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The Application of GRA Method Base on Choquet Integral Using Spherical Fuzzy Information in Decision Making Problems

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Abstract − Spherical fuzzy set is the generalized structure over existing structures of fuzzy sets to deals with uncertainty and imprecise information in decision making problems. Viewing the effectiveness of the spherical fuzzy set, we developed a decision-making algorithm to deal with multi-criteria decision-making problems. In this paper, we extend operational laws to propose spherical fuzzy Choquet integral weighted averaging (SFCIWA) operator based on spherical fuzzy numbers. Further, the proposed SFCIWA operator is applied to multi-attribute group decision-making problems. Also, we propose the GRA method to aggregate the spherical fuzzy information. To implement the proposed models, we provide some numerical applications of group decision-making problems. Also compared with the previous model, we conclude that the proposed technique is more effective and reliable.

Keywords − *Choquet integral, Spherical fuzzy Choquet integral weighted averaging (SFCIWA) operator, GRA method, Spherical fuzzy sets, Decision making technique.*

1. Introduction

Multi-criteria group decision making problems have importance in most kinds of fields such as economics, engineering and management. Generally, it has been assumed that the information which accesses the alternatives in term of criteria and weight are expressed in real numbers. But due to the complexity of the system day-by-day, it is difficult for the decision makers to make a perfect decision, as most of the preferred value during the decision-making process imbued with uncertainty. In order to handle the uncertainties and fuzziness, intuitionistic fuzzy set [11] theory is one of the prosperous extensions of the fuzzy set theory [45], which is characterized by the degree of membership and degree of non-membership has been presented. Fuzzy set theory is extended in many ways by different authors but to modelling imprecision IFS theory is much impressive. IFS theory attracts many authors because of its important in handling uncertainty and different aggregation operators are defined to aggregate information. For study the aggregation operators for IFSs, we refer to [25, 28, 41, 42].

But there are several cases where the decision maker may provide the degree of membership and nonmembership of a particular attribute in such a way that their sum is greater than one. For example, suppose a man expresses his preferences towards the alternative in such a way that degree of their satisfaction is 0.6 and degree of rejection is 0.8. Clearly its sum is greater than one. Therefore, Yager

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[43, 44] introduced the concept of another set called Pythagorean fuzzy set. Pythagorean fuzzy set is more powerful tool to solve uncertain problems. Like intuitionistic fuzzy operators, Pythagorean fuzzy operators also become an interesting and important area for research, after the advent of Pythagorean fuzzy sets theory. Yager and Abbasov [43] introduced many aggregation operators to tackle MADM tangle in PyFS environment. The superiority and inferiority ranking (SIR) MABAC technique to tackle MADM problems in Pythagorean fuzzy environment is discussed by Peng and Yang [31]. Zhang [48] proposed an approach for multi-criteria Pythagorean fuzzy decision analysis based on the closeness index-ranking methods. Khan et al. [23] proposed the Pythagorean fuzzy Dombi aggregation operators and discussed their applications in decision making problems.

As for human nature only, satisfaction and dissatisfaction degree is quite insufficient and needs abstain and refusal degree too but Yager Pythagorean concept not covers this problem. This problem is solved by Coung [15] defining a new structure called picture fuzzy set (PFS) which also includes degree of neutral membership with a condition that the sum of triplet should remain with in unit interval and covers all aspects of human nature and quite applicable in real life problems and very near to the human nature. Cuong [17] in 2014 introduced the concept of picture soft sets, the relation of compositions and the distance between picture fuzzy numbers. Singh [38] in 2015, proposed the idea related to correlation coefficients for picture fuzzy sets. The concepts like convex combination of PFNs, alpha-cuts of PFS, picture fuzzy relations are introducing by Cuong [16] in 2015. Generalized picture fuzzy distance measure are developed by Son [39] in 2016, and discussed their applications. Ashraf et al. [1] proposed the geometric aggregation operators for picture fuzzy information and in [2] proposed the concept of picture fuzzy linguistic set. Khan et al. [24] proposed the concept of generalized picture fuzzy soft set (GPFSSs) and illustrate the applications of GPFSSs in decision making problems. For more study about decision making techniques we refer to [3, 4, 26, 27, 32–35]

Ashraf et al. [5] proposed the novel concept of spherical fuzzy set by applying extra condition on sum of their memberships as square sum of the membership degrees oscillate from 0 to 1. Ashraf and Abdullah [6] proposed the series of aggregation operators for spherical fuzzy environment and in [7] proposed the notion of spherical linguistic fuzzy set and developed their applications using GRA technique. For more study about spherical fuzzy sets, we refer to [8–10, 21, 22, 36, 47]

In this paper, our aim to develop the GRA technique with unknown weight information using spherical fuzzy information to deal with uncertainty in decision making problems. To do this, the article structured is follows as:

Basic definitions and result about Choquet integral, Pythagorean fuzzy sets and picture fuzzy sets are present in Sec. 2. In Sec. 3, we introduced the notion of Spherical fuzzy sets. In Sec. 4 we proposed the GRA method for spherical fuzzy MAGDM problems with incomplete weight data. In Section 5, we strengthen our proposed algorithmic method with a descriptive example. Last Sections contains the conclusion of the work.

2. Preliminary

This section consists of some basic concepts of Pythagorean fuzzy set (PyFS), picture fuzzy set (PFS) and also give some discussion related to fuzzy measure and Choquet integral.

Definition 2.1. [44]A PyFS $\Im_{\breve{u}}$ on the universe of discourse $\mathbb{Z} \neq \phi$ is defined as;

$$\Im_{\breve{u}} = \{\langle L_{\breve{e}_{\breve{u}}}(k), M_{\breve{e}_{\breve{u}}}(k) |\ k \in \mathbb{Z}\rangle\}.$$

A PyFS in a set $\mathbb{Z}$ is defined by $L_{\breve{e}_{\breve{u}}}(k) : \mathbb{Z} \to \Theta$ and $M_{\breve{e}_{\breve{u}}}(k) : \mathbb{Z} \to \Theta$ are the positive and negative membership grades of each $k \in \mathbb{Z}$, respectively. Furthermore $L_{\breve{e}_{\breve{u}}}(k)$ and $M_{\breve{e}_{\breve{u}}}(k)$ satisfy $0 \leq L^2_{\breve{e}_{\breve{u}}}(k) + M^2_{\breve{e}_{\breve{u}}}(k) \leq 1$ for all $k \in \mathbb{Z}$.

Definition 2.2. [15]A PFS $\Im_{\breve{u}}$ on the universe of discourse $\mathbb{Z} \neq \phi$ is defined as;

$$\Im_{\breve{u}} = \{\langle L_{\breve{e}_{\breve{u}}}(k), M_{\breve{e}_{\breve{u}}}(k), O_{\breve{e}_{\breve{u}}}(k) |\ k \in \mathbb{Z}\rangle\}.$$

A PFS in a set $\mathbb{Z}$ is defined by $L_{\breve{e}_{\breve{u}}}(k) : \mathbb{Z} \to \Theta$, $M_{\breve{e}_{\breve{u}}}(k) : \mathbb{Z} \to \Theta$ and $O_{\breve{e}_{\breve{u}}}(k) : \mathbb{Z} \to \Theta$ are the positive grade, neutral grade and negative grade of each $k \in \mathbb{Z}$, respectively. Furthermore $L_{\breve{e}_{\breve{u}}}(k)$, $M_{\breve{e}_{\breve{u}}}(k)$ and $O_{\breve{e}_{\breve{u}}}(k)$ satisfy $0 \leq L_{\breve{e}_{\breve{u}}}(k) + M_{\breve{e}_{\breve{u}}}(k) + O_{\breve{e}_{\breve{u}}}(k) \leq 1$ for all $k \in \mathbb{Z}$.

2.1. Fuzzy measure and Choquet integral

The concept of fuzzy measure are developed by Sugeno in 1974 [48] which instead of additivity property only make a monotonicity . It is a powerful tool for modeling interaction phenomena in decision making for MADM problems, it does not required assumption that criteria or preferences are free from one another. Criteria can be dependent in the Choquet integral model [14,31], where on each combination of criteria a fuzzy measure is used to define a weight, thus making it possible to model the interaction existing among criteria. Concept of fuzzy measure, discrete Choquet integral, λ-fuzzy measure and Pythagorean fuzzy Choquet integral operators are presented in this subsection as follows;

Definition 2.3. [14] Let the universe of discourse $\mathbb{Z} = \{k_1, ..., k_n\} \neq \phi$ and $p(\mathbb{Z})$ denote the power set of $\mathbb{Z}$. Then, a function $L_{\breve{e}_{\breve{u}}} : p(\mathbb{Z}) \to \Theta$ is called a fuzzy measure $L_{\breve{e}_{\breve{u}}}$ on $\mathbb{Z}$, if satisfy the following conditions;

1) $L_{\breve{e}_{\breve{u}}}(\phi) = 0$, $L_{\breve{e}_{\breve{u}}}(\mathbb{Z}) = 1$.

2) If $\Im_{\breve{u}_1}, \Im_{\breve{u}_2} \in p(\mathbb{Z})$ and $\Im_{\breve{u}_1} \subseteq \Im_{\breve{u}_2}$ then $L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1}) \leq L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_2})$.

It is mandatory to consider the adage of continuity when $\mathbb{Z}$ is infinite, it is enough to assume a finite universe of discourse in genuine exercise. For decision attribute set $\{k_1, k_2, ..., k_n\}$, $L_{\breve{e}_{\breve{u}}}(\{k_1, k_2, ..., k_n\})$ can be deem as the degree of subjective importance. Thus, weights of any set of attributes can also be obtained with the separate weights of attributes. Instinctively, we say that the following about any pair of criteria sets $\Im_{\breve{u}_1}, \Im_{\breve{u}_2} \in p(\mathbb{Z})$, $\Im_{\breve{u}_1} \cap \Im_{\breve{u}_2} = \phi$; $\Im_{\breve{u}_1}$and $\Im_{\breve{u}_2}$ are assumed to be without interaction (or to be independent) and called it additive measure if

$$L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1} \cup \Im_{\breve{u}_2}) = L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1}) + L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_2}). \tag{1}$$

$\Im_{\breve{u}_1}$and $\Im_{\breve{u}_2}$ reveals a positive synergetic interaction among them (or are complementary) and called a super additive measure if

$$L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1} \cup \Im_{\breve{u}_2}) > L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1}) + L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_2}). \tag{2}$$

$\Im_{\breve{u}_1}$and $\Im_{\breve{u}_2}$ reveals a negative synergetic interaction among them (or are redundant or substitutive) and said to be a sub-additive measure if

$$L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1} \cup \Im_{\breve{u}_2}) < L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1}) + L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_2}). \tag{3}$$

From the Definition 2.3 it is hard to find the fuzzy measure, therefore, Sageno defined the following measure to confirm a fuzzy measure in MAGDM problems:

$$L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1} \cup \Im_{\breve{u}_2}) = L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1}) + L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_2}) + \lambda L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1}) L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_2}) \tag{4}$$

$\lambda \in [-1, \infty)$, $\Im_{\breve{u}_1} \cap \Im_{\breve{u}_2} = \phi$. The interaction between the attributes is determine by the parameter λ. Simply an additive measure is obtained when $\lambda = 0$ in Equation 4. Sub additive and super additive measures is obtained, respectively for negative and positive λ. Meanwhile, if all the elements in $\mathbb{Z}$ are independent, and we have

$$L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}}) = \sum_{p=1}^{n} L_{\breve{e}_{\breve{u}}}(\{k_p\}) \tag{5}$$

If $\mathbb{Z}$ is a finite set, then $\cup_{\mathrm{p}=1}^{n} k_{\mathrm{p}} = \mathbb{Z}$. The λ-fuzzy measure $L_{\breve{e}_{\breve{u}}}$ satisfies following Equation6

$$L_{\breve{e}_{\breve{u}}}(\mathbb{Z}) = L_{\breve{e}_{\breve{u}}}\left(\cup_{p=1}^{n} k_i\right) = \begin{cases} \frac{1}{\lambda}\left(\prod_{p=1}^{n}\left[1 + \lambda L_{\breve{e}_{\breve{u}}}(k_p)\right] - 1\right) & \text{if } \lambda \neq 0 \\ \sum_{p=1}^{n} L_{\breve{e}_{\breve{u}}}(k_p) & \text{if } \lambda = 0 \end{cases} \tag{6}$$

where $k_{\mathrm{p}} \cap r_{\check{\mathrm{d}}} = \phi$ for all p,$\check{\mathrm{d}}$= $1, ..., n$ and $p \neq \check{\mathrm{d}}$. It should be noted that $L_{\breve{e}_{\breve{u}}}(k_{\mathrm{p}})$ for a subset with a single member k_{p} is called a fuzzy density, and can be signified as $L_{\breve{e}_{\breve{u}}} = L_{\breve{e}_{\breve{u}}}(k_{\mathrm{p}})$.

Particularly for every subset $\Im_{\breve{u}_1} \in p(\mathbb{Z})$, we have

$$L_{\breve{e}_{\breve{u}}}(\Im_{\breve{u}_1}) = \begin{cases} \frac{1}{\lambda}\left(\prod_{p=1}^{n}\left[1+\lambda L_{\breve{e}_{\breve{u}}}(k_p)\right]-1\right) & \text{if } \lambda \neq 0 \\ \sum_{p=1}^{n} L_{\breve{e}_{\breve{u}}}(k_p) & \text{if } \lambda = 0 \end{cases} \tag{7}$$

A uniquely value of λ is determined from $L_{\breve{e}_{\breve{u}}}(\mathbb{Z}) = 1$, based on Equation2 which is equivalent to solving

$$\lambda + 1 = \prod_{p=1}^{n}\left[1+\lambda L_{\breve{e}_{\breve{u}}}\right] \tag{8}$$

It can be seen that λ are uniquely obtained by $L_{\breve{e}_{\breve{u}}}(\mathbb{Z}) = 1$.

Definition 2.4. [14] Let g and $L_{\breve{e}_{\breve{u}}}$ be a positive real-valued function on the fuzzy measure $\mathbb{Z}$, respectively. Then, the discrete Choquet integral of g with respect to $L_{\breve{e}_{\breve{u}}}$ is defined by

$$C_{\mu}(g) = \sum_{p=1}^{n} g_{\rho(p)}[L_{\breve{e}_{\breve{u}}}(A_{\rho(p)}) - L_{\breve{e}_{\breve{u}}}(A_{\rho(\text{p-1})})] \tag{9}$$

where $\rho(p)$ shows a permutation on $\mathbb{Z}$ such that $g_{\rho(1)} \geq g_{\rho(2)} \geq ... \geq g_{\rho(\text{n})}$, $A_{\rho(\text{n})} = \{1,2,...,p\}$, $A_{\rho(0)} = \phi$.

Up to a reordering of the elements it can be noticed that the discrete Choquet integral is a linear expression. Moreover, when the fuzzy measure is additive it identifies with the weighted mean (discrete Lebesgue integral). And OWA operator the Choquet integral operator coincides in some conditions.

3. Some Operations on Spherical Fuzzy Set

The notion of SFS and their operational laws are defined in this section.

Definition 3.1. [5]A SFS $\Im_{\breve{u}}$ on the universe of discourse $\mathbb{Z} \neq \phi$ is defined as;

$$\Im_{\breve{u}} = \{\langle L_{\breve{e}_{\breve{u}}}(k), M_{\breve{e}_{\breve{u}}}(k), O_{\breve{e}_{\breve{u}}}(k) |\ k \in \mathbb{Z}\rangle\}. \tag{10}$$

Where $L_{\breve{e}_{\breve{u}}}(k) : \mathbb{Z} \to \Theta$, $M_{\breve{e}_{\breve{u}}}(k) : \mathbb{Z} \to \Theta$ and $O_{\breve{e}_{\breve{u}}}(k) : \mathbb{Z} \to \Theta$ are the positive grade, neutral grade and negative grade of each $k \in \mathbb{Z}$, respectively. Furthermore $L_{\breve{e}_{\breve{u}}}(k)$, $M_{\breve{e}_{\breve{u}}}(k)$ and $O_{\breve{e}_{\breve{u}}}(k)$ satisfy $0 \leq L^2_{\breve{e}_{\breve{u}}}(k) + M^2_{\breve{e}_{\breve{u}}}(k) + O^2_{\breve{e}_{\breve{u}}}(k) \leq 1$ for all $k \in \mathbb{Z}$. $\chi_{\Im_{\breve{u}}}(k) = \sqrt{1 - \left(L^2_{\breve{e}_{\breve{u}}}(k) + M^2_{\breve{e}_{\breve{u}}}(k) + O^2_{\breve{e}_{\breve{u}}}(k)\right)}$ is called refusal degree of k in $\mathbb{Z}$, for SFS $\{\langle L_{\breve{e}_{\breve{u}}}(k), M_{\breve{e}_{\breve{u}}}(k), O_{\breve{e}_{\breve{u}}}(k) |\ k \in \mathbb{Z}\rangle\}$, which is triple components $\langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}}\rangle$ is called SF number and each SF number can be presented as $E = \langle L_e, M_e, O_e\rangle$, where L_e, M_e and $O_e \in \Theta$, under the condition $0 \leq L^2_e + M^2_e + O^2_e \leq 1$.

Definition 3.2. Let $\Im_{\breve{u}_1} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}}\rangle$ and $\Im_{\breve{u}_2} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}}\rangle$ are two SFNs define on the universe of discourse $\mathbb{Z} \neq \phi$, some operations on SFNs are defined as follows:

(a) $\Im_{\breve{u}_1} \subseteq \Im_{\breve{u}_2} \quad iff \quad \forall r \in R$,

$$L_{\breve{e}_{\breve{u}}} \leq \Im_{\breve{u}_2}, M_{\breve{e}_{\breve{u}}} \leq M_{\breve{e}_{\breve{u}}} \text{ and } O_{\breve{e}_{\breve{u}}} \geq O_{\breve{e}_{\breve{u}}} \tag{11}$$

(b) $\Im_{\breve{u}_1} = \Im_{\breve{u}_2} \quad iff$

$$\Im_{\breve{u}_1} \subseteq \Im_{\breve{u}_2} \text{ and } \Im_{\breve{u}_2} \subseteq \Im_{\breve{u}_1} \tag{12}$$

(c) Union

$$\Im_{\breve{u}_1} \cup \Im_{\breve{u}_2} = \langle \max(L_{\breve{e}_{\breve{u}}}, L_{\breve{e}_{\breve{u}}}),\ \min(M_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}),\ \min(O_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}})\rangle; \tag{13}$$

(d) Intersection

$$\Im_{\breve{u}_1} \cap \Im_{\breve{u}_2} = \langle \min(L_{\breve{e}_{\breve{u}}}, L_{\breve{e}_{\breve{u}}}),\ \min(M_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}),\ \max(O_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}})\rangle; \tag{14}$$

(e) Compliment

$$\Im^c_{\breve{u}} = \langle O_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, L_{\breve{e}_{\breve{u}}}\rangle. \tag{15}$$

Definition 3.3. Let $\Im_{\breve{u}_1} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}}\rangle$ and $\Im_{\breve{u}_2} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}}\rangle$ are two SFNs define on the universe of discourse $\mathbb{Z} \neq \phi$, some operations on SFNs are defined as follows with $\tau \geq 0$.

(1) $\Im_{\breve{u}_1} \oplus \Im_{\breve{u}_2} = \left\{\sqrt{L^2_{\breve{e}_{\breve{u}}} + L^2_{\breve{e}_{\breve{u}}} - L^2_{\breve{e}_{\breve{u}}} \cdot L^2_{\breve{e}_{\breve{u}}}},\ M_{\breve{e}_{\breve{u}}} \cdot M_{\breve{e}_{\breve{u}}},\ O_{\breve{e}_{\breve{u}}} \cdot O_{\breve{e}_{\breve{u}}}\right\}$.

(2) $\tau \cdot \Im_{\breve{u}} = \left\{\sqrt{1 - (1 - L^2_{\breve{e}_{\breve{u}}})^{\tau}},\ (M_{\breve{e}_{\breve{u}}})^{\tau},\ (O_{\breve{e}_{\breve{u}}})^{\tau}\right\}$.

3.1. Comparison Rules for SFNs

For ranking the SFNs, different functions are introduced in this section described as.

Definition 3.4. Let $\Im_{\breve{u}} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$ be any SFNs. Then
(1) Score function is defined as $sc(\Im_{\breve{u}}) = \frac{(L_{\breve{e}_{\breve{u}}}+1-M_{\breve{e}_{\breve{u}}}+1-O_{\breve{e}_{\breve{u}}})}{3} = \frac{1}{3}(2 + L_{\breve{e}_{\breve{u}}} - M_{\breve{e}_{\breve{u}}} - O_{\breve{e}_{\breve{u}}})$.
(2) Accuracy function is defined as $acu(\Im_{\breve{u}}) = L_{\breve{e}_{\breve{u}}} - O_{\breve{e}_{\breve{u}}}$.
(3) Certainty function is defined as $cr(\Im_{\breve{u}}) = L_{\breve{e}_{\breve{u}}}$.

Ranking of SFNs described from Definition 3.4 .

Definition 3.5. Let $\Im_{\breve{u}_1} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$ and $\Im_{\breve{u}_2} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$ are two SFNs define on the universe of discourse $\mathbb{Z} \neq \phi$. Then Ranking of SFNs described from Definition 3.4 ,
(1) If $sc(\Im_{\breve{u}_1}) \succ sc(\Im_{\breve{u}_2})$,then $\Im_{\breve{u}_1} \succ \Im_{\breve{u}_2}$.
(2) If $sc(\Im_{\breve{u}_1}) \approx sc(\Im_{\breve{u}_2})$,and $acu(\Im_{\breve{u}_1}) \succ acu(\Im_{\breve{u}_2})$,then $\Im_{\breve{u}_1} \succ \Im_{\breve{u}_2}$.
(3) If $sc(\Im_{\breve{u}_1}) \approx sc(\Im_{\breve{u}_2})$, $acu(\Im_{\breve{u}_1}) \approx acu(\Im_{\breve{u}_2})$ and $cr(\Im_{\breve{u}_1}) \succ cr(\Im_{\breve{u}_2})$, then $\Im_{\breve{u}_1} \succ \Im_{\breve{u}_2}$.
(4) If $sc(\Im_{\breve{u}_1}) \approx sc(\Im_{\breve{u}_2})$, $acu(\Im_{\breve{u}_1}) \approx acu(\Im_{\breve{u}_2})$ and $cr(\Im_{\breve{u}_1}) \approx cr(\Im_{\breve{u}_2})$, then $\Im_{\breve{u}_1} \approx \Im_{\breve{u}_2}$.

Definition 3.6. Let any collections $\Im_{\breve{u}_{\mathrm{p}}} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$, $p \in N$ be the SFNs and $SFWA : SFN^n \times SFN^n \to SFN$, then $SFWA$ describe as,

$$SFWA\left(\Im_{\breve{u}_1}, \Im_{\breve{u}_2}, ..., \Im_{\breve{u}_n}\right) = \sum_{p=1}^{n} \tau_p \Im_{\breve{u}_p}, \tag{16}$$

In which $\tau = \{\tau_1, ..., \tau_n\}^T$ be the weight vector of $\Im_{\breve{u}_{\mathrm{p}}} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$, $p \in N$, with $\tau_{\mathrm{p}} \geq 0$ and $\sum_{\mathrm{p}=1}^{n} \tau_{\mathrm{p}} = 1$.

Theorem 3.7. Let any collections $\Im_{\breve{u}_{\mathrm{p}}} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$, $p \in N$ be the SFNs. Then operational properties of SFNs can obtained by utilizing the Definition 3.6 as.

$$SFWA\left(\Im_{\breve{u}_1}, \Im_{\breve{u}_2}, ..., \Im_{\breve{u}_n}\right) = \left\{\sqrt{1 - \Pi_{p=1}^{n}(1 - L_{\breve{e}_{\breve{u}}}^2)^{\tau_p}}, \Pi_{p=1}^{n}(M_{\breve{e}_{\breve{u}}})^{\tau_p}, \Pi_{p=1}^{n}(O_{\breve{e}_{\breve{u}}})^{\tau_p}\right\}. \tag{17}$$

Definition 3.8. Let any collections $\Im_{\breve{u}_{\mathrm{p}}} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$, $p \in N$ be the SFNs and $SFOWA : SFN^n \times SFN^n \to SFN$, then $SFOWA$ describe as,

$$SFOWA\left(\Im_{\breve{u}_1}, \Im_{\breve{u}_2}, ..., \Im_{\breve{u}_n}\right) = \sum_{p=1}^{n} \tau_p \Im_{\breve{u}_{\rho(p)}}, \tag{18}$$

In which $\tau = \{\tau_1, \tau_2, ..., \tau_n\}$ be the weight vector of $\Im_{\breve{u}_{\mathrm{p}}} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$, $p \in N$, with $\tau_{\mathrm{p}} \geq 0$ and $\sum_{\mathrm{p}=1}^{n} \tau_{\mathrm{p}} = 1$ and $\rho(p)$ indicates a permutation on $\mathbb{Z}$.

Theorem 3.9. Let any collections $\Im_{\breve{u}_{\mathrm{p}}} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$, $p \in N$ be the SFNs. Then operational properties of SFNs can obtained by utilizing the Definition 3.8 as,

$$SFOWA\left(\Im_{\breve{u}_1}, \Im_{\breve{u}_2}, ..., \Im_{\breve{u}_n}\right) = \left\{\sqrt{1 - \Pi_{p=1}^{n}(1 - L_{\breve{e}_{\breve{u}}}^2)^{\tau_p}}, \Pi_{p=1}^{n}(M_{\breve{e}_{\breve{u}}})^{\tau_p}, \Pi_{p=1}^{n}(O_{\breve{e}_{\breve{u}}})^{\tau_p}\right\}. \tag{19}$$

Theorem 3.10. Let any collections $\Im_{\breve{u}_{\mathrm{p}}} = \langle L_{\breve{e}_{\breve{u}}}, M_{\breve{e}_{\breve{u}}}, O_{\breve{e}_{\breve{u}}} \rangle$, $p \in N$ be the SFNs and λ be a fuzzy measure on $\mathbb{Z}$. Based on fuzzy measure, a spherical fuzzy Choquet integral weighted averaging ($SFCIWA$) operator of dimension n is a mapping $SFCIWA : SFN^n \times SFN^n \to SFN$ such that

$$SFCIWA\left(\Im_{\breve{u}_1}, \Im_{\breve{u}_2}, ..., \Im_{\breve{u}_n}\right) = \left\{\begin{array}{c} \sqrt{1 - \Pi_{p=1}^{n}(1 - L_{\breve{e}_{\breve{u}}}^2)^{\lambda\left(A_{\rho(\mathrm{p})}\right) - \lambda\left(A_{\rho(\mathrm{p}\text{-}1)}\right)}}, \\ \Pi_{p=1}^{n}(M_{\breve{e}_{\breve{u}}})^{\lambda\left(A_{\rho(\mathrm{p})}\right) - \lambda\left(A_{\rho(\mathrm{p}\text{-}1)}\right)}, \\ \Pi_{p=1}^{n}(O_{\breve{e}_{\breve{u}_{\rho(p)}}})^{\lambda\left(A_{\rho(\mathrm{p})}\right) - \lambda\left(A_{\rho(\mathrm{p}\text{-}1)}\right)} \end{array}\right\}$$

where $\rho(p)$ indicates a permutation on $\mathbb{Z}$ and $A_{\rho(\mathrm{n})} = \{1, ..., p\}$, $A_{\rho(0)} = \phi$.

Definition 3.11. Let $\mathbb{Z} \neq \phi$ be the universe of discourse and any two spherical fuzzy sets $\Im_j$, $\Im_l$. Then normalized Hamming distance $f_{NHD}(\Im_j, \Im_l)$ is given as for all $k \in \mathbb{Z}$,

$$f_{NHD}(\Im_j, \Im_l) = \frac{1}{n} \sum_{p=1}^{n} \left(\begin{array}{c} \left|L_{\Im_j}(k_p) - L_{\Im_l}(k_p)\right| + \left|M_{\Im_j}(k_p) - M_{\Im_l}(k_p)\right| + \\ \left|O_{\Im_j}(k_p) - O_{\Im_l}(k_p)\right| \end{array} \right). \tag{20}$$

Definition 3.12. Let $\mathbb{Z} \neq \phi$ be the universe of discourse and any two spherical fuzzy sets $\Im_j$, $\Im_l$. Then normalized Euclidean distance $f_{NED}(\Im_j, \Im_l)$ is given as for all $k \in \mathbb{Z}$,

$$f_{NED}(\Im_j, \Im_l) = \sqrt{\frac{1}{n}\sum_{p=1}^{n}\left(\begin{array}{c}(L_{\Im_j}(k_p) - L_{\Im_l}(k_p))^2 + (M_{\Im_j}(k_p) - M_{\Im_l}(k_p))^2 + \\ (O_{\Im_j}(k_p) - O_{\Im_l}(k_p))^2\end{array}\right)}. \tag{21}$$

4. GRA method for multiple attribute decision making with incomplete weight information in Spherical fuzzy setting

Suppose that $A = \{b_1, ..., b_n\}$, n alternatives and $C = \{d_1, ..., c_m\}$, m alternatives,weight vector for parameter is $\nu = (\nu_1, ..., \nu_m)$, where $\nu_k \geq 0$ $(k = 1, ..., m)$, $\Sigma_{k=1}^{n}\nu_k = 1$. Assume that the DM give information about weights of criteria may be denotes in the following form, for $j \neq k$,

(1) If $\{\nu_j \geq \nu_k\}$ (weak ranking).
(2) If $\{\nu_j - \nu_k \geq \lambda_j (> 0)\}$, (strict ranking).
(3) If $\{\nu_j \geq \lambda_j \nu_k\}$, $0 \leq \lambda_j \leq 1$,(multiple ranking).
(4) If $\{\lambda_j \leq \nu_j \leq \lambda_j + \delta_j\}$, $0 \leq \lambda_j \leq \lambda_j + \delta_j \leq 1$, (interval ranking).

Δ denoted the set of the known information about the attribute weights provided by the experts. The decision maker $f_k (k = 1, ..., l)$ give the following decision matrix;

$$R^k = \left[\Im^{(k)}_{\breve{u}_{pq}}\right]_{m\times n} = \begin{array}{c} \\ b_1 \\ b_2 \\ \vdots \\ b_m \end{array}\left(\begin{array}{cccc} d_1 & d_2 & \cdots & d_n \\ \Im^{(k)}_{u_{11}} & \Im^{(k)}_{u_{12}} & \cdots & \Im^{(k)}_{u_{1n}} \\ \Im^{(k)}_{u_{21}} & \Im^{(k)}_{u_{22}} & \cdots & \Im^{(k)}_{u_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ \Im^{(k)}_{u_{m1}} & \Im^{(k)}_{u_{m2}} & \cdots & \Im^{(k)}_{u_{mn}} \end{array}\right)$$

where $\Im^{(k)}_{\breve{u}_{pq}} = \left(L^{(k)}_{\breve{u}_{pq}}, M^{(k)}_{\breve{u}_{pq}}, O^{(k)}_{\breve{u}_{pq}}\right)$ is an SFN representing the performance rating of the alternative $a_p \in A$ with respect to the attribute $c_p \in C$ provided by the decision makers d_k. To extend GRA method in the process of group decision making, we first need to fuse all individual decision matrices into a collective matrix by using SFCIW operator.

Step:1 Suppose that we have m alternative, $A = \{b_1, b_2, ..., b_m\}$,and n attributes $C_q (q = 1, 2, ..., n)$, now we invited each expert d$_k$ $(k = 1, 2, ..., r)$ to express their individual preference according to each by an spherical fuzzy numbers $\Im^{(k)}_{\breve{u}_{pq}} = \left(L^{(k)}_{\breve{u}_{pq}}, M^{(k)}_{\breve{u}_{pq}}, O^{(k)}_{\breve{u}_{pq}}\right)$ $(p = 1, 2, ..., m; q = 1, 2, ..., n, r = 1, 2, ..., k)$ expressed by the experts f_r. Then, we obtain a decision making matrices, $D^s = \left[E^{(s)}_{ip}\right]_{m\times n}$ $(s = 1, 2, ..., r)$ for decision. But there are two types of criteria, such as benefit and cost criteria, then we convert the decision matrices, $D^s = \left[E^{s}_{ip}\right]_{m\times n}$ into the normalized spherical fuzzy decision matrices, $R^r = \left[\Im^{(r)}_{\breve{u}_{pq}}\right]_{m\times n}$, by the following rules; $\Im^{(r)}_{\breve{u}_{pq}} = \begin{cases} \Im^{r}_{\breve{u}_{pq}}, \text{ for benefit criteria } A_p \\ \overline{\Im^{(r)}_{\breve{u}_{pq}}}, \text{ for cost criteria } A_p, \end{cases}$ $j = 1, 2, ..., n$, and $\overline{\Im^{(r)}_{\breve{u}_{pq}}}$ is the complement of $\Im^{(r)}_{\breve{u}_{pq}}$. If all the criteria have the same type, then there is no need of normalization.

Step:2 Confirm the fuzzy density $L_{\breve{e}_{\breve{u}}} = L_{\breve{e}_{\breve{u}}}(a_p)$ of each expert. According to Eq.(8), parameter λ_1 of expert can be determined.

Step:3 $\Im^{(r)}_{\breve{u}_{pq}}$ is reordered such that $\Im^{(r)}_{\breve{u}_{pq}} \geq \Im^{(r-1)}_{\breve{u}_{pq}}$. Using the SF Choquet integral average operator;

$$SFCIWA\left(\Im^{(1)}_{\breve{u}_{pq}}, \Im^{(2)}_{\breve{u}_{pq}}, ..., \Im^{(r)}_{\breve{u}_{pq}}\right) = \left\{\begin{array}{c} \sqrt{1 - \Pi^{r}_{p=1}(1 - L_{\breve{e}_{\breve{u}}})^{\lambda(A_{\rho(p)}) - \lambda(A_{\rho(p-1)})}}, \\ \Pi^{r}_{p=1}(M_{\breve{e}_{\breve{u}}})^{\lambda(A_{\rho(p)}) - \lambda(A_{\rho(p-1)})}, \\ \Pi^{r}_{p=1}(O_{\breve{e}_{\breve{u}_{\rho(p)}}})^{\lambda(A_{\rho(p)}) - \lambda(A_{\rho(p-1)})} \end{array}\right\}$$

to aggregate all the spherical fuzzy decision matrices $R^r = \left[\Im^{(r)}_{\breve{u}_{pq}}\right]_{m\times n}$ $(r = 1, ..., k)$ into a collective spherical fuzzy decision matrix $R = \left[\Im^{(r)}_{\breve{u}_{pq}}\right]_{m\times n}$ where $\Im^{(r)}_{\breve{u}_{pq}} = \left(L^{(r)}_{\breve{u}_{pq}}, M^{(r)}_{\breve{u}_{pq}}, O^{(r)}_{\breve{u}_{pq}}\right)$ $(p =$

$1,...,m; q = 1,...,n, r = 1,...,k$), where $\rho(p)$ indicates a permutation on $\mathbb{Z}$ and $A_{\rho(n)} = \{1,...,p\}$, $A_{\rho(0)} = \phi$ and $L_{\breve{e}_{\breve{u}}}(a_p)$ can be calculated by Eq. (9).

Step:4 $L^+ = \{L_1^+, L_2^+, ..., L_m^+\}$ and $P^- = \{P_1^-, P_2^-, ..., P_m^-\}$ are the SFPIS and SFNIS, respectively.

$$L_p^+ = \max_{\mathrm{q}} sc_{pq} \tag{22}$$

and

$$P_p^- = \min_{q} sc_{pq}, \tag{23}$$

where $L^+ = \left(L^+_{\breve{u}_{\mathrm{p}}}, I^+_{\breve{u}_{\mathrm{p}}}, N^+_{\breve{u}_{\mathrm{p}}}\right)$ and $P^- = \left(P^-_{\breve{u}_{\mathrm{p}}}, I^-_{\breve{u}_{\mathrm{p}}}, N^-_{\breve{u}_{\mathrm{p}}}\right)$ $p = 1,..,m$.

Step:5 Calculate the distance between the alternative a_{p} and the SFPIS L^+, and SFNIS P^-, respectively;

$$f(e_j, e_k) = \frac{1}{n}\sum_{p=1}^{n}\left(\left|P_{e_j}(a_p) - P_{e_k}(a_p)\right| + \left|I_{e_j}(a_p) - I_{e_k}(a_p)\right| + \left|N_{e_j}(a_p) - N_{e_k}(a_p)\right|\right). \tag{24}$$

This distance is known to be Normalized Hamming distance [1] $d(e_j, e_k)$, and construct an spherical fuzzy positive-ideal separation matrix D^+ and Spherical fuzzy negative-ideal separation matrix D^- as follows;

$$\begin{matrix} f\left(\Im_{u_{11}}, L_1^+\right) & f\left(\Im_{u_{12}}, L_2^+\right) & . & . & . & f\left(\Im_{u_{1n}}, L_n^+\right) \\ f\left(\Im_{u_{21}}, L_1^+\right) & f\left(\Im_{u_{22}}, L_2^+\right) & . & . & . & f\left(\Im_{u_{1n}}, L_n^+\right) \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ f\left(\Im_{u_{m1}}, L_1^+\right) & f\left(\Im_{u_{m2}}, L_2^+\right) & . & . & . & f\left(\Im_{u_{mn}}, L_n^+\right) \end{matrix} \tag{25}$$

and

$$\begin{matrix} f\left(\Im_{u_{11}}, P_1^-\right) & f\left(\Im_{u_{12}}, P_2^-\right) & . & . & . & f\left(\Im_{u_{1n}}, P_n^-\right) \\ f\left(\Im_{u_{21}}, L_1^+\right) & f\left(\Im_{u_{22}}, P_2^-\right) & . & . & . & f\left(\Im_{u_{1n}}, P_n^-\right) \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ f\left(\Im_{u_{m1}}, P_1^-\right) & f\left(\Im_{u_{m2}}, P_2^-\right) & . & . & . & f\left(\Im_{u_{mn}}, P_n^-\right) \end{matrix} \tag{26}$$

Step:6 Grey coefficient for each alternative calculated from PIS and NIS by utilizing following below equation. The grey coefficient for each alternative calculated from PIS is provided as

$$\xi_{pq}^+ = \frac{\min_{1\le p\le m} min_{1\le q\le n} d\left(\Im_{\breve{u}_{pq}}, L_p^+\right) + \rho \max_{1\le p\le m} max_{1\le q\le n} d\left(\Im_{\breve{u}_{pq}}, L_p^+\right)}{d\left(\Im_{\breve{u}_{pq}}, L_p^+\right) + \rho \max_{1\le p\le m} max_{1\le q\le n} d\left(\Im_{\breve{u}_{pq}}, L_p^+\right)}. \tag{27}$$

Where $p = 1,2,3,...,m$ and $q = 1,2,3,...,n$.Similarly, the grey coefficient of each alternative calculated from NIS is provided as

$$\xi_{pq}^- = \frac{\min_{1\le p\le m} min_{1\le q\le n} d\left(\Im_{\breve{u}_{pq}}, P_k^-\right) + \rho \max_{1\le p\le m} max_{1\le q\le n} d\left(\Im_{\breve{u}_{pq}}, P_k^-\right)}{d\left(\Im_{\breve{u}_{pq}}, P_k^-\right) + \rho \max_{1\le p\le m} max_{1\le q\le n} d\left(\Im_{\breve{u}_{pq}}, P_k^-\right)}. \tag{28}$$

Where $p = 1,2,3,...,m$ and $q = 1,2,3,...,n$ and the identification coefficient $\rho = 0.5$.

Step:7 Calculating the grey coefficient degree for each alternative from PIS and NIS by utilizing following below equation, respectively,

$$\xi_p^+ = \sum_{q=1}^{n} \nu_q \xi_{pq}^+ \tag{29}$$

$$\xi_p^- = \sum_{q=1}^{n} \nu_q \xi_{pq}^-$$

The basic principle of the Grey method is that the chosen alternative should have the "largest degree of grey relation" from the PIS and the "smallest degree of grey relation" from the NIS. Obviously, for the weights are known, the smaller ξ_p^- and the larger ξ_p^+, the finest alternative a_p is. But incomplete information about weights of alternatives is known. So, in this circumstances the ξ_p^- and ξ_p^+, information about weight calculated initially. So we provide following optimization models for multiple objective to calculate the information about weight,

$$(OM1)\begin{cases} \min \xi_p^- = \sum_{q=1}^{n} \nu_q \xi_{pq}^- \; p = 1, 2, ..., m \\ \max \xi_p^+ = \sum_{q=1}^{n} \nu_q \xi_{pq}^+ \;\; p = 1, 2, ..., m \end{cases} \tag{30}$$

Since each alternative is non-inferior, so there exists no preference relation on the all the alternatives. Then, we aggregate the above optimization models with equal weights into the following optimization model of single objective,

$$(OM2)\begin{cases} \min \xi_p = \sum_{p=1}^{m}\sum_{q=1}^{n} \left(\xi_{pq}^- - \xi_{pq}^+\right) \nu_q \end{cases} \tag{31}$$

To finding solution of OM2, we obtain optimal solution $\nu = (\nu_1, \nu_2, ..., \nu_m)$, which utilized as weights informations of provided alternatives. Then, we obtain ξ_p^+ $(p = 1, 2, ..., m)$ and ξ_p^- $(p = 1, 2, ..., m)$ as utilizing above formula, respectively.

Step:8 Relative degree calculated for each alternative utilizing the following equation from PIS and NIS,

$$\xi_p = \frac{\xi_p^+}{\xi_p^- + \xi_p^+} \left(p = 1, 2, ..., m\right). \tag{32}$$

Step:9 Ranking all the alternatives $a_p(p = 1, 2, ..., m)$ and select finest one(s) in accordance with ξ_p $(p = 1, 2, ..., m)$. If any alternative has the highest ξ_p value, then it is finest alternative according to the criteria.

Step:10 End.

5. Descriptive Example

The technique proposed in this paper is illustrated by a numerical examples with Spherical fuzzy information in this section. Suppose a panel of three experts is arranged for selection from four possible emerging technology enterprises $\check{Z}_i$ $(i = 1, 2, 3, 4)$. So panel select optimal alternative from given four alternatives,

(1) Technical advancement is denoted by B_1;
(2) Potential market risk is denoted by B_2;
(3) Industrialization infrastructure, human resources and financial condition is denoted by B_3;
(4) Employment creation and the development of science and technology is denoted by B_4.

Step:1 From the results obtained with each emerging technology enterprise, the three experts offering their own opinions which are shown in tables 1-3.

Table-1.: Spherical fuzzy information D^1

	$\breve{A}_1$	$\breve{A}_2$	$\breve{A}_3$	$\breve{A}_4$
$\check{Z}_1$	$\langle 0.3, 0.8, 0.5\rangle$	$\langle 0.8, 0.4, 0.3\rangle$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.3, 0.3, 0.4\rangle$
$\check{Z}_2$	$\langle 0.2, 0.6, 0.7\rangle$	$\langle 0.3, 0.9, 0.1\rangle$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.5, 0.4, 0.2\rangle$
$\check{Z}_3$	$\langle 0.4, 0.8, 0.4\rangle$	$\langle 0.5, 0.8, 0.2\rangle$	$\langle 0.2, 0.3, 0.7\rangle$	$\langle 0.6, 0.6, 0.1\rangle$
$\check{Z}_4$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.6, 0.6, 0.3\rangle$	$\langle 0.3, 0.6, 0.5\rangle$	$\langle 0.4, 0.2, 0.3\rangle$

Table-2.: Spherical fuzzy information D^2

	$\breve{A}_1$	$\breve{A}_2$	$\breve{A}_3$	$\breve{A}_4$
$\check{Z}_1$	$\langle 0.1, 0.5, 0.7\rangle$	$\langle 0.6, 0.3, 0.4\rangle$	$\langle 0.3, 0.8, 0.6\rangle$	$\langle 0.6, 0.3, 0.2\rangle$
$\check{Z}_2$	$\langle 0.4, 0.4, 0.8\rangle$	$\langle 0.5, 0.7, 0.1\rangle$	$\langle 0.4, 0.2, 0.7\rangle$	$\langle 0.6, 0.1, 0.6\rangle$
$\check{Z}_3$	$\langle 0.2, 0.9, 0.3\rangle$	$\langle 0.7, 0.1, 0.4\rangle$	$\langle 0.3, 0.6, 0.4\rangle$	$\langle 0.6, 0.2, 0.4\rangle$
$\check{Z}_4$	$\langle 0.3, 0.4, 0.8\rangle$	$\langle 0.4, 0.6, 0.5\rangle$	$\langle 0.4, 0.1, 0.8\rangle$	$\langle 0.6, 0.3, 0.3\rangle$

Table-3.: Spherical fuzzy information D^3

	$\breve{A}_1$	$\breve{A}_2$	$\breve{A}_3$	$\breve{A}_4$
$\breve{Z}_1$	$\langle 0.4, 0.2, 0.8\rangle$	$\langle 0.4, 0.4, 0.3\rangle$	$\langle 0.5, 0.4, 0.6\rangle$	$\langle 0.5, 0.1, 0.4\rangle$
$\breve{Z}_2$	$\langle 0.2, 0.5, 0.7\rangle$	$\langle 0.6, 0.5, 0.4\rangle$	$\langle 0.2, 0.5, 0.8\rangle$	$\langle 0.7, 0.2, 0.4\rangle$
$\breve{Z}_3$	$\langle 0.6, 0.4, 0.5\rangle$	$\langle 0.9, 0.3, 0.1\rangle$	$\langle 0.3, 0.1, 0.9\rangle$	$\langle 0.6, 0.2, 0.6\rangle$
$\breve{Z}_4$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.8, 0.5, 0.2\rangle$	$\langle 0.3, 0.8, 0.4\rangle$	$\langle 0.5, 0.3, 0.5\rangle$

Since C_1, C_3 are cost-type criteria and C_2, C_4 are benefit-type criteria. So we have need to normalized the Spherical fuzzy information. Normalized Spherical fuzzy information are shown in table-4,5,6.:

Table-4.: Normalized Spherical fuzzy information R^1

	$\breve{A}_1$	$\breve{A}_2$	$\breve{A}_3$	$\breve{A}_4$
$\breve{Z}_1$	$\langle 0.5, 0.8, 0.3\rangle$	$\langle 0.8, 0.4, 0.3\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.3, 0.3, 0.4\rangle$
$\breve{Z}_2$	$\langle 0.7, 0.6, 0.2\rangle$	$\langle 0.3, 0.9, 0.1\rangle$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.5, 0.4, 0.2\rangle$
$\breve{Z}_3$	$\langle 0.4, 0.8, 0.4\rangle$	$\langle 0.5, 0.8, 0.2\rangle$	$\langle 0.7, 0.3, 0.2\rangle$	$\langle 0.6, 0.6, 0.1\rangle$
$\breve{Z}_4$	$\langle 0.8, 0.3, 0.5\rangle$	$\langle 0.6, 0.6, 0.3\rangle$	$\langle 0.5, 0.6, 0.3\rangle$	$\langle 0.4, 0.2, 0.3\rangle$

Table-5.: Normalized Spherical fuzzy information R^2

	$\breve{A}_1$	$\breve{A}_2$	$\breve{A}_3$	$\breve{A}_4$
$\breve{Z}_1$	$\langle 0.7, 0.5, 0.1\rangle$	$\langle 0.6, 0.3, 0.4\rangle$	$\langle 0.6, 0.8, 0.3\rangle$	$\langle 0.6, 0.3, 0.2\rangle$
$\breve{Z}_2$	$\langle 0.8, 0.4, 0.4\rangle$	$\langle 0.5, 0.7, 0.1\rangle$	$\langle 0.7, 0.2, 0.4\rangle$	$\langle 0.6, 0.1, 0.6\rangle$
$\breve{Z}_3$	$\langle 0.3, 0.9, 0.2\rangle$	$\langle 0.7, 0.1, 0.4\rangle$	$\langle 0.4, 0.6, 0.3\rangle$	$\langle 0.6, 0.2, 0.4\rangle$
$\breve{Z}_4$	$\langle 0.8, 0.4, 0.3\rangle$	$\langle 0.4, 0.6, 0.5\rangle$	$\langle 0.8, 0.1, 0.4\rangle$	$\langle 0.6, 0.3, 0.3\rangle$

Table-6.: Normalized dSpherical fuzzy information R^3

	$\breve{A}_1$	$\breve{A}_2$	$\breve{A}_3$	$\breve{A}_4$
$\breve{Z}_1$	$\langle 0.8, 0.2, 0.4\rangle$	$\langle 0.4, 0.4, 0.3\rangle$	$\langle 0.6, 0.4, 0.5\rangle$	$\langle 0.5, 0.1, 0.4\rangle$
$\breve{Z}_2$	$\langle 0.7, 0.5, 0.2\rangle$	$\langle 0.6, 0.5, 0.4\rangle$	$\langle 0.8, 0.5, 0.2\rangle$	$\langle 0.7, 0.2, 0.4\rangle$
$\breve{Z}_3$	$\langle 0.5, 0.4, 0.6\rangle$	$\langle 0.9, 0.3, 0.1\rangle$	$\langle 0.9, 0.1, 0.3\rangle$	$\langle 0.6, 0.2, 0.6\rangle$
$\breve{Z}_4$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.8, 0.5, 0.2\rangle$	$\langle 0.4, 0.8, 0.2\rangle$	$\langle 0.5, 0.3, 0.5\rangle$

Assume that the information about attribute weights, given by experts, is partly known; $\Delta = \left\{\begin{array}{c} 0.2 \leq w_1 \leq 0.25, \\ 0.15 \leq w_2 \leq 0.2, \\ 0.28 \leq w_3 \leq 0.32, \\ 0.35 \leq w_4 \leq 0.4 \end{array}\right\}$, $w_p \geq 0$, $p = 1, 2, 3, 4$, $\sum\limits_{p=1}^{4} w_p = 1$ Then, we utilize the developed approach to get the most desirable alternative(s).

Step:2 We firstly determine fuzzy density of each decision maker, and its λ parameter. Suppose that $L_{\breve{e}_{\breve{u}}}(b_1) = 0.30$, $L_{\breve{e}_{\breve{u}}}(b_2) = 0.40$, $L_{\breve{e}_{\breve{u}}}(A_3) = 0.50$. Then λ of expert can be determined: $\lambda = -0.45$. By Eq.(6), we have $L_{\breve{e}_{\breve{u}}}(b_1, b_2) = 0.65$, $L_{\breve{e}_{\breve{u}}}(b_1, A_3) = 0.73$, $L_{\breve{e}_{\breve{u}}}(b_2, A_3) = 0.81$, $L_{\breve{e}_{\breve{u}}}(b_1, b_2, A_3) = 1$.

Step:3 According to Definition 3.5, $\Im^{(k)}_{\breve{u}_{pq}}$ is reordered such that $\Im^{(k)}_{\breve{u}_{pq}} \geq \Im^{(k-1)}_{\breve{u}_{pq}}$. Then Utilize the Spherical fuzzy Choquet integral weighted operator

$$SFCIWA\left(\Im_{\breve{u}_1}, \Im_{\breve{u}_2}, ..., \Im_{\breve{u}_n}\right) = \left\{\begin{array}{c} \sqrt{1 - \Pi_{p=1}^{n}(1 - L_{\breve{e}_{\breve{u}}})^{\lambda\left(A_{\rho(p)}\right) - \lambda\left(A_{\rho(p\text{-}1)}\right)}}, \\ \Pi_{p=1}^{n}(M_{\breve{e}_{\breve{u}}})^{\lambda\left(A_{\rho(p)}\right) - \lambda\left(A_{\rho(p\text{-}1)}\right)}, \\ \Pi_{p=1}^{n}(O_{\breve{e}_{\breve{u}_{\rho(p)}}})^{\lambda\left(A_{\rho(p)}\right) - \lambda\left(A_{\rho(p\text{-}1)}\right)} \end{array}\right\}$$

to aggregate all the Spherical fuzzy decision matrices $R^k = \left[\Im^{(k)}_{\breve{u}_{pq}}\right]_{m \times n}$ into a collective Spherical fuzzy decision matrix as follows:

Table-7.: Collective Spherical fuzzy information

	$\breve{A}_1$	$\breve{A}_2$	$\breve{A}_3$	$\breve{A}_4$
$\breve{Z}_1$	$\langle 0.702, 0.417, 0.225\rangle$	$\langle 0.638, 0.361, 0.331\rangle$	$\langle 0.634, 0.545, 0.391\rangle$	$\langle 0.498, 0.204, 0.313\rangle$
$\breve{Z}_2$	$\langle 0.740, 0.488, 0.254\rangle$	$\langle 0.498, 0.670, 0.162\rangle$	$\langle 0.740, 0.311, 0.335\rangle$	$\langle 0.616, 0.193, 0.374\rangle$
$\breve{Z}_3$	$\langle 0.411, 0.654, 0.361\rangle$	$\langle 0.748, 0.274, 0.200\rangle$	$\langle 0.755, 0.260, 0.265\rangle$	$\langle 0.600, 0.278, 0.304\rangle$
$\breve{Z}_4$	$\langle 0.770, 0.331, 0.418\rangle$	$\langle 0.651, 0.562, 0.311\rangle$	$\langle 0.629, 0.354, 0.311\rangle$	$\langle 0.515, 0.265, 0.358\rangle$

Step:4 Utilize Equations 22 and 23 we get the positive-ideal and negative-ideal solution respectively are:

$$L^{+} = \{\langle 0.702, 0.417, 0.225\rangle, \langle 0.748, 0.274, 0.200\rangle, \langle 0.755, 0.260, 0.265\rangle, \langle 0.616, 0.193, 0.374\rangle\}$$

$$L^{-} = \{\langle 0.411, 0.654, 0.361\rangle, \langle 0.498, 0.670, 0.162\rangle, \langle 0.634, 0.545, 0.391\rangle, \langle 0.515, 0.265, 0.358\rangle\}$$

Step:5 Utilize equation (25) and (26) to get the positive-ideal separation matrix and negative-ideal separation matrix respectively as follow;

Table-8.:
Positive-ideal separation matrix

$D^{+} =$

	$\breve{A}_1$	$\breve{A}_2$	$\breve{A}_3$	$\breve{A}_4$
$\breve{Z}_1$	0.0000	0.0822	0.1327	0.0475
$\breve{Z}_2$	0.0345	0.1710	0.0338	0.0000
$\breve{Z}_3$	0.1656	0.0000	0.0000	0.0429
$\breve{Z}_4$	0.0865	0.1241	0.0662	0.0472

Table-9.: Negative-ideal separation matrix

$D^{-} =$

	$\breve{A}_1$	$\breve{A}_2$	$\breve{A}_3$	$\breve{A}_4$
$\breve{Z}_1$	0.1656	0.1547	0.0000	0.0309
$\breve{Z}_2$	0.1502	0.0000	0.0989	0.0472
$\breve{Z}_3$	0.0000	0.1710	0.1327	0.0378
$\breve{Z}_4$	0.1842	0.1025	0.0687	0.0000

Step:6 Utilize equations (27) and (28) we get the grey relational coefficient matrices in which each alternative is calculated from PIS and NIS as follow:

$$\left[\zeta_{ij}^{+}\right] = \begin{bmatrix} 1.0000 & 0.5098 & 0.3918 & 0.6428 \\ 0.7125 & 0.3333 & 0.7166 & 1.0000 \\ 0.3405 & 1.0000 & 1.0000 & 0.6658 \\ 0.4970 & 0.4079 & 0.5636 & 0.6443 \end{bmatrix}$$

$$\left[\zeta_{ij}^{-}\right] = \begin{bmatrix} 0.3573 & 0.3731 & 1.0000 & 0.7487 \\ 0.3801 & 1.0000 & 0.4821 & 0.6611 \\ 1.0000 & 0.3500 & 0.4096 & 0.7090 \\ 0.3333 & 0.4732 & 0.5151 & 1.0000 \end{bmatrix}$$

Step:7 We used the model (M2) to establish the single-objective programming model:

$$\min \xi(w) = -0.0659w_1 - 0.2392w_2 - 0.5377w_3 + 0.2088w_4$$

After their solution, the weight vector of attributes are:

$$w = (0.273, 0.368, 0.227, 0.1300)$$

From the PIS and NIS, we obtain grey relational coefficient of each alternative:

$$\xi_1^{+} = 0.6331, \xi_2^{+} = 0.6098, \xi_3^{+} = 0.7745, \xi_4^{+} = 0.4974,$$

$$\xi_1^{-} = 0.5590, \xi_2^{-} = 0.6671, \xi_3^{-} = 0.5869, \xi_4^{-} = 0.5120.$$

Step:8 Utilize equation 32, we obtain the relative relational degree of each alternative from PIS and NIS as follows:

$$\begin{aligned}
\xi_1 &= \frac{\xi_1^+}{\xi_1^- + \xi_1^+} = \frac{0.6331}{0.5590 + 0.6331} = 0.5310 \\
\xi_2 &= \frac{\xi_2^+}{\xi_2^- + \xi_2^+} = \frac{0.6098}{0.6671 + 0.6098} = 0.4775 \\
\xi_3 &= \frac{\xi_3^+}{\xi_3^- + \xi_3^+} = \frac{0.7745}{0.5869 + 0.7745} = 0.5688 \\
\xi_4 &= \frac{\xi_4^+}{\xi_4^- + \xi_4^+} = \frac{0.4974}{0.5120 + 0.4974} = 0.4927
\end{aligned}$$

Step:9 The ranking order, according to the relative relational degree are:

$$\check{Z}_3 > \check{Z}_1 > \check{Z}_4 > \check{Z}_2,$$

and best alternative is $\check{Z}_3$.

6. Conclusion

In this paper, we proposed decision making approach to deal with spherical fuzzy information. As spherical fuzzy set is the generalization of all the existing structure of fuzzy sets, so an algorithm based on GRA approach to deal with uncertainty and inaccurate information in decision making problems using spherical fuzzy environments. Final, a numerical application is illustrated to shows the how our proposed technique is effective and reliable to deal with uncertainty. In future, we use TOPSIS, VIKOR, TODAM approaches to deal with uncertainty using spherical fuzzy information. .

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