC) α in the analysis for the SCSASIL. This analysis of the SCSASIL has been based on the NPC α.

For each of the regions I, II and III, the one-dimensional wave equation of electric field [1] is given by

$$\left[\frac{\partial^2}{\partial x^2}+n_i^2(x)k_o^2-\beta_z^2\right]F_{yi}(x)=0,\ \text{i=I, II, III} \tag{1}$$

where $n_i(x)$, k_o, β_z and $F_i(x)$ are respectively index of refraction, free space wave number and phase factor and transverse electric field phasor given in the time-dependent electric field $e(x,z)=F_i(x)\exp[j(\omega t-\beta z)]$ with time-harmonic dependence of the type $e^{j\omega t}$, in i[th] region, i=I, II or III [6].

Energy states of the charged particles such as electrons or holes are described by $F_i(x)=E_i(x)$ for even field and $F_i(x)=e_i(x)$ for odd field. $E_i(x)$ and $e_i(x)$ are given by

$$E_{yI} = A_I\exp[\alpha_I(x+a)], \tag{2}$$

$$E_{yII} = A\cos\alpha_{II}x, \tag{3}$$

$$E_{yIII} = A_{III}\exp[-\alpha_{III}(x-a)] \tag{4}$$

and

$$e_{yI} = B_I\exp[\alpha'_I(x+a)], \tag{5}$$

$$e_{yII} = B\sin\alpha'_{II}x, \tag{7}$$

$$e_{yIII} = B_{III}\exp[-\alpha'_{III}(x-a)] \tag{8}$$

where α_i is propagation constant (PC) of i[th] region, i=I, II or III. They are given by

* Transverse resonant condition is given by $4\varphi-4\alpha_{II}a=2m\pi$, m = 0, 1, 2, 3,... (m is mode number). It can be shown that $E_u=E_d$ by transverse resonant condition: When the field $E_{yII}=E_ue^{-j\alpha_{II}x}e^{-j\beta_z z}$ arrives at point A (x=-2a) in Fig.2, if $\beta_z z=2\varphi$ then the field E_{yII} becomes $E_{yIIA}=E_ue^{j\alpha_{II}2a}e^{-j2\varphi}$, which has the amplitude E_u. This field arrives at point B, it has the amplitude E_d and the phase $e^{j\alpha_{II}2a}e^{-j2\varphi}$ and so this field becomes $E_{yIIB}=E_de^{j\alpha_{II}2a}e^{-j2\varphi}e^{j\alpha_{II}2a}e^{-j2\varphi}=E_de^{j\alpha_{II}4a}e^{-j4\varphi}=E_d$.
by means of transverse resonant condition.

$$\alpha_{II} = \sqrt{(\frac{\omega n_{II}}{c})^2 - \beta_z^2}\ , \tag{9}$$

$$\alpha_I = \sqrt{\beta_z^2 - (\frac{\omega n_I}{c})^2}\ , \tag{10}$$

$$\alpha_{III} = \sqrt{\beta_z^2 - (\frac{\omega n_{III}}{c})^2} \tag{11}$$

for the SCSASIL for even fields. E_{yI}, E_{yIII}, e_{yI} and e_{yIII} are called evanescent fields which have a role in confining of charged carriers to the AR.

The field probabilities of the AR for the even and odd fields are normalized to 1 by the expressions

$$I_{II} = \int_{-a}^{a} \left|E_{yII}(x)\right|^2 dx = 1\ , \tag{12}$$

$$I'_{II} = \int_{-a}^{a} \left|e_{yII}(x)\right|^2 dx = 1 \tag{13}$$

giving the amplitudes (Ams)

$$A = \sqrt{\frac{2\alpha_{II}}{2\zeta + \sin 2\zeta}} \tag{14}$$

$$B = \sqrt{\frac{2\alpha'_{II}}{2\zeta' - \sin 2\zeta'}} \tag{15}$$

where exponential sign $(')$ represents the quantity belonging to odd field[1,2,3].

The special variables ζ, η for even field and ζ', η' for odd field are respectively given by $\zeta = \alpha_{II} a = V\cos\zeta$, $\eta = \eta_I = V\sin\zeta$ and $\zeta' = \alpha_{II}' a = V\sin\zeta$, $\eta' = \eta_I' = \alpha_I' a = V\cos\zeta$. These parameters satisfy the eigenvalue equations [1,2,3] $\tan\zeta = \eta/\zeta$, $\cot\zeta = \eta'/\zeta'$ of the waveguide for even and odd fields, respectively. $\zeta \to \zeta + 2\pi k$, $\eta \to \eta + 2\pi k$ and $\zeta' \to \zeta' + 2\pi k$, $\eta' \to \eta' + 2\pi k$, k=0, 1, 2, 3,… are valid for solution (see Figure 2).

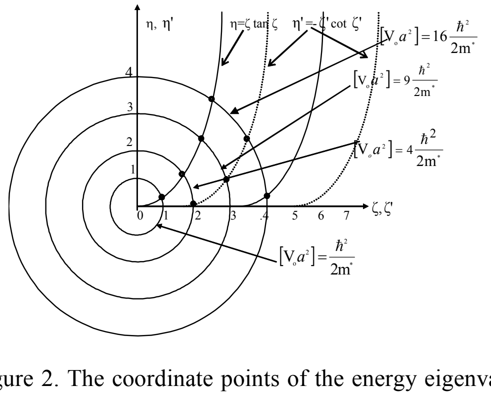

Figure 2. The coordinate points of the energy eigenvalue of the charged carriers in the normalized coordinate system ζ - η in the SCSASIL (dotted lines belong to the odd field).

V is called normalized frequency (NF) [1,2] and is given by $V_a = \frac{1}{\sqrt{1-\alpha}}[m\pi + \arctan\sqrt{\frac{\alpha + a_p}{1-\alpha}}]$, m=0, 1, 2,…. in a SCSASIL [7] and $V = \sqrt{\zeta^2 + \eta^2} = \sqrt{\zeta'^2 + \eta'^2} = a k_o NA$ in a SCSSSIL for a_p=0 ($n_{III} = n_I = n_{I,III}$). NA is numeric aperture as $NA = \sqrt{n_{II}^2 - n_{I,III}^2}$ in a SCSSSIL [1,2]. a_p is here asymmetric factor (AF) [8]. The phase constant (PhC) β_z in a SCSASIL is found by

$$\beta_z = k_{II}\sqrt{1 - (\frac{\alpha_{II}}{k_{II}})^2} = k_o\sqrt{n_{II}^2 - (1-\alpha)NA^2}\ . \tag{16}$$

The effective refractive index (ERI) of the AR, the PhC and phase velocity (PV) are respectively given by $n_{ef} = n_{II}\sqrt{1 - 2\Delta(1-\alpha)}$, $\beta_z = k_o n_{ef}$ and velocity $v = c/n_{ef}$ [3]. Here $\Delta = (n_{II}^2 - n_I^2)/2n_{II}^2$ is called normalized refractive index difference (NID). Cut off frequency (CF) f_c for even field and f'_c for odd field is obtained from $V = n\pi$, n=0, 1, 2, 3,… . [5].

2. PROPAGATION CONSTANTS IN THE SCSASIL AND SCSSSIL

The PCs α_I, α_{II}, α_{III} and α'_I, α'_{II}, α'_{III} and abscissa ζ, ordinates η, η_{III} and η', η'_{III} in the regions I, II and III of the SCSIL are formulated by

$$\alpha_I = k_o NA\sqrt{\alpha}\ , \tag{17}$$

$$\alpha_{II} = k_o NA\ \sqrt{(1-\alpha)}\ , \tag{18}$$

$$\alpha_{III} = k_o NA\sqrt{(1+a_p) - (1-\alpha)}\ , \tag{19}$$

$$\zeta = a k_o NA\ \sqrt{(1-\alpha)}\ , \tag{20}$$

$$\eta_I = \eta = a k_o NA\sqrt{\alpha}\ , \tag{21}$$

$$\eta_{III} = a k_o NA\sqrt{(1+a_p) - (1-\alpha)} \tag{22}$$

for even field and

$$\alpha'_I = k_o NA\sqrt{1-\alpha}\ , \tag{23}$$

$$\alpha'_{II} = k_o NA\ \sqrt{\alpha}\ , \tag{24}$$

$$\alpha'_{III} = k_o NA\sqrt{(1+a_p) - \alpha}\ , \tag{25}$$

$$\zeta' = a k_o NA\ \sqrt{\alpha}\ , \tag{26}$$

$$\eta'_I = a k_o NA\sqrt{1-\alpha}\ , \tag{27}$$

$$\eta'_{III} = a k_o NA\sqrt{(1+a_p) - \alpha} \tag{28}$$

for odd field in a SCSASIL.

3. FIELD PROBABILITY RATIOS IN THE SCSASIL AND SCSSSIL

The percent $\overline{R}_a$ of the AR loss (PARL) for even field [$\overline{r}_a$ for odd field] in the region II in a SCSASIL is given by $\overline{R}_a = I_\ell / I_{II}$ [$\overline{r}_a = I'_\ell / I'_{II}$], where $I_\ell = I_I + I_{III}$, $I'_\ell = I'_I + I'_{III}$ are the total of lost fields, as shown in Figure 3. Here I_{II} and I_ℓ are respectively field probabilities in AR and CLs. For even field the ratio of total evanescent field probability [1,2,3] to the AR field is given by

$$\overline{R}_a = \frac{I_\ell}{I_{II}} = \frac{\int_{-\infty}^{-a} \left|E_{yI}(x)\right|^2 dx + \int_{a}^{\infty} \left|E_{yIII}(x)\right|^2 dx}{\int_{-a}^{a} \left|E_{yII}(x)\right|^2 dx} = A^2 \cos^2\zeta\left(\frac{1}{2\alpha_I} + \frac{1}{2\alpha_{III}}\right) \tag{29}$$

and for odd field

$$\overline{r}_a = \frac{I'_\ell}{I'_{II}} = \frac{\int_{-\infty}^{-a} \left|e_{yI}(x)\right|^2 dx + \int_{a}^{\infty} \left|e_{yIII}(x)\right|^2 dx}{\int_{-a}^{a} \left|e_{yII}(x)\right|^2 dx} = B^2 \sin^2\zeta\left(\frac{1}{2\alpha'_I} + \frac{1}{2\alpha'_{III}}\right) \tag{30}$$

in a SCSASIL [1,2,3,7].

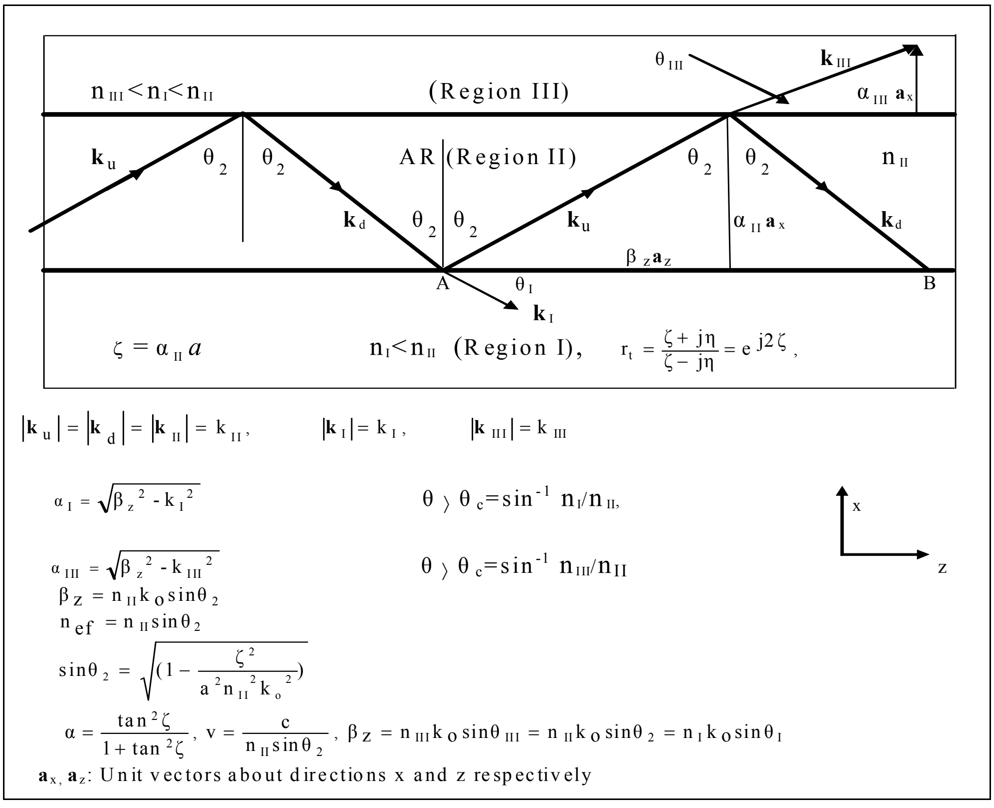

Figure 2 The regions of the SCSASIL confining the light closely around the AR through action of the total internal reflection.

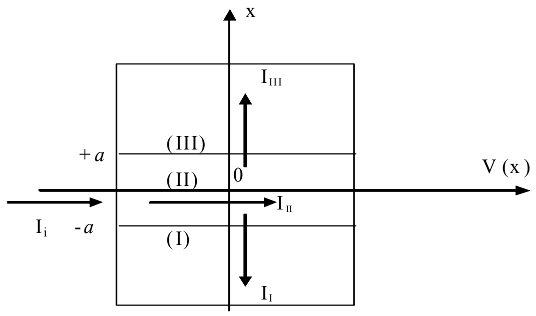

Figure 3. The field probabilities

Denoting input probability with $I_i = I_{II} + I_\ell$, $I'_i = I'_{II} + I'_\ell$, the percent of device loss (PDL) $\overline{K}_a$ and $\overline{q}_a$ as total evanescent field probabilities to the input probabilities I_ℓ / I_i and I'_ℓ / I'_i, for even and odd fields in a SCSASIL [1,2,3,7] are respectively defined as

$$\frac{I_\ell}{I_i} = \overline{K}_a = \frac{\int_{-\infty}^{-a} |E_{yI}(x)|^2 dx + \int_{a}^{\infty} |E_{yIII}(x)|^2 dx}{1 + \int_{-\infty}^{-a} |E_{yI}(x)|^2 dx + \int_{a}^{\infty} |E_{yIII}(x)|^2 dx}$$

$$= \frac{1}{1 + \dfrac{1}{\dfrac{2\zeta(1-\alpha)}{2\zeta + \sin 2\zeta}\left(\dfrac{1}{2\eta_I} + \dfrac{1}{2\eta_{III}}\right)}} \tag{31}$$

$$\frac{I'_\ell}{I'_i} = \overline{q}_a = \frac{\int_{\infty}^{-a} |e_{yI}(x)|^2 dx + \int_{a}^{\infty} |e_{yIII}(x)|^2 dx}{1 + \int_{\infty}^{-a} |e_{yI}(x)|^2 dx + \int_{a}^{\infty} |e_{yIII}(x)|^2 dx}$$

$$= \frac{1}{1 + \dfrac{1}{\dfrac{2\zeta\alpha}{2\zeta - \sin 2\zeta}\left(\dfrac{1}{2\eta'} + \dfrac{1}{2\eta'_{III}}\right)}} \,. \tag{32}$$

Absorption constants of the regions I, II and III of a SCSASIL for even field are given [2] by

$$\frac{I_I}{I_i} = F_I = \frac{\int_{-\infty}^{-a} |E_{yI}(x)|^2 dx}{1 + \int_{-\infty}^{-a} |E_{yI}(x)|^2 dx + \int_{a}^{\infty} |E_{yIII}(x)|^2 dx} ,$$

$$= \frac{A^2 \cos^2\zeta}{2\alpha_I [1 + A^2 \cos^2\zeta (\dfrac{1}{2\alpha_I} + \dfrac{1}{2\alpha_{III}})]} \tag{33}$$

$$\frac{I_{II}}{I_i} = F_{II} = \frac{1}{1 + \int_{-\infty}^{-a} |E_{yI}(x)|^2 dx + \int_{a}^{\infty} |E_{yIII}(x)|^2 dx} ,$$

$$= \frac{1}{[1 + A^2 \cos^2\zeta (\dfrac{1}{2\alpha_I} + \dfrac{1}{2\alpha_{III}})]} \tag{34}$$

$$\frac{I_{III}}{I_i} = F_{III} = \frac{\int_{a}^{\infty} |E_{yIII}(x)|^2 dx}{1 + \int_{-\infty}^{-a} |E_{yI}(x)|^2 dx + \int_{a}^{\infty} |E_{yIII}(x)|^2 dx}$$

$$= \frac{A^2 \cos^2\zeta}{2\alpha_{III} [1 + A^2 \cos^2\zeta (\dfrac{1}{2\alpha_I} + \dfrac{1}{2\alpha_{III}})]} \tag{35}$$

and for odd field

$$F'_I = \frac{I'_I}{I'_i} = \frac{\int_{-\infty}^{-a} |e_{yI}(x)|^2 dx}{1 + \int_{-\infty}^{-a} |e_{yI}(x)|^2 dx + \int_{a}^{\infty} |e_{yIII}(x)|^2 dx} , \tag{36}$$

$$= \frac{B^2 \sin^2\zeta}{2\alpha'_I [1 + B^2 \sin^2\zeta (\dfrac{1}{2\alpha'_I} + \dfrac{1}{2\alpha'_{III}})]}$$

$$= \frac{aB^2 \sin^2\zeta}{2\eta'_I [1 + aB^2 \sin^2\zeta (\dfrac{1}{2\eta'_I} + \dfrac{1}{2\eta'_{III}})]}$$

$$F'_{II} = \frac{I'_{II}}{I'_i} = \frac{1}{1 + \int_{-\infty}^{-a} |e_{yI}(x)|^2 dx + \int_{a}^{\infty} |e_{yIII}(x)|^2 dx} ,$$

$$= \frac{1}{1 + B^2 \sin^2\zeta (\dfrac{1}{2\alpha'_I} + \dfrac{1}{2\alpha'_{III}})} \tag{37}$$

$$F'_{III} = \frac{I'_{III}}{I'_i} = \frac{\int_{a}^{\infty} |e_{yIII}(x)|^2 dx}{1 + \int_{-\infty}^{-a} |e_{yI}(x)|^2 dx + \int_{a}^{\infty} |e_{yIII}(x)|^2 dx}$$

$$= \frac{B^2 \sin^2\zeta}{2\alpha'_{III} [1 + B^2 \sin^2\zeta (\dfrac{1}{2\alpha'_I} + \dfrac{1}{2\alpha'_{III}})]} \,. \tag{38}$$

The absorption constant (AC) for any region is also called confinement factor (CnF) giving that $F_I + F_{II} + F_{III} = 1$, $F'_I + F'_{II} + F'_{III} = 1$, $\overline{K}_a + F_{II} = 1$, and $\overline{q}_a + F'_{II} = 1$.

4. RESULTS

The coordinate variables η, ζ for energy eigenvalues for charged carriers in the perpendicular coordinate system $\zeta - \eta$, phase constant, phase velocity, effective index of active region, percent of the active region and device losses, absorption constants or confinement factors, propagation constants for the regions have been obtained in terms of normalized propagation constants for even and odd fields in a SCSASIL.

The quantities in Table 1 are computed numerically by using found formulas for given data such as λ=53.2 nm, *a*=800 A^o, n_I=9.125, n_{II}=9.128, n_{III}=9.123 in a SCSASIL for even and odd fields. Thus, the validities of found formulas have been tested, numerically. For example, by using values of the quantities in Table 1, it is possible to see that $F_I + F_{II} + F_{III}$=1. Since effective refractive index belong to active region of the SCSASIL is constant, that the quantities such as phase constant, phase velocity does not change for each of even and odd fields have observed. Since effective refractive index belong to active region of the SCSASIL is constant as 9.12730124710783, the phase constant β_z and the phase velocity v are respectively constants as 1.077979794925373x10^9 1/m and 3.286842319300691x10^7 m/s for each of even and odd fields in a SCSASIL.

Table 1. Quantities for data λ=53.2 nm, *a*=800 A^o, n_I=9.125, n_{II}=9.128, n_{III}=9.123 in a SCSASIL for each of even and odd fields.

	ASYMMETRIC FOR EVEN		ASYMMETRIC FOR ODD	
Quantity	**Symbol**	**Value**	**Symbol**	**Value**
NF	V	2.21098592204107	V	2.09697010488701
NA	NA	0.23400641016862	NA	0.23400641016862
NID	Δ	4.381100788014320x10^{-4}	Δ	4.381100788014320x10^{-4}
NPC	α	0.767053004185813	α	0.767053004185813
PC	α_I (1/m)	2.420520013886074x10^7	α'_I (1/m)	1.333903915312632x10^7
PC	α_{II} (1/m)	1.333903915312404x10^7	α'_{II} (1/m)	2.420520013886648x10^7
PC	α_{III} (1/m)	3.309028072929887x10^7	α'_{III} (1/m)	2.621077889934053x10^7
Ams	A (V/m)	2.992220821953748x10^3	B (V/m)	3.265213125452570x10^3
ERI	n_{ef}	9.12730124710783	n_{ef}	9.12730124710783
zeta	ζ	1.0671231322503	ζ'	1.93641601110932
eta	$\eta = \eta_I$	1.93641601110932	$\eta' = \eta'_I$	1.06712313225011
Eta$_{III}$	η_{III} (1/m)	2.64722245834391	η'_{III} (1/m)	2.09686231194724
AC (CnF)	F_I	0.04009219196865	F'_I	0.31356331880134
AC (CnF)	F_{II}	0.93058078181040	F'_{II}	0.52685983678140
AC (CnF)	F_{III}	0.02932702622095	F'_{III}	0.15957684441726
PARL	R_a	0.07459773460457	r_a	0.89803801730097
PDL	K_a	0.06941921818960	q_a	0.47314016321864
PV	v (m/s)	3.286842319300691x10^7	v	3.286842319300691x10^7
PhC	β_z (1/m)	1.077979794925373x10^9	β_z	1.077979794925373x10^9
CF	f_c (Hz)	8.012601016565813x10^{15}	f'_c (Hz)	1.602520203313163x10^{16}
AF	a_p	0.66648404828450	a_p	0.66648404828450

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