

A Markov Chain Modelling Of The Earthquakes Occuring In Turkey

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ABSTRACT

In this study, it is aimed to estimate seismic risk by Markov chain for Turkey, located between the longitudes of $36^{\circ}42'N$ and the latitudes of $26^{\circ}45'E$, using the earthquake data from the year 1901 to 2006. For this purpose, the most possible transition matrix was found by the maximum entropy principle and then the earthquakes in Turkey were tried to predict. Also, in a simulation study it was tested whether the prediction model is correct and some first passage time distributions are geometric. Besides, it was observed that for the earthquakes having magnitude $M \geq 4$ and the time interval $\Delta t = 0,07$ year, the method yielded an 81,1% aftcast success rate for the entire catalog.

Key Words: Markov Chain, Seismic Hazard, Entropy, Earthquake in Turkey.

1. INTRODUCTION

Turkey is a country which frequently comes across the earthquakes in various magnitudes since it takes part on the Alpine-Himalayan (Mediterranean) seismic belt, one of the important seismic belts of the world. Nowadays it is accepted that it is impossible either to know where and when earthquakes occur, or to predict surely in advance their magnitudes, and to prevent these devastating natural events. However, the statistical studies existing in the fields of geophysical, geological and earthquake engineering show that we can only probabilistically estimate the parameters of possible earthquakes and the severity of ground motions they created, by the years ahead. While the occurrence of earthquakes can not be prevented, based on these estimates it seems possible to take various measures against earthquakes so that casualties and damage are reduced to some extent. Keeping this in mind, it is aimed to predict seismic hazard by Markov chain, by means of earthquake occurrence data (magnitude $M \geq 4$ and from the year 1901 to 2006) of Turkey between the longitudes of $36^{\circ}42'N$ and the latitudes of $26^{\circ}45'E$.

In a recent study, using the data of years 1904-1992 for the North Anatolian fault line earthquakes, two models

are utilized and their results are compared, the Poisson model having the assumption that for the seismic risk analysis, the earthquakes are independent from the times and places that they occurred, and the Markov model based on the assumption that the earthquakes indicate a dependence on the time dimension in connection with the extreme value statistics and the elastic rebound theory. According to this study, the stochastic models examining the earthquake occurrence only in time domain give different risk estimates for earthquakes having magnitudes $4.5 \leq M \leq 6.5$, while risk estimates for the earthquakes having magnitudes $M > 6.5$ yield approximately the same results [14]. In another study, Özel and İnal [11] model the number of aftershocks occurred within one month in Turkey for the 94 destructive earthquakes between the years 1903 and 2005, having surface wave magnitudes $M_s \geq 5$, by the compound Poisson process. Also in a study by Gürten and Kasap [6], it is tried to predict the years of earthquake recurrence having various magnitudes. When compared to the studies in which the Poisson models used, it has been seen that the method used in [6] generally gives better results for small magnitude earthquakes, while the Poisson models give more good results for the earthquakes having large magnitude.

In the introduction section, the research problem is stated. The methodology used for the analysis is given in Section 2. In Section 3, the earthquakes occurring on

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Turkey are modelled by the Markov Chain and then the obtained results are interpreted. Finally, some conclusions and suggestions are contained in Section 4.

2. METHODOLOGY

In this section, the techniques that shall be used for the analysis will be given. Accordingly, we will summarize the methodology used for estimating the seismic risk.

2.1. Markov Chain

Modern probability theory studies random (stochastic) processes for which the knowledge of previous outcomes influences predictions for future experiments. In this principle, it is thought when we observe a sequence of chance experiments, all of the past outcomes could influence our predictions for the next experiment [5]. In 1907, A. A. Markov began the study of an important new type of chance process. In this process, the outcome of a given experiment can affect the outcome of the next experiment [5, 1]. In other

$$P\{X_{n+1} = j \mid X_0 = i_0, X_1 = i_1, \dots, X_{n-1} = i_{n-1}, X_n = i\} = P\{X_{n+1} = j \mid X_n = i\} = P_{ij,n}$$

for all j 's and i 's, and $n \geq 0$. By this definition, a Markov chain is a sequence of random variables such that for any n , the "next" state of the process X_{n+1} is independent of the "past" states X_0, X_1, \dots, X_{n-1} ; that is, the strong Markov property is to hold at randomly chosen times [3]. The probability P_{ij} is called (*one step*) *transition probability* from state i to state j . When the transition probabilities satisfy the condition, $P_{ij,n} = P_{ij}$, for all $n \geq 0$, i.e., they are independent of the time parameter n , then the Markov chain $X = \{X_n : n = 0, 1, 2, \dots\}$ is said to be *time-homogeneous*, or *stationary* [5,1]:

$$P\{X_{n+1} \mid X_n = i\} = P_{ij,n} = P_{ij} \quad ; \quad i, j \in S.$$

For the Markov chains, the transition probabilities are arranged in a matrix form and the resulting matrix is called the *transition matrix* of the chain. The elements of a transition matrix hold the following conditions:

a) for any two states $i, j \in S$, $P_{ij} \geq 0$; and

b) for all $i \in S$, $\sum_j P_{ij} = 1$.

As it can be easily seen from the next theorem and following corollary, the joint distribution X_0, X_1, \dots, X_m can be completely specified for every m once the initial distribution and the transition matrix P are known [3].

words, Markov chains are the stochastic processes whose futures are conditionally independent of their pasts provided that their present values are known [3].

Let $X = \{X_n : n = 0, 1, 2, \dots\}$ be a stochastic process that has a finite or countable infinite state space S . When $X_n = i$, we say that 'the process is in state i at time n '. The probability that the process is in state j in the next time provided that its present state is i , is denoted by P_{ij} .

Let $i_0, i_1, \dots, i_{n-1}, i, j$ be the states of the process and $n \geq 0$. The stochastic process $X = \{X_n : n = 0, 1, 2, \dots\}$ is called a Markov chain provided that

Theorem 2.1.1 Let $X = \{X_n : n \in N\}$ be a Markov chain. For any $m, n \in N ; m \geq 1$ and $i_1, i_2, \dots, i_m \in S$,

$$P\{X_{n+1} = i_1, X_{n+2} = i_2, \dots, X_{n+m} = i_m \mid X_n = i_0\} = P_{i_0 i_1} P_{i_1 i_2} \dots P_{i_{m-1} i_m}$$

Corollary 2.1.1 For the Markov chain, let the initial probability distribution π_0 be given on the state space S ; i.e., let $P\{X_0 = i\} = \pi_0(i)$ be for all $i \in S$. Then for $m \in N$ and $m \in N$ $i_0, i_1, i_2, \dots, i_m \in S$, we have

$$P\{X_0 = i_0, X_1 = i_1, \dots, X_m = i_m\} = \pi_0(i_0) P_{i_0 i_1} P_{i_1 i_2} \dots P_{i_{m-1} i_m}$$

In some cases, it is needed to calculate the probabilities for the transitions between distant times for Markov chain. Thus, the following definition is given.

Definition 2.1.1. For any $m \in N$, n -step *transition probability* from state i to state j is given by

$$P\{X_{m+n} = j \mid X_m = i\} = P_{ij}^{(n)} ; i, j \in S, n \in N.$$

Among the Markov chain characteristics, the first passage times play an important role. For any two states, the first passage time probability in n steps is defined as follows and this probability is related to the ever reaching probability.

Definition 2.1.2. For any two states i and j , the *first passage time probability* from i to j in n steps, $f_{ij}^{(n)}$ is defined as

$$f_{ij}^{(n)} = \begin{cases} p_{ij}; n = 1 \\ \sum_{b \in E - \{j\}} p_{ij} f_{bj}^{(n-1)}; n = 2, 3, \dots \end{cases}$$

Definition 2.1.3. The value $f_{ij} = \sum_{n=1}^{\infty} f_{ij}^{(n)}$ is called *ever reaching probability*, or *reaching probability in every step* from state i to state j [3].

The following theorem reflects how to calculate the steady state probabilities for the process.

Theorem 2.1.2. If $X = \{X_n : n = 0, 1, 2, \dots\}$ is an irreducible aperiodic finite state Markov chain, the system of equations

$$\begin{aligned} \pi^1 P &= \pi^1 \\ \pi^1 \underline{1} &= 1 \end{aligned}$$

has a unique positive solution. This solution is called the *limit distribution* of Markov chain.

Definition 2.1.4. An important indicator of the first passage times is the *mean first passage time* and for an irreducible recurrent Markov chain, this quantity is calculated as

$$\mu_{ij} = 1 + \sum_{k \neq j} p_{ik} \mu_{kj} \quad \text{or} \quad \mu_{ii} = \frac{1}{\pi_i} \quad [3].$$

2.2. Entropy

2.2.1. Introduction

Entropy measures the uncertainty of a collection of events while probability measures uncertainty about the occurrence of a single event [8]. In other words, entropy is a measure of the uncertainty level for a system. Occurrence probability of any event is an indicator of whether or not this event occurs at a certain level of uncertainty [4]. According to Shannon, who has done studies on entropy, it can be mentioned to learn about an event only in the case in which it includes an uncertainty. Accordingly, the higher the likelihood of occurrence of events does not bring more information, on the contrary, the occurrence of unlikely events carry more information [12].

For discrete random variables entropy is defined as follows:

Definition 2.2.1. Let X be a random variable having the values $\{x_1, x_2, \dots, x_n\}$ and corresponding probabilities

$$p(x_i) = p(X = x_i) = p_i ; i = 1, 2, \dots, n.$$

The entropy of discrete random variable X is defined by

$$H(X) = H(p) = -c \sum_{i=1}^n p_i \log p_i$$

where c is an arbitrary positive constant and is taken as $c = 1$ when the logarithm base is 2. In addition, in calculations it is assumed that $\log 0 = 0$ [9].

2.2.2. Maximum Entropy Principle of Jaynes

Shannon suggests making the entropy measure maximum and choosing the distribution simultaneously consistent with the average constraints. Let X be a random variable having the values x_1, x_2, \dots, x_n with corresponding probabilities p_1, p_2, \dots, p_n , respectively. *Maximum Entropy Principle* is a natural extension of the Laplace's famous 'insufficient reason principle' which assumes that uniform distribution is the most satisfactory candidate of our knowledge when we don't know anything about the random variable X

except $p_i \geq 0$ ($i = 1, 2, \dots, n$) and $\sum_{i=1}^n p_i = 1$.

According to Jaynes, if a distribution is chosen such that its entropy is less than maximum entropy, this reduction in entropy might have come from some additional information used consciously or unconsciously. However, in the case in which such information is not given, it would not be right to use the distribution having less entropy. Thus, only the distribution having the maximum entropy should be used [4].

2.2.3. Entropy and Markov Chains

Let $i, j \in S$ be the states of Markov chain, p_i be the probability of i , and $p_i(j) = p_{ij}$ be the conditional probability of j given i . For the Markov chains the entropy is denoted by $H(S)$ and is defined by

$$H(S) = -\sum_i p_i \sum_j p_i(j) \log_2 p_i(j).$$

3. APPLICATION TO EARTHQUAKE DATA

3.1. Aim and Content of the Application

In this section, it is intended to estimate the seismic risk by using Markov chains on the basis of the statistical analysis of the earthquakes in Turkey. For this purpose, a similar study that was done by Nava et al. [6] for Japan is done for Turkey and the results obtained are evaluated.

3.2. Application Data and Procedure

In the seismic risk assessment, generally it should be chosen a threshold magnitude M_r , such that $M \geq M_r$ for the large and destructive earthquakes. For all regions the threshold magnitude is taken as $M_r = 4$, in conformity with our observations that there might be many people died and homeless in fact in the case of an earthquake of magnitude 4 in the East Anatolia Region owing to weak building structure. Therefore, in our study, we have used the seismic data having magnitude $M \geq 4$ of the earthquakes in Turkey between the longitudes of $36^{\circ}42'N$ and the latitudes of $26^{\circ}45'E$. Historical data belong to the years 1901-2006 and are received from Bogazici

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Then by the consideration of earthquake zones map in geographic information system (GIS) [13], and the seismic activity maps to Turkey and its vicinity in the Integrated Homogeneous Earthquake Catalog [7], and Turkey's fault lines (North Anatolia, Eastern Anatolia, Western Anatolia), Turkey is divided into four areas as follows:

- Region 1, if latitude $\geq 39,5$;
- Region 2, if latitude $< 39,5$ and longitude ≤ 31 ;
- Region 3, if latitude $< 39,5$ and longitude ≥ 36 ; and
- Region 4, if latitude $< 39,5$ and $31 < \text{longitude} < 36$.

See Map 3.1.



Map 3.1 Separation of regions of Turkey for the study.

Given a seismic catalog and a starting time, during each time interval Δt , the state of each r th region S_r can have one of two values: 0 or 1, corresponding, respectively, to the absence or presence in it of the earthquakes with magnitude larger than or equal to the threshold value M_r^0 . In this study, since Turkey is divided into four regions, there are $2^4=16$ states which can be encountered. Hence the set of all possible states is $S = \{0,1,2,\dots,15\}$.

For a given interval Δt , if there are no earthquakes in any region, we write 0000 for the state 0, if there is earthquake(s) only in region 1, we write 1000 for the state 1, if there is earthquake(s) only in region 2, we write 0100 for the state 2, ..., and if there is (are) earthquake(s) in all regions, we write 1111 for the state 15, and the regions and corresponding states can be shown as follows:

State	=	R e g i o n			
0	=	0	0	0	0
1	=	1	0	0	0
2	=	0	1	0	0
3	=	1	1	0	0
4	=	0	0	1	0
5	=	1	0	1	0
6	=	0	1	1	0
7	=	1	1	1	0
8	=	0	0	0	1
9	=	1	0	0	1
10	=	0	1	0	1
11	=	1	1	0	1
12	=	0	0	1	1
13	=	1	0	1	1
14	=	0	1	1	1
15	=	1	1	1	1

A parameter that should be chosen in such an application is the time interval Δt which is used to determine the system states. For a too small Δt , state 0 (no earthquakes in any region) will be the most frequent one, so that the transition 0 to 0 will be dominant, and other probabilities different from $p_{0,0}$ may be so small as to have no forecasting value. Conversely, for a too large Δt , state 15 (earthquakes in all regions) will be more frequent than any other, so that the transition 15 to 15 will be dominant, and all probabilities different from $p_{15,15}$ may be so small as to have no forecasting value.

In addition, for a given catalog length, increasing Δt diminishes the number of sampled transitions, and makes estimates of p_{ij} less robust [10].

In order to determine the parameter Δt , it has been benefited from Maximum Entropy Principle and has been observed that the most suitable transition matrix corresponding to the year $\Delta t = 0,07$ is the most fitted to our objective. From the data, the matrix of transition frequencies and transition matrix are estimated as follows:

The matrix of transition frequencies:

287	72	67	14	29	8	10	6	9	2	7	3	3	2	0	1
73	31	13	13	8	10	3	13	1	2	2	2	1	1	0	1
59	20	34	13	16	4	10	15	6	1	4	3	0	0	0	4
20	8	15	18	7	5	10	20	1	1	4	4	1	2	4	2
25	14	7	10	5	3	5	8	4	1	1	0	1	0	1	1
11	5	10	3	2	5	1	6	2	0	1	3	0	1	0	2
9	4	11	14	6	3	6	7	2	2	2	4	0	0	1	2
4	5	18	19	5	9	12	16	1	1	0	6	2	2	4	6
12	4	2	3	1	0	3	1	0	0	0	0	0	1	0	1
1	3	3	1	1	0	1	2	0	2	1	1	1	0	0	0
8	2	4	2	1	1	2	2	0	1	0	1	0	0	1	0
5	1	3	6	2	1	4	4	1	0	2	4	1	0	2	1
2	3	0	1	1	0	0	1	0	1	0	0	0	0	1	1
1	0	0	2	1	0	3	0	0	1	0	0	1	1	0	0
1	0	0	1	0	1	3	3	1	2	0	4	1	0	3	2
1	2	2	2	1	2	0	6	0	1	0	2	0	0	4	0

The transition matrix:

0,5519	0,1385	0,1288	0,0269	0,0558	0,0154	0,0192	0,0115	0,0173	0,0038	0,0135	0,0058	0,0058	0,0038	0,0000	0,0019
0,4195	0,1782	0,0747	0,0747	0,0460	0,0575	0,0172	0,0747	0,0057	0,0115	0,0115	0,0115	0,0057	0,0057	0,0000	0,0057
0,3122	0,1058	0,1799	0,0688	0,0847	0,0212	0,0529	0,0794	0,0317	0,0053	0,0212	0,0159	0,0000	0,0000	0,0000	0,0212
0,1639	0,0656	0,1230	0,1475	0,0574	0,0410	0,0820	0,1639	0,0082	0,0082	0,0328	0,0328	0,0082	0,0164	0,0328	0,0164
0,2907	0,1628	0,0814	0,1163	0,0581	0,0349	0,0581	0,0930	0,0465	0,0116	0,0116	0,0000	0,0116	0,0000	0,0116	0,0116
0,2115	0,0962	0,1923	0,0577	0,0385	0,0962	0,0192	0,1154	0,0385	0,0000	0,0192	0,0577	0,0000	0,0192	0,0000	0,0385
0,1233	0,0548	0,1507	0,1918	0,0822	0,0411	0,0822	0,0959	0,0274	0,0274	0,0274	0,0548	0,0000	0,0000	0,0137	0,0274
0,0364	0,0455	0,1636	0,1727	0,0455	0,0818	0,1091	0,1455	0,0091	0,0091	0,0000	0,0545	0,0182	0,0182	0,0364	0,0545
0,4286	0,1429	0,0714	0,1071	0,0357	0,0000	0,1071	0,0357	0,0000	0,0000	0,0000	0,0000	0,0000	0,0357	0,0000	0,0357
0,0588	0,1765	0,1765	0,0588	0,0588	0,0000	0,0588	0,1176	0,0000	0,1176	0,0588	0,0588	0,0588	0,0000	0,0000	0,0000
0,3200	0,0800	0,1600	0,0800	0,0400	0,0400	0,0800	0,0800	0,0000	0,0400	0,0000	0,0400	0,0000	0,0000	0,0400	0,0000
0,1351	0,0270	0,0811	0,1622	0,0541	0,0270	0,1081	0,1081	0,0270	0,0000	0,0541	0,1081	0,0270	0,0000	0,0541	0,0270
0,1818	0,2727	0,0000	0,0909	0,0909	0,0000	0,0000	0,0909	0,0000	0,0909	0,0000	0,0000	0,0000	0,0000	0,0909	0,0909
0,1000	0,0000	0,0000	0,2000	0,1000	0,0000	0,3000	0,0000	0,0000	0,0000	0,1000	0,0000	0,0000	0,1000	0,1000	0,0000
0,0455	0,0000	0,0000	0,0455	0,0000	0,0455	0,1364	0,1364	0,0455	0,0909	0,0000	0,1818	0,0455	0,0000	0,1364	0,0909
0,0435	0,0870	0,0870	0,0870	0,0435	0,0870	0,0000	0,2609	0,0000	0,0435	0,0000	0,0870	0,0000	0,0000	0,1739	0,0000

3.3. Markov Chain Analysis

In the first part of the application, after the estimation of transition matrix, it has been proceeded to the stages of analyzing the information obtained and the interpretation. In the stages of analysis, Microsoft Excel, WinQSB-Markov Process, Q-Basic, and Matlab programs are used.

3.3.1. Chi-Square Analysis

For the goodness-of-fit test of the transition matrix, it has been conducted a chi-square analysis after the simulation study. In the simulation study with the same total frequency, we obtained the following expected frequencies, and observed frequencies from data:

Expected Frequencies														Observed Frequencies																					
281	67	70	11	38	8	3	9	11	1	7	0	5	1	0	1	287	72	67	14	29	8	10	6	9	2	7	3	3	2	0	1				
69	31	19	13	8	5	3	14	2	2	4	4	2	0	0	1	73	31	13	13	8	10	3	13	1	2	2	2	1	1	0	1				
58	25	46	17	15	6	14	12	4	1	8	6	0	0	0	1	59	20	34	13	16	4	10	15	6	1	4	3	0	0	0	4				
22	4	15	18	9	3	6	20	1	1	5	2	1	3	3	2	20	8	15	18	7	5	10	20	1	1	4	4	1	2	4	2				
32	17	9	13	6	5	5	6	1	0	1	0	3	0	3	1	25	14	7	10	5	3	5	8	4	1	1	0	1	0	1	1	1			
9	4	6	2	1	4	0	6	1	0	1	3	0	1	0	2	11	5	10	3	2	5	1	6	2	0	1	3	0	1	0	2	2			
9	3	12	10	7	3	3	6	0	2	1	2	0	0	1	3	9	4	11	14	6	3	6	7	2	2	2	4	0	0	1	2	1			
2	8	17	21	6	3	15	15	2	0	0	2	1	1	7	6	4	5	18	19	5	9	12	16	1	1	0	6	2	2	4	6	6			
9	6	1	1	0	0	4	1	0	0	0	0	0	0	0	2	12	4	2	3	1	0	3	1	0	0	0	0	0	1	0	1	0	1	1	
1	4	5	0	1	0	0	0	0	1	1	1	0	0	0	0	1	3	3	1	1	0	1	2	0	2	1	1	1	0	0	0	0	0	0	
9	1	4	2	3	0	2	3	0	1	0	1	0	0	3	0	8	2	4	2	1	1	2	2	0	1	0	1	0	0	1	0	0	1	0	
1	0	6	5	3	2	4	2	1	0	1	2	0	0	4	1	5	1	3	6	2	1	4	4	1	0	2	4	1	0	2	1	1	1	1	
4	3	0	1	3	0	0	2	0	1	0	0	0	0	0	2	2	3	0	1	1	0	0	1	0	1	0	0	0	0	0	1	1	0	1	1
3	0	0	1	0	0	0	0	0	0	0	0	0	0	1	2	0	0	0	2	1	0	3	0	0	0	1	0	0	1	0	0	1	1	0	0
3	0	0	1	0	0	3	4	1	2	0	6	3	0	2	1	1	0	0	1	0	1	3	3	1	2	0	4	1	0	3	2	1	0	3	2
1	4	3	0	2	1	0	6	0	2	0	3	0	0	1	0	1	2	2	2	1	2	0	6	0	1	0	2	0	0	4	0	4	0	4	0

H_0 : Estimated transition matrix fits the data.

H_1 : Estimated transition matrix does not fit the data.

From the chi-square analysis, we have

$\chi^2_{Cal} =$	75,53206
$\chi^2_{90,0.05} =$	113,145
Conclusion:	H_0 Accept

Moreover, by the consideration of the observed and expected frequencies, we conclude that, we have an 81,1 % aftcast (forecast of data already used to evaluate

the hazard) success rate in the average for the entire catalog (period). Some of the first passage time distributions observed are given below and their goodness-of-fit to the geometric distribution is tested by chi-square analysis.

H_0 : Transitions have a geometric distribution with success probability p .

H_1 : Transitions do not have a geometric distribution with success probability p .

Table 3.1 The observed distributions of the first passage time from state i to state j .

$i = 1, j = 1$

Period	Frequency
1	31
2	15
3	21
4	13
5	7
6	11
7	8
8	9
9	6
10	8
11	2
12	7
13	4
14	3
15	4
16	2
17	1
18	4
19	3
26	3
28	1
29	2
31	1
32	1
33	1
34	1
51	1
52	1
57	1
60	1

Mean =	8,618497
p =	0,11603

$\chi^2_{Cal} =$	17,03349
$\chi^2_{11,0.05} =$	19,6751
Conclusion:	H_0 Accept

$i = 2, j = 2$

Period	Frequency
1	34
2	26
3	20
4	16
5	13
6	10
7	11
8	5
9	5
10	5
11	10
12	1
13	4
14	4
15	2
16	2
17	2
18	2
19	1
20	1
23	1
25	3
26	2
30	2
47	1
49	1
50	1
53	1
59	1
65	1

Mean =	7,87766
p =	0,126941

	16,44363
$\chi^2_{11,0.05} =$	19,6751
Conclusion:	H_0 Accept

$i = 0, j = 1$

Period	Frequency
1	20
2	14
3	14
4	10
5	9
6	4
7	9
8	5
9	4
10	3
11	6
12	4
13	3
14	4
15	2
17	1
18	1
19	1
24	1
26	1
28	2
31	1
32	1
37	1
39	1
52	1

Mean =	7,788618
p =	0,128392

$\chi^2_{Cal} =$	15,60683
$\chi^2_{10,0.05} =$	18,3
Conclusion:	H_0 Accept

3.3.2. The Probabilities of at least k earthquakes occurrence

From the chi-square analysis, it has been seen that the distributions of the first return times to the states 1 and 2 are geometric distributions with success probabilities $p_1 = 0,1160$ and $p_2 = 0,1269$, respectively.

Let X be the number of periods in any year in which earthquakes occur. Since there are approximately

$\frac{365}{25,5} \cong 15$ periods in a year, we have

$$X \sim b(x;15; p) = \begin{cases} \binom{15}{x} p^x q^{15-x}, & x = 0,1,2,\dots,15 \\ 0; & d.h. \end{cases}$$

Table 3.2 Probabilities of at least k periods of earthquakes occurrence in a year for the states 1 and 2.

States	k	Probability of at least k periods of earthquakes occurrence
1	1	0,8428
	2	0,5332
	3	0,2487
	4	0,0869
	5	0,0232
2	1	0,8695
	2	0,5848
	3	0,2951
	4	0,1126
	5	0,0330

3.3.3. Regional Transition Probabilities

From the transition probability matrix, it is possible to obtain the conditional probabilities of earthquake occurrence in region L given that the system is in state i as follows:

$$p_{iL} = \Pr(L | i) = \sum_{j \supset L} p_{ij} \tag{3.1}$$

where $j \supset L$ indicates that state j involves earthquake occurrence in region L, and in general

$$\sum_L p_{iL} \neq 1 \text{ [10].}$$

WinQSB package program stated that the system can reach to this steady state period on the average, after 22 periods (a period in excess of approximately 1,5 years). This limit distribution can be interpreted as, in the long-run there will be no earthquakes in all the regions in 34,6% of the time, there will be earthquake(s) only in the region 1 in 11,6% of the time, ..., and there will be earthquake(s) (affecting) in all the regions in 1,6% of

In consequence, the matrix of transition probabilities from states to regions is obtained as follows:

	Region(L)			
State (j)	0,207692	0,207692	0,113462	0,051923
	0,419540	0,270115	0,212644	0,057471
	0,317460	0,439153	0,259259	0,095238
	0,491803	0,631148	0,418033	0,155738
	0,430233	0,383721	0,279070	0,104651
	0,480769	0,500000	0,326923	0,173077
	0,493151	0,643836	0,342466	0,178082
	0,581818	0,736364	0,509091	0,200000
	0,357143	0,357143	0,250000	0,071429
	0,529412	0,529412	0,294118	0,294118
	0,360000	0,480000	0,280000	0,120000
	0,459459	0,702703	0,405405	0,297297
	0,636364	0,363636	0,363636	0,272727
	0,300000	0,700000	0,600000	0,300000
0,590909	0,727273	0,590909	0,590909	
0,652174	0,695652	0,565217	0,304348	

From the above matrix, for example, looking at the line 9, we can observe that in the case in which an earthquake occurs only in the region 4 in any period (with length $\Delta t = 0,07$ year) the probabilities of earthquake occurrences in each region for the next period are low. Hence, in a sense we can achieve the result that any earthquake in the region 4 does not trigger much more the earthquakes which may occur in the other regions. Besides, the aftcasts of regional activity have a 92,35% success rate in the average and those of activity in the highest probability region about 93,52% success rate.

3.3.4. Limit distribution

The limit distribution of the Markov chain is found to be:

$$\pi' = (0,3455 \ 0,1160 \ 0,1260 \ 0,0815 \ 0,0573 \ 0,0348 \ 0,0487 \ 0,0736 \ 0,0187 \ 0,0114 \ 0,0167 \ 0,0248 \ 0,0073 \ 0,0067 \ 0,0148 \ 0,0160)$$

the time, where the length of a period is $\Delta t = 0,07$ year.

For Markov chains, the ratio $\pi(k)/\pi(j)$ can be interpreted as the expected number of visits to k between two visits to j [3]. Under this interpretation, we can evaluate the following matrix:

1,0000	2,9795	2,7411	4,2389	6,0243	9,9371	7,0871	4,6910	18,5050	30,3310	20,7220	13,9280	47,0140	51,7760	23,2770	21,5310
0,3356	1,0000	0,9200	1,4227	2,0219	3,3352	2,3787	1,5745	6,2108	10,1800	6,9551	4,6746	15,7790	17,3780	7,8126	7,2264
0,3648	1,0870	1,0000	1,5465	2,1978	3,6253	2,5855	1,7114	6,7510	11,0660	7,5600	5,0811	17,1520	18,8890	8,4921	7,8549
0,2359	0,7029	0,6466	1,0000	1,4212	2,3442	1,6719	1,1067	4,3654	7,1554	4,8886	3,2856	11,0910	12,2140	5,4913	5,0793
0,1660	0,4946	0,4550	0,7036	1,0000	1,6495	1,1764	0,7787	3,0717	5,0349	3,4398	2,3119	7,8041	8,5946	3,8639	3,5740
0,1006	0,2998	0,2758	0,4266	0,6062	1,0000	0,7132	0,4721	1,8622	3,0523	2,0853	1,4016	4,7311	5,2104	2,3425	2,1667
0,1411	0,4204	0,3868	0,5981	0,8500	1,4021	1,0000	0,6619	2,6111	4,2798	2,9239	1,9652	0,0038	7,3057	3,2844	3,0380
0,2132	0,6351	0,5843	0,9036	1,2842	2,1183	1,5108	1,0000	3,9447	6,4658	4,4174	2,9690	10,0220	11,0370	4,9621	4,5898
0,0540	0,1610	0,1481	0,2291	0,3256	0,5370	0,3830	0,2535	1,0000	1,6391	1,1198	0,7527	2,5406	2,7980	1,2579	1,1635
0,0330	0,0982	0,0904	0,1398	0,1986	0,3276	0,2337	0,1547	0,6101	1,0000	0,6832	0,4592	1,5500	1,7070	0,7674	0,7099
0,0483	0,1438	0,1323	0,2046	0,2907	0,4795	0,3420	0,2264	0,8930	1,4637	1,0000	0,6721	2,2687	2,4986	1,1233	1,0390
0,0718	0,2139	0,1968	0,3044	0,4325	0,7135	0,5089	0,3368	1,3286	2,1778	1,4879	1,0000	3,3756	3,7175	1,6713	1,5459
0,0213	0,0634	0,0583	0,0902	0,1281	0,2114	0,1507	0,0998	0,3936	0,6452	0,4408	0,2962	1,0000	1,1013	0,4951	0,4580
0,0193	0,0575	0,0529	0,0819	0,1164	0,1919	0,1369	0,0906	0,3574	0,5858	0,4002	0,2690	0,9080	1,0000	0,4496	0,4158
0,0430	0,1280	0,1178	0,1821	0,2588	0,4269	0,3045	0,2015	0,7950	1,3030	0,8902	0,5983	2,0197	2,2243	1,0000	0,9250
0,0464	0,1384	0,1273	0,1969	0,2798	0,4615	0,3292	0,2179	0,8595	1,4087	0,9625	0,6469	2,1836	2,4048	1,0811	1,0000

For example, the element in the 5th row and 13th column of this matrix can be interpreted as ‘we expect that Markov chain passes approximately 8 times to state 4 between the transitions to state 12’. In terms of earthquakes, it is possible to interpret it as follows: between the two earthquakes occurred only in the region 3, it is expected approximately 8 earthquakes in the regions 3 and 4 occurring simultaneously.

3.3.5 Estimated Distribution of Earthquakes in Turkey in Future Times

In this section, using 2006 as the beginning year, it has been made the predictions of earthquakes in Turkey in the future years. Hence, using the initial distribution from the observations of the year 2006,

$$\pi'_0 = (0,0000 \ 0,0714 \ 0,1429 \ 0,0714 \ 0,0714 \ 0,0714 \ 0,2143 \ 0,1429 \ 0,0714 \ 0,0714 \ 0,0000 \ 0,0000 \ 0,0000 \ 0,0714 \ 0,0000 \ 0,0000)$$

and n , the number of periods after the year 2006 such that $2006 + 0,07 \text{ year} \times n$, the distribution of earthquakes in the period n , π'_n is given by

The following table gives the estimated distribution of earthquakes in the next five periods from the beginning of 2006.

$$\pi'_n = \pi'_0 P^n ; n=1,2,\dots$$

Table 3.3 Estimated distributions for the first five periods from 01.01.2006.

2006+ 0,07year	0,196	0,009	0,133	0,130	0,006	0,040	0,009	0,010	0,019	0,002	0,003	0,003	0,001	0,002	0,002	0,002
2006+ 0,14year	0,278	0,103	0,127	0,101	0,006	0,004	0,006	0,009	0,002	0,001	0,002	0,003	0,001	0,001	0,002	0,002
2006+ 0,21year	0,312	0,109	0,126	0,009	0,006	0,004	0,005	0,008	0,002	0,001	0,002	0,003	0,001	0,001	0,002	0,002
2006+ 0,28year	0,329	0,113	0,126	0,009	0,006	0,004	0,005	0,008	0,002	0,001	0,002	0,003	0,001	0,001	0,002	0,002
2006+ 0,35year	0,337	0,114	0,126	0,008	0,006	0,004	0,005	0,008	0,002	0,001	0,002	0,003	0,001	0,001	0,002	0,002

According to the estimation of first period, between 01.01.2006-25.01.2006 ($\Delta t = 0,07$ year, about 25.5 days), the probability that there are no earthquakes

having magnitude $M \geq 4$ in any region is 19,57%, earthquake(s) only in region 1 is 0,9%, ..., and earthquake(s) (affecting) all the regions is 0,2%.

3.3.6. Mean first passage times

Table 3.4. Mean first passage times from state i to state j .

i	j	μ_{ij}
0	0	2,8943
	1	8,7826
	2	8,3918
	3	14,9951
	4	17,4127
	5	31,8400
	6	23,1067
	7	17,3527
	8	52,5358
	9	100,5710
	10	59,5740
	11	47,2916
	12	137,5020
	13	166,1210
	14	83,0862
15	65,6818	

i	j	μ_{ij}
1	0	3,6340
	1	8,6235
	2	8,8134
	3	14,0408
	4	17,6448
	5	30,1896
	6	22,8126
	7	15,9944
	8	53,2260
	9	99,5470
	10	59,5750
	11	46,3994
	12	137,1280
	13	165,3290
	14	82,2702
15	64,8729	

i	j	μ_{ij}
2	0	4,1465
	1	9,3821
	2	7,9335
	3	13,8617
	4	16,9005
	5	31,2373
	6	21,7871
	7	15,6772
	8	51,6817
	9	99,9203
	10	58,9788
	11	45,9639
	12	137,9290
	13	166,3180
	14	81,8157
15	63,6081	

i	j	μ_{ij}
3	0	5,1782
	1	10,1413
	2	8,3973
	3	12,2688
	4	17,4258
	5	30,2134
	6	20,4106
	7	13,7982
	8	52,9625
	9	98,7370
	10	57,9853
	11	43,9827
	12	136,0650
	13	163,1560
	14	77,7992
15	62,8307	

i	j	μ_{ij}
4	0	4,2858
	1	8,8975
	2	8,7858
	3	13,1842
	4	17,4363
	5	30,7029
	6	21,6128
	7	15,3720
	8	51,0702
	9	99,0530
	10	59,5519
	11	46,4228
	12	136,1390
	13	165,9490
	14	80,7712
15	63,9915	

i	j	μ_{ij}
5	0	4,7225
	1	9,6795
	2	7,8143
	3	13,7644
	4	17,7437
	5	28,7614
	6	22,1244
	7	14,8430
	8	51,3060
	9	100,3380
	10	58,8883
	11	43,5794
	12	137,7620
	13	162,7510
	14	80,8809
15	61,9856	

i	j	μ_{ij}
6	0	5,2759
	1	10,1828
	2	8,1564
	3	11,7359
	4	16,9679
	5	30,3053
	6	20,5125
	7	14,6733
	8	51,9054
	9	97,1055
	10	58,1904
	11	43,2138
	12	137,2060
	13	165,8810
	14	79,3764
15	62,3866	

i	j	μ_{ij}
7	0	5,8779
	1	10,4870
	2	8,0734
	3	11,6700
	4	17,6180
	5	28,7800
	6	19,6040
	7	13,5770
	8	52,8160
	9	98,2560
	10	59,7480
	11	42,4110
	12	134,5400
	13	162,6900
	14	76,5060
15	59,9150	

i	j	μ_{ij}
8	0	3,7168
	1	9,0650
	2	8,8879
	3	13,4210
	4	17,7380
	5	31,9420
	6	20,6820
	7	16,4330
	8	53,5580
	9	100,3500
	10	60,0060
	11	46,7070
	12	138,0400
	13	160,5900
	14	81,5920
15	63,0780	

i	j	μ_{ij}
9	0	5,4353
	1	8,8310
	2	7,9901
	3	13,5290
	4	17,3310
	5	31,6380
	6	21,2340
	7	14,5260
	8	53,6480
	9	87,7870
	10	56,2830
	11	43,2820
	12	128,5700
	13	166,4700
	14	80,7160
15	64,1440	

i	j	μ_{ij}
10	0	4,3038
	1	9,7385
	2	8,0818
	3	13,6310
	4	17,6970
	5	30,6600
	6	21,0400
	7	15,5170
	8	53,3090
	9	96,1350
	10	59,9750
	11	44,3130
	12	137,2400
	13	166,4200
	14	78,6680
15	64,5420	

i	j	μ_{ij}
11	0	5,3911
	1	10,5760
	2	8,8239
	3	11,9500
	4	17,5270
	5	30,6630
	6	19,7300
	7	14,3830
	8	51,9210
	9	98,9400
	10	56,6140
	11	40,3110
	12	133,1500
	13	165,9700
	14	75,4940
15	61,8590	

Table 3.4. (Continued) Mean first passage times from state i to state j .

i	j	μ_{ij}	i	j	μ_{ij}	i	j	μ_{ij}	i	j	μ_{ij}
12	0	4,9972	13	0	5,6684	14	0	6,2425	15	0	6,0479
	1	8,0235		1	11,034		1	11,0640		1	10,2710
	2	9,6276		2	9,5711		2	9,6167		2	8,8162
	3	13,3950		3	11,1200		3	13,0650		3	12,7920
	4	17,0620		4	16,6090		4	18,6190		4	17,9310
	5	31,3520		5	31,6010		5	29,8630		5	28,4310
	6	22,6060		6	15,0240		6	18,6400		6	21,5160
	7	14,7810		7	15,9110		7	13,3450		7	11,8700
	8	53,6700		8	53,2780		8	50,9460		8	53,1790
	9	89,9360		9	98,1170		9	88,5100		9	93,9630
	10	60,1770		10	53,2460		10	59,5470		10	59,9640
	11	45,0950		11	44,5540		11	35,3540		11	39,8140
	12	136,0700		12	137,0400		12	128,7800		12	135,2900
	13	166,2800		13	149,8400		13	166,0300		13	165,6000
	14	73,2720		14	71,8060		14	67,3690		14	66,2230
15	58,5940	15	63,7450	15	56,6730	15	62,3180				

For example, if it needs to interpret $\mu_{01} = 8,7826$, so it is expected to pass 8,78 periods (about 9 periods), i.e., $(8,78 * 25,5 = 223,96$ days) until the first earthquake occurrence only in the region 1, given that there no earthquakes in any region.

4. CONCLUSIONS AND SUGGESTIONS

In the earthquakes Erzincan, December 26, 1939, one of the largest earthquakes of the 20th century and Marmara, August 17, 1999 thousands of our citizens lost their lives and tens of thousands wounded, hundreds of thousands of buildings were destroyed. The experiences we have in the past are revealed that we will face with these type destructive earthquakes in the future. At this point what can be done is to try to minimize the effects of catastrophic earthquakes by their estimates to be obtained. For this purpose, we have tried to do a study for Turkey which is similar to Nava et al. (2005) that have been done for Japan and used a different statistical perspective.

In this study, we have done some statistical analysis and predictions for the earthquake data having the magnitudes $M \geq 4$ in Turkey between the longitudes of $36^{\circ}42'N$ and the latitudes of $26^{\circ}45'E$ from the year 1901 to 2006. For this reason, first, using the maximum entropy principle the best possible transition matrix was estimated for the data and its appropriateness was statistically supported with the chi-square analysis. Later, it was found that the chain reached steady state after 22 periods on the average. From the limit distribution, it was observed that in the long-run there will be no earthquakes in all the regions in 34,6% of the time, there will be earthquake(s) only in the region 1 in 11,6% of the time, ..., and there will be earthquake(s) (affecting) in all the regions in 1,6% of the time. Later on, starting at the beginning of 2006, the

distributions of earthquake predictions were made for the next five periods, i.e., roughly for 128 days.

From the earthquake data during the time interval between 1.1.2006-25.1.2006 (about 25,5 days), it was observed that the proportion of days with no earthquake occurrences in all regions is 19,57%, the proportion of days with earthquake occurrence only in the region 1 is 0,9% , ..., the proportion of days with earthquake occurrence in all regions is 0,2%. A related simulation study to time interval with the same length gave an 81,1% aftcast success. Moreover, a 42,9% forecast success of the earthquakes having magnitude $M \geq 4$ was gained for the time after the 1901-2006 periods.

As can be seen from the analysis and results obtained, we can conclude that the earthquakes occurring in Turkey can be modeled successfully by Markov chains.

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