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On Fuzzy Regular-*I*-Closed Sets, Fuzzy Semi-*I*-Regular Sets, Fuzzy AB_I-Sets and Decompositions of Fuzzy Regular-*I*-Continuity, Fuzzy A_I - Continuity

Fadhıl ABBAS[1], Cemil YILDIZ[1,♠]

[1]*Gazi University, Department of Mathematics, Ankara, Turkey*

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ABSTRACT

The concepts of fuzzy regular-*I*-closed set and fuzzy semi-*I*-regular set in fuzzy ideal topological spaces are investigated and some of their properties are obtained.

Key words: Topological, Spaces, Fuzzy, Regular, Sets

1. INTRODUCTION

The fundamental concept of a fuzzy set was introduced in Zadeh [1]. Subsequently, Chang [12] defined the notion of fuzzy topology. An alternative definition of fuzzy topology was given by Lowen [15]. Yalvac [4] introduced the concepts of fuzzy set and function on fuzzy spaces. In general topology,by introducing the notion of ideal, [16], and several other authors carried out such analysis. There has been an extensive study on the importance of ideal in general topology in the paper of Janković & Hamlet [14]. Sarkar [7] introduced the notions of fuzzy ideal and fuzzy local function in fuzzy set theory. Mahmoud [3] investigated one application of fuzzy set theory. Hatir & Jafari [13] and Nasef & Hatir [8] defined fuzzy semi-*I*-open set and fuzzy pre-*I*-open set via fuzzy ideal.

2. PRELIMINARIES

Through this paper, X represents a nonempty fuzzy set and fuzzy subset A of X, denoted by $A \leq X$, then is characterized by a membership function in the sense of Zadeh [1]. The basic fuzzy sets are the empty set, the whole set the class of all fuzzy sets of X which will be denoted by 0_X, 1_X and I^X, respectively. A subfamily τ of I^X is called a fuzzy topology due to Chang [12] . Moreover, the pair (X,τ) will be meant by a fuzzy topological space, on which no separation axioms are assumed unless explicitly stated. The fuzzy closure, the fuzzy interior and the fuzzy complement of any set in A in (X,τ) are denoted by Cl(A), Int(A) and 1_X -A, respectively. A fuzzy set which is a fuzzy point WONG [17] with support $x \in X$ and the value $\lambda \in (0,1]$ will be denoted by x_λ . The value of a fuzzy set A for some $x \in X$ will be denoted by A(x). Also, for a fuzzy point x_λ and a fuzzy set A we shall write $x_\lambda \in A$ to mean that $\lambda \leq A(x)$. For any two fuzzy sets A and B in (X,τ), $A \leq B$ if and only if $A(x) \leq B(x)$ for all $x \in X$. A fuzzy set in (X,τ)

♠Corresponding author, e-mail: cyildiz@gazi.edu.tr

is said to be quasi-coincident with a fuzzy set B, denoted by AqB, if there exists $x \in X$ such that $A(x)+B(x)>1$ [11]. A fuzzy set V in (X,τ) is called a q-neigbourhood (q-nbd, for short) of a fuzzy point x_λ if and only if there exists a fuzzy open set U such that $x_\lambda qU \leq V$ [11]. We will denote the set of all q-nbd of x_λ in (X,τ) by $Nq(x_\lambda)$. A nonempty collection of fuzzy sets *I* of a set X is called a fuzzy ideal ([7]) on X if and only if (1) $A \in I$ and $B \leq A$, then $B \in I$ (heredity), (2) if $A \in I$ and $B \in I$, then $A \vee B \in I$ (finite additivity). The triple (X,τ,I) means fuzzy ideal topological space with a fuzzy ideal *I* and fuzzy topology τ. For (X,τ,I) the fuzzy local function of $A \leq X$ with respect to τ and *I* is denoted by $A^*(\tau, I)$ (briefly A^*) [7]. The fuzzy local function $A^*(\tau, I)$ of A is the union of all fuzzy points x_λ such that if $N \in Nq(x_\lambda)$ and $E \in I$ then there is at least one $y \in X$ for which $N(y)+A(y)-1>E(y)$ [7] . Fuzzy closure operator of a fuzzy set A in (X,τ, I) is defined as $Cl^*(A)=A \vee A^*$ [7]. In (X,τ, I), the collection $\tau^*(I)$ means an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta=\{U-E : U\in\tau, E\in I\}$ as a base [7].

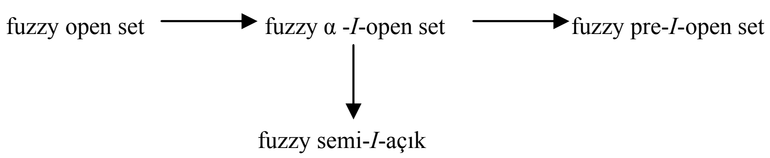
Lemma 1. Let (X,τ, I) be a fuzzy ideal topological space and A,B fuzzy subsets of X. Then the following properties hold:

a) If $A \leq B$,then $A^* \leq B^*$,
b) $A^* = Cl(A^*) \leq Cl(A)$,
c) $(A^*)^* \leq A^*$,
d) $(A \vee B)^* = A^* \vee B^*$ ([7]).

Definition 1. A fuzzy subset A of a fuzzy ideal topological space (X,τ, I) is said to be

a) fuzzy-*I*-open [3] if $A \leq Int(A^*)$,
b) fuzzy α -*I*-open [2] if $A \leq Int(Cl^*(Int(A)))$,
c) fuzzy semi-*I*-open [13] if $A \leq Cl^*(Int(A))$,
d) fuzzy pre-*I*-open [8] if $A \leq Int(Cl^*(A))$,
e) fuzzy α *-*I*-open [2] if $Int(A)= Int(Cl^*(Int(A)))$,
f) fuzzy t-*I*-set [8] if $Int(A)= Int(Cl^*(A))$,

Nasef & Hatir et al. [8] gave the following diagram using some of the expressions of Definitioin 1:



Diyagram I

3. FUZZY REGULAR -*I*-CLOSED SETS

We give some new definitions for fuzzy sets and some theorems in any ideal fuzzy topological spaces.

Definition 2. A fuzzy subset A of a fuzzy ideal topological space (X, τ, I) is said to be

a) fuzzy *-dense -in-itself if $A \leq A^*$,
b) fuzzy τ^*-closed if $A^* \leq A$,
c) fuzzy *-perfect if $A=A^*$.

Definition 3. A fuzzy subset A of a fuzzy ideal topological space (X,τ, I) is said to be fuzzy regular-*I*-closed if $A=(Int(A))^*$.

We denote by FR*I*C(X) the family of all fuzzy regular-*I*-closed subsets of (X,τ, I).

Theorem 1. In a fuzzy ideal topological space (X,τ,I), the following statements hold:

a) Every fuzzy regular-*I*-closed set is fuzzy *-perfect set,
b) Every fuzzy *-perfect set is fuzzy τ^*-closed set,
c) Every fuzzy τ^*-closed set is fuzzy t-*I*-set,

Proof. a) Let A be a fuzzy regular-*I*-closed set. Then, we have $A=(Int(A))^*$.Since $Int(A) \leq A$, $(Int(A))^* \leq A^*$.Then we have $A=(Int(A))^* \leq A^*$.Since $A=(Int(A))^*$, $A^*=((Int(A))^*)^* \leq (Int(A))^*=A$. Therefore, we obtain $A=A^*$.This shows that A is fuzzy *-perfect set.

b) Let A be a fuzzy *-perfect set. Then, we have $A=A^*$. Therefore, we obtain $A^* \leq A$. This shows that A is fuzzy τ^*-closed set.

c) Let A be a fuzzy τ^*-closed set. Then, we have $A^* \leq A \Rightarrow A \vee A^* \leq A \vee A \Rightarrow Cl^*(A) \leq A \Rightarrow Int(Cl^*(A)) \leq Int(A)$.Since $A \leq Cl^*(A) \Rightarrow Int(A) \leq Int(Cl^*(A))$. Therefore,we obtain $Int(A) = Int(Cl^*(A))$. This shows that A is fuzzy t-*I*-set.

Remark 1. The converses of Theorem 1 need not be true as the following examples show.

Example 1. Let X={a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,4 A(b)=0,7 A(c)=0,5
B(a)=0,6 B(b)=0,3 B(c)=0,5

We put $\tau = \{0_X,1_X,A\}$. If we take $I=\{0_X\}$, then B is fuzzy *-perfect set but not fuzzy regular-*I*-closed set.

Example 2. Let X={a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,1 A(b)=0,3 A(c)=0,5
B(a)=0,4 B(b)=0,6 B(c)=0,7

We put $\tau = \{0_X,1_X,A\}$. If we take $I=P(X)$, then B is fuzzy τ^*-closed set but not fuzzy *-perfect set.

Example 3. Let X={a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,2 A(b)=0,3 A(c)=0,1
B(a)=0,5 B(b)=0,6 B(c)=0,7

We put $\tau = \{0_X, 1_X, A\}$. If we take $I=\{0_X\}$, then B is fuzzy t-*I*-set but not fuzzy τ*-closed set

For the relationship related to several sets defined above, we have the following diagram:

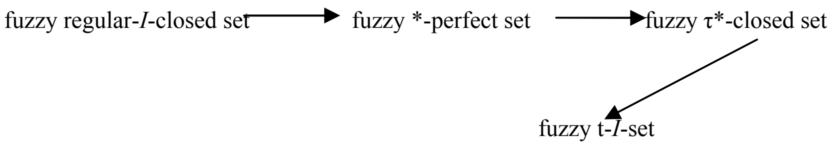


Diagram II

Remark 2. Since every fuzzy open set is fuzzy α -*I*-open, fuzzy regular-*I*-closed and fuzzy α-*I*-open (fuzzy open) set are independent concepts as show in the following examples;

Example 4. Let X={a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,3 A(b)=0,2 A(c)=0,4
B(a)=0,7 B(b)=0,8 B(c)=0,6

We put $\tau = \{0_X, 1_X, A\}$. If we take $I=\{0_X\}$, then B is fuzzy regular-*I*-closed set but not fuzzy α -*I*-open (fuzzy open) set.

Example 5. In Example(3) A is fuzzy α -*I*-open (fuzzy open) set but not fuzzy regular-*I*-closed set.

Theorem 2. For a fuzzy subset A of a fuzzy ideal topological space (X, τ, *I*), the following property holds: A is a fuzzy regular-*I*-closed set if and only if A is a fuzzy semi-*I*-open set and a fuzzy *-perfect set.

Proof. Necessity. The proof is obvious [Theorem 1 a), Diagram II].
Sufficiency. Let A be a fuzzy semi-*I*-open set and a fuzzy *-perfect set. Since A is a fuzzy semi-*I*-open set, there is $A \le Cl^*(Int(A))$. Respectively, by using Lemma 1 a), d) and c)

$$A \le (Cl^*(Int(A)))^* = (Int\,(A) \vee (Int\,(A))^*)^* = (Int\,(A))^* \vee ((Int\,(A))^*)^* \le (Int\,(A))^*,$$

and hence we have $A^* \le (Int(A))^*$. On the other hand, since $Int\,(A) \le A$, we already have $(Int(A))^* \le A^*$ using Lemma 1a). Therefore, we obtain that $A^* = (Int(A))^*$. Furthermore by hypothesis, since A is also a fuzzy *-perfect set, we have $A = A^*$. So, $A = A^* = (Int(A))^*$; that is, $A = (Int(A))^*$ and hence A is a fuzzy regular-*I*-closed set.

4. FUZZY SEMI-*I*-REGULAR SETS

We give some new definitions for fuzzy sets and some examples for those fuzzy sets in any ideal fuzzy topological spaces.

Definition 4. A fuzzy subset A of a fuzzy ideal topological space (X, τ, *I*) is said to be fuzzy semi-*I*-regular if A is both a fuzzy t-*I*-set and a fuzzy semi-*I*-open set.

We will denote the family of all fuzzy semi-*I*-regular sets of (X, τ, *I*) by FS*I*R(X), if there is no chance for confusion with the fuzzy ideal.

Remark 3. Note first that fuzzy t-*I*-sets and fuzzy semi-*I*-open sets are independent concepts as shown in the following examples.

Example 6. Let X={a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,3 A(b)=0,1 A(c)=0,6
B(a)=0,5 B(b)=0,2 B(c)=0,7

We put $\tau = \{0_X, 1_X, A\}$. If we take $I=\{0_X\}$, then B is fuzzy semi-*I*-open set but not fuzzy t-*I*-set.

Example 7. In Example 2. B is fuzzy t-*I*-set but not fuzzy semi-*I*-open set.

Theorem 3. In a fuzzy ideal topological space (X, τ, *I*), the following properties hold:

a) Every fuzzy regular-*I*-closed set is a fuzzy semi-*I*-regular set.
b) Every fuzzy semi-*I*-regular set is a fuzzy semi-*I*-open set.
c) Every fuzzy semi-*I*-regular set is a fuzzy t-*I*-set.

Proof. a) Let A be a fuzzy regular-I-closed set. According to Diagram II, A is both a fuzzy t-*I*-set and a fuzzy semi-*I*-open set. So, A is a fuzzy semi-*I*-regular set.
b), c) The proof is obvious as seen by Definition 4.

Remark 4. The converses of Theorem 3 need not be true as the following examples show.

Example 8. Let X={a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,1 A(b)=0,3 A(c)=0,8
B(a)=0,7 B(b)=0,6 B(c)=0,9

We put $\tau = \{0_X, 1_X, A\}$. If we take *I*=P(X), then A is fuzzy semi-*I*-regular set but not fuzzy regular-*I*-closed set.

Example 9. In Example 6. B is fuzzy semi-*I*-open set but not fuzzy semi-*I*-regular set.

Example 10. In Example 2. B is fuzzy t-*I*-set but not fuzzy semi-*I*-regular set.

By Theorem 3 and Diagram II, we obtain the following diagram:

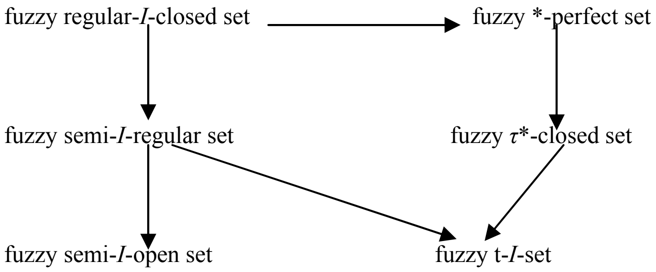


Diagram III

Remark 5. Since every fuzzy *-perfect set is a fuzzy τ*-closed set and every fuzzy semi-*I*-regular set is a fuzzy semi-*I*-open set, a fuzzy τ*-closed (hence fuzzy*-perfect) set and a fuzzy semi-*I*-open (hence fuzzy semi-*I*-regular) set are independent concepts as shown in the following examples.

Example 11. In Example 3. A is fuzzy semi-*I*-regular (hence fuzzy semi-*I*-open) set but not a fuzzy τ*-closed set.

Example 12. Let X={a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,2 A(b)=0,6 A(c)=0,5
B(a)=0,8 B(b)=0,4 B(c)=0,5

We put $\tau = \{0_X, 1_X, A\}$. If we take $I=\{0_X\}$, then B is fuzzy *-perfect (fuzzy τ*-closed) set but not a fuzzy semi-*I*-open (hence fuzzy semi-*I*-regular) set.

Remark 6. Since every fuzzy open set is a fuzzy α -*I*-open set and a fuzzy pre-*I*-open set, a fuzzy pre-*I*-open (hence resp. fuzzy α -*I*-open and fuzzy open) set and a fuzzy semi-*I*-regular set are independent concepts as shown in the following examples.

Example 13. In Example 1. A is a fuzzy open (hence fuzzy α -*I*-open and fuzzy pre-*I*-open) set, but not fuzzy semi-*I*-regular set.

Example 14. Let X={a, b, c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,3 A(b)=0,1 A(c)=0,2
B(a)=0,7 B(b)=0,2 B(c)=0,6

We put $\tau = \{0_X, 1_X, A\}$. If we take $I=\{0_X\}$, then B is a fuzzy semi-*I*-regular set but not fuzzy pre-*I*-open (hence fuzzy α -*I*-open) set.

Diagram III can be expanded to the following diagram using Remark 5 and Remark 6.

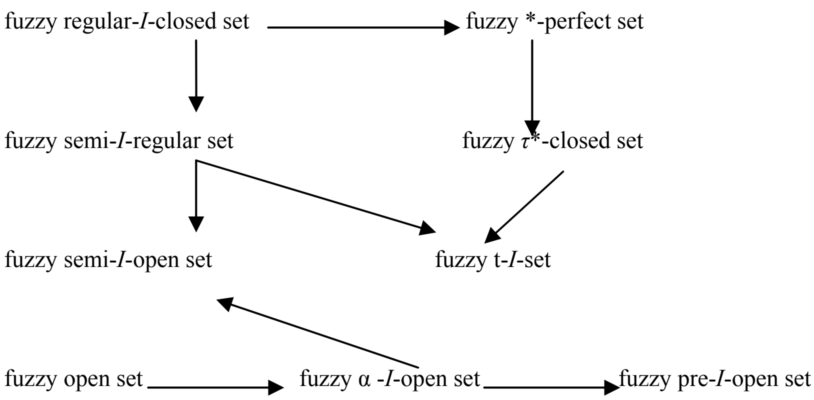


Diagram IV

5. FUZZY AB_I –SETS

We give some another new definitions for fuzzy sets and the relationships between them. In addition, we give some diagram for those fuzzy sets.

Definition 5. A fuzzy subset A of a fuzzy ideal topological space (X, τ, *I*) is said to be a fuzzy A_I -set (resp. fuzzy B_I -set , fuzzy *I*-local closed set) if A $=U \wedge V$, where $U\in\tau$ and V is fuzzy regular-*I*-closed (resp. fuzzy t-*I*-set, fuzzy *-perfect).

Definition 6. A fuzzy subset A of a fuzzy ideal topological space (X, τ, *I*) is said to be a fuzzy AB_I -set if $A = U \wedge V$, where $U\in\tau$ and V is fuzzy semi-*I*-regular set.

We will denote the family of all fuzzy AB_I -sets in (X, τ, *I*) by FAB_I (X) if there is no chance for confusion with the ideal.

Theorem 4. In a fuzzy ideal topological space (X, τ, *I*), the following properties hold:

a) Every fuzzy open set is a fuzzy AB_I -set.

b) Every fuzzy semi-*I*-regular set is a fuzzy AB_I -set.

c) Every fuzzy AB_I -set is a fuzzy B_I -set.

d) Every fuzzy A_I -set is a fuzzy AB_I -set.

Proof. a), b) Since $X\in \tau \wedge FSI R(X)$, the statements are clear.

c) Since every fuzzy regular-*I*-closed set is fuzzy semi-*I*-regular it is obvious by Diagram III.

d) Since every fuzzy semi-*I*-regular set is a fuzzy t-*I*-set it is obvious by Diagram III.

Remark 7. The converses of Theorem 4. need not be true as shown in the following examples.

Example 15. In Example 8. B is fuzzy AB_I -set but not fuzzy open set.

Example 16. In Example 1. B is fuzzy AB_I -set but not fuzzy semi-*I*-regular set.

Example 17. In Example 2. B is fuzzy B_I -set but not fuzzy AB_I -set .

Example 18. In Example 8. B is fuzzy AB_I -set but not fuzzy A_I -set .

By using Theorem 4 and Remark 7 we have the following diagram:

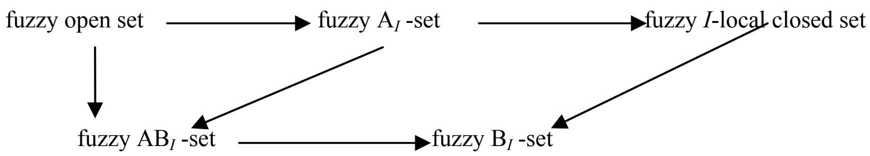


Diagram V

Remark 8. A fuzzy AB_I -set and a fuzzy *I*-local closed set are independent concepts as shown in the following examples.

Example 19. In Example 3. B is fuzzy AB_I -set but not fuzzy *I*-local closed set.

Example 20. In Example 12. B is fuzzy *I*-local closed set but not fuzzy AB_I -set .

Remark 9. As every fuzzy α-*I*-open set is fuzzy pre-*I*-open, a fuzzy pre-*I*-open (hence Fuzzy α-*I*-open) set and a fuzzy AB_I -set are independent concepts as the following examples show.

Example 21. In Example 1. B is fuzzy pre-*I*-open (hence fuzzy α-*I*-open) set but not fuzzy AB_I -set .

Example 22. In Example 14. B is fuzzy AB_I -set but not fuzzy pre-*I*-open (hence fuzzy α-*I*-open) set.

Theorem 5. For a fuzzy subset A of a fuzzy ideal topological space (X, τ, *I*),the following property holds: Every fuzzy AB_I -set is fuzzy semi-*I*-open.

Proof. Let A be a fuzzy AB_I -set. Then according to Definition 6, $A = U \wedge V$, where $U\in\tau$ and V is fuzzy semi-*I*-regular set. By Definition 4, V is also fuzzy semi-*I*-open set. Since V is fuzzy semi-*I*-open,

$$A = U \wedge V \leq U \wedge Cl^* (Int (V)) \leq Cl^* (U \wedge Int (V)) = Cl^* (Int (U \wedge V)) = Cl^* (Int (A))$$

and hence $A \leq Cl^* (Int (A))$ by using Definition 1 c) This shows that A is fuzzy semi-*I*-open.

Remark 10. The converse of Theorem 4 need not be true as shown by the following example.

Example 23. In Example 6. B is fuzzy semi-*I*-open set but not fuzzy AB_I -set.

By using Diagrams I and V with Remarks 8 ,9,10 and Theorem 5 we have the following diagram:

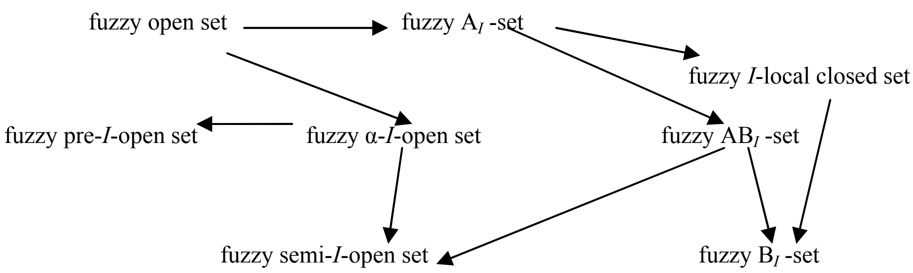


Diagram VI

Theorem 6. For a fuzzy subset A of a fuzzy ideal topological space (X, τ, *I*),the following properties are equivalent:

a) A is a fuzzy open set,

b) A is a fuzzy α-*I*-open set and a fuzzy AB_I -set,

c) A is a fuzzy pre-*I*-open set and a fuzzy AB_I -set.

Proof. We prove only the implication c) $\Longrightarrow$ a), the other implications

a) $\Longrightarrow$ b) and b) $\Longrightarrow$ c) being obvious from Diagram VI.

c) $\Longrightarrow$ a). Let A be a fuzzy pre-*I*-open set and a fuzzy AB_I -set. Then, since A is a fuzzy pre-*I*-open set, we have $A \leq Int(Cl^*(A))$. Furthermore, because A is a fuzzy AB_I -set, we have $A = U \wedge V$, where U is fuzzy open and V is a fuzzy semi-*I*-regular set. Since Cl* is a fuzzy Kuratowski closure operation,

$$A \leq Int(Cl^*(A)) = Int(Cl^*(U \wedge V)) \leq Int(Cl^*(U) \wedge Cl^*(V)) = Int(Cl^*(U)) \wedge Int(Cl^*(V))$$

and hence

$$A \leq Int(Cl^*(U)) \wedge Int(Cl^*(V)) \quad (1.5)$$

Additionally, since V is a fuzzy semi-*I*-regular set, V is also a fuzzy t-*I*-set. Thus, $Int(V) = Int(Cl^*(V))$. Using this in (1.5), we have

$A \leq Int(Cl^*(U)) \wedge Int(Cl^*(V)) = Int(Cl^*(U)) \wedge Int(V)$.

Therefore, we obtain that $A \leq Int(Cl^*(U)) \wedge Int(V)$. Besides, because $A \leq U$, we have

$$A = U \wedge A \leq U \wedge [Int(Cl^*(U)) \wedge Int(V)] = [U \wedge Int(Cl^*(U))] \wedge Int(V) = U \wedge Int(V)$$

and $A \leq U \wedge Int(V)$. Since U is an fuzzy open set, we have

$$A \leq U \wedge Int(V) = Int(U \wedge V) = Int(A).$$

Thus $A \in \tau$.

6. DECOMPOSITIONS OF FUZZY REGULAR-*I*-CONTINUITY

We give some decompositions of fuzzy regular-I-continuity and some examples for those continuity in any ideal fuzzy topological spaces.

Definition 7. A function f : (X, τ, *I*) → (Y, Ψ) is said to be fuzzy *-perfectly continuous (resp. fuzzy semi-*I*-regular continuous, fuzzy semi-*I*-continuous [13], FR*I*C-continuous , fuzzy contra*-continuous) if for every V∈ Ψ, $f^{-1}(V)$ is fuzzy *-perfect (resp. fuzzy semi-*I*-regular, fuzzy semi-*I*-open, fuzzy regular-*I*-closed, fuzzy τ*-closed) set of (X, τ, *I*).

Theorem 7. For a function f : (X, τ, *I*) → (Y, Ψ) the following statements hold:

a) Every FR*I*C-continuous is fuzzy *-perfectly continuous,
b) Every FR*I*C-continuous is fuzzy semi-*I*-regular continuous,
c) Every fuzzy *-perfectly continuous is fuzzy contra*-continuous,
d) Every fuzzy semi-*I*-regular continuous is fuzzy semi-*I*-continuous.

Proof. This follows from Theorem1 , Theorem 2 and Definition 7.

Remark 11. The converses of Theorem 7. need not be true as shown in the following examples.

Example 24. Let X={a, b, c},Y={x, y, z} and A, B be fuzzy subsets defined as follows:

A(a)=0.7 A(b)=0.4 A(c)=0.8
B(x)=0.3 B(y)=0.6 B(z)=0,2

Let τ={0_X,1,A}, Ψ ={0_Y,1_Y,B} and *I*={0_X }. Then the function f : (X, τ, *I*) → (Y, Ψ) defined by f(a)=x, f(b)=y and f(c)=z then f is fuzzy *-perfectly continuous but not FR*I*C-continuous.

Example 25. Let X={a, b, c},Y={x, y, z} and A, B be fuzzy subsets defined as follows:

A(a)=0.1 A(b)=0.4 A(c)=0.3
B(x)=0.3 B(y)=0.5 B(z)=0,2

Let τ={0_X,1_X,A}, Ψ ={0_Y,1_Y,B} and *I*={0_X }. Then the function f : (X, τ, *I*) → (Y, Ψ) defined by f(a)=x, f(b)=y and f(c)=z then f is fuzzy semi-*I*-regular continuous but not FR*I*C-continuous.

Example 26. Let X={a, b, c},Y={x, y, z} and A, B be fuzzy subsets defined as follows:

A(a)=0.8 A(b)=0.2 A(c)=0.4
B(x)=0.9 B(y)=0.4 B(z)=0,7

Let τ={0_X,1_X,A}, Ψ ={0_Y,1_Y,B} and *I*={0_X }. Then the function f : (X, τ, *I*) → (Y, Ψ) defined by f(a)=x,

f(b)=y and f(c)=z then f is fuzzy contra*-continuous but not fuzzy *-perfectly continuous.

Example 27. Let X={a, b, c},Y={x, y, z} and A, B be fuzzy subsets defined as follows:

A(a)=0.2 A(b)=0.7 A(c)=0.1

B(x)=0.6 B(y)=0.8 B(z)=0,3

Let $\tau=\{0_X,1_X,A\}$, $\Psi=\{0_Y,1_Y,B\}$ and $I=\{0_X\}$. Then the function $f:(X,\tau,I)\rightarrow(Y,\Psi)$ defined by f(a)=x, f(b)=y and f(c)=z then f is fuzzy semi-*I*-continuous but not fuzzy semi-*I*-regular continuous.

We have the following diagram using Diagram IV and Theorem 7 and Remark 11:

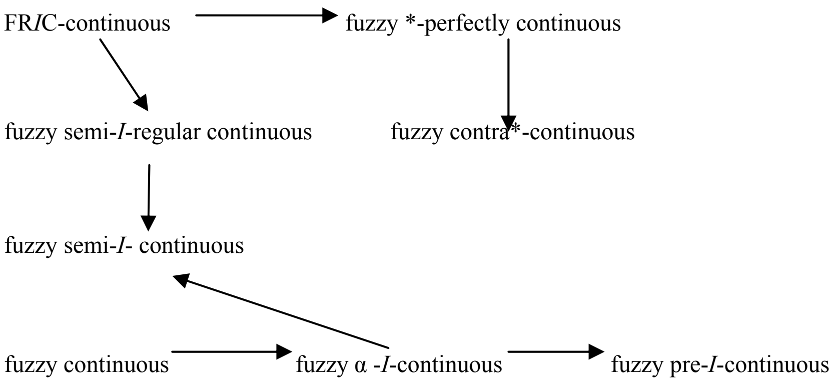


Diagram VII

7. DECOMPOSITIONS OF FUZZY A_I - CONTINUITY

We give some decompositions of fuzzy A_I- continuity and some diagram for those continuity in any ideal fuzzy topological spaces.

Theorem 8. For a fuzzy subset A of a fuzzy ideal topological space (X,τ,I),the following property holds: A is a fuzzy A_I -set if and only if A is a fuzzy semi-*I*-open set and a fuzzy *I*-local closed set.

Proof. This is obvious from Definition 5 and Theorem 2.

Definition 8. A function $f:(X,\tau,I)\rightarrow(Y,\Psi)$ is said to be fuzzy A_I -continuous (resp. fuzzy AB_I -continuous, fuzzy *I*-LC-continuous ,fuzzy B_I -continuous) if for every $V\in\Psi$, $f^{-1}(V)$ is fuzzy A_I -set (resp. fuzzy AB_I -set, fuzzy *I*-local closed set, fuzzy B_I -set) of (X,τ,I)

Theorem 9. For a function $f:(X,\tau,I)\rightarrow(Y,\Psi)$, the following properties hold:

a) If f is fuzzy continuous, then f is fuzzy AB_I -continuous;

b) If f is fuzzy semi-*I*-regular continuous, then f is fuzzy AB_I -continuous;

c) If f is fuzzy AB_I -continuous, then f is fuzzy B_I -continuous;

d) If f is fuzzy A_I -continuous, then f is fuzzy AB_I -continuous.

Proof. The proof is obvious from Theorem 4 .

We have the following diagram using Diagram VI , Definition 8 and Theorem 9:

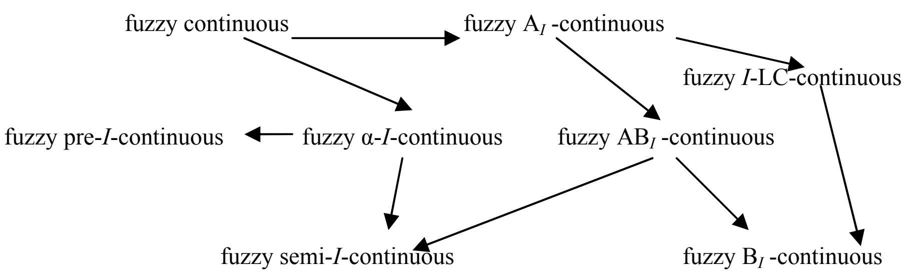


Diagram VIII

Theorem 10. For a function $f : (X, \tau, I) \rightarrow (Y, \Psi)$, the following properties are equivalent:
a) f is fuzzy continuous;
b) f is fuzzy α-*I*-continuous and fuzzy AB_I –continuous;
c) f is fuzzy pre-*I*-continuous and fuzzy AB_I -continuous.

Proof. This is an immediate consequence of Theorem 6.

Theorem 11. For a function $f : (X, \tau, I) \rightarrow (Y, \Psi)$, the following property holds: f is fuzzy A_I -continuous if and only if f is fuzzy semi-*I*-continuous and fuzzy *I*-LC-continuous.

Proof. This is an immediate consequence of Theorem 8.

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