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Determination of Sample Size Selecting from Strata Under Nonlinear Cost Constraint by Using Goal Programming and Kuhn-Tucker Methods

S. Tuğba ŞAHİN♠

Gazi University, Arts and Science Faculty, Department of Statistics Teknikokullar, 06500, Ankara

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Abstract

Although sampling methods are various, most frequently used method is Stratified Random Sampling in practice, especially, in case of heterogeneous population structure. One of the most important points, which should be considered, in the use of stratified random sampling method is how many units of samples should be selected from which stratum. Determination of optimum sample size to be selected from strata allows the sample to represent the population properly and increases precision of the obtained estimations. Kuhn-Tucker Method, which is accepted as a basic method for determination of sample sizes to be selected from strata in stratified random sampling, and the goal programming method, which can take into consideration the researcher's multi-objectives, will be used in this study. It will be tried to minimize variance of sample mean statistics by using these methods under the non-linear cost constraint and superiorities of these methods over each other will be discussed under the light of the results obtained from the conducted simulation study.

Keywords: *Goal programming, Kuhn-Tucker Method, nonlinear cost function, stratified random sampling.*

1. INTRODUCTION

A well established sampling plan plays an important role to make the results obtained from statistical studies useful and reflect the reality. A well established sampling plan and samples representing population well produce more reliable statistical results.

Although sampling methods are various, most frequently used method is stratified random sampling in practice, especially, in case of heterogeneous population structure. The most important problem in stratified random sampling method is to allocate the specified sample size (n) for strata and to decide how many units of sample are selected from which stratum. In fact, the problem under consideration is an optimization problem. Many studies were conducted to specify sample size to be selected from strata under the linear cost constraint with a fixed budget. First of these studies was conducted by Neyman in 1934 [1]. Neyman used the Lagrange multiplier method to find the sample sizes to minimize variance of sample mean statistics in stratified random sampling under the linear cost constraint given in Equation (1).

$$c = c_0 + \sum_{h=1}^{L} t_h n_h \tag{1}$$

In Equation (1), c stands for the total budget allocated for the research while c_0 represents fixed costs and t_h represents the cost for selecting a unit from h th stratum. This cost function is the linear cost function in which travel to each stratum has an effect on the cost function as an increase of one unit.

The method of Lagrange multiplier is generally used in specifying sample sizes to be selected from strata. However, the use of the method of Lagrange multiplier causes certain negative effects from the point of view of sampling. They are the fact that the constraint of

♠Corresponding author, e-mail: sinemsahin@gazi.edu.tr

$n_h \le N_h$ is neglected in the method of Lagrange multiplier and the fact that the results like $n_h > N_h$ or $n > N$ are encountered in the solutions. In this study, Kuhn-Tucker method was accepted as basic method to eliminate such negative effects. When our cost function is linear, specifying values of n_h, which will minimize variance of sample mean statistics, is very easy. If the cost function is not linear, in other words, if travel to each stratum does not cause an increase of one unit in cost function, specifying values of n_h will be quite complex.

The literature contains numerous nonlinear optimization techniques that could be used to minimize variance of sample mean statistics in stratified random sampling under nonlinear cost constraints. Various nonlinear optimization methods such as the Kuhn-Tucker method, Geometric Programming, Dynamic Programming and Separable Programming could be used to solve the problem. However, all these methods allow the minimization or maximization of only single objective.

In this study, Kuhn-Tucker and goal programming methods were used for optimum allocation of the sample size of n selected from the population for strata under non-linear cost constraint.

The reason for the use of the Kuhn-Tucker method: In stratified random sampling, the Lagrange multipliers is the classical method used in allocating the sample size into strata. The Lagrange multipliers method is used in solving optimization methods with equal constraints. Since it doesn't take unequal constraints into consideration, the Lagrange multipliers method has certain negative effects concerning sampling. For instance, the use of the Lagrange multiplier method might yield results such as $n_h > N_h$ or $n > N$. The Kuhn-Tucker method takes unequal constraints, which cannot be included in the Lagrange multipliers method, into account. Therefore the use of this method eliminates such negative results as $n_h > N_h$ and $n > N$. Moreover, the Kuhn-Tucker method encompasses the Lagrange multipliers method. As a result, this study uses the Kuhn-Tucker method as its principal method for comparison. The Kuhn-Tucker method is one of the leading methods that have a single objective function (the variance of sample mean statistics for this study).

The reason for the use of Goal Programming: In general, nonlinear programming or nonlinear optimization deals with the solution of single-objective models (models in which a single objective is best accomplished under given constraints). Goal programming, however, focus on multi-objectives. Basic consideration in goal programming technique is to convert a multi-objective problem into a problem with a single objective to solve it. Within the framework of the problem at hand, our first goal is to minimize the variance of mean sample statistics, the second goal is to minimize the part of the budget used for the research and the third and final goal is to make sure that the sum of sizes of samples selected from strata equals the total sample size ($n = \sum_{h=1}^{L} n_h$).

Solutions of the models, which were created by using these methods, were obtained with simulation study. Variances of sample mean statistics obtained from these two methods were compared and superiorities of these methods over each other were discussed.

Non-linear cost function is in the form of $c = c_0 + \sum_{h=1}^{L} t_h n_h^{\alpha}$, $\alpha > 0$ [2, 3, 4]. Herein, c stands for total budget allocated for the research. c_0 represents fixed costs. t_h represents the cost for selecting a unit from h th stratum. α represents effect of travel to strata on the cost function.

Specifying size of the sample to be selected from strata is the most frequently considered problem in the literature. However, most of the conducted studies are on specifying sample size to minimize variance of sample mean statistics under linear cost constraint [5-14].

2. KUHN-TUCKER METHOD

Kuhn-Tucker method is used in solving non-linear optimization problems by including both of equity and inequity constraints into the model. Every constraint is made an equity constraint to create a general Lagrange function while the model is solved by this method. General structure of the problem is as seen in Model (2).

$$\begin{aligned} &Max\, f(x) \\ &g_i(x) - b_i \le 0 \qquad i = 1, 2, \ldots, m \end{aligned} \tag{2}$$

Kuhn-Tucker(K-T) conditions are used in solution of Model (2). First of all, constraints in Model (2) are put into equity form by using slack variable ($s_i^2 \ge 0$). Suitable points are obtained from the required conditions of this function. The problem of specifying sizes of sample to be selected from strata to minimize variance of sample mean statistics are modeled according to Kuhn-Tucker method in the form of (3). Value of objective function of Model (3) may be converted into the maximum problem to make Model (3) similar to Model (2).

$$\min V(\overline{x}_{tb}) = \min\left(\sum_{h=1}^{L} \frac{W_h^2 S_h^2}{n_h} - \sum_{h=1}^{L} \frac{W_h^2 S_h^2}{N_h} \right)$$

$$c=\sum_{h=1}^{L} t_h n_h^{\alpha}\ ,\quad n_h \le N_h\ ,\quad h=1,2,\ldots,L \qquad (3)$$

As seen, there are $L+1$ constraints in total. General Lagrange function is expressed as seen in Equation (4).

$$L(n_h,\lambda,s)=-\sum_{h=1}^{L}\frac{W_h^2S_h^2}{n_h}+\sum_{h=1}^{L}\frac{W_h^2S_h^2}{N_h}-\lambda_1\left(\sum_{h=1}^{L}t_h n_h^{\alpha}-c\right)-\sum_{h=1}^{L}\lambda_{h+1}\left(n_h+s_h^2-N_h\right) \qquad (4)$$

Required conditions for Kuhn-Tucker method are expressed as the following:

1. $$\frac{\partial L(n_h,\lambda,s)}{\partial n_h}=\frac{W_h^2S_h^2}{n_h^2}-\lambda_1\alpha t_h n_h^{\alpha-1}-\lambda_{h+1}=0$$

2. $$\frac{\partial L(n_h,\lambda,s)}{\partial \lambda_1}=\sum_{h=1}^{L}t_h n_h^{\alpha}-c=0$$

$$\frac{\partial L(n_h,\lambda,s)}{\partial \lambda_i}=n_i+s_i^2-N_i=0 \quad \text{or}$$

$$n_i \le N_i \quad i=2,\ldots,L+1$$

3. $$\frac{\partial L(n_h,\lambda,s)}{\partial s_i}=-2\lambda_{i+1}s_i=\lambda_{i+1}(n_i-N_i)=0$$

$i=1,\ldots,L$

4. λ_1 Free (equity constraint), λ_2, λ_3, …, $\lambda_{L+1}\ge 0$ (inequity constraints)

As seen, totally $3L+1$ equations are obtained from the required conditions of Kuhn-Tucker. Generalization cannot be made by using the obtained equations; however, a possible solution is sought for different situations of λ_i s.

3. GOAL PROGRAMMING

Researchers face multi-objective decision making problems in practices. Goal programming method was begun to be used by Charnes and Cooper in 1961 for the first time in solving multi-objective problems [15]. Most of the constrained optimization problems have only one objective function. Also, the objective function is aimed only to be maximized or minimized. Because these models have only a single objective, they are used in a limited area.

Goal programming method is a technique responding the decision maker in solution of multi-objective problems. Also, it is the multi-objective decision making technique, which tries to optimize objectives more than one conflicting with each other under certain conditions. In goal programming model, all goals specified are included in the model. Sum of the potential deviations from the goals is tried to be minimized. These deviations may be positive, being above the goal, or negative, being below the goal.

Basic consideration in goal programming technique is to convert a multi-objective problem into a problem with a single objective to solve it. Optimum solution for all conflicting objectives may not be found with goal programming. Efficient solutions obtained in solution of the problem do not always need to be optimum. Goal programming tries to find a solution in a way to correct the goals of the model. Therefore, ordering, which reflect the objectives in order of importance, are required in solving multi-objective decision making models with the approach of goal programming. This order is called as priority. Consequently, compromised solutions, which basically consider priority of each objective, may be found. First of all, deviation from the goal with 1^{st} priority is minimized as much as possible. Then, deviation from the following goal in order of priority is minimized as much as possible. This continues until deviation from the goal with the least priority is minimized as much as possible. Then, efficient solutions satisfying all goals are sought.

Goal programming is used in many areas because it considers minimizing objectives more than one [16-25]. There are many methods for goal programming. In this study, non-linear goal programming method was used in which variables get a continuous value in specifying sizes of sample to be selected from strata under non-linear cost constraint. Simplex approximation was used while this method was used and Method of Approximation Programming (MAP) was employed to obtain a solution with the simplex method [26].

d_i^- : The negative deviation from b_i

d_i^+ : The positive deviation from b_i

x_j : j^{th} decision variable

b_i : The associated right-hand-side value and reflect the value that $f_i(\overline{x})$ must satisfy

c_{ij} : The coefficient associated with variable j in the i^{th} objective

$\rho_k(d^-,d^+)$: Linear function of the goal with k^{th} priority

Thus, the model of the linear goal programming with priority is expressed as the following:

$$Mina = \left\{ \rho_1(d^-, d^+), \rho_2(d^-, d^+), ..., \rho_k(d^-, d^+) \right\}$$

$$\sum_{j=1}^{n} c_{ij} x_j + d_i^- - d_i^+ = b_i \quad i = 1, 2, ..., m$$

$$x,\ d^-,\ d^+ > 0$$

If the desired values of the goals, in other words right-hand side constants b_i, are not known, they must be specified by the researcher.

MAP method was used by Griffith and Stewart for the first time for solving single-objective non-linear problems and then, it was expanded to the solution of non-linear goal programming problems by Ignizio [26, 27]. MAP method is used as GS method also in the literature. Basic approach of Griffith and Stewart (GS) method may be given as the following:

1st step: Decision model is formulated.
2nd step: An initial solution was chosen arbitrarily.
3rd step: Vicinities (limits) are defined relating to this initial solution.
4th step: Linear approximation of the non-linear model is developed.
5th step: Optimization of linear approximation of the model is solved with multi-dimensional simplex method according to the vicinity defined in 3rd step.
6th step: Because the initial point is chosen arbitrarily, the solution obtained in the 5th step takes us to a better or worse point. In the fist case, we go to the 3rd step while in the second case, the method is ceased or linear approximation model is considered in smaller vicinity.

In GS approximation, Taylor series expansion of each $f_i(\overline{x}_s)$ is used around a given $\overline{x}_s$ initial point. Then, the non-linear function is converted into a linear function by neglecting non-linear terms in the series. To do this, $f_i(\overline{x}_s)$ s should be derivable. Each objective in multi-objective decision models may be written as the following:

$$G_i = f_i(\overline{x}) + d_i^- - d_i^+ = b_i$$

$\overline{x}_s$: A possible solution for the set of objectives, $\hat{G}_i$: linear approximation of non-linear G_i objective function. Thus:

$$\hat{G}_i = f_i(\overline{x}_s) + \sum_{j=1}^{J} \frac{\partial f(\overline{x}_s)}{\partial x_j}(x_j - x_{s,j}) + d_i^- - d_i^+ = b_i \quad (5)$$

Linear approximation in Equation (5) may be re-written with vector notation as below:

$$\hat{G}_i = f_i(\overline{x}_s) + \left[\overline{\nabla} f_i(\overline{x}_s)\right]^{'} (\overline{x} - \overline{x}_s) + d_i^- - d_i^+ = b_i \quad (6)$$

We can replace the term of $\overline{x} - \overline{x}_s$ in Equation (6) with a new $\overline{y}$ vector.

$$\overline{y} = \overline{x} - \overline{x}_s \quad (7)$$

As a result, $\overline{y}$ vector in Equation (7) represents a variation between a new $\overline{x}_s$ solution point and the $\overline{x}$ point. However, components of $\overline{y}$ may be free from the point of view of their signs. Therefore, problem may occur in using the simplex method. To eliminate this problem, variable of $\overline{y}$ may be expressed in two positive variables.

$$\overline{y} = \overline{u} - \overline{v} \quad \left(\overline{u} \geq 0 \ , \overline{v} \geq 0\right) \quad (8)$$

From Equations (7) and (8), Equation (6) may be written as below.

$$\hat{G}_i = f_i(\overline{x}_s) + \left[\overline{\nabla} f_i(\overline{x}_s)\right]^{'} (\overline{u} - \overline{v}) + d_i^- - d_i^+ = b_i \quad (9)$$

The obtained Equation (9) is a function, which is made linear. Thus, Equation (9) will be used as approximation of a non-linear function to a linear form.

Two issues should be taken under consideration in using linear approximation. The first one is; each component of $\overline{x} = (x_1, x_2, \ . \ . \ . \ , x_J)$ must have upper and lower limits. If U_j is upper limit of x_j component, then

$$0 \leq x_j \leq U_j \quad (10)$$

This value of U_j may be found by considering the set of objectives. The second issue is whether $\hat{G}_i$ (linear approximation of non-linear G_i) is good approximation in vicinity of $\overline{x}_s$ or not.

Let d_j be the maximum distance in which x_j can move. Thus, d_j will help us in defining vicinity. Because

$$-d_j \leq y_j \leq d_j \qquad j = 1,\ 2,\ .\ .\ .\ ,\ J \quad (11)$$

$$y_j = u_j - v_j$$

$$-d_j \leq u_j - v_j \leq d_j \qquad j = 1, 2,\ .\ .\ .\ ,\ J$$

$$u_j \geq 0\ ,\ v_j \geq 0 \quad (12)$$

Then, Equation (12) may be written as below:

$$0 \le u_j \le d_j \qquad j = 1,2, \ldots, J \qquad (13)$$

$$0 \le v_j \le d_j \qquad j = 1,2, \ldots, J \qquad (14)$$

Remind from Equations (7) and (10) that

$$0 \le x_j \le U_j \qquad j = 1,2, \ldots, J$$

and

$$\overline{y} = \overline{x} - \overline{x}_s \quad \text{or} \quad y_j = x_j - x_{s,j}$$

and combine these equations with Equations (13) and (14):

$$0 \le u_j \le \min\{d_j, U_j - x_{s,j}\} \qquad (15)$$

$$0 \le v_j \le \min\{d_j, x_{s,j}\} \qquad (16)$$

As a result, upper limits' set consisting of Equations (15) and (16) is the most superior objectives' set. With this the most superior objectives' set, x_j decision variables cannot pass any upper limit. Also, it enables the region where G_i is approached to stay relatively small. The most superior objectives have higher priority than other all objectives with priority. Equations (15) and (16) may be written in the following form:

$$u_j + d_t^- - d_t^+ = \min\left\{d_j,\ U_j - x_{s,j}\right\} \qquad (17)$$

$$v_j + d_q^- - d_q^+ = \min\left\{d_j,\ x_{s,j}\right\} \qquad (18)$$

Cooper and Steinberg recommended the interval in Equation (19) for d_j [28].

$$U_j/10 \le d_j \le U_j/5 \qquad (19)$$

If recovery cannot be obtained in optimum solution of the linear model, d_j value may be decreased properly. Because Equations (17) and (18) are the most superior objectives, we want to minimize d_t^+ in Equation (17) and d_q^+ in Equation (18). Herein, the most clear problem is selection of d_j.

Under the light of the definitions above, let us model the problem of specifying sizes of sample to be selected from strata under non-linear cost constraint, which is considered in this study, according to the goal programming method.

Our first goal is to minimize variance of sample mean statistics. The second goal is to minimize the used part of the allocated budget for the research. Our final goal is to make total of sample size selected from strata equal to sample size of n. Also, because right-hand side resource must be specified in goal programming problems, right-hand side resource may be taken as the desired variance as specified by the researcher according to his opinion for variance of sample mean statistics:

$$\frac{precision^2}{reliability^2} = \frac{d^2}{z^2} = D^2$$

If population consists of two strata, then goal programming model is expressed as below:

$$f_1 : \frac{1}{N^2}\frac{N_1^2 S_1^2}{n_1} + \frac{1}{N^2}\frac{N_2^2 S_2^2}{n_2} \le D^2 = \frac{d^2}{z^2}$$

variance of sample mean statistics

$$f_2 : t_1 n_1^{\alpha} + t_2 n_2^{\alpha} \le c' \quad \text{non-linear cost function}$$

$$f_3 : n_1 + n_2 = n$$

We may write each goal as below:

$$G_1 : \frac{1}{N^2}\frac{N_1^2 S_1^2}{n_1} + \frac{1}{N^2}\frac{N_2^2 S_2^2}{n_2} + d_1^- - d_1^+ = D^2$$

$$G_2 : t_1 n_1^{\alpha} + t_2 n_2^{\alpha} + d_2^- - d_2^+ = c'$$

$$G_3 : n_1 + n_2 + d_3^- - d_3^+ = n$$

$$min\left(\left\{d_1^+\right\}, \left\{d_2^+\right\}, \left\{d_3^- + d_3^+\right\}\right)$$

Once the non-linear model has been established as seen above, limits of the variables are specified. d_j, maximum distance in which n_j can move, and the initial point $\overline{n}_s$ are chosen arbitrarily.

$$\left.\begin{aligned} n_1 &\le n-1 \\ n_2 &\le n-1 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} U_1 &= n-1 \\ U_2 &= n-1 \end{aligned}\right\} \Rightarrow \begin{aligned} \frac{U_1}{10} &\le d_1 \le \frac{U_1}{5} \\ \frac{U_2}{10} &\le d_2 \le \frac{U_2}{5} \end{aligned}$$

Once the initial point, $\overline{n}_s = (n_{1s}, n_{2s})$, has been chosen arbitrarily according to the researcher's foresight, non-linear goal programming method is made linear as seen below.

$$\hat{G}_1 : f_1(\overline{n}_s) + \left[\nabla f_1(\overline{n}_s)\right]'(\overline{u} - \overline{v}) + d_1^- - d_1^+ = D^2$$

$$f_1(\bar{n}_s)+\begin{pmatrix}-\dfrac{N_1^2S_1^2}{N^2n_1^2}\\ -\dfrac{N_2^2S_2^2}{N^2n_2^2}\end{pmatrix}'_{\bar{n}_s}\begin{pmatrix}u_1-v_1\\ u_2-v_2\end{pmatrix}+d_1^- - d_1^+ = D^2$$

$$f_1(\bar{n}_s)-\frac{N_1^2S_1^2}{N^2n_1^2}(u_1-v_1)-\frac{N_2^2S_2^2}{N^2n_2^2}(u_2-v_2)+d_1^- - d_1^+ = D^2$$

$$\hat{G}_2 : f_2(\bar{n}_s)+\left[\nabla f_2(\bar{n}_s)\right]'(\bar{u}-\bar{v})+d_2^- - d_2^+ = c'$$

$$f_2(\bar{n}_s)+\begin{pmatrix}\alpha t_1 n_1^{\alpha-1}\\ \alpha t_2 n_2^{\alpha-1}\end{pmatrix}'_{\bar{n}_s}\begin{pmatrix}u_1-v_1\\ u_2-v_2\end{pmatrix}+d_2^- - d_2^+ = c'$$

$$f_2(\bar{n}_s)+\alpha t_1 n_1^{\alpha-1}(u_1-v_1)+\alpha t_2 n_2^{\alpha-1}(u_2-v_2)+d_2^- - d_2^+ = c'$$

$$\hat{G}_3 : f_3(\bar{n}_s)+\left[\nabla f_3(\bar{n}_s)\right]'(\bar{u}-\bar{v})+d_3^- - d_3^+ = n$$

$$f_3(\bar{n}_s)+\begin{pmatrix}1\\ 1\end{pmatrix}'_{\bar{n}_s}\begin{pmatrix}u_1-v_1\\ u_2-v_2\end{pmatrix}+d_3^- - d_3^+ = n$$

$$f_3(\bar{n}_s)+(u_1-v_1)+(u_2-v_2)+d_3^- - d_3^+ = n$$

$$\hat{G}_4 : u_1+d_4^- - d_4^+ = min\{d_1, U_1-n_{1s}\}$$

$$\hat{G}_5 : v_1+d_5^- - d_5^+ = min\{d_1, n_{1s}\}$$

$$\hat{G}_6 : u_2+d_6^- - d_6^+ = min\{d_2, U_2-n_{2s}\}$$

$$\hat{G}_7 : v_2+d_7^- - d_7^+ = min\{d_2, n_{2s}\}$$

The model, which has been made linear to specify sizes of sample to be selected from the strata under non-linear cost constraint for the population consisting of two strata, is expressed as below.

$$min\left(\{d_4^+ + d_5^+ + d_6^+ + d_7^+\},\{d_1^+\},\{d_2^+\},\{d_3^- + d_3^+\}\right)$$

$$f_1(\bar{n}_s)-\frac{N_1^2S_1^2}{N^2n_1^2}(u_1-v_1)-\frac{N_2^2S_2^2}{N^2n_2^2}(u_2-v_2)+d_1^- - d_1^+ = D^2$$

$$f_2(\bar{n}_s)+\alpha t_1 n_1^{\alpha-1}(u_1-v_1)+\alpha t_2 n_2^{\alpha-1}(u_2-v_2)+d_2^- - d_2^+ = c'$$

$$f_3(\bar{n}_s)+(u_1-v_1)+(u_2-v_2)+d_3^- - d_3^+ = n$$

$$u_1+d_4^- - d_4^+ = min\{d_1, U_1-n_{1s}\}$$

$$v_1+d_5^- - d_5^+ = min\{d_1, n_{1s}\}$$

$$u_2+d_6^- - d_6^+ = min\{d_2, U_2-n_{2s}\}$$

$$v_2+d_7^- - d_7^+ = min\{d_2, n_{2s}\}$$

The optimization problem, which is made linear as seen above, is solved with multi-dimensional simplex method. Models can be established in similar way for higher strata numbers.

4. TO MINIMIZE VARIANCE OF SAMPLE MEAN STATISTICS WITH SIMULATION STUDY BY USING GOAL PROGRAMMING AND KUHN-TUCKER METHOD UNDER THE CONSTRAINT OF $c' = \sum_{h=1}^{L} t_h n_h^{\alpha}$

In this chapter, considering the constraint of $c' = \sum_{h=1}^{L} t_h n_h^{\alpha}$ ($\alpha > 0$), simulation study was conducted relating to the cost constraint, which is a function of α. Kuhn-Tucker and goal programming method were used to find which α values produce more efficient variance of sample mean statistics as well as superiorities of the methods over each other were investigated. Monte Carlo simulation with 1000 repetitions to find the variance of sample mean statistics with the help of this cost constraint. In the simulation study, population size (N), size of sample to be selected from population (n), budget (c'), travel cost from a stratum to another stratum (t_h) and α were given at the beginning. The models to be used to minimize variance of sample mean statistics under non-linear cost constraint are as the following.

Model 1: The model established for goal programming

$$min\left(\left\{d_1^+\right\},\left\{d_2^+\right\},\left\{d_3^- + d_3^+\right\}\right)$$

$$\frac{1}{N^2}\frac{N_1^2 S_1^2}{n_1} + ... + \frac{1}{N^2}\frac{N_h^2 S_h^2}{n_h} + d_1^- - d_1^+ = D^2$$

$$t_1 n_1^\alpha + ... + t_h n_h^\alpha + d_2^- - d_2^+ = c'$$

$$n_1 + ... + n_h + d_3^- - d_3^+ = n \quad h = 1, \ . \ . \ . \ , L$$

Model 2: The model established for Kuhn-Tucker method

$$\min V(\overline{x})_2$$

$$\sum_{h=1}^{L} n_h = n$$

$$\sum_{h=1}^{L} t_h n_h^\alpha \leq c'$$

$$r_h \leq n_h \leq k_h \quad h = 1, \ . \ . \ . \ , L$$

In actual practices, it is hard to specify stratum variances and sample size to be selected from the population. Therefore, stratum variances are estimated with the help of a pilot sample and then, sample size to be selected from population may be specified. We mentioned before that right hand side resource must be specified to solve problems in goal programming. Therefore, the researcher will specify the right hand side resource of variance of sample mean statistics. The researcher may specify the desired variance with a certain precision and reliability based on previous studies or according to his competency. Increase in precision will decrease variance of sample mean statistics and more precise estimations are obtained.

In addition, since the researcher determines the initial points and the maximum distances that n_h s can change in the goal programming method arbitrarily, it is very difficult to compare the Kuhn-Tucker and Goal Programming methods theoretically in this study. Failing to identify the initial point correctly can result in a very inaccurate solution. This study takes into consideration the size of each stratum (N_h) and has initial points that correspond to values in the middle of strata sizes. The Kuhn-Tucker method relies on trying all possible conditions of Lagrange multiplier λ_i s that are used in formulating the general Lagrange function. For these reasons, it is almost impossible to compare the two methods theoretically because a theoretical comparison will not yield precise results but instead produce different results according to each initial point and different λ_i conditions. Since virtually all nonlinear optimization methods contain intuitional or arbitrary initial points, the best way to compare two nonlinear optimization techniques is to conduct a simulation whereby all possible iterations in the solution area will be tested. The simulation conducted in this study is especially significant since it clearly displays the superiorities of each method that couldn't be theoretically demonstrated.

The following steps were followed in solving the models.

Step 1: Since sizes of strata are known at the beginning of the study, units on the strata are obtained by producing N_h amount of data with different means and variances from Normal Distribution. Because the strata were aimed to be homogeneous, Normal Distribution data, which have quite different stratum means, but have small stratum variances so that inside the stratum is homogeneous, were used.
Step 2: Stratum variances were calculated with the help of the specified units.
Step 3: Models 1 and 2, which were established under non-linear cost constraint, were solved with the help of the simulation program and variance values and CPU solution times were obtained. One of the most important problems in solving non-linear models is to specify the initial point required for the solution. In this study, initial points were specified by considering stratum sizes.
Step 4: Steps 1-2-3 were repeated 1000 times.
Step 5: Mean of the values obtained from 1000 repetitions was calculated to generalize the results and then, optimum sample sizes to be selected from strata, variance of sample mean statistics and CPU solution times were obtained.

When there are two or three strata, it is possible to conduct a manual solution using the Kuhn-Tucker or Goal Programming methods. However, as the number of strata goes up, it becomes more and more difficult to solve models using the two methods. There are also programmes such as Lindo, Lingo and Gino that are used in the solution of models using the Kuhn-Tucker and Goal Programming methods. These programmes make the solution of a single model possible yet they do not allow for a simulation. Therefore, the simulation intended to compare different models included in this study was done using a computer code developed in MATLAB.

The simulation study was conducted under different values of travel costs to the strata. The simulation study was conducted for the cases that the population consists of two, three, four and five strata. Because the results could be generalized for four different stratum numbers, no more stratum numbers were considered. The results are summarized in Tables 1-10. In the tables, n_{ih} stands for the sample size, which was obtained from i^{th} model and should be selected from hth stratum while i. Cons. Budg. shows the used part of the budget allocated for the research when solution is conducted with ith model. However, the cases in which possible solutions could not be ensured are expressed in bold.

Table 1: If two strata exist, $N = 500$, $N_1 = 300$, $N_2 = 200$, $d^2 = 15$, $z = 1.96$, $D^2 = 3.904$, $n = 100$, $c' = 500$, $t_1 = 1$ and $t_2 = 1$

	The results obtained from goal programming					The results obtained from Kuhn-Tucker method				
α	n_{11}	n_{12}	$V(\overline{x})_1$	cpu_1	1. Cons. Budg.	n_{21}	n_{22}	$V(\overline{x})_2$	cpu_2	2. Cons. Budg.
0.2	59.9259	40.0741	0.00797	72.6094	4.3594	66.4350	33.5650	0.00815	171.4219	4.3338
0.4	59.8200	40.1800	0.00801	72.2344	9.5187	66.4680	33.5320	0.00819	171.0313	9.4340
0.5	59.7521	40.2479	*0.00757*	70.1410	14.0741	65.1000	34.9000	*0.00771*	170.1848	13.9761
0.6	59.6984	40.3016	0.00800	72.8594	20.8174	66.4070	33.5930	0.00818	171.4844	20.6340
0.8	60.0820	39.9180	0.00799	72.0938	45.5804	66.5490	33.4510	0.00817	169.0625	45.3196
1	59.9930	40.0070	0.00801	74.4375	100	66.5180	33.4820	0.00819	169.6563	100
1.2	59.4315	40.5685	0.00801	74.3125	219.6101	66.4930	33.5070	0.00819	167.4844	221.5664
1.5	**52.9511**	**34.2319**	0.00946	77.4375	**585.5960**	**51.1820**	**26.1450**	0.01114	181.7500	499.8493
1.8	**21.0856**	**29.8000**	0.02049	77.3281	**692.0338**	**27.1240**	**14.2380**	0.02252	172.8281	499.4029
2	**15.0667**	**29.8000**	0.02718	77.8906	**1115.04**	**17.8550**	**13.5680**	0.02999	183.0313	**502.8916**

Table 2: If two strata exist, $N = 500$, $N_1 = 400$, $N_2 = 100$, $d^2 = 20$, $z = 1.96$, $D^2 = 5.20$, $n = 100$, $c' = 1000$, $t_1 = 1$ and $t_2 = 50$

	The results obtained from goal programming					The results obtained from Kuhn-Tucker method				
α	n_{11}	n_{12}	$V(\overline{x})_1$	cpu_1	1. Cons. Budg.	n_{21}	n_{22}	$V(\overline{x})_2$	cpu_2	2. Cons. Budg.
0.2	80.0862	19.9138	0.00798	83.3438	93.3523	89.6830	10.3170	0.00897	176.1094	82.1986
0.4	76.9092	23.0908	0.00805	84.7031	181.2084	89.6760	10.3240	0.00898	176.3125	133.2469
0.5	79.8969	20.1031	*0.00782*	83.6230	233.1210	89.7000	10.3000	*0.00880*	177.1290	169.9391
0.6	77.9281	22.0719	0.00802	85.2813	333.7377	89.6630	10.3370	0.00898	174.0469	217.8963
0.8	81.7607	18.2393	0.00803	83.4531	544.1297	89.6400	10.3600	0.00900	175.2656	361.0077
1	83.5063	16.4937	0.00810	76.1563	908.1934	89.6200	10.3800	0.00899	178.4531	608.6200
1.2	91.4891	8.5109	0.00963	77.1840	878.7992	**88.7000**	**9.9000**	0.00925	176.1341	**1000.5**
1.5	**98.9907**	**0.4512**	0.03583	83.5625	**1000.1**	**40.1180**	**6**	0.03332	187.1094	988.9494
1.8	**89.8000**	**1.2512**	0.05922	81.1047	**3355.02**	**24**	**4**	0.04193	185.1216	911.3431
2	**82.7923**	**5.2851**	0.02614	79.1310	**8251.17**	**18.1000**	**4**	0.06165	183.1200	**1127.61**

Table 3: If two strata exist, $N=500$, $N_1=350$, $N_2=150$, $d^2=0.055$, $z=1.96$, $D^2=0.0143$, $n=100$, $c'=3500$, $t_1=50$ and $t_2=1$

	The results obtained from goal programming					The results obtained from Kuhn-Tucker method				
α	n_{11}	n_{12}	$V(\bar{x})_1$	cpu_1	1. Cons. Budg.	n_{21}	n_{22}	$V(\bar{x})_2$	cpu_2	2. Cons. Budg.
0.2	61.5136	38.4864	0.00827	88.6094	116.0383	79.8660	20.1340	0.00856	171.4688	121.8952
0.4	74.1757	25.8243	0.00806	86.7031	283.6174	79.9330	20.0670	0.00857	170.7188	291.7622
0.5	66.7660	33.2340	*0.00803*	86.6094	414.3172	79.8680	20.1320	*0.00851*	170.2188	451.3314
0.6	72.3142	27.6858	0.00804	86.6250	659.7195	79.8520	20.1480	0.00859	167.0156	698.4361
0.8	35.8545	64.1455	0.01212	87.9688	904.1116	79.9010	20.0990	0.00859	165.4688	1674.5
1	48.5219	51.4781	0.00986	82.4531	2477.6	**70**	**17.8670**	0.01009	163.4219	**3517.9**
1.2	32.3490	67.6510	0.00763	81.9063	3399.08	**33.1170**	**60.4590**	0.01498	185.3594	3471.8
1.5	**12.4437**	**87.4920**	**0.03850**	73.2969	3013.2	**16**	**39.2840**	0.03063	204.1094	3446.2
1.8	**13.9906**	**59.6000**	**0.03654**	75.1819	**7342.3**	**10**	**22**	0.03360	187.1354	3415.6
2	**3.7477**	**85.3552**	**1.15266**	98.7656	**7987.7**	**8**	**16.8140**	0.06442	182.1875	3482.7

Table 4: If two strata exist, $N=500$, $N_1=200$, $N_2=300$, $d^2=15$, $z=1.96$, $D^2=3.904$, $n=100$, $c'=4000$, $t_1=25$ and $t_2=35$

	The results obtained from goal programming					The results obtained from Kuhn-Tucker method				
α	n_{11}	n_{12}	$V(\bar{x})_1$	cpu_1	1. Cons. Budg.	n_{21}	n_{22}	$V(\bar{x})_2$	cpu_2	2. Cons. Budg.
0.2	39.4349	60.5651	0.00798	84.2656	131.6600	33.6350	66.3650	0.00816	165.6563	131.4954
0.4	36.3212	63.6788	0.00804	82.4688	289.5569	33.5330	66.4670	0.00817	165.0938	289.4365
0.5	33.8115	66.1885	*0.00717*	82.2500	430.1162	33.5590	66.4410	*0.00799*	164.9375	430.1150
0.6	30.2006	69.7994	0.00810	82.1250	640.2479	33.7660	66.2340	0.00815	165.5000	639.7793
0.8	37.3318	62.6682	0.00806	87.0938	1411.2	33.5180	66.4820	0.00820	165.0938	1420.3
1	41.4459	58.5541	0.00801	81.2813	3085.5	33.5450	66.4550	0.00818	164.4844	3164.5
1.2	**53.6262**	**18.6233**	0.02640	81.1094	**4142.8**	**21.1230**	**41.1430**	0.01429	163.0313	**4000.4**
1.5	**20.1000**	**18.8260**	0.03089	79.5469	**5111.8**	**14.6370**	**17.6970**	0.02932	177.9375	**4005.6**
1.8	**21.5485**	**1.4106**	0.02581	80.1460	**6346.9**	**9**	**11**	0.04904	173.1888	3926.5
2	**23.2020**	**10.3768**	0.39750	82.5313	**17227.04**	**7.3010**	**8.9110**	0.06126	164.7188	**4111.8**

Table 5: If three strata exist, $N = 10000$, $N_1 = 2000$, $N_2 = 3000$, $N_3 = 5000$, $d^2 = 20$, $z = 1.96$, $D^2 = 5.206$, $n = 250$, $c' = 600$, $t_1 = 1$, $t_2 = 1$ and $t_3 = 1$

	The results obtained from goal programming						The results obtained from Kuhn-Tucker method					
α	n_{11}	n_{12}	n_{13}	$V(\bar{x})_1$	cpu_1	1 Cons. Budg.	n_{21}	n_{22}	n_{23}	$V(\bar{x})_2$	cpu_2	2. Cons. Budg.
0.2	54.5623	75.2175	120.2202	0.00391	81.53	7.2042	30.3749	62.7066	156.9185	0.00424	178.95	7.0160
0.4	55.2662	74.9087	119.8251	0.00394	80.15	17.3811	30.3732	62.6513	156.9755	0.00426	177.76	16.7073
0.5	55.7726	74.7288	119.4986	*0.00390*	80.78	27.0442	30.3968	62.7347	156.8685	*0.00423*	179.40	25.9586
0.6	55.0128	75.5546	119.4326	0.00392	80.81	42.0996	30.3154	62.7022	156.9824	0.00424	177.62	40.4957
0.8	51.7445	77.9879	120.2676	0.00391	80.21	102.2760	30.3656	62.6714	156.9630	0.00425	178.37	99.8371
1	52.2178	76.3912	121.3910	0.00395	80.29	250	30.3962	62.6918	156.9120	0.00427	179.43	250
1.2	65.3490	82.9819	101.6691	0.00405	80.75	**607.7959**	**28.9916**	**59.7999**	**149.6917**	0.00445	183.37	599.9839
1.5	**73.7972**	**5.4000**	**50.1930**	0.02208	79.85	**1002.1**	**12.2231**	**23.8702**	**57.9031**	0.01126	173.51	599.9651
1.8	**74.6000**	**5.4000**	**43.6787**	0.02284	80.71	**3266.4**	**8.1202**	**13.7433**	**29.4068**	0.02025	181.35	594.9546
2	**74.6000**	**5.4000**	**49.6300**	0.02215	82.04	**8057.5**	**9.9089**	**12.9871**	**18.2517**	0.02457	224.81	599.9756

Table 6: If three strata exist, $N = 10000$, $N_1 = 2000$, $N_2 = 3000$, $N_3 = 5000$, $d^2 = 15$, $z = 1.96$, $D^2 = 3.904$, $n = 200$, $c' = 6000$, $t_1 = 15$, $t_2 = 45$ and $t_3 = 30$

	The results obtained from goal programming						The results obtained from Kuhn-Tucker method					
α	n_{11}	n_{12}	n_{13}	$V(\bar{x})_1$	cpu_1	1. Cons. Budg.	n_{21}	n_{22}	n_{23}	$V(\bar{x})_2$	cpu_2	2. Cons. Budg.
0.2	38.2386	55.4012	106.3602	0.00492	84.68	207.8218	24.5884	50.1954	125.2162	0.00531	178.29	205.7626
0.4	34.4243	57.1950	108.3807	0.00494	83.09	484.3263	24.5467	50.2696	125.1837	0.00533	178.54	476.6858
0.5	35.8860	58.0852	106.0288	*0.00491*	82.46	741.7297	24.5531	50.2977	125.1492	*0.00530*	178.53	729.0808
0.6	33.3014	61.6950	105.0036	0.00493	81.64	1146.3	24.5212	50.2051	125.2737	0.00531	177.57	1118.3
0.8	42.2565	40.5942	117.1493	0.00519	83.60	2526.3	24.5499	50.2628	125.1873	0.00535	176.98	2656.9
1	47.2288	42.6132	110.1580	0.00512	77.64	5930.8	**23.1348**	**47.2666**	**117.5327**	0.00565	178.89	6000
1.2	**59.6000**	**10.0391**	**60.4000**	0.01367	81.28	**6856.3**	**11.8696**	**23.1151**	**56.0072**	0.01162	172.76	5999.8
1.5	**59.6000**	**7.4406**	**60.4000**	0.01737	79.04	**21897.4**	**14.1825**	**12.6375**	**22.3839**	0.02101	233.89	5999.8
1.8	**59.6000**	**10.4000**	**50.4238**	0.01419	78.95	**61395.2**	**9.0027**	**8.1281**	**13.5287**	0.03391	216.03	5999.8
2	**59.6000**	**10.4000**	**56.9900**	0.01360	81.3281	**155585.4**	**7.1805**	**6.5276**	**10.5023**	0.04303	215.2656	5999.8

***Table 7*:** If four strata exist, $N=1000$, $N_1=100$, $N_2=200$, $N_3=300$, $N_4=400$, $d^2=20$, $z=1.96$, $D^2=5.206$, $n=100$, $c'=1000$, $t_1=1$, $t_2=1$, $t_3=1$ and $t_4=1$

	The results obtained from goal programming							The results obtained from Kuhn-Tucker method						
α	n_{11}	n_{12}	n_{13}	n_{14}	$V(\bar{x})_1$	cpu_1	1. Cons. Budg.	n_{21}	n_{22}	n_{23}	n_{24}	$V(\bar{x})_2$	cpu_2	2. Cons. Budg.
0.2	11.6620	16.4847	25.8373	46.0160	0.00921	85.10	7.4529	5.3446	14.8393	29.8520	49.9641	0.00973	165.25	7.2721
0.4	11.6715	15.3638	26.2182	46.7465	0.00930	84.09	14.0030	5.3617	14.8893	29.8537	49.8953	0.00975	163.50	13.5712
0.5	11.7328	14.8165	26.4429	47.0078	*0.00919*	85.15	19.2730	5.3751	14.8867	29.7877	49.9505	*0.00966*	165.42	18.7021
0.6	11.1577	15.4032	26.7265	46.7126	0.00928	85.67	26.6295	5.3558	14.9177	29.9243	49.8022	0.00975	164.81	25.9140
0.8	13.1829	14.2111	26.2442	46.3618	0.00947	84.60	51.4058	5.3459	14.8959	29.7930	49.9652	0.00978	163.14	50.4652
1	12.3246	15.5130	26.0819	46.0805	0.00929	83.10	100	5.3748	14.8930	29.7786	49.9536	0.00974	165.35	100
1.2	11.7264	15.1722	26.3065	46.7949	0.00934	81.87	196.8957	5.3563	14.8997	29.8951	49.8489	0.00976	163.56	200.9894
1.5	14.7988	17.2227	25.2845	42.6940	0.00928	82.93	534.5096	5.4127	14.8442	29.8729	49.8702	0.00974	165.50	585.2358
1.8	21.6330	25.1580	24.8463	28.3627	0.01032	78.15	**1321.9**	**10.6851**	**17.8399**	**23.6727**	**29.7707**	0.01139	251	997.1487
2	**23.4000**	**29.4000**	**13.7360**	**25.6000**	0.01356	81.85	**2256**	**8.4000**	**13.2503**	**17.5019**	**21.1418**	0.01586	239.51	999.4226

***Table 8*:** If four strata exist, $N=6000$, $N_1=500$, $N_2=1500$, $N_3=2000$, $N_4=2000$, $d^2=15$, $z=1.96$, $D^2=3.904$, $n=200$, $c'=6000$, $t_1=40$, $t_2=30$, $t_3=20$ and $t_4=10$

	The results obtained from goal programming							The results obtained from Kuhn-Tucker method						
α	n_{11}	n_{12}	n_{13}	n_{14}	$V(\bar{x})_1$	cpu_1	1. Cons. Budg.	n_{21}	n_{22}	n_{23}	n_{24}	$V(\bar{x})_2$	cpu_2	2. Cons. Budg.
0.2	20.8958	36.5880	66.3393	76.1769	0.00500	84.65	205.1605	6.9756	43.8051	74.6437	74.5756	0.00523	169.21	193.9499
0.4	10.8546	41.8949	69.0709	78.1796	0.00499	85.06	403.4899	6.9772	43.8402	74.6470	74.5356	0.00522	167.95	391.4667
0.5	13.8823	44.8132	67.7512	73.5533	*0.00488*	84.95	600.2493	6.9659	43.9237	74.4975	74.6129	*0.00520*	168.95	563.3994
0.6	17.9376	39.5912	67.9811	74.4901	0.00492	84.71	883.0817	6.9678	43.9581	74.5774	74.4967	0.00524	168.01	817.2237
0.8	16.8837	42.1778	68.1577	72.7808	0.00490	85.75	1877.1	6.9658	43.9611	74.5173	74.5558	0.00523	171.39	1751.9
1	11.8215	40.8603	69.8039	77.5143	0.00496	81.53	3869.9	6.9702	43.8968	74.5731	74.5599	0.00522	170.13	3832.8
1.2	**44.5663**	**45.7008**	**47.1889**	**50.9382**	0.00583	80.12	**9912.2**	**5.6142**	**32.4102**	**55.2438**	**56.5820**	0.00688	164.06	5999.7
1.5	**40.2333**	**40.4000**	**40.4000**	**40.4000**	0.00701	81.46	**25615.1**	**5.6386**	**15.1061**	**22.3414**	**29.3569**	0.01388	164.54	5999.6
1.8	**23.8444**	**40.4000**	**40.4000**	**40.4000**	0.00712	81.37	**58794.7**	**3.8896**	**9.5109**	**13.3573**	**17.2521**	0.02278	164.46	5999.4
2	**15.6500**	**40.4000**	**40.4000**	**40.4000**	0.00732	80.53	**107726.5**	**3.2859**	**7.4929**	**10.4050**	**13.1067**	0.02945	165.54	5999.4

Table 9: If five strata exist, $N=12000$, $N_1=500$, $N_2=1500$, $N_3=2000$, $N_4=3000$, $N_5=5000$, $d^2=15$, $z=1.96$, $D^2=3.904$, $n=3000$, $c'=10000$, $t_1=1$, $t_2=1$, $t_3=1$, $t_4=1$ and $t_5=1$

	The results obtained from goal programming								The results obtained from Kuhn-Tucker method							
α	n_{11}	n_{12}	n_{13}	n_{14}	n_{15}	$V(\bar{x})_1$	cpu_1	1. Cons. Budg.	n_{21}	n_{22}	n_{23}	n_{24}	n_{25}	$V(\bar{x})_2$	cpu_2	2. Cons. Budg.
0.2	383.8852	469.3990	549.0027	723.8627	873.8504	0.0002901	81.73	17.8477	23.0248	182.9513	317.1620	685.2190	1791.6429	0.0003531	214.56	16.0358
0.4	389.7346	468.4325	547.6591	722.1248	872.0490	0.0002908	81.23	63.9487	23.0595	183.1433	317.6459	685.3889	1790.7624	0.0003530	214.34	55.1987
0.5	390.4496	467.0902	546.4128	723.3082	872.7392	*0.0002896*	80.09	121.1841	22.9835	182.8854	317.4297	685.3558	1791.3456	*0.0003517*	214.32	104.6378
0.6	394.0425	458.4847	544.5004	729.1695	873.8029	0.0002904	83.67	229.8087	23.0836	182.7230	317.5309	686.1007	1790.5618	0.0003529	221.04	200.8568
0.8	388.1765	449.5937	530.9316	749.2129	882.0853	0.0002912	81.56	828.2816	23.0915	181.9827	315.1197	678.1884	1801.6177	0.0003532	213.96	762.7089
1	380.6586	459.5457	538.6283	735.5823	885.5851	0.0002976	82.01	3000	23.0277	182.8430	317.2660	685.1607	1791.7026	0.0003529	215.46	3000
1.2	519.1677	560.5407	582.2710	638.4316	699.5890	0.0003416	75.90	**10797.4**	**19.8913**	**156.4575**	**270.6815**	**583.9678**	**1527.0064**	0.0004270	217.46	9999.6
1.5	**619.2000**	**769.2000**	**228.9295**	**0.8000**	**150.8000**	0.07928	76.59	**42057.6**	**6.2727**	**39.6574**	**67.5611**	**146.7108**	**379.7887**	0.00186	213.73	9999.2
1.8	599.9969	599.9990	600.0004	600.0033	600.0004	0.0003831	81.43	**500774.5**	**3.7291**	**16.8254**	**27.4395**	**57.9506**	**146.8292**	0.00463	213.89	9999.1
2	599.9978	599.9991	600.0001	600.0020	600.0010	0.0003845	77.43	**1800000**	**15.2575**	**31.6147**	**37.7754**	**49.4929**	**69.9348**	0.00499	215.25	9999.7

Table 10: If five strata exist, $N=12000$, $N_1=500$, $N_2=1500$, $N_3=2000$, $N_4=3000$, $N_5=5000$, $d^2=40$, $z=1.96$, $D^2=10.412$, $n=1000$, $c'=25000$, $t_1=10$, $t_2=20$, $t_3=30$, $t_4=40$ and $t_5=50$

	The results obtained from goal programming								The results obtained from Kuhn-Tucker method							
α	n_{11}	n_{12}	n_{13}	n_{14}	n_{15}	$V(\bar{x})_1$	cpu_1	1. Cons. Budg.	n_{21}	n_{22}	n_{23}	n_{24}	n_{25}	$V(\bar{x})_2$	cpu_2	2. Cons. Budg.
0.2	75.0570	113.6166	114.1282	150.0917	547.1065	0.00105	84.39	438.0407	8.9476	61.9672	106.3287	228.4535	594.3030	0.00118	200.14	435.3464
0.4	81.3125	103.0263	121.9716	153.5278	540.1618	0.00104	85.92	1309.7	8.9351	61.9087	106.1899	228.3533	594.6130	0.00119	200.68	1316.9
0.5	79.2506	104.2901	122.0161	157.4363	537.0069	*0.00103*	82.18	2285.2	8.9596	61.9427	106.4002	228.6803	594.0172	*0.00108*	201.32	2320.3
0.6	79.1165	105.5777	127.3873	153.2341	534.6844	0.00108	79.96	4000.8	8.9546	61.8709	106.3984	228.2108	594.5653	0.00120	199.34	4117.8
0.8	42.3120	64.7840	111.3294	194.1176	587.4570	0.00106	79.92	12977.5	8.9463	61.8421	106.4447	228.5068	594.2601	0.00119	200.07	13222
1	**200.2000**	**200.2000**	**200.2000**	**200.2000**	**99.6000**	0.00219	81.51	25000	**5.9247**	**35.9314**	**61.9546**	**131.5188**	**342.0546**	0.00205	203.90	24999.9
1.2	**200.2000**	**200.2000**	**200.2000**	**200.2000**	**16.7667**	0.01087	81.39	**59250.6**	**10.3352**	**21.1204**	**30.7460**	**56.5519**	**129.0020**	0.00465	204.35	24890.7
1.5	**100.8454**	**100.2317**	**100.4790**	**100.3852**	**9.1250**	0.08367	81.48	**102022**	**11.8989**	**22.1846**	**23.8122**	**29.2027**	**40.1087**	0.00843	206.90	24999.3
1.8	**50.5259**	**50.2295**	**50.4916**	**50.4331**	**4.1759**	0.08497	79.60	**116712.8**	**7.5169**	**12.8554**	**13.6792**	**16.5465**	**22.0206**	0.01510	206.07	24999.1
2	**25.2665**	**25.4220**	**25.0001**	**25.0002**	**2.0714**	0.08319	75.50	**63274.6**	**5.9107**	**9.7882**	**10.4284**	**12.4855**	**16.2690**	0.02016	205.42	24997.6

5. A COMPARISON OF GOAL PROGRAMMING AND KUHN-TUCKER METHODS

Different situations of population sizes, stratum sizes, costs of travels to strata, sample size of n to be allocated for strata and precision were considered in simulation studies. Possible solutions were sought for different situations of α showing effect of traveling to strata on cost constraint. The cost constraint used in the study is non-linear in the form of $c' = \sum t_h n_h^{\alpha}$.

As α value becomes higher, number of the possible solutions decreases for both of Model 1, which was solved by goal programming, and Model 2, which was solved by Kuhn-Tucker method. As α value becomes higher, the constraints cannot be satisfied for both of the models. As seen in tables, the most efficient α values, which provide possible solutions for Model 1, which was solved by goal programming, and Model 2, which was solved by Kuhn-Tucker method, is $\alpha = 0.5$. If it is $\alpha = 0.5$, variance of sample mean statistics is obtained as the smallest among the possible solutions no matter what cost for traveling to strata is. Also, as α values producing possible solution become higher, the used part of the budget allocated for the research increases.

Model 1 which was solved by goal programming obtained smaller variance of sample mean statistics for α values having possible solutions rather than Model 2 which was solved by Kuhn-Tucker method. However, it should be kept in mind that there is not a single objective in goal programming. Kuhn-Tucker method aims only to minimize variance of sample mean statistics while goal programming method aims to minimize variance of sample mean statistics and the budget used in the research at the same time to allocate sample size for strata. As seen in Table 1, when costs for traveling to strata are close or same, goal programming and Kuhn-Tucker methods use the budget allocated for the research in almost same way. As seen in Table 2, differences exist between costs for traveling to strata while Kuhn-Tucker method generally uses lesser part of the budget allocated for the research compared with goal programming. In goal programming, as α value increases, the used part of the budget shows higher tendency to increase although sample size is shared well compared with Kuhn-Tucker method. We may generalize this as the fact that in goal programming the used part of the budget is higher for higher α values compared with Kuhn-Tucker method. Generally, goal programming achieves smaller variance of sample mean statistics and Kuhn-Tucker method achieves smaller cost; however, vice versa has been encountered rarely. For example, in Table 3, while it is $\alpha = 0.8$, goal programming found the value of $V(\bar{x})_1$ as 0.01212 and the budget used for the research as 904.1116; however, Kuhn-Tucker method found the value of $V(\bar{x})_2$ as 0.00859, and the budget used for the study as 1674.5. Also, the situation in which the desired variance is small was considered in Table 3. In other words, goal value of the 1^{st} priority for goal programming was made small as much as possible. Thus, because variance of sample mean statistics is lower than 0.0143 when it is α =1.5, 1.8 and 2 for goal programming, the 1^{st} priority could not be ensured in goal programming. Situations occurred in which goal programming found smaller values for both of variance of sample mean statistics and the used budget for the research; however, such situations were not encountered in case of Kuhn-Tucker method. For example, goal programming method found lower values for both of variance of sample mean statistics and the used budget for the research compared with Kuhn-Tucker method when it is α =1.2 in Table 1, α =0.2, 0.4, 0.5 and 0.6 in Table 3, α =0.8 and 1 in Table 4, α =0.8 in Table 6, α =1.2 in Table 7, α =0.4, 0.5, 0.6 and 0.8 in Table 10. Furthermore, any occasion in which goal programming method does not produce solution while Kuhn-Tucker method produces solution was not encountered. Moreover, in some cases, Kuhn-Tucker method failed to produce any solution while goal programming produced solution. For example, all goals were satisfied and possible solutions were produced in goal programming when it is α =1.2 in Table 2, α =1 and 1.2 in Table 3, α =1 in Table 6; however, Kuhn-Tucker method could not produce possible solutions. Also, according to the results obtained by simulation, goal programming solves problems in shorter time compared with Kuhn-Tucker method.

5. CONCLUSION

As a result, smaller variances of sample mean statistics were obtained in goal programming method compared with Kuhn-Tucker method. Goal programming method can produce smaller variances of sample mean statistics and the budget at the same time compared with Kuhn-Tucker method. Goal programming method produced possible solutions by satisfying all goals in cases that Kuhn-Tucker method failed to produce any solution. Also, goal programming method produces solution in shorter time. As a result of the aforementioned reasons, it is recommended to use goal programming method in the problem of allocation of sample size for strata in optimum way because it considers both of objectives more than one and produces smaller variance of sample mean statistics which is the only objective in Kuhn-Tucker method.

For a further research; under linear and nonlinear cost constraints, comparing of the variance of sample mean statistic for The Separate Ratio Estimation, The Combined Ratio Estimation and Stratified Cluster Sampling using Stratified Random Sampling can be investigated.

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