

A COMBINED MATHEMATICAL MODELING AND ANALYTIC HIERARCHY PROCESS APPROACH FOR SPORTS SCHEDULING PROBLEMS

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Keywords	Abstract
<i>Sports scheduling, match importance, mathematical modeling, timetabling, AHP</i>	<i>Malaysian football is witnessing a decrease in the number of supporters at their stadiums. Therefore, league administrators give the higher priority to this issue. We hypothesize that spectators would prefer to watch more important matches at stadiums and no matter when they are. We propose to define an importance level for each fixture. The Analytic Hierarchy Process (AHP) is utilised to obtain these importance levels. We also develop an integer programming model to assign fixtures to the timeslots in a way to maximize the number of supporters that attend matches. The outcome of the first process is used as input to this optimization model. We apply the methodology to a real case from the Super League Season in Malaysia and we believe that we produced a superior schedule which will maximize gate receipts.</i>

SPOR ÇİZELGELEME PROBLEMİ İÇİN MATEMATİKSEL MODEL VE ANALİTİK HİYERARŞİ SÜRECİ İLE BÜTÜNLEŞİK BİR YAKLAŞIM

Anahtar Kelimeler	Öz
<i>Müsabaka çizelgeleme, maç önem derecesi, matematiksel model, zaman çizelgeleme, AHP</i>	<i>Malezya futbolunda stadyumda maç izleyen taraftar sayısında önemli azalış gözlenmesi, lig yöneticilerini bu konuyu öncelikli olarak ele almaya yöneltmiştir. Taraftarların özellikle önemli maçları, ne zaman çizelgelendiklerine bakılmaksızın stadyumda izleme eğilimleri maçların yeni bir anlayışla çizelgelenmesi fikrini ortaya çıkmıştır. Bu çalışmada öncelikle maçların, Analitik Hiyerarşi Süreci kullanılarak ağırlıklandırılması için bir model geliştirilmiş, aynı zamanda enbüyük sayıda taraftarın gelmesini sağlayacak çizelgeyi oluşturmak amacıyla yazarlar tarafından daha önce geliştirilen ve gerçek bir veri seti üzerinde henüz uygulanamamış bir tamsayı matematiksel model de söz konusu yöntemle birlikte kullanılmıştır. Yöntem, Malezya Süper Ligindeki maçları çizelgelemek için kullanılmış ve söz konusu dönem için maçlara gelen toplam taraftar sayısının arttığı bir çözüme ulaşılmıştır. Önerilen bütünleşik yaklaşımın, diğer spor dallarında da kullanılabileceği öngörülmektedir.</i>

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1. Introduction

In Malaysia, football supporters seem to prefer to watch games on TV rather than watch them live at a stadium (Abdul-Hamid and Kendall, 2008). Therefore, league administrators give the higher priority to the issue of maximizing the number of supporters that attend matches. We hypothesize that spectators would prefer to watch matches at stadiums if they are more important and we develop a mathematical model to schedule the games in order to achieve this objective. The problem is the practical task of creating a schedule for a sports tournament or league.

In an effort to determine the importance of matches, AHP (Analytic Hierarchy Process) are utilized in this work. The Analytic Hierarchy Process introduced by Thomas Saaty (1980) is an effective tool for dealing with complex decision making, and may aid the decision maker to set priorities and make the best decision. By reducing complex decisions to a series of pairwise comparisons, and then synthesizing the results, the AHP helps to capture both subjective and objective aspects of a decision.

The paper has 5 sections. Section 2 gives literature work for sports scheduling and presents the AHP methodology. Section 3 presents the problem and the methodology to solve this problem. In section 4 we present our application and summarize our work in section 5.

2. Related Work

2.1. Sports Scheduling

In general, sports scheduling problems consist of determining opponents, date and venue of each game of a league (Gunnec and Demir, 2019). A calendar for two leagues has to be constructed; besides the usual restrictions on the alternation of home- and away-games, one has to consider the fact that some pairs of teams in the two leagues share the same facilities and cannot play home-games simultaneously. The Traveling Tournament Problem is also a problem of scheduling round robin leagues which minimizes the total travel distance maintaining some constraints on consecutive home and away matches (Bhattacharyya and Bhattacharyya, 2016). de Werra, Descombes and Masson (1990) define the problem in terms of graph theory. A league of $2n$ teams is represented by a complete graph. Each game which has to be played by teams i and j is associated with an edge $[i, j]$. Each

team meets once every other team. A game between team i and team j takes place in the home-city of either team i or team j ; in the first case, it is a home-game for i and an away-game for team j . A round for a league of $2n$ teams consists of a collection of $n(2n-1)$ games. Generally breaks are undesirable and it is required to construct schedules where for each team home-games and away-games are alternating as regularly as possible.

There are many papers about sports scheduling problems. Professional soccer leagues have been scheduled by Bartsch, Drexel, and Kröger (2006) by providing models. Duran et al. (2007) used an integer linear programming model for the similar problem. Kendall (2008) schedules English fixtures over holiday periods in such a way to minimize travel distances. Briskorn and Drexel (2009) worked on round robin tournament scheduling and they used Integer Programming to solve problem instances for small-to-medium-size.

Besides soccer, solving real world sports problems can be found in other sports. For example, Wright (2007), formulates and solves the problem faced every year by the Devon Cricket League in England. In his earlier work, Wright (2006) describes the problem faced every year by Basketball New Zealand in scheduling the National Basketball League fixtures.

Since we deal with stadium attendance, we discuss the papers with similar topic. Buraimo, Forrest and Simmons (2009).model match attendance to decide when to schedule midweek games. Duran and Shi (2008) work on the importance of a match in a tournament by using logistic regression and Monte Carlo simulation.

Recently, Gunnec and Demir (2019), study a sports scheduling problem. The objective is to minimize carry-over effects in round robin tournaments. They consider tournaments that allow minimum number of breaks for each team, and an integer programming model is formulated, they also provide an efficient heuristic algorithm to solve this problem. The mathematical model in this work is not solved due to the complexity and the size of the model. Therefore a heuristic approach is proposed.

There are papers on other kind sports scheduling problems such as basketball tournaments, Duran, Duran, Marenco, Mascialino and Pablo (2019) deal with professional basketball leagues and the main scheduling objective is to reduce the teams' total

travel distance. They have two-staged mathematical model where the first one defines the sequences in which each team plays the other teams and the second one assigns the days on which each game is played.

More information on sports scheduling in general can be found in the annotated bibliography of sports scheduling (Kendall, Knust, Ribeiro & Urrutia. 2010).

Abdul-Hamid, N. and Kendall, G. (2008) and Abdul-Hamid, N., Kendall, G. and Sagir, M.(2009) discuss the issue of maximizing stadium attendance by introducing the mathematical model without any real case application in their conference papers. This paper is the extension of their work by applying the mathematical model to a real case and also introduces Analytic Hierarchy Process to prioritize different games.

Therefore considering the existing literature, it is obvious that there is no scientific journal paper to schedule the games in such a way that the supporters that attend matches are going to be maximized. To achieve this objective, our purpose is to maximize the assignment of important games to weekdays and to maximize the assignment of unimportant matches to weekends.

2.2. Analytic Hierarchy Process and applications in sports

The Analytic Hierarchy Process has been used extensively for the last 30 years. There are many papers related to just reviews of AHP applications in some specific area such as Amos, Chan, and Ameyaw (2019). They recently published a review on the applications of AHP in construction. Vaidya and Kumar (2006) review the applications of AHP covering 150 application papers from the area of planning, selecting a best alternative, resource allocations, resolving conflict and optimization. Hundreds of papers are published about single decisions such as evaluating of a web site (Kabassi, 2018).

As for the application of AHP in sports, Partovi and Corredora (2002) used QFD (Quality Function Deployment) techniques along with AHP. The approach proposes a model for prioritizing and designing rule changes for the game of soccer. Gholamian, Fatemi Ghomi and Ghazanfari (2007) developed a new ranking system for the judgment matrix in the AHP. Fuzzy rules and fuzzy reasoning methods are used. The numerical example of a world

cup soccer tournament is used to clarify the performance of the developed system comparing with the AHP method in ranking the sparse judgment matrices.

Saaty and Sağır (2015) consider the intangible criteria that influence the outcome of the Summer Olympics by using the Analytic Network Process, and apply the idea to evaluate the medals won and the country scores in the 2012 London Olympics. Different events of the same category game could have different properties therefore the medals should not be just counted but should be prioritized. With minor modifications, this systematic approach for ranking countries can be used for any Summer Olympics.

These are just a few examples about the AHP applications and the reader easily can find many more in the literature.

3. Problem Definition and Methodology

The Malaysian Super League (MSL) is structured as a double round robin tournament. Each team plays against each other once at home and once away. Six matches are played in a given timeslot, and there are 26 timeslots for the season. The first and the second rounds, each has 13 timeslots. When a team plays at home, it would play away on the next game. If a team plays at home on the first day of the season, they play away on the last day of the season (and vice versa). The current schedules are produced manually by the Football Association of Malaysia (FAM).

In this work, our purpose is to maximize the assignment of important games to weekdays and to maximize the assignment of unimportant matches to weekends. The rationale is that the important matches attract supporters to the stadium no matter they are played on weekdays or weekends. Therefore, by allocating important matches during weekdays, the weekend slots can be assigned to unimportant matches on the assumption that supporters have free time to watch matches at stadium. So our first attempt is to decide the importance level of each fixture and we employ AHP by a group of experts in order to obtain these priorities. Once we get these priorities, our purpose is then to assign the prioritized games to timeslots in a way to maximize the gate receipts.

3.1. Assigning Weights Using AHP

AHP is a theory of relative measurement with absolute scales applied to measure both tangible and intangible criteria that are homogeneous based on the judgment of experts. AHP enables the linking of measurements with human values. It derives priorities from informed judgments which correspond to relative measurements obtained after understanding and is used rather than developing understanding from measurements obtained prior to doing analysis (Saaty, 1980).

The decision structures geometrically take on the form of a hierarchy in the AHP. It is comprised of a goal, levels of elements and connections between the elements. In a hierarchy, influence flows down from the top of the structure. A set of pairwise comparison matrices is constructed.

To make pairwise comparisons, we need a scale of numbers that indicates how many times more important or dominant one element is over another element with respect to the criterion or property with respect to which they are compared. This scale is defined as Fundamental Scale (Saaty, 2008). To use the scale, the smaller element is considered to be the unit and one estimates how many times more important (more dominant) the other is by using a number from the Fundamental Scale from 1-9 scale and if activity i has one of these nonzero numbers assigned to it when compared with activity j , then j has the reciprocal value when compared with i . There are many AHP papers in the literature the reader easily can find the details about the scale and its applications. In Russo and Camanho (2015)'s review paper, the articles selected for review all refer to a case study and they are related mainly to industries. They focus on the importance of the context and how to treat the problem, offering detailed information about the methods and the mathematics involved.

3.2. Criteria

The criteria are determined by the experts from Football Association of Malaysia and categorized under four main groups, these being *Championship*, *Switching position*, *Derby effect* and *Relegation*. Under each, there are sub-criteria. *Championship* has two, whether the match is between any two of *top three teams*, or between *fourth and sixth teams* in the league standings. Similarly, *Switching position* as the second main criterion has two sub-criteria as *switching among top three teams* and *switching*

between the fourth to sixth teams in the league standings.

In contrast to the possibility of becoming league champion, the teams with a chance of being relegated will also be considered important in terms of having more supporters. Thus, *Relegation* becomes our third criteria. The sub-criteria for relegation are *relegate among the last three teams* and *relegate among the fifth to tenth teams* in the league standings.

Finally, there is also a *derby effect* among the teams who are located close to each other geographically. This means that the supporters from both teams can go and watch the game at the stadium. For our specific problem, we consider three specific regions of which the teams from these regions could possibly have a derby effect when they play each other. These regions are the *northern region*, *eastern region*, and *central and southern regions*.

Here we use the leading software Expert Choice supporting AHP (<http://www.expertchoice.com/>) to prioritize each game between two teams. Expert Choice is a computerized business management tool combines both quantitative and qualitative information. The software is sensitive to real life variables that influence decisions such as changes in the criteria. In order to provide a better understanding, here we show a few screenshots from the software.

Figure 1 represents 3-level AHP model for this problem with the goal, main criteria and sub criteria levels. The main criteria are *championship*, *switching position*, *derby affect* and *relegate*.

Once the criteria are defined, the AHP uses pairwise comparisons of a knowledgeable person (Professional Footballers' Association of Malaysia in this case) to determine the importance of criteria in a decision. Figure 2 reflects the paired comparisons of the main criteria which is self-explanatory, while Figure 3 shows an example of paired comparison of the sub-criteria. As an example, according to Figure 2, on line 1, for a match, *championship* possibility is four times more attractive than having a *derby effect* for the supporters in terms of decision to go to the games. The selection numbers on the left side of any row show that the criterion on the left side is more important than the ones on the right side. In contrast, if a number from the selection of numbers on the right side of any row is selected, this means that the criterion on the right side is more important than the left hand side. For example, on line 3, the games that

have a possibility of switching the teams in the league table are three times more important than the left side criterion of this 3rd line.

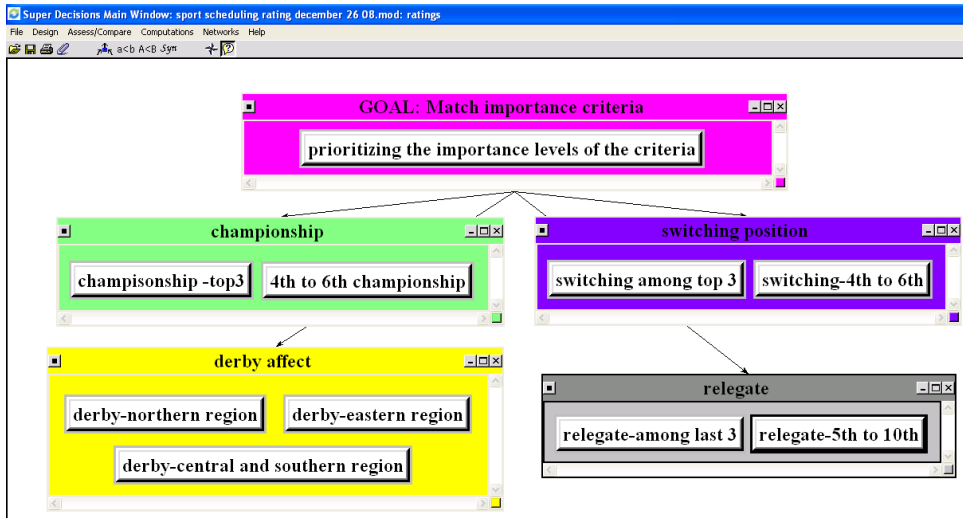


Figure 1. A Screen View Of The AHP Model

Cluster comparisons for "GOAL: Match importance criteria". The interface shows a comparison between "championship" and "derby affect".

Rank	Left Criterion	Scale (1-9)	Right Criterion
1.	championship	>=9.5 9 8 7 6 5 4 3 2	derby affect
2.	championship	>=9.5 9 8 7 6 5 4 3 2	relegate
3.	championship	>=9.5 9 8 7 6 5 4 3 2	switching position
4.	derby affect	>=9.5 9 8 7 6 5 4 3 2	relegate
5.	derby affect	>=9.5 9 8 7 6 5 4 3 2	switching position
6.	relegate	>=9.5 9 8 7 6 5 4 3 2	switching position

Figure 2. A Screen View of the Paired Comparison of Main Criteria

Comparisons wrt "prioritizing the importance levels of the criteria" node in "championship" cluster. The interface shows a comparison between "4th to 6th championship" and "championship -top3".

Rank	Left Criterion	Scale (1-9)	Right Criterion
1.	4th to 6th championship	>=9.5 9 8 7 6 5 4 3 2	championship -top3

Figure 3. A Screen View of the Paired Comparison of Sub-Criteria

Icon	Name	Normalized by Cluster	Limiting
No Icon	4th to 6th championship	0.80000	0.277951
No Icon	championship -top3	0.20000	0.069488
No Icon	derby-central and southern region	0.16667	0.051148
No Icon	derby-eastern region	0.16667	0.051148
No Icon	derby-northern region	0.66667	0.204591
No Icon	prioritizing the importance levels of the~	0.00000	0.000000
No Icon	relegate-5th to 10th	0.16667	0.025442
No Icon	relegate-among last 3	0.83333	0.127211
No Icon	switching among top 3	0.66666	0.128681
No Icon	switching-4th to 6th	0.33334	0.064341

Figure 4. Criteria Weights

In the Analytic Hierarchy Process, the inconsistency of judgments is also measured by an index based on the principal eigenvalue of the positive reciprocal matrix of judgments.

Once the criteria weights are obtained, the alternatives are evaluated in terms of each criterion. There are two procedures to evaluate alternatives. When there are many alternatives, a rating is used to prioritize them. In the rating procedure, instead of comparing alternatives under each criterion (second type evaluation), each alternative is rated with respect to that single criterion. For example, in Figure 5 the match between Kedah and Perlis is rated as *highly attractive* under *championship among top three* teams criterion. By combining individual criteria ratings, the final importance level of Kedah and Perlis game is obtained as 0.02248. The second column in Figure 5 lists the overall outcome for all alternatives.

	Priorities	championship -top 0.069488	4th to 6th champio 0.277951	switching among to 0.128681	derby-northern regio 0.204591	switching-4th to 6th 0.064341	derby-central and s 0.051148	derby-eastern regio 0.051148	relegate-5th to 10th 0.025442	relegate-among 0.127211
KEDAH-PERLIS	0.022481	highly attractive	not attractive	highly attractive	highly attractive	not attractive	not attractive	not attractive	not attractive	not attract
PENANG-PAHANG	0.011066	not attractive	average	not attractive	not attractive	not attractive	not attractive	not attractive	average	not attract
SARAWAK-PDRM	0.009095	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	average	not attract
JOHOR FC - TERE	0.013612	average	average	not attractive	not attractive	highly attractive	not attractive	not attractive	not attractive	not attract
N.SEMBILAN NAZ	0.021256	average	highly attractive	average	not attractive	not attractive	highly attractive	not attractive	not attractive	not attract
PERAK-DPMM	0.010984	not attractive	average	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attract
PAHANG-KEDAH	0.010984	not attractive	average	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attract
PDRM-PULAU PIN	0.013316	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	average	highly attract
TERENGGANU-SA	0.009013	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attract
SELANGOR-JOHO	0.011148	not attractive	not attractive	not attractive	not attractive	highly attractive	not attractive	not attractive	not attractive	not attract
DPMM-N.SEMBILA	0.010984	not attractive	average	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attract
UPB MY TEAM-PE	0.009221	not attractive	not attractive	not attractive	not attractive	average	not attractive	not attractive	not attractive	not attract
PERLIS-PAHANG	0.009303	not attractive	not attractive	not attractive	not attractive	average	not attractive	not attractive	average	not attract
KEDAH-PDRM	0.009095	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	average	not attract
PULAU PINANG-TI	0.011274	not attractive	average	not attractive	not attractive	average	not attractive	not attractive	average	not attract
SARAWAK-SELAN	0.009095	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	average	not attract
JOHOR-DPMM	0.011066	not attractive	average	not attractive	not attractive	not attractive	not attractive	not attractive	average	not attract
N.SEMBILAN NAZ	0.009178	not attractive	not attractive	not attractive	not attractive	not attractive	average	not attractive	not attractive	not attract
PDRM-PERLIS	0.011230	not attractive	not attractive	not attractive	not attractive	highly attractive	not attractive	not attractive	average	not attract
TERENGGANU-KE	0.013119	not attractive	average	not attractive	not attractive	highly attractive	not attractive	not attractive	not attractive	not attract
SELANGOR-PENA	0.011148	not attractive	not attractive	not attractive	not attractive	highly attractive	not attractive	not attractive	not attractive	not attract
DPMM-SARAWAK	0.020348	not attractive	highly attractive	not attractive	not attractive	not attractive	not attractive	highly attractive	not attractive	not attract
UPB MYTEAM-JOH	0.009013	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attract
PERAK-N.SEMBILA	0.019067	not attractive	highly attractive	average	not attractive	not attractive	not attractive	not attractive	not attractive	not attract
KEDAH-SELANGO	0.009921	average	not attractive	average	not attractive	not attractive	not attractive	not attractive	not attractive	not attract
PERLIS-TERENGG	0.013119	not attractive	average	not attractive	not attractive	highly attractive	not attractive	not attractive	not attractive	not attract
PAHANG-PDRM	0.011230	not attractive	not attractive	not attractive	not attractive	highly attractive	not attractive	not attractive	average	not attract
PENANG-DPMM	0.009095	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	average	not attract
SARAWAK-UPB M	0.013234	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	highly attract
JOHOR-PERAK	0.011148	not attractive	not attractive	not attractive	not attractive	highly attractive	not attractive	not attractive	not attractive	not attract
PAHANG-TERENG	0.012889	not attractive	average	not attractive	not attractive	average	not attractive	highly attractive	not attractive	not attract
SELANGOR-PERLI	0.020786	not attractive	highly attractive	not attractive	not attractive	highly attractive	not attractive	not attractive	not attractive	not attract
DPMM-KEDAH	0.009013	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attractive	not attract
UPB MYTEAM-PUL	0.009095	not attractive	not attractive	not attractive	average	not attractive	not attractive	not attractive	not attractive	highly attract

Figure 5. A Screen View of the Outcome

3.3. Mathematical Model

The proposed mathematical model is presented below. The problem characteristics are as follows:

1. There are six matches (games) per timeslot.
2. Each match is assigned to a timeslot.
3. A team cannot have more than two consecutive home match.
4. A team cannot have more than two consecutive away matches.
5. Each team is scheduled 24 timeslots and is also scheduled a *bye* in one round.
6. If a team plays home during the first round, it must play away during the second round.

Sets and Data

$N = \{1, \dots, n_{tot}\}$ the set of teams
 $T = \{1, \dots, t_k\}$ the set of timeslots
 $T_e = \{1, \dots, t_k - 2\}$ the set of the first $t_k - 2$ timeslots
 T_{wd} the set of weekday timeslots
 T_{we} the set of weekend timeslots
 T_{r_1} the set of first round timeslots
 T_{r_2} the set of second round timeslots
 l_{q_i, q_j} the importance level of the game where team q_i plays home against team q_j (the parameters are obtained from the AHP analysis)

Subject to

The total number of scheduled matches must be six for each timeslot.

$$\sum_{q_i \in N} \sum_{\substack{q_j \in N \\ q_j \neq q_i}} x_{q_i, q_j, t} = 6 \quad \forall t \tag{2}$$

Every fixture has to be scheduled once.

$$\sum_{t \in T} x_{q_i, q_j, t} = 1 \quad \forall q_i, q_j \in N : q_j \neq q_i \tag{3}$$

A team cannot have more than two consecutive home games.

$$\sum_{\substack{q_j \in N \\ q_j \neq q_i}} (x_{q_i, q_j, t} + x_{q_i, q_j, t+1} + x_{q_i, q_j, t+2}) \leq 2 \quad \forall q_i \in N, \forall t \in T_e \tag{4}$$

Decision Variables

$$x_{q_i, q_j, t} = \begin{cases} 1, & \text{if team } q_i \text{ play home with team } q_j \text{ in timeslot } t \\ 0, & \text{otherwise} \end{cases}$$

Objective function

Maximise

$$f = \sum_{q_i \in N} \sum_{\substack{q_j \in N \\ q_j \neq q_i}} l_{q_i, q_j} \left(\sum_{t_1 \in T_{wd}} x_{q_i, q_j, t_1} - \sum_{t_2 \in T_{we}} x_{q_i, q_j, t_2} \right) \tag{1}$$

In this work, our purpose is to maximize the assignment of important games to weekdays and unimportant games to weekends. According to (1), if team q_i and q_j play a weekday game, the first term in parenthesis takes value 1 (the second term has to be 0 in this case). Therefore to maximize this function, the games correspond to high level of importance are assigned to weekdays in a way we want. Because as we explained before, an important game already attracts supporters to the stadium no matter that is played on weekdays or weekends. Therefore, by allocating important matches during weekdays as much as possible, the weekend slots can be used to schedule unimportant matches and since the supporters have free time on weekends, they are expected to watch matches at stadium even if the game is not important.

A team cannot have more than two consecutive away games.

$$\sum_{\substack{q_i \in N \\ q_i \neq q_j}} (x_{q_i, q_j, t} + x_{q_i, q_j, t+1} + x_{q_i, q_j, t+2}) \leq 2 \quad \forall q_j \in N, \forall t \in T_e \quad (5)$$

Each team must play every other team only once in the first round.

$$\sum_{t \in T_{r1}} (x_{q_i, q_j, t} + x_{q_j, q_i, t}) = 1 \quad \forall q_i, q_j \in N : q_j \neq q_i \quad (6)$$

Each team must play every other team only once in the second round.

$$\sum_{t \in T_{r2}} (x_{q_i, q_j, t} + x_{q_j, q_i, t}) = 1 \quad \forall q_i, q_j \in N : q_j \neq q_i \quad (7)$$

The games scheduled must follow the “mirror” condition.

$$\sum_{\substack{q_j \in N \\ q_j \neq q_i}} (x_{q_i, q_j, t} - x_{q_j, q_i, (k-t+1)}) = 0 \quad \forall q_i \in N, \forall t \in T_{r1} \quad (8)$$

Each team can only play at most one game per timeslot.

$$\sum_{\substack{q_j \in N \\ q_j \neq q_i}} (x_{q_i, q_j, t} + x_{q_j, q_i, t}) \leq 1 \quad \forall q_i \in N, \forall t \in T \quad (9)$$

4. A Real Life Application

Data from the Super League Season has been used to set the priorities of the season's fixtures using AHP. Appendix 1 shows the fixtures and priorities that are used in this exercise. The mathematical model is input into ILOG OPL version 4.2.

For further clarification, we reiterate the whole objective of this study. There are 13 teams, playing a double round robin tournament, meaning that 156 matches have to be scheduled in 26 timeslots. The objective function is to maximize the matches with higher priorities during weekdays, leaving the lower priorities for weekends, on the assumption that supporters would have free time to watch matches during weekends.

The optimal solution is obtained in 7 seconds when run with ILOG CPLEX version 10, on an Intel Pentium 4, 2.4GHz with 1 GB RAM.

The schedule is tabulated into a readable format and presented in Table 2. The rows represent teams, the columns represent timeslots and the bordered

timeslots (T2, T3, T6, T9, T12) correspond to weekdays. The numbers in cells are team numbers. For example a fixture of Kedah against 12th team which is UPB Myteam is scheduled at timeslot T1 which falls on weekend. As seen from Table 3 shows a snapshot of fixtures' priorities, Kedah-UPB Myteam game has a low priority (0,0090) and it is convenient to assign this fixture to a weekend time slot. Similarly, Johor against Pulau Pinang (the fixture priority is 0,0090 and low) is scheduled in T4 which is weekend as expected. On the other hand a game of Perlis against 13th team which is Perak is scheduled at timeslot T3 which falls on a weekday as seen in Table 2. Since Perlis-Perak has one of the highest priorities as 0,0276, being scheduled in T3 which is weekday as seen in Table 2 on 2nd row (13 refers to Perak) is very suitable. Similarly, a fixture of Perak against Kedah (the priority is 0,0225 and considered as high) is scheduled in timeslot T2 which falls on a weekday as seen in the last row of Table 2. As seen from Table 2 and Appendix 1, most of the games have been assigned to time slots in such a way to maximize the objective function according to their

priorities. Colored ones illustrates a part of these assignments.

Table 2
Proposed schedule

Teams		Timeslots												
		T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13
Team1	KEDAH	12			6	11		7	9			4		8
Team2	PERLIS			13		6		4	8		12		1	
Team3	PULAU PINANG		2	1		9			5		13			
Team4	PAHANG		5	10		12			13				7	3
Team5	SARAWAK	13			2			7			1	8		
Team6	PDRM	7		8			5			3	4			12
Team7	JOHOR			11	3		2			1	9			
Team8	TERENGGANU		7				4	12			10		3	
Team9	N.SEMBILAN	4			8		10	6				2		5
Team10	SELANGOR	3	6			5			7		2	11		13
Team11	DPMM	8	9		4			3		5			6	2
Team12	UPB MYTEAM			5	13		3		11			7	10	
Team13	PERAK		1			7	11			8		6	9	

unimportant games scheduled at weekend
 important games scheduled at weekday

Table 3
Snapshot of sorted priorities

Fixture	Priorities
1 Kedah- UPB Myteam	0,0090
2 Johor- Pulau Pinang	0,0090
3 Perlis - Perak	0.0276
4 Perak - Kedah	0.0225
5 Sarawak-Perlis	0,0090

We analyze the proposed schedule by comparing it against the original schedule (see Appendix 2). In the original schedule, we find that most of the lower priorities matches are held during weekdays such as the fixture of Sarawak- Perlis (the priority is 0,0090) is scheduled at T12 (weekday). On the other hand this fixture is scheduled at weekend (T4) in the proposed solution. Perak-Kedah is scheduled at weekend (T8) in the current schedule our schedule assigns this important game to T2 which is weekday as expected. Thus, we believe that we produce superior schedules to maximize gate receipts.

5. Conclusion and Future Work

In this paper, we address the problem of decreasing gate receipts in Malaysian football and apply a mathematical model previously developed by the authors with the objective of maximizing the number of supporters. Besides we use Analytic Hierarchy Process approach to prioritize the fixtures. We assign the important games to weekdays while assigning the unimportant ones to weekends. It is based on the assumption that if the match is important, supporters will go to stadium to watch it no matter when it is played. Since weekday timeslots are not so attractive for supporters unless the game is important, we use this advantage to get them to the stadiums. We had discussions with the Football Association of Malaysia (FAM) with respect to implementing this approach. They have expressed significant interest in the idea of assigning the important games to weekdays and unimportant ones to weekends. However, they anticipate that there might be questions from the strong teams as they are frequently scheduled to play weekday matches.

Our discussions with the FAM representatives also highlight the current constraint that they are currently facing, which is, sharing of stadium between teams from the same states. For example, *KL, KL Plus* and *Felda* share *Stadium Bolasepak KL* in *Cheras*. *Terengganu* and *Terengganu PBDKL T-Team* are sharing *Terengganu Stadium*. Our next step for the future work is to include the new constraint into our model and produce new schedules. In future, we are also planning to investigate other methods of measuring the match importance.

Conflict of interest

The authors declare no conflict of interest.

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Appendices

Appendix 1 : Priorities of Each Fixture

Home	Away												
	KEDAH	PERLIS	PULAU PINANG	PAHANG	SARAWAK	PDRM	JOHOR	TERENGGANU	N.SEMBILAN	SELANGOR	DPMM	UPB MYTEAM	PERAK
KEDAH	0.0000	0.0225	0.0158	0.0110	0.0090	0.0091	0.0157	0.0131	0.0157	0.0099	0.0090	0.0090	0.0225
PERLIS	0.0225	0.0000	0.0166	0.0093	0.0090	0.0112	0.0208	0.0131	0.0112	0.0208	0.0093	0.0112	0.0276
PULAU PINANG	0.0158	0.0166	0.0000	0.0111	0.0132	0.0133	0.0090	0.0113	0.0090	0.0111	0.0091	0.0139	0.0158
PAHANG	0.0110	0.0093	0.0111	0.0000	0.0090	0.0112	0.0208	0.0111	0.0090	0.0112	0.0090	0.0090	0.0208
SARAWAK	0.0090	0.0090	0.0132	0.0090	0.0000	0.0091	0.0090	0.0090	0.0090	0.0091	0.0203	0.0132	0.0090
PDRM	0.0091	0.0112	0.0133	0.0112	0.0091	0.0000	0.0092	0.0208	0.0092	0.0129	0.0100	0.0116	0.0091
JOHOR	0.0157	0.0208	0.0090	0.0208	0.0090	0.0092	0.0000	0.0136	0.0157	0.0111	0.0111	0.0090	0.0111
TERENGGANU	0.0131	0.0131	0.0113	0.0111	0.0090	0.0208	0.0136	0.0000	0.0112	0.0208	0.0091	0.0091	0.0208
N.SEMBILAN	0.0157	0.0112	0.0090	0.0090	0.0090	0.0092	0.0157	0.0112	0.0000	0.0213	0.0110	0.0092	0.0191
SELANGOR	0.0099	0.0208	0.0111	0.0112	0.0091	0.0129	0.0111	0.0208	0.0213	0.0000	0.0090	0.0107	0.0210
DPMM	0.0090	0.0093	0.0091	0.0090	0.0203	0.0100	0.0111	0.0091	0.0110	0.0090	0.0000	0.0132	0.0110
UPB MYTEAM	0.0090	0.0112	0.0139	0.0090	0.0132	0.0116	0.0090	0.0091	0.0092	0.0107	0.0132	0.0000	0.0092
PERAK	0.0225	0.0276	0.0158	0.0208	0.0090	0.0091	0.0111	0.0208	0.0191	0.0210	0.0110	0.0092	0.0000

Appendix 2 : The Original Schedule

Among lowest priority matches – scheduled on weekdays

Timeslots

		T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13
Team1	KEDAH	2		6		10		12		9		5		
Team2	PERLIS			4		8		11		13		7		
Team3	PULAU PINANG	4		8		11		13		7			1	2
Team4	PAHANG		1			6	8	10		12		9		
Team5	SARAWAK	6		10		12		9			3		2	4
Team6	PDRM		3		2			8		11		13		
Team7	JOHOR	8		11		13			5		1		4	6
Team8	TERENGGANU		5		1					10		12		
Team9	N.SEMBILAN	10		12			7		3		2		6	8
Team10	SELANGOR		7		3		2		6			11		
Team11	DPMM	13	9		5		1		4		8			
Team12	UPB MYTEAM		13		7		3		2		6		10	11
Team13	PERAK				9		5		1		4		8	10