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Free Vibration Analysis of Shear Deformable Beams by Discrete Singular Convolution Technique

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ABSTRACT

In this study, free vibration of shear deformable beams was investigated. Discrete singular convolution (DSC) method is used for free vibration problem of numerical solution of shear deformable beams. Numerical results are presented and compared with that available in the literature. It is shown that reasonable accurate results are obtained.

Key Words: *Shear Deformable Beam, Free Vibration, Discrete Singular Convolution.*

1. INTRODUCTION

Beams are widely used as structural component in various engineering applications. Therefore, free vibration analysis of such structures is a most important task for engineer in the design stage of civil, mechanical, aerospace and railroad applications. The shear deformation theory was first demonstrated by Timoshenko [1] for elastic beams. After this, various shear deformation theory were proposed for elastic beams. There are many studies in the literature on theory and analysis of shear deformable beams. The analysis of shear deformable beams and plates have been of interest to researchers for many years since they are found in wide application of various problems in mechanical, aeronautical and structural engineering [2-5]. The majority of the available publications are based on the analytical and numerical solution of shear deformable beams [6-25]. In the past years, discrete singular convolution (DSC) method has become increasingly popular in the numerical solution of initial and boundary value problems [26-30]. These methods can yield accurate solutions with relatively much fewer grid points. It has been also successfully employed for different solid, fluid mechanic and heat transfer problems [31-42]. The main objective of this study is to give a numerical solution of free vibration analysis of Timoshenko beams.

2. DISCRETE SINGULAR CONVOLUTION (DSC)

Discrete singular convolution (DSC) method is a relatively new numerical technique in applied mechanics. The method of discrete singular convolution (DSC) was proposed to solve linear and nonlinear differential equations by Wei [26], and later it was introduced to solid and fluid mechanics by Wei [27,29,30,11] Wei et al. [18, 32], Zhao et al.[33, 34, 36], and Civalek [37-41]. For more details of the mathematical background and application of the DSC method in solving problems in engineering, the readers may refer to some recently published reference [26-35]. In the context of distribution theory, a singular convolution can be defined by [35]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \qquad (1)$$

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Where T is a kind of singular kernel such as Hilbert, Abel and delta type, and $\eta(t)$ is an element of the space of the given test functions. In the present approach, only singular kernels of delta type are chosen. This type of kernel is defined by [33]

$$T(x) = \delta^{(r)}(x); \qquad (r = 0,1,2,...) \tag{2}$$

where subscript r denotes the rth-order derivative of distribution with respect to parameter x. In order to illustrate the DSC approximation, consider a function $F(x)$. In the method of DSC, numerical approximations of a function and its derivatives can be treated as convolutions with some kernels. According to DSC method, the r*th* derivative of a function $F(x)$ can be approximated as [36]

$$F^{(r)}(x) \approx \sum_{i=-M}^{M} \delta_{\Delta,\sigma}^{(r)}(x_i - x_k) f(x_k); \quad (r=0,1,2,...). \tag{3}$$

where Δ is the grid spacing, σ is the DSC parameter, x_k are the set of discrete grid points which are centered around x, and $2M+1$ is the effective kernel, or computational bandwidth. It is also known, the regularized Shannon kernel (RSK) delivers very small truncation errors when it use the above convolution algorithm. The regularized Shannon kernel (RSK) is given by [29]

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right]; \quad \sigma > 0 \tag{4}$$

The researchers have generally used the regularized delta Shannon kernel by this time. The required derivatives of the DSC kernels can be easily obtained using the formulation below

$$\delta_{\Delta,\sigma}^{(r)}(x - x_j) = \frac{d^r}{dx^r}\left[\delta_{\Delta,\sigma}(x - x_j)\right]\Big|_{x = x_i} \tag{5}$$

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3. SOLUTION OF GOVERNING EQUATIONS

The governing equations for free vibration of Timoshenko beam can be written as

$$kGA\frac{d^2W}{dx^2} - kGA\frac{d\theta}{dx} + \rho A\omega^2 W = 0, \qquad (6)$$

$$EI\frac{d^2\theta}{dx^2} + kGA\frac{dW}{dx} - kGA\theta + \rho I\omega^2\theta = 0, \qquad (7)$$

where k shear coefficient, θ is the angular displacement, G is the shear modulus, W is the vertical displacement, and ω is the angular frequency. By using DSC discretization the Eqs. (6-7) take the form

$$kGA\sum_{j=1}^{N}\delta^{(2)}_{\pi/\Delta,\sigma}(\Delta x)W(x_i) - kGA\sum_{j=1}^{N}\delta^{(1)}_{\pi/\Delta,\sigma}(\Delta x)\theta(x_i) = -\rho A\omega^2 W_i, \qquad (8)$$

$$EI\sum_{j=1}^{N}\delta^{(2)}_{\pi/\Delta,\sigma}(\Delta x)\theta(x_i) + kGA\sum_{j=1}^{N}\delta^{(1)}_{\pi/\Delta,\sigma}(\Delta x)W(x_i) = -\rho I\omega^2\theta_i, \qquad (9)$$

Two-types of boundary conditions are considered. These are:

Clamped (C)

$$\theta = 0 \text{ and } W = 0 \qquad (10)$$

Simply supported (S)

$$M = 0 \text{ and } W = 0 \qquad (11)$$

In these equations V and M are the shear and moment resultants and given by

$$V = kGh\left(\frac{\partial W}{\partial x} - \theta\right), \quad M = EI\frac{\partial\theta}{\partial x} \qquad (12,13)$$

After implementation of the given boundary conditions, Eqs. (8) and (9) can be expressed by

$$[\mathbf{R}]\{\mathbf{U}\} = \omega^2\{\mathbf{U}\}, \qquad (14)$$

where $\mathbf{U}$ is the displacements vector, $\mathbf{R}$ is the stiffness matrix. The frequency values for Timoshenko beam are given by the following non-dimensional form

$$\Omega^2 = \omega L^2\sqrt{\frac{\rho A}{EI}} \qquad (15)$$

where ρ is the mass density, A the cross-sectional area, I the second moment of area of cross-section, E the Young's modulus, L is the length of the beam, ω is the circular frequency.

4. RESULTS

The results given in this section are aimed at illustrating the numerical accuracy of the proposed DSC method. The obtained results are listed in Table 1-3. First three frequency parameters of simply supported beam are given in Table 1 for *h/L*=0.02. Where *h/L* is the thickness-to-length ratio of beam. It is observed that a good agreement between the present calculated results and the results of literature [8] has been obtained. The results obtained classical beam theory (CBT) has also been presented. Comparison of fundamental frequency of C-C (both end clamped) Timoshenko beam for different *h*/*L* ratio is listed in Table 2. For a validation, the present results are compared with other published results by using pseudo spectral method [11], the analytical solution using third-order shear deformation theory (TSDT) and the classical beam theory (CBT) by Şimşek and Kocatürk [16]. Table 1 and Table 2 show that good convergence and accuracy of the solutions are obtained by increasing the grid numbers for all cases. It is seen that good results are obtained for beam by using *N*=15 and *M*=16. Non-dimensional frequencies of S-S (both end simply supported) Timoshenko beam for different geometric parameter are given in Table 3. In general, the frequencies decrease with the increasing of *h/L* ratios.

Table 1. Comparison of frequency parameters of S-S Timoshenko beam for *h/L*=0.02 (k=5/6; $\upsilon = 0.3$ as Poisson's ratio).

Mode	CBT	Ref. 8 (N=35)	Ref. 14 (N=35)	DSC N=11	DSC N=15	DSC N=18
1	3.1415	3.14053	3.1405	3.1405	3.1405	3.1405
2	6.2831	6.27471	6.2747	6.2747	6.2747	6.2747
3	9.4247	9.39632	9.3963	9.3965	9.3963	9.3963

Table 2. Comparison of fundamental frequency of C-C Timoshenko beam (k=5/6; $\upsilon = 0.3$ as Poisson's ratio).

h/*L*	Ref. 8 (N=35)	Ref. 14 (N=35)	Ref. 16	DSC N=13	DSC N=15	DSC N=21
0.002	4.7299	4.7308	4.7299	4.7302	4.7302	4.7302
0.01	4.7284	4.7287	4.7284	4.7286	4.7286	4.7286
0.02	4.7235	4.7236	4.7235	4.7238	4.7236	4.7236
0.05	4.6899	4.6899	4.6902	4.6902	4.6899	4.6899

Table 3. Frequency parameters of S-S Timoshenko beam (k=5/6; $\upsilon = 0.3$; N=15).

Mode	*h*/*L*=0.002	*h*/*L*=0.01	*h*/*L*=0.02	*h*/*L*=0.1	*h*/*L*=0.2
1	3.1415	3.1413	3.1405	3.1156	3.0453
2	6.2831	6.2811	6.2747	6.2313	5.6715
3	9.4245	9.4176	9.3963	9.2553	7.8395

5. CONCLUSIONS

In this study, using the DSC method, a numerical approach for the free vibration analysis of shear deformable beam is presented. Several examples were worked to demonstrate the convergence of the method. Excellent convergence behavior and accuracy in comparison with exact results or results obtained by other numerical methods were obtained. Although not provided here, the method is also useful in providing vibration solutions of Euler beam. The present study is being further developed to overcome the convergence problems encountered in the nonlinear vibration analysis of beams.

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