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INTRODUCTION TO TEMPORAL INTUITIONISTIC FUZZY APPROXIMATE REASONING

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Abstract. In this study; temporal intuitionistic fuzzy negation, temporal intuitionistic fuzzy triangular norm and temporal intuitionistic fuzzy triangular conorm have been researched. The aim of this study is to define negator, t-norm and t-conorms, which is the generalization of negation, conjunctions and disconjunctions in the temporal intuitionistic fuzzy sets and to examine the De Morgan relations between these concepts. The thing to note here is that conjunctions generalized with $t-$norm and $t-$conorm is changed depending on time. We will carry concept of implication and coimplication to temporal intuitionistic fuzzy sets. With the new implication definitions, a causal structure will be established which will match the variable structure of the systems depending on the position and time variables. It is evident that successful results will be achieved in this type of system, which is being dealt with by this new structure.

1. Introduction

The notion of fuzzy logic was firstly defined by Zadeh in 1965 [10]. Then; intuitionistic fuzzy sets (shortly IFS) were defined by K.Atanassov in 1986 [1]. Intuitionistic fuzzy sets form a generalization of the notion of fuzzy sets. The concept of temporal intuitionistic fuzzy sets is defined by Atanassov in 1991 [2]. In this concept; the membership and non-membership degrees are described based on the time-moment and time-element. The temporal intuitionistic fuzzy set theory create a new perspective in various application areas such as: Weather, economy, image, video processing, etc.

In this study, firstly definition of temporal intuitionistic fuzzy sets has been given. Then, temporal intuitionistic fuzzy negation, temporal intuitionistic fuzzy triangular norm and temporal intuitionistic fuzzy triangular conorm have been researched. The aim of this study is to define negator, t-norm and t-conorms,

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which is the generalization of negation, conjunctions and disconjunctions in the temporal intuitionistic fuzzy sets and to examine the De Morgan relations between these concepts. The thing to note here is that conjunctions generalized with $t-$norm and $t-$conorm is changed depending on time. The changing conjunctive idea that depends on time has a meaning only when the connected objects change depending on time. Therefore these conjunctions can be used on temporal intuitionistic fuzzy sets.

In this study; we will carry concept of implication and coimplication to temporal intuitionistic fuzzy sets. The definition of the intuitionistic implication is based on the notation from fuzzy set theory introduced by Fodor, Roubens [26]. These concepts, which are used to establish the IF-THEN structure with a clearer reasoning in the fuzzy set and in the intuitionistic fuzzy set theory, are known to be the basic elements in the systems studied by fuzzy and intuitionistic fuzzy set theories. With the new implication definitions given below, a causal structure will be established which will match the variable structure of the systems depending on the position and time variables. It is evident that successful results will be achieved in this type of system, which is being dealt with by this new structure. When these two concepts are established, the necessity of satisfying the "modus ponens" conditions in the classical logic will be taken into consideration. At this point, implications and coimplication definitions will be moved to the temporal intuitionistic fuzzy set space in the studies light, which has been done previously and successfully in practice. Many researchers have been researched in this field ([8],[12],[13],[22],[23],[24],[25],[27])

2. Preliminaries

Definition 1. [1] *An intuitionistic fuzzy set on a non-empty set X given by a set of ordered triples $A = \{(x, \mu_A(x), \eta_A(x)) : x \in X\}$ where $\mu_A(x) : X \to I = [0,1]$, $\eta_A(x) : X \to I$, are functions such that $0 \leq \mu(x) + \eta(x) \leq 1$ for all $x \in X$. For $x \in X$, $\mu_A(x)$ and $\eta_A(x)$ represent the degree of membership and degree of non-membership of x to A respectively. For each $x \in X$; intuitionistic fuzzy index of x in A is defined as follows $\pi_A(x) = 1 - \mu_A(x) - \eta_A(x)$. π_A is the called degree of hesitation or indeterminacy. Let denote the set of all intuitionistic fuzzy sets defined on X by IFS^X*

Definition 2. [1] *Let $A, B \in IFS^X$. Then,*

(i) $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ *and* $\eta_A(x) \geq \eta_B(x)$ *for* $\forall x \in X$,

(ii) $A = B \Leftrightarrow A \subseteq B$ *and* $B \subseteq A$,

(iii) $\bar{A} = \{(x, \eta_A(x), \mu_A(x)) : x \in X\}$,

(iv) $\bigcap A_i = \left\{\left(x, \wedge\mu_{A_i}(x), \vee\eta_{A_i}(x)\right) : x \in X\right\}$,

(v) $\bigcup A_i = \left\{\left(x, \vee\mu_{A_i}(x), \wedge\eta_{A_i}(x)\right) : x \in X\right\}$.

Definition 3. [2] *Let X be an universe and T be a non-empty time set. We call the elements of T as "time moments". Based on the definition of IFS, a temporal* *intuitionistic fuzzy set (TIFS) is defined as the following:*

$$A(T) = \{(x, \mu_A(x,t), \eta_A(x,t)) : X \times T\}$$

where:
a. $A \subseteq X$ *is a fixed set,*
b. $\mu_A(x,t) + \eta_A(x,t) \leq 1$ *for every* $(x,t) \in X \times T$,
c. $\mu_A(x,t)$ *and* $\eta_A(x,t)$ *are the degrees of membership and non-membership, respectively, of the element* $x \in X$ *at the time moment* $t \in T$.

For brevity, we write A instead of $A(T)$. The hesitation degree of an TIFS is defined as $\pi_A(x,t) = 1 - \mu_A(x,t) - \eta_A(x,t)$. Obviously, every ordinary IFS could be regarded as TIFS for which T is a singleton set. All operations and operators on IFS could be defined for TIFSs.

By $TIFS^{(X,T)}$, we denote to the set of all temporal intuitionistic fuzzy sets defined on X and time set T. Obviously, each intuitionistic fuzzy sets could be expressed as temporal intuitionistic fuzzy set via a singular time set. In additionally, all operations and operators defined for intuitionistic fuzzy sets could be defined for temporal intuitionistic fuzzy sets.

Definition 4. [2] *Let*

$$A(T') = \{(x, \mu_A(x,t), \eta_A(x,t)) : X \times T'\}$$

and

$$B(T'') = \{(x, \mu_B(x,t), \eta_B(x,t)) : X \times T''\}$$

where T' *and* T'' *have finite number of distinct time-elements or they are time intervals. Then;*

$A(T') \cap B(T'') =$

$$\left\{(x,\ min(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)),\ max(\bar{\eta}_A(x,t), \bar{\eta}_B(x,t))) : (x,t) \in X \times \left(T' \cup T''\right)\right\}$$

and

$A(T') \cup B(T'') =$

$$\left\{(x,\ max(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)),\ min(\bar{\eta}_A(x,t), \bar{\eta}_B(x,t))) : (x,t) \in X \times \left(T' \cup T''\right)\right\}$$

Also from definition of subset in intuitionistic fuzzy sets, subsets of temporal intuitionistic fuzzy sets can be defined as the following:

$$A(T') \subseteq B(T'') \Leftrightarrow \bar{\mu}_A(x,t) \geq \bar{\mu}_B(x,t) \text{ and } \bar{\eta}_A(x,t) \leq \bar{\eta}_B(x,t)$$

for every $(x,t) \in X \times (T' \cup T'')$ *where*

$$\bar{\mu}_A(x,t) = \begin{cases} \mu_A(x,t), & if \quad t \in T' \\ 0, & if \quad t \in T'' - T' \end{cases}$$

$$\bar{\mu}_B(x,t) = \begin{cases} \mu_B(x,t), & if \quad t \in T'' \\ 0, & if \quad t \in T' - T'' \end{cases}$$

$$\bar{\eta}_A(x,t) = \begin{cases} \eta_A(x,t), & if \quad t \in T' \\ 1, & if \quad t \in T'' - T' \end{cases}$$

$$\bar{\eta}_B(x,t) = \begin{cases} \eta_B(x,t), & \text{if} \quad t \in T'' \\ 1, & \text{if} \quad t \in T' - T'' \end{cases}$$

It is obviously seen that if $T' = T''$; $\bar{\mu}_A(x,t) = \mu_A(x,t)$, $\bar{\mu}_B(x,t) = \mu_B(x,t)$, $\bar{\eta}_A(x,t) = \eta_A(x,t)$, $\bar{\eta}_B(x,t) = \eta_B(x,t)$. *[2]*

Let J *be an index set and* T_i *is a time set for each* $i \in J$. *Let define that* $T = \bigcup_{i \in J} T_i$. *Now we extend union and intersection of temporal intuitionistic fuzzy sets to the family* $F = \left\{A_i(T_i) = \left(x, \mu_{A_i}(x,t), \eta_{A_i}(x,t)\right) : x \in X \times T_i,\ i \in J\right\}$ *as:*

$$\begin{aligned} \bigcup_{i \in J} A(T_i) &= \left\{\left(x,\ \max_{i \in J}\left(\bar{\mu}_{A_i}(x,t)\right), \min_{i \in J}\left(\bar{\eta}_{A_i}(x,t)\right) :\ (x,t) \in X \times T\right)\right\}, \\ \bigcap_{i \in J} A(T_i) &= \left\{\left(x,\ \min_{i \in J}\left(\bar{\mu}_{A_i}(x,t)\right), \max_{i \in J}\left(\bar{\eta}_{A_i}(x,t)\right) :\ (x,t) \in X \times T\right)\right\} \end{aligned}$$

where

$$\bar{\mu}_{A_i}(x,t) = \begin{cases} \mu_{A_i}(x,t), & \text{if} \quad t \in T_i \\ 0, & \text{if} \quad t \in T - T_i \end{cases}$$

and

$$\bar{\eta}_{A_i}(x,t) = \begin{cases} \eta_{A_i}(x,t), & \text{if} \quad t \in T_i \\ 1, & \text{if} \quad t \in T - T_i \end{cases}.$$

Definition 5. *The set of all intuitionistic fuzzy pair is defined as*

$$IFP^* = \{(x,y) \in\ [0,1] \times [0,1];\ x + y \leq 1\}$$

The order relation $\leq$ *on this set is defined by* $(x_1, y_1) \leq (x_2, y_2) \Leftrightarrow x_1 \leq x_2,\ y_1 \geq y_2$ *for* $\forall (x_1, y_1), (x_2, y_2) \in IFP^*$. *Also* $\widetilde{1} = (1,0)$ *and* $\widetilde{0} = (0,1)$.

Let $x : T \to [0,1]$, $y : T \to [0,1]$ *are functions such that* $x(t) + y(t) \leq 1$ *for each time moment* $t \in T$. *Then temporal intuitionistic pair set on time set* T *defined as follows:*

$$TIFP_T^* = \{(x(t), y(t)) :\ t \in T\}$$

$0_T, 1_T \in TIFP_T^*$ *which are defined such as* $0_T = (x_{0_T}(t), y_{0_T}(t)) = (0,1)$ *and* $1_T = (x_{1_T}(t), y_{1_T}(t)) = (1,0)$ *for each time moment* $t \in T$ *and are called overall zero and overall one. On the other hand* $0_t, 1_t \in TIFP_T^*$, *which are defined such as* $0_t = (x_{0_t}(t), y_{0_t}(t)) = (0,1)$ *and* $1_t = (x_{1_t}(t), y_{1_t}(t)) = (1,0)$ *for a fixed time moment* $t \in T$, *are called temporal zero and temporal one at time moment* t.

3. Temporal Intuitionistic Fuzzy Negation, t-norm and t-conorm

In this section firstly; we will carry negation, $t-$norm and $t-$conorm definitions to temporal intuitionistic fuzzy sets. Then, the basic relations between these definitions will be researched.

Definition 6. *Let T be a time set, the decreasing mapping $N_t : TIFP_T^* \times T \to TIFP_T^*$ which is satisfied following the condition $N_t(0_t, t) = 1_t$ and $N_t(1_t, t) = 0_t$ at fixed time moment $t \in T$ is called temporal intuitionistic fuzzy negation at fixed time moment t .*

Definition 7. *If N_t is satisfied*
a. $N_t(N_t(a(t), t), t) = a(t)$ for all time moment $t \in T$ and all $a(t) \in TIFP_T^*$, it is called temporal intuitionistic fuzzy strong negation at time moment t,
b. $x(t) = 0_t \Leftrightarrow N_t(x(t), t) = 1_t$ for fixed time moment $t \in T$, it is called temporal intuitionistic fuzzy non-filling negation at time moment t,
c. $x(t) = 1_t \Leftrightarrow N_t(x, t) = 0_t$ for $t \in T$ and all $a \in IF^*$, it is called temporal intuitionistic fuzzy non-vanishing negation at time moment t.

Remark 1. *According to the this definition, it would be seen that the negation operator may change with the time parameter. It would be more correct to define temporal intuitionistic fuzzy negation on a temporal intuitionistic fuzzy pair, even if it is true with a classical approach which defined with intuitionistic fuzzy pair. Despite the fact that the cases to be handled by the negation operator can change according to the time makes it necessary for the negation operator to change depending on the time.*

Definition 8. *The mapping $N_t : TIFP_T^* \times T \to [0, 1]$ defined by $N_t((x_1(t), x_2(t)), t) = (x_2, x_1)$ for all $(x_1, x_2) \in IF^*$ is called standard temporal intuitionistic fuzzy negator.*

The following proposition is also valid for temporal intuitionistic fuzzy negations as well as fuzzy and intuitionistic fuzzy negations.

Proposition 1. *The equation $N_t\,(N_t(0_t, t), t) = 0_t$ is satisfied for any temporal intuitionistic fuzzy strong negator N_t.*

Proof. From the temporal intuitionistic fuzzy negation definition;

$$N_t(0_t, t) = 1_t,\ N_t(1_t, t) = 0_t,\ N_t(N_t(0_t, t), t) = 0_t.$$

□

Definition 9. *Let T be a time set. If the mapping $T_t : (TIFP_T^* \times TIFP_T^{**}) \times T \to TIFP_T^*$ is satisfied following condition for a fixed time moment $t \in T$, it is called temporal intuitionistic fuzzy triangular norm ($t-$norm) at time moment t:*

- T1. *$T_t((x(t), y(t)), t) = T_t((y(t), x(t)), t)$ for every $x, y \in TIFP_T^*$ at fixed the time moment $t \in T$ (symmetry),*
- T2. *$T_t((x_1(t), y_1(t)), t) \leq T_t((x_2(t), y_2(t)), t)$ for every $x_1(t)$, $y_1(t)$, $x_2(t)$, $y_2(t) \in TIFP_T^*$ such that $x_1(t) \leq x_2(t)$ and $y_1(t) \leq y_2(t)$ at fixed the time moment $t \in T$ (monotonicity),*
- T3. *$T_t((T_t((x(t), y(t)), t)), z(t), t) = T_t((x(t), T_t((z(t), y(t)), t)), t)$ for every $x(t), y(t), z(t) \in TIFP_T^*$ at fixed the time moment $t \in T$ (associativity),*

T4. $T_t\left(\left(x\left(t\right), 1_t\right), t\right) = x\left(t\right)$ *for every* $x\left(t\right) \in TIFP_T^*$ *(boundary condition).*

Definition 10. *Let* T *be a time set. If the mapping* $S_t : (TIFP_T^* \times TIFP_T^*) \times T \rightarrow TIFP_T^*$ *is satisfied following condition at time moment* $t \in T$ *and , it is called temporal triangular conorm (or* $s-$*norm) at time moment* t *:*

S1. $S_t\left(\left(x\left(t\right), y\left(t\right)\right), t\right) = S_t\left(\left(y\left(t\right), x\left(t\right)\right), t\right)$ *for every* $x, y \in TIFP_T^*$ *at fixed the time moment* $t \in T$ *(symmetry),*

S2. $S_t\left(\left(x_1\left(t\right), y_1\left(t\right)\right), t\right) \leq S_t\left(\left(x_2\left(t\right), y_2\left(t\right)\right), t\right)$ *for every* $x_1\left(t\right),\ y_1\left(t\right),\ x_2\left(t\right),$ $y_2\left(t\right) \in TIFP_T^*$ *such that* $x_1\left(t\right) \leq x_2\left(t\right)$ *and* $y_1\left(t\right) \leq y_2\left(t\right)$ *at fixed the time moment* $t \in T$*(monotonicity),*

S3. $S_t\left(\left(S_t\left(\left(x\left(t\right), y\left(t\right)\right), t\right)\right), z\left(t\right), t\right) = S_t\left(\left(x\left(t\right), S_t\left(\left(z\left(t\right), y\left(t\right)\right), t\right)\right), t\right)$ *for every* $x\left(t\right), y\left(t\right), z\left(t\right) \in TIFP_T^*$ *at fixed the time moment* $t \in T$*(associativity),*

S4. $S_t\left(\left(x\left(t\right), 0_t\right), t\right) = x\left(t\right)$ *for every* $x\left(t\right) \in TIFP_T^*$ *at fixed the time moment* $t \in T$ *(boundary condition).*

The thing to note here is that conjunctions generalized with $t-$norm and $t-$conorm is changed depending on time. The changing conjunctive idea that depends on time has a meaning only when the connected objects change depending on time. Therefore these conjunctions could be used on temporal intuitionistic fuzzy sets.

Proposition 2. *Let*

$$A = \left\{\left(x, \mu_A\left(x,t\right),\ \eta_A\left(x,t\right)\right) : \left(x,t\right) \in\ X \times T'\right\}$$

and

$$B = \left\{\left(x, \mu_B\left(x,t\right),\ \eta_B\left(x,t\right)\right) : \left(x,t\right) \in\ X \times T''\right\}$$

be two TIFSs where T' *and* T'' *are time set. Then the following mappings are* $t-$*norm and* $t-$*conorm for* $\left(x,t\right) \in\ X \times T' \cup T''$*:*

(1) $T_{\min}^t\left[\left(A,B\right), t\right] = \left(\min\left(\bar{\mu}_A\left(x,t\right),\ \bar{\mu}_B\left(x,t\right)\right),\ \max\left(\bar{\eta}_A\left(x,t\right),\ \bar{\eta}_B\left(x,t\right)\right)\right)$,

(2) $T_0^t\left[\left(A,B\right), t\right] = \begin{cases} \left(\bar{\mu}_A\left(x,t\right), \bar{\eta}_A\left(x,t\right)\right) & , \quad \left(\mu_B\left(x,t\right), \eta_B\left(x,t\right)\right) = \widetilde{1} \\ \left(\bar{\mu}_B\left(x,t\right), \bar{\eta}_B\left(x,t\right)\right) & , \quad \left(\mu_A\left(x,t\right), \eta_A\left(x,t\right)\right) = \widetilde{1} \\ \widetilde{0} & , \quad otherwise \end{cases}$,

(3) $T_1^t\left[\left(A,B\right), t\right] = \left(\max\left\{0, \left(\bar{\mu}_A\left(x,t\right) +\ \bar{\mu}_B\left(x,t\right)\right)\right\}, \min\left\{1, \bar{\eta}_A\left(x,t\right) +\ \bar{\eta}_B\left(x,t\right)\right\}\right)$,

(4) $T_2^t\left[\left(A,B\right), t\right] = \left(\bar{\mu}_A\left(x,t\right) \bar{\mu}_B\left(x,t\right), \bar{\eta}_A\left(x,t\right) +\ \bar{\eta}_B\left(x,t\right) - \bar{\eta}_A\left(x,t\right)\ \bar{\eta}_B\left(x,t\right)\right)$,

(5) $T_3^t\left[\left(A,B\right), t\right] =$

$$\left(\log_t\left(1 + \frac{\left(t^{\left(\bar{\mu}_A\left(x,t\right)\right)} - 1\right)\left(t^{\left(\bar{\mu}_B\left(x,t\right)\right)} - 1\right)}{t - 1}\right), 1 - \log_t\left(1 + \frac{\left(t^{\left(1-\bar{\eta}_A\left(x,t\right)\right)} - 1\right)\left(t^{\left(1-\bar{\eta}_B\left(x,t\right)\right)} - 1\right)}{t - 1}\right)\right),$$

(6) $S_{\max}^t\left[\left(A,B\right), t\right] = \left(\max\left(\bar{\mu}_A\left(x,t\right),\ \bar{\mu}_B\left(x,t\right)\right),\ \min\left(\bar{\eta}_A\left(x,t\right),\ \bar{\eta}_B\left(x,t\right)\right)\right)$,

(7) $S_0^t\left[\left(A,B\right), t\right] = \begin{cases} \left(\bar{\mu}_A\left(x,t\right), \bar{\eta}_A\left(x,t\right)\right) & , \quad B = 1_t \\ \left(\bar{\mu}_B\left(x,t\right), \bar{\eta}_B\left(x,t\right)\right) & , \quad A = 0_t \\ \widetilde{1} & , \quad otherwise \end{cases}$,

(8) $S_1^t\left[\left(A,B\right), t\right] = \left(\min\left\{1, \bar{\mu}_A\left(x,t\right) +\ \bar{\mu}_B\left(x,t\right)\right\}, \max\left\{0, \left(\bar{\eta}_A\left(x,t\right) + \bar{\eta}_B\left(x,t\right)\right)\right\}\right)$,

(9) $S_2^t\left[\left(A,B\right), t\right] = \left(\bar{\mu}_A\left(x,t\right) +\ \bar{\mu}_B\left(x,t\right) - \bar{\mu}_A\left(x,t\right)\ \bar{\mu}_B\left(x,t\right), \bar{\eta}_A\left(x,t\right) \bar{\eta}_B\left(x,t\right)\right)$,

(10) $S_3^t\left[(A,B),t\right] =$

$$\left(1-\log_t\left(1+\frac{\left(t^{\left(1-\bar{\mu}_A(x,t)\right)}-1\right)\left(t^{\left(1-\bar{\mu}_B(x,t)\right)}-1\right)}{t-1}\right), \log_t\left(1+\frac{\left(t^{\left(\bar{\eta}_A(x,t)\right)}-1\right)\left(t^{\left(\bar{\eta}_B(x,t)\right)}-1\right)}{t-1}\right)\right)$$

Proposition 3. *Following inequalities are satisfied for each T^t temporal intuitionistic fuzzy t−norm and S^t temporal intuitionistic fuzzy t−conorm*

(1) $T_0^t \le T^t \le T_{\min}^t$,
(2) $S_{\max}^t \le S^t \le S_0^t$.

Proof. 1. Let's prove on a single T time set without disturbing the generality. Firstly; let's show that $T_0^t \le T^t$.

In case of $(\mu_B(x,t), \eta_B(x,t)) = \widetilde{1}$ or $(\mu_A(x,t), \eta_A(x,t)) = \widetilde{1}$ (let's accept $(\mu_B(x,t), \eta_B(x,t)) = \widetilde{1}$ without loss of the generality) the following equation is easily obtained.

$$T_0^t\left[(A,B),t\right] = (\mu_A(x,t), \eta_A(x,t)) = T^t\left[(A,B),t\right]$$

In other cases, because of $T_0^t\left[(A,B),t\right] = \widetilde{0}$, $T_0^t\left[(A,B),t\right] \le T^t\left[(A,B),t\right]$ inequality is clearly obtained. Let's show that $T^t\left[(A,B),t\right] \le T_{\min}^t\left[(A,B),t\right]$. Because of $T^t\left[(A,B),t\right] \le T^t\left[\left(A,\widetilde{1}\right),t\right] = (\mu_A(x,t), \eta_A(x,t))$ and

$$\begin{aligned} T^t\left[(A,B),t\right] &= T^t\left[(B,A),t\right] \le T^t\left[\left(B,\widetilde{1}\right),t\right] = (\mu_B(x,t), \eta_B(x,t)) \\ T^t\left[(A,B),t\right] &\le \left(\min\left(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)\right), \max\left(\bar{\eta}_A(x,t), \bar{\eta}_B(x,t)\right)\right) \\ &= T_{\min}^t\left[(A,B),t\right] \end{aligned}$$

inequality is easily obtained. The other expression could be similarly proven. □

Definition 11. *As stated in* [9]*, Let $T^* : TIFP_T^* \times TIFP_T^* \to [0,1]$ and $S^* : TIFP_T^* \times TIFP_T^* \to [0,1]$ be respectively intuitionistic fuzzy t− norm and t− conorm on $TIFP_T^*$ and at fixed time moment $t \in T$ such that*

$$T^*(x(t), y(t)) \le N\left(S^*\left((N(x(t)), N(y(t))\right)\right)\right)$$

where N is intuitionistic fuzzy standard negation. Then the mapping T_t defined as follows

$$T_t((A,B),t) = \left(T^*\left(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)\right), S^*\left(\bar{\eta}_A(x,t), \bar{\eta}_B(x,t)\right)\right)$$

is a temporal intuitionistic fuzzy t− norm and it is called t-representable temporal intuitionistic fuzzy t− norm.

Similarly; the mapping S_t defined as follows

$$S_t((A,B),t) = \left(S^*\left(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)\right), T^*\left(\bar{\eta}_A(x,t), \bar{\eta}_B(x,t)\right)\right)$$

is a temporal intuitionistic fuzzy t− conorm and it is called t-representable temporal intuitionistic fuzzy t− conorm.

Looking at the definitions of $t-$norm and $t-$conorm given above, it is seen that they are $t-$representable temporal intuitionistic fuzzy $t-$norm and $t-$conorm. The temporal intuitionistic fuzzy De Morgan triplet defined with approach described in [10] as follows:

Definition 12. *A triplet (S_t, T_t, N_t) is called temporal intuitionistic fuzzy De Morgan triplet if T_t is temporal intuitionistic fuzzy $t-$ norm, S_t is temporal intuitionistic fuzzy $t-$ conorm, N_t is temporal intuitionistic fuzzy negator and if they fulfill De Morgan's law*

$$S_t\left(\left(A,B\right),t\right) = N_t\left(T_t\left(\left(N_t\left(A,t\right),N_t\left(B,t\right)\right),t\right),t\right)$$

or equivalently

$$T_t\left(\left(A,B\right),t\right) = N_t\left(S_t\left(\left(N_t\left(A,t\right),N_t\left(B,t\right)\right),t\right),t\right).$$

Proposition 4. *$T^t_{\min}$ and $S^t_{\max}$ together with N_t generate a De Morgan Triplet.*

Proof.

$$\begin{aligned}
S^t_{\max}\left(\left(A,B\right),t\right) &= N_t\left(T^t_{\min}\left(N_t\left(A,t\right),N_t\left(B,t\right)\right),t\right)\\
S^t_{\max}\left[\left(A,B\right),t\right] &= \left(\max\left(\bar{\mu}_A\left(x,t\right),\bar{\mu}_B\left(x,t\right)\right),\min\left(\bar{\eta}_A\left(x,t\right),\bar{\eta}_B\left(x,t\right)\right)\right)\\
T^t_{\min}\left[\left(A,B\right),t\right] &= \left(\min\left(\bar{\mu}_A\left(x,t\right),\bar{\mu}_B\left(x,t\right)\right),\max\left(\bar{\eta}_A\left(x,t\right),\bar{\eta}_B\left(x,t\right)\right)\right)\\
T^t_{\min}\left[\left(N_t\left(A\right),N_t\left(B\right),t\right)\right] &= \left(\min\left(\bar{\eta}_A\left(x,t\right),\bar{\eta}_B\left(x,t\right)\right),\max\left(\bar{\mu}_A\left(x,t\right),\bar{\mu}_B\left(x,t\right)\right)\right)
\end{aligned}$$

$$N_t\left(T^t_{\min}\left(N_t\left(A,t\right),N_t\left(B,t\right),t\right),t\right) = \left(\max\left(\bar{\mu}_A\left(x,t\right),\bar{\mu}_B\left(x,t\right),\min\left(\bar{\eta}_A\left(x,t\right),\bar{\eta}_B\left(x,t\right)\right)\right)\right)$$

$$S^t_{\max}\left(\left(A,B\right),t\right) = N_t\left(T^t_{\min}\left(N_t\left(A,t\right),N_t\left(B,t\right),t\right),t\right)$$

□

Proposition 5. *T^t_i and S^t_i $\quad(i = 1,2,3)$ together with N_t generate a De Morgan Triplet.*

Proof. for $i = 1$;
T^t_1 and S^t_1 together with N_t generate a De Morgan Triplet.

$$\begin{aligned}
S^t_1\left(\left(A,B\right),t\right) &= N_t\left(T^t_1\left(N_t\left(A\right),N_t\left(B\right)\right)\right)\\
S^t_1\left[\left(A,B\right),t\right] &= \left(\min\left\{1,\bar{\mu}_A\left(x,t\right)+\bar{\mu}_B\left(x,t\right)\right\},\max\left\{0,\left(\bar{\eta}_A\left(x,t\right)+\bar{\eta}_B\left(x,t\right)\right)\right\}\right)\\
T^t_1\left[\left(A,B\right),t\right] &= \left(\max\left\{0,\left(\bar{\mu}_A\left(x,t\right)+\bar{\mu}_B\left(x,t\right)\right)\right\},\min\left\{1,\bar{\eta}_A\left(x,t\right)+\bar{\eta}_B\left(x,t\right)\right\}\right)
\end{aligned}$$

$$T^t_1\left(N_t\left(A\right),N_t\left(B\right)\right) = \left(\max\left\{0,\left(\bar{\eta}_A\left(x,t\right)+\bar{\eta}_B\left(x,t\right)\right)\right\},\min\left\{1,\bar{\mu}_A\left(x,t\right)+\bar{\mu}_B\left(x,t\right)\right\}\right)$$

$$N_t\left(T^t_1\left(N_t\left(A\right),N_t\left(B\right)\right)\right) = \left(\min\left\{1,\bar{\mu}_A\left(x,t\right)+\bar{\mu}_B\left(x,t\right)\right\},\max\left\{0,\left(\bar{\eta}_A\left(x,t\right)+\bar{\eta}_B\left(x,t\right)\right)\right\}\right)$$

Consequently (for $i = 1$);
$S^t_1\left(\left(A,B\right),t\right) = N_t\left(T^t_1\left(N_t\left(A\right),N_t\left(B\right)\right)\right)$
for $i = 2$;
T^t_2 and S^t_2 together with N_t generate a De Morgan Triplet.
$S^t_2\left(\left(A,B\right),t\right) = N_t\left(T^t_2\left(N_t\left(A\right),N_t\left(B\right)\right)\right)$
$S^t_2\left[\left(A,B\right),t\right] = \left(\bar{\mu}_A\left(x,t\right)+\bar{\mu}_B\left(x,t\right)-\bar{\mu}_A\left(x,t\right)\bar{\mu}_B\left(x,t\right),\bar{\eta}_A\left(x,t\right)\bar{\eta}_B\left(x,t\right)\right)$

$T_2^t[(A,B),t] = (\bar{\mu}_A(x,t)\,\bar{\mu}_B(x,t), \bar{\eta}_A(x,t) + \bar{\eta}_B(x,t) - \bar{\eta}_A(x,t)\,\bar{\eta}_B(x,t))$
$T_2^t(N_t(A), N_t(B)) = (\bar{\eta}_A(x,t)\,\bar{\eta}_B(x,t), \bar{\mu}_A(x,t) + \bar{\mu}_B(x,t) - \bar{\mu}_A(x,t)\,\bar{\mu}_B(x,t))$

$$N_t\left(T_2^t(N_t(A), N_t(B))\right) = (\bar{\mu}_A(x,t) + \bar{\mu}_B(x,t) - \bar{\mu}_A(x,t)\,\bar{\mu}_B(x,t), \bar{\eta}_A(x,t)\,\bar{\eta}_B(x,t))$$

Consequently (for $i=2$);
$S_2^t((A,B),t) = N_t(T_2^t(N_t(A), N_t(B)))$
for $i=3$; T_3^t and S_3^t together with N_t generate a De Morgan Triplet.
$S_3^t[(A,B),t] = S_3^t((A,B),t) = N_t(T_3^t(N_t(A), N_t(B)))$

$$\left(1-\log_t\left(1+\frac{\left(t^{(1-\bar{\mu}_A(x,t))}-1\right)\left(t^{(1-\bar{\mu}_B(x,t))}-1\right)}{t-1}\right), \log_t\left(1+\frac{\left(t^{(\bar{\eta}_A(x,t))}-1\right)\left(t^{(\bar{\eta}_B(x,t))}-1\right)}{t-1}\right)\right)$$

$T_3^t[(A,B),t] =$

$$\left(\log_t\left(1+\frac{\left(t^{(\bar{\mu}_A(x,t))}-1\right)\left(t^{(\bar{\mu}_B(x,t))}-1\right)}{t-1}\right), 1-\log_t\left(1+\frac{\left(t^{(1-\bar{\eta}_A(x,t))}-1\right)\left(t^{(1-\bar{\eta}_B(x,t))}-1\right)}{t-1}\right)\right)$$

$T_3^t(N_t(A), N_t(B)) =$

$$\left(\log_t\left(1+\frac{\left(t^{(\bar{\eta}_A(x,t))}-1\right)\left(t^{(\bar{\eta}_B(x,t))}-1\right)}{t-1}\right), 1-\log_t\left(1+\frac{\left(t^{(1-\bar{\mu}_A(x,t))}-1\right)\left(t^{(1-\bar{\mu}_B(x,t))}-1\right)}{t-1}\right)\right)$$

$N_t(T_3^t(N_t(A), N_t(B))) =$

$$\left(1-\log_t\left(1+\frac{\left(t^{(1-\bar{\mu}_A(x,t))}-1\right)\left(t^{(1-\bar{\mu}_B(x,t))}-1\right)}{t-1}\right), \log_t\left(1+\frac{\left(t^{(\bar{\eta}_A(x,t))}-1\right)\left(t^{(\bar{\eta}_B(x,t))}-1\right)}{t-1}\right)\right)$$

Consequently (for $i=3$);
$S_3^t((A,B),t) = N_t(T_3^t(N_t(A), N_t(B)))$ □

4. Temporal Intuitionistic Fuzzy Implicator

In this section firstly; we will carry concepts of implication and coimplication to temporal intuitionistic fuzzy sets. These concepts, which are used to establish the IF-THEN structure with a clearer reasoning in the fuzzy set and in the intuitionistic fuzzy set theory, are known to be the basic elements in the systems studied by fuzzy and intuitionistic fuzzy set theories. With the new implication definitions given below, a causal structure will be established which will match the variable structure of the systems depending on the position and time variables. It is evident that successful results will be achieved in this type of system, which is being dealt with by this new structure. When these two concepts are established, the necessity of satisfying the "modus ponens" conditions in the classical logic will be taken into consideration. At this point, definitions of implication and coimplication will be moved to the temporal intuitionistic fuzzy set space in the studies light, which has been done previously and successfully in practice.

Definition 13. *If a function* $I_t : (TIFP_T^* \times TIFP_T^*) \times T \to IFP^*$ *is satisfied* *following condition, I is called temporal intuitionistic fuzzy implication at time moment t*

I-1: *(Boundary Conditions):*

a: $I_t((0_t, a(t)), t) = \widetilde{1}$ *for all* $a(t) \in TIFP_t^*$ *at fixed time moment* t,

b: $I_t((a(t), 1_t), t) = \widetilde{1}$ *for all* $a(t) \in TIFP_t^*$ *at fixed time moment* t,

c: $I_t((1_t, 0_t), t) = \widetilde{0}$,

I-2: I_t *is decreasing in first variable i.e.*

If $x \leq y$ *then* $I_t((y, z), t) \leq I_t((x, z), t)$ *for each* $x = (x_1(t), x_2(t))$, $y = (y_1(t), y_2(t))$, $z = (z_1(t), z_2(t)) \in IF^* \times T$ *and time moment* $t \in T$,

I-3: I_t *is increasing in second variable i.e.*

If $y \leq z$ *then* $I_t((x, y), t) \leq I_t((x, z), t)$ *for each* $x = (x_1(t), x_2(t))$, $y = (y_1(t), y_2(t))$, $z = (z_1(t), z_2(t)) \in IF^* \times T$ *and time moment* $t \in T$.

As this definition shows, the intuitionistic fuzzy pairs to be subjected to the implication process need to change depending on the time. For this reason, the following implication examples will be given based on membership and non-membership values in temporal intuitionistic fuzzy sets. These implications have been obtained by modifying existing implications in the literature according to temporal intuitionistic fuzzy sets.

Proposition 6. *Let*

$$A(T') = \{(x, \mu_A(x,t), \eta_A(x,t)) : X \times T'\}$$

and

$$B(T'') = \{(x, \mu_B(x,t), \eta_B(x,t)) : X \times T''\}$$

where T' *and* T'' *have finite number of distinct time-elements or they are time intervals. Then the followings are temporal intuitionistic fuzzy implication at time moment* $t \in T = T' \cup T''$.

1. *Kleene- Dienes:*

$$I_t^1(((\mu_A(x,t), \eta_A(x,t)), (\mu_B(x,t), \eta_B(x,t))), t)$$
$$= (\max\{\bar{\eta}_A(x,t), \bar{\mu}_B(x,t)\}, \min\{\bar{\mu}_A(x,t), \bar{\eta}_B(x,t)\})$$

(This implication is defined by Parvathi and Geeta in [14]*)*

2. *Reichenbach:*

$$I_t^2(((\mu_A(x,t), \eta_A(x,t)), (\mu_B(x,t), \eta_B(x,t))), t) =$$
$$(\bar{\eta}_A(x,t) + \bar{\mu}_B(x,t) - \bar{\eta}_A(x,t)\,\bar{\mu}_B(x,t), \bar{\mu}_A(x,t)\,\bar{\eta}_B(x,t))$$

3. *Gödel:*

$$I_t^3(((\mu_A(x,t), \eta_A(x,t)), (\mu_B(x,t), \eta_B(x,t))), t) =$$
$$\begin{cases} (1,0) & , \quad 1 - \bar{\eta}_A(x,t) \leq \bar{\mu}_B(x,t) \\ (\bar{\mu}_B(x,t), \bar{\eta}_B(x,t)) & , \quad 1 - \bar{\mu}_A(x,t) \leq \bar{\eta}_B(x,t) \\ (\bar{\mu}_B(x,t), 0) & , \quad \text{otherwise} \end{cases}$$

4. *Lukasiewicz:*
$I_t^4\left(\left(\left(\mu_A(x,t),\,\eta_A(x,t)\right),\left(\mu_B(x,t),\,\eta_B(x,t)\right)\right),t\right)=$

$$\left(\min\left\{1,\bar{\eta}_A(x,t)+\bar{\mu}_B(x,t)\right\},\max\left\{0,\left(\bar{\mu}_A(x,t)+\bar{\eta}_A(x,t)-1\right)\right\}\right)$$

5. *Yager:*
$I_t^5\left(\left(\left(\mu_A(x,t),\,\eta_A(x,t)\right),\left(\mu_B(x,t),\,\eta_B(x,t)\right)\right),t\right)=$

$$\left(\left(\bar{\mu}_B(x,t)\right)^{1-\bar{\eta}_A(x,t)},1-\left(1-\bar{\eta}_B(x,t)\right)^{\bar{\mu}_A(x,t)}\right)$$

6. *Mamdani:* $I_t^6\left(\left(\left(\mu_A(x,t),\,\eta_A(x,t)\right),\left(\mu_B(x,t),\,\eta_B(x,t)\right)\right),t\right)=$

$$\left(\min\left\{1-\bar{\eta}_A(x,t),\bar{\mu}_B(x,t)\right\},\max\left\{1-\bar{\mu}_A(x,t),\bar{\eta}_B(x,t)\right\}\right)$$

If I_t is a implication and N_t is a temporal fuzzy strong negation at time moment t then the function

$$\widetilde{I}_t\left(\left(\left(x_1,x_2\right),\left(y_1,y_2\right)\right),t\right)=I_t\left(N_t\left(\left(y_1,y_2\right),t\right),N_t\left(\left(x_1,x_2\right),t\right),t\right)$$

is an implication at time moment t.

Proof. Let I_t be a temporal intuitionistic fuzzy implication at time moment $t\in T$. Then we should show that the mapping $\widetilde{I}_t$ satisfy the conditions I1,I2,I3.

I1:

a. $\widetilde{I}_t\left(\left(0_t,a(t)\right),t\right)=I_t\left(N_t\left(a(t),t\right),N_t\left(0_t,t\right)\right)=I_t\left(\left(N_t\left(a(t),t\right),1_t\right),t\right)$. Since I_t satisfy the condition I-1(a) I-1(b), it is obtained that $I_t\left(\left(N_t\left(a(t),t\right),1_t\right),t\right)=\widetilde{1}$. So it is obtained that $\widetilde{I}_t\left(\left(0_t,a(t)\right),t\right)=\widetilde{1}$ for all $a(t)=\left(a_1(t),a_2(t)\right)\in IF^*$ at fixed time moment t.

b. $\widetilde{I}_t\left(\left(a(t),1_t\right),t\right)=I_t\left(\left(N_t\left(1_t,t\right),N_t\left(a(t),t\right)\right),t\right)=I_t\left(\left(0_t,N_t\left(a(t),t\right)\right),t\right)$. Since I_t satisfy the condition I-1(a), it is obtained that $I_t\left(\left(\left(0_t,N_t\left(a(t),t\right)\right),t\right)\right)=\widetilde{1}$. So it is obtained that $\widetilde{I}_t\left(\left(a(t),0_t\right),t\right)=\widetilde{1}$ for all $a(t)=\left(a_1(t),a_2(t)\right)\in IF^*$ at fixed time moment t.

c. Since I_t satisfy the condition I-1(c), the following equation is obtained as: $\widetilde{I}_t\left(\left(0_t,1_t\right),t\right)=I_t\left(\left(N_t\left(0_t,t\right),N_t\left(1_t,t\right)\right),t\right)=I_t\left(\left(1_t,0_t\right),t\right)=\widetilde{0}$

I2: Let $x(t)=\left(x_1(t),x_2(t)\right)$ and $y(t)=\left(y_1(t),y_2(t)\right)$ are two temporal intuitionistic fuzzy pair such that $x(t)\leq y(t)$ at the time moment t. Since I_t satisfy the condition I3 and $N_t\left(y(t)\right)\leq N_t\left(x(t)\right)$, it is clearly obtained that

$$\widetilde{I}_t\left(\left(y(t),z(t)\right),t\right)=I_t\left(N_t\left(\left(z_1,z_2\right),t\right),N_t\left(\left(y_1,y_2\right),t\right)\right)$$
$$\leq I_t\left(N_t\left(\left(z_1,z_2\right),t\right),N_t\left(\left(x_1,x_2\right),t\right)\right)=\widetilde{I}_t\left(\left(x(t),z(t)\right),t\right).$$

I3: Let $y(t)=\left(y_1(t),y_2(t)\right)$ and $z(t)=\left(z_1(t),z_2(t)\right)$are two temporal intuitionistic fuzzy pair such that $y(t)\leq z(t)$ at the time moment t. Since I_t satisfy the condition I2 and $N_t\left(z(t)\right)\leq N_t\left(y(t)\right)$, it is clearly obtained that

$$\widetilde{I}_t\left(\left(x(t),y(t)\right),t\right)=I_t\left(N_t\left(\left(y_1,y_2\right),t\right),N_t\left(\left(x_1,x_2\right),t\right)\right)$$
$$\leq I_t\left(N_t\left(\left(z_1,z_2\right),t\right),N_t\left(\left(x_1,x_2\right),t\right)\right)=\widetilde{I}_t\left(\left(x(t),z(t)\right),t\right).$$

□

As stated in [15] , the coimplication, which is the dual of the implication concept, is transferred to temporal intuitionistic fuzzy sets as follows.

Definition 14. *If a function $I_t^c : (TIFP_t^* \times TIFP_t^*) \times T \rightarrow IFP^*$ is satisfied following condition, I_t^c is called temporal intuitionistic fuzzy coimplication at time moment t*

CI-1: *(Boundary Conditions):*

a: $I_t^c((a(t), 0_t), t) = \widetilde{0}$ *for all* $a(t) = (a_1(t), a_2(t)) \in IF^*$ *at time moment* t,

b: $I_t^c((1_t, a(t)), t) = \widetilde{0}$ *for all* $a(t) = (a_1(t), a_2(t)) \in IF^*$ *at time moment* t,

c: $I_t^c((0_t, 1_t), t) = \widetilde{1}$,

CI-2: I_t^c *is decreasing in first variable i.e.*
If $x \leq y$ *then* $I_t^c((y, z), t) \leq I_t^c((x, z), t)$ *for each* $x = (x_1(t), x_2(t))$, $y = (y_1(t), y_2(t))$, $z = (z_1(t), z_2(t)) \in IF^* \times T$ *and time moment* $t \in T$,

CI-3: I_t^c *is increasing in second variable i.e.*
If $y \leq z$ *then* $I_t^c((x, y), t) \leq I_t^c((x, z), t)$ *for each* $x = (x_1(t), x_2(t))$, $y = (y_1(t), y_2(t))$, $z = (z_1(t), z_2(t)) \in IF^* \times T$ *and time moment* $t \in T$.

The relationship between temporal intuitionistic fuzzy implication and temporal intuitionistic fuzzy coimplication is shown below.

Proposition 7. *A function $I_t^c : (TIFP_t^* \times TIFP_t^*) \times T \rightarrow IFP^*$ is a temporal coimplication at time moment t if and only if the function*

$$I_t((x(t), y(t)), t) = N_t((I_t^c(N_t(x(t), t), N_t(y(t)), t)), t)$$

is a temporal intuitionistic fuzzy implication at time moment t for any temporal intuitionistic fuzzy strong negation N_t and each $x(t) = (x_1(t), x_2(t))$, $y(t) = (y_1(t), y_2(t)) \in TIFP_t^*$.

Proof. $\Rightarrow$: Let I_t^c be a coimplication at time moment $t \in T$. Then we should show that the conditions I1,I2,I3 are satisfied.

I1:

a. $I_t((0_t, a(t)), t) = N_t(I_t^c(N_t(0_t, t), N_t(a(t)), t))$. From CI-1(b), it is obtained that
$N_t(I_t^c(1_t, N_t(a(t)), t)) = N_t\left(\widetilde{0}, t\right) = \widetilde{1}$. So it is obtained that $I_t((0_t, a(t)), t) = \widetilde{1}$
for all $a(t) = (a_1(t), a_2(t)) \in IF^*$ at time moment t.

b. $I_t((a(t), 1_t), t) = N_t(I_t^c(N_t(a(t), t), N_t(1_t, t), t)) = N_t(I_t^c(N_t(a(t), t), 0_t, t))$. From CI-1(a), it is obtained that $N_t(I_t^c((N_t(a(t)), 0_t), t)) = N_t(0_t, t) = \widetilde{1}$ for all $a(t) = (a_1(t), a_2(t)) \in IF^*$ at fixed time moment t.

c. From CI-1(c), $I_t((1_t, 0_t), t) = N_t(I_t^c(N_t(1_t, t), N_t(0_t, t), t)) = N_t(I_t^c(0_t, 1_t, t))$ $= N_t\left(\widetilde{1}, t\right) = \widetilde{0}$

I2: Let $x(t)$ and $y(t) \in TIFP_t^*$ such that $x(t) \leq y(t)$ at the time moment t, From CI-2 and $N_t(y(t), t) \leq N_t(x(t), t)$, the inequality

$$I_t^c((N_t(x(t), t), N_t(z(t), t)), t) \leq I_t^c((N_t(y(t), t), N_t(z(t), t)), t)$$

is satisfied for any $z(t) \in TIFP_t^*$ at fixed time moment t. Since N_t is temporal intuitionistic fuzzy strong negation at the time moment t, the inequality

$$N_t(I_t^c((N_t(y(t), t), N_t(z(t), t)), t)) \leq N_t(((N_t(x(t), t), N_t(z(t), t)), t))$$

is obtained. So it is clearly understood that the inequality

$$I_t((y(t), z(t)), t) \leq I_t((x(t), z(t)), t)$$

is satisfied at the time moment t with the above assumptions.

I3: Let be $y(t)$ and $z(t) \in TIFP_t^*$ such that $y(t) \leq z(t)$ at fixed time moment t. From CI-3 and $N_t(z(t), t) \leq N_t(y(t), t)$, it is obtained that

$$I_t^c((N_t(x(t), t), N_t(z(t)), t), t) \leq I_t^c((N_t(x(t), t), N_t(y(t), t)), t).$$

Since N_t is temporal intuitionistic strong negation at the time moment t, the inequality

$$N_t(I_t^c((N_t(x(t), t), N_t(y(t), t)), t), t) \leq N_t(I_t^c((N_t(x(t), t), N_t(z(t), t)), t))$$

is obtained. So it is clearly understood that the inequality

$$I_t(x(t), y(t)) \leq I_t(x(t), z(t))$$

is satisfied at the time moment t with the above assumptions. □

Theorem 1. *Let S_t be a temporal intuitionistic fuzzy t-conorm and N_t be a temporal intuitionistic fuzzy strong negation at time moment t. Then, the mapping defined as $I_{S_t}((x(t), y(t)), t) = S_t((N_t(x(t), t), y(t)), t)$ for each $x(t), y(t) \in IF^*$ is a temporal intuitionistic fuzzy implication.*

Proof. **I1-**

a. $I_{S_t}((0_t, x(t)), t) = S_t((N_t(0_t, t), y(t)), t) = S_t((1_t, y(t)), t) = \widetilde{1}$,

b. $I_{S_t}((x(t), 1_t), t) = S_t((N_t(a_t, t), 1_t), t) = \widetilde{1}$,

c. $I_{S_t}((1_t, 0_t), t) = S_t((N_t(1_t, t), 1_t), t) = S_t((0_t, 0_t), t) = \widetilde{0}$.

I2- Let be $x(t)$ and $y(t) \in TIFP_t^*$ such that $x(t) \leq y(t)$ at the time moment t. Then $N_t(y(t), t) \leq N_t(x(t), t)$. From *S2*, $S_t((N_t(y(t), t), z(t)), t) \leq S_t((N_t(x(t), t), z(t)), t)$ for each $z(t) \in TIFP_t^*$. Thus

$$I_{S_t}((y(t), z(t)), t) \leq I_{S_t}((x(t), z(t)), t).$$

I3- Let be $y(t)$ and $z(t) \in TIFP_t^*$ such that $y(t) \leq z(t)$ at the time moment t. From *S2*, $I_{S_t}((x(t), y(t)), t) = S_t((N_t(x(t), t), y(t)), t) \leq S_t((N_t(x(t), t), z(t)), t)$ $= I_{S_t}((x(t), z(t)), t)$ for each $z(t) \in TIFP_t^*$. Thus it is obtained that

$$I_{S_t}((x(t), y(t)), t) \leq I_{S_t}((x(t), z(t)), t).$$

□

Definition 15. *Let S_t be a temporal intuitionistic fuzzy t-conorm and N_t be a temporal intuitionistic fuzzy strong negation at fixed time moment t. Then I_{S_t} : $(TIFP_T^* \times TIFP_T^*) \times T \to IFP^*$ is called temporal intuitionistic fuzzy $S-$implication.*

Example 1. *I_t^1 is a $S-$implication produced with $S_{\max}^t$ and temporal intuitionistic fuzzy standard negation*

Theorem 2. *Let T_t be a temporal intuitionistic fuzzy t-norm and N_t be a temporal intuitionistic fuzzy strong negation at time moment t. Let define the family of TIFPs such as $Z_{(x(t),y(t))} = \{z(t) = (z_x(t), z_y(t)) \in TIFP_T^* : T_t((x(t), z(t)), t) \leq y(t)\}$ for each $x(t), y(t) \in TIFP_T^*$. Then, the mapping defined as $I_{T_t}((x(t), y(t)), t) = (\sup(z_x), \inf(z_y))$ is a temporal intuitionistic fuzzy implication.*

Proof. **I1-**

a. Since $T_t((0_t, z(t)), t) = 0_t \leq y(t)$ for each $z(t) = (z_x(t), z_y(t)) \in TIFP_T^*$, So 1_t can be chosen as $z(t)$. Then, it is obtained that $I_{T_t}((0_t, y(t)), t) = \widetilde{1}$,

b. From T4, $T_t((x(t), 1_t), t) = x(t) \leq 1_t$. So 1_t can be chosen as $z(t)$. Then, it is obtained that $I_{T_t}((0_t, x(t)), t) = \widetilde{1}$.

c. Since *the equation* $T_t((1_t, z(t)), t) = z(t) \leq 0_t$ has a only one solution as $z(t) = 0_t$,it is clearly understood that $I_{T_t}((1_t, 0_t), t) = \widetilde{1}$.

I2- Let be $x(t)$ and $y(t) \in TIFP_t^*$ such that $x(t) \leq y(t)$ at the time moment t. We must show that $I_{T_t}((y(t), z(t)), t) \leq I_{T_t}((x(t), z(t)), t)$. From T2, The inequality $T_t((x(t), z^*(t)), t) \leq z(t)$ is satisfied for each $z^*(t) \in TIFP_T^*$ which satisfy the inequality $T_t((y(t), z^*(t)), t) \leq z(t)$.Then $Z_{(y(t),z(t))} \subseteq Z_{(x(t),z(t))}$. Then it is clearly understood from the definition of I_{T_t}

$$I_{T_t}((y(t), z(t)), t) \leq I_{T_t}((x(t), z(t)), t).$$

I3- Let be $y(t)$ and $z(t) \in TIFP_t^*$ such that $y(t) \leq z(t)$ at the time moment t. We must show that $I_{T_t}((x(t), y(t)), t) \leq I_{T_t}((x(t), z(t)), t)$. The inequality $T_t((x(t), z^*(t)), t) \leq z(t)$ is satisfied for each $z^*(t) \in TIFP_T^*$ which satisfy the inequality $T_t((x(t), z^*(t)), t) \leq y(t)$.Then $Z_{(x(t),z(t))} \subseteq Z_{(y(t),z(t))}$. Then it is clearly understood from the definition of I_{T_t}

$$I_{T_t}((x(t), y(t)), t) \leq I_{T_t}((x(t), z(t)), t).$$

□

Definition 16. *Let T_t be a temporal intuitionistic fuzzy t-conorm and N_t be a temporal intuitionistic fuzzy strong negation at fixed time moment t. Then I_{T_t} : $(TIFP_T^* \times TIFP_T^*) \times T \to IFP^*$ is called temporal intuitionistic fuzzy $R-$implication.*

Proposition 8. *Let I_{T_t} be a temporal intuitionistic fuzzy $R-$implication produced any T_t temporal intuitionistic fuzzy t-conorm and N_t temporal intuitionistic fuzzy strong negation at fixed time moment t. Then $I_{T_t}((x(t), x(t)), t) = \widetilde{1}$ for each $x(t) \in TIFP_t^*$.*

Proof. From T4, $T_t((x(t), 1_t), t) = x(t)$. Then it is understood that $1_t \in Z_{(x(t),x(t))}$. So $I_{T_t}(x(t), x(t)) = \widetilde{1}$ □

Remark 2. *The concepts, which we have given in our work until this section, have always been defined for a single time moment. If these concepts are defined in all of their clusters when these concepts are defined, these concepts are called the overall intuitionistic fuzzy (negation, t-norm, t-conorm, implication and coimplication). It is often essential to produce a final conclusion from a concept that is overall intuitionistic fuzzy. This could be done using the aggregation function. The following theorem offers a way for this final conclusion.*

Theorem 3. *Let $T = \{t_1, t_2, ...t_n\}$ be a finite time set which has $n \geq 2$ elements, N_{t_i} be a overall intuitionistic fuzzy negation and $f : (TIFP_T^*)^n \to IFP^*$ $(n \geq 2)$ be a function satisfied following conditions:*

(1) $f(0_T, 0_T, ..., 0_T) = \widetilde{0}$ *and* $f(1_T, 1_T, ..., 1_T) = \widetilde{1}$
(2) $f(a(t_1), a(t_2), ..., a(t_n)) \leq f(b(t_1), b(t_2), ..., b(t_n))$ *for any pair* $(a(t_1), a(t_2), \ldots, a(t_n))$ *and* $(b(t_1), b(t_2), ..., b(t_n))$ *of* $n-$*tuples in* $(TIFP_T^*)^n$ *such that* $a(t_i) \leq b(t_i)$ $(i \in \{1, 2, ..., n\})$
(3) f *is a continuous function.*

Then the mapping $N : TIFP_T^* \to IFP^*$ *defined as*

$$N(x(t_i)) = f(N_{t_1}(x(t_i), t_1), N_{t_2}(x(t_i), t_2), ..., N_{t_n}(x(t_i), t_n))$$

$(i \in \{1, 2, ..., n\})$ *is a intuitionistic fuzzy negation on* $TIFP_T^*$

Proof. For every $x(t_i), y(t_i) \in TIFP_T^*$ and $i \in \{1, 2, ..., n\}$ such that $x(t_i) \leq y(t_i)$, the inequality $N_{t_j}(y(t_i), t_j) \leq N_{t_j}(x(t_i), t_j)$ is obtained for each $i, j \in \{1, 2, ..., n\}$ from the definition of overall intuitionistic fuzzy negation. Then, following inequality is clearly obtained from the definition of f for each $i \in \{1, 2, ..., n\}$:

$$N(y(t_i)) = f(N_{t_1}(y(t_i), t_1), N_{t_2}(y(t_i), t_2), ..., N_{t_n}(y(t_i), t_n))$$
$$\leq f(N_{t_1}(x(t_i), t_1), N_{t_2}(x(t_i), t_2), ..., N_{t_n}(x(t_i), t_n)) = N(x(t_i))$$

Hence it is clearly understood that N is decreasing. On the other hand,

$$\begin{aligned} N(0_T(t_i)) &= f(N_{t_1}(0_T(t_i), t_1), N_{t_2}(0_T(t_i), t_2), ..., N_{t_n}(0_T(t_i), t_n)) \\ &= f\left(\widetilde{1}, \widetilde{1}, ..., \widetilde{1}\right) = \widetilde{1}, \\ N(1_T(t_i)) &= f(N_{t_1}(1_T(t_i), t_1), N_{t_2}(1_T(t_i), t_2), ..., N_{t_n}(1_T(t_i), t_n)) \\ &= f\left(\widetilde{0}, \widetilde{0}, ..., \widetilde{0}\right) = \widetilde{0}. \end{aligned}$$

□

Theorem 4. *Let $T = \{t_1, t_2, ...t_n\}$ be a finite time set which has $n \geq 2$ elements, T_{t_i} be a overall intuitionistic fuzzy* $t-$*norm and $f : (TIFP_T^*)^n \to IFP^*$ $(n \geq 2)$ be a function satisfied following conditions:*

(1) $f(a(t_i), a(t_i), ..., a(t_i)) = a(t_i)$ *for* $a(t_i) \in TIFP_T^*$,

(2) *$f(a(t_1), a(t_2), ..., a(t_n)) \leq f(b(t_1), b(t_2), ..., b(t_n))$ for any pair $(a(t_1), a(t_2), ..., a(t_n))$ and $(b(t_1), b(t_2), ..., b(t_n))$ of $n-$tuples in $(TIFP_T^*)^n$ such that $a(t_i) \leq b(t_i)$ $(i \in \{1, 2, ..., n\})$.*

(3) *f is a continuous function.*

Then the mapping $T : TIFP_T^* \rightarrow IFP^*$ defined as

$$T(x(t_i), y(t_i)) = f(T_{t_1}((x(t_i), y(t_i)), t_1), T_{t_2}((x(t_i), y(t_i)), t_2), ..., T_{t_n}((x(t_i), y(t_i)), t_n))$$

$(i \in \{1, 2, ..., n\})$ is a intuitionistic fuzzy $t-norm$ on $TIFP_T^*$.

Proof. T1. Since the equation $T_{t_j}((x(t_i), y(t_i)), t_j) = T_{t_j}((x(t_i), y(t_i)), t_j)$ holds for every $x, y \in TIFP_T^*$ and $i, j \in \{1, 2, ..., n\}$, the following equation

$$\begin{aligned} & T((x(t_i), y(t_i))) \\ = \; & f(T_{t_1}((x(t_i), y(t_i)), t_1), T_{t_2}((x(t_i), y(t_i)), t_2), ..., T_{t_n}((x(t_i), y(t_i)), t_n)) \\ = \; & f(T_{t_1}((y(t_i), x(t_i)), t_1), T_{t_2}((y(t_i), x(t_i)), t_2), ..., T_{t_n}((y(t_i), x(t_i)), t_n)) \\ = \; & T((y(t_i), x(t_i))) \end{aligned}$$

is obtained for each $i \in \{1, 2, ..., n\}$.

T2. Since T_{t_j} is a overall intuitionistic fuzzy $t-$norm, the inequality

$$T_{t_j}((x_1(t_i), y_1(t_i)), t_j) \leq T_{t_j}((x_2(t_i), y_2(t_i)), t_j)$$

is satisfied for every $i, j \in \{1, 2, ..., n\}$ and every $x_1(t_i), y_1(t_i), x_2(t_i), y_2(t_i) \in TIFP_T^*$ such that $x_1(t_i) \leq x_2(t_i)$ and $y_1(t_i) \leq y_2(t_i)$.

From the definition of f , the following inequality is obtained:

$$\begin{aligned} & f(T_{t_1}((x_1(t_i), y_1(t_i)), t_1), T_{t_2}((x_1(t_i), y_1(t_i)), t_2), ..., T_{t_n}((x_1(t_i), y_1(t_i)), t_n)) \\ \leq \; & f(T_{t_1}((x_2(t_i), y_2(t_i)), t_1), T_{t_2}((x_2(t_i), y_2(t_i)), t_2), ..., T_{t_n}((x_2(t_i), y_2(t_i)), t_n)) \end{aligned}$$

Then it is obtained that $T((x_1(t_i), y_1(t_i))) \leq T((x_2(t_i), y_2(t_i)))$.

T3. Since T_{t_j} is a overall intuitionistic fuzzy $t-$norm, the equality

$$T_{t_j}\left(\left(T_{t_j}((x(t_i), y(t_i)), t_j)\right), z(t_i), t_j\right) = T_{t_j}\left(\left(x(t_i), T_{t_j}((z(t_i), y(t_i)), t_j)\right), t_j\right)$$

is satisfied for every $i, j \in \{1, 2, ..., n\}$ and every $x(t_i), y(t_i) \in TIFP_T^*$. Then we must show that

$$T((T((x(t_i), y(t_i)), t_j)), z(t_i), t_j) = T((x(t_i), T((z(t_i), y(t_i)), t_j)), t_j)$$

Let $T((x(t_i), y(t_i))) = a(t_i)$, $T((z(t_i), y(t_i))) = b(t_i)$. Hence the following equation is obtained

$$T_{t_j}((a(t_i), z(t_i)), t_j) = T_{t_j}((x(t_i), b(t_i)), t_j)$$

Then

$$\begin{aligned} & T(a(t_i), z(t_i)) \\ = \; & f(T_{t_1}((a(t_i), z(t_i)), t_1), T_{t_2}((a(t_i), z(t_i)), t_2), ..., T_{t_n}((a(t_i), x(t_i)), t_n)) \\ = \; & f(T_{t_1}((x(t_i), b(t_i)), t_1), T_{t_2}((x(t_i), b(t_i)), t_2), ..., T_{t_n}((x(t_i), b(t_i)), t_n)) \end{aligned}$$

$$= \quad T\left(x\left(t_i\right), b\left(t_i\right)\right)$$

T4. Since T_{t_j} is a overall intuitionistic fuzzy $t-$norm, the equality

$$T_{t_j}\left(\left(x\left(t_i\right), 1_t\right), t_j\right) = x\left(t_i\right)$$

for every $x\left(t_i\right) \in TIFP_T^*$ and for every $i, j \in \{1, 2, ..., n\}$. Then it is easily obtained that

$$\begin{aligned} & f\left(T_{t_1}\left(\left(x\left(t_i\right), 1_t\right), t_1\right), T_{t_2}\left(\left(x\left(t_i\right), 1_t\right), t_2\right), ..., T_{t_n}\left(\left(x\left(t_i\right), 1_t\right), t_n\right)\right) \\ = \quad & f\left(x\left(t_i\right), x\left(t_i\right), ..., x\left(t_i\right)\right) = x\left(t_i\right) \end{aligned}$$

□

Theorem 5. *Let $T = \{t_1, t_2, ...t_n\}$ be a finite time set which has $n \geq 2$ elements, S_{t_i} be a overall intuitionistic fuzzy $s-norm$ and $f : \left(TIFP_T^*\right)^n \rightarrow IFP^*$ $(n \geq 2)$ be a function satisfied following conditions:*

1. *$f\left(a\left(t_i\right), a\left(t_i\right), ..., a\left(t_i\right)\right) = a\left(t_i\right)$ for $a\left(t_i\right) \in TIFP_T^*$,*
2. *$f\left(a\left(t_1\right), a\left(t_2\right), ..., a\left(t_n\right)\right) \leq f\left(b\left(t_1\right), b\left(t_2\right), ..., b\left(t_n\right)\right)$ for any pair $(a\left(t_1\right), a\left(t_2\right),$ $..., a\left(t_n\right))$ and $\left(b\left(t_1\right), b\left(t_2\right), ..., b\left(t_n\right)\right)$ of $n-$tuples in $\left(TIFP_T^*\right)^n$ such that $a\left(t_i\right) \leq b\left(t_i\right)$ $\left(i \in \{1, 2, ..., n\}\right)$,*
3. *f is a continuous function.*

Then the mapping $S : TIFP_T^* \rightarrow IFP^*$ defined as

$$S\left(x\left(t_i\right), y\left(t_i\right)\right) = f\left(S_{t_1}\left(\left(x\left(t_i\right), y\left(t_i\right)\right), t_1\right), S_{t_2}\left(\left(x\left(t_i\right), y\left(t_i\right)\right), t_2\right), ..., S_{t_n}\left(\left(x\left(t_i\right), y\left(t_i\right)\right), t_n\right)\right)$$

$\left(i \in \{1, 2, ..., n\}\right)$ is a intuitionistic fuzzy $s-norm$ on $TIFP_T^*$.

Proof. It could be proven as previous theorem. □

Theorem 6. *Let $T = \{t_1, t_2, ...t_n\}$ be a finite time set which has $n \geq 2$ elements, I_{t_i} be a overall intuitionistic fuzzy implication and $f : \left(TIFP_T^*\right)^n \rightarrow IFP^*$ $(n \geq 2)$ be a function satisfied following conditions:*

1. *$f\left(a\left(t_i\right), a\left(t_i\right), ..., a\left(t_i\right)\right) = a\left(t_i\right)$ for $a\left(t_i\right) \in TIFP_T^*$,*
2. *$f\left(a\left(t_1\right), a\left(t_2\right), ..., a\left(t_n\right)\right) \leq f\left(b\left(t_1\right), b\left(t_2\right), ..., b\left(t_n\right)\right)$ for any pair $(a\left(t_1\right), a\left(t_2\right)$ $, ..., a\left(t_n\right))$ and $\left(b\left(t_1\right), b\left(t_2\right), ..., b\left(t_n\right)\right)$ of $n-$tuples in $\left(TIFP_T^*\right)^n$ such that $a\left(t_i\right) \leq b\left(t_i\right)$ $\left(i \in \{1, 2, ..., n\}\right)$,*
3. *f is a continuous function.*

Then the mapping $I : TIFP_T^* \rightarrow IFP^*$ defined as

$$I\left(x\left(t_i\right), y\left(t_i\right)\right) = f\left(I_{t_1}\left(\left(x\left(t_i\right), y\left(t_i\right)\right), t_1\right), I_{t_2}\left(\left(x\left(t_i\right), y\left(t_i\right)\right), t_2\right), ..., I_{t_n}\left(\left(x\left(t_i\right), y\left(t_i\right)\right), t_n\right)\right)$$

$\left(i \in \{1, 2, ..., n\}\right)$ is a intuitionistic fuzzy implication on $TIFP_T^*$.

Proof. **I-1**: (Boundary Conditions):

a. Since $I_t\left(\left(0_T, a\left(t_i\right)\right), t\right)=\widetilde{1}$ for all $a\left(t\right) \in TIFP_t^*$ and every time moment $t \in T$,The equation is satisfied

$$\begin{aligned} I\left(0_T, a\left(t_i\right)\right) &= f\left(I_{t_1}\left(\left(0_T, a\left(t_i\right)\right), t_1\right), I_{t_2}\left(\left(0_T, a\left(t_i\right)\right), t_2\right), \ldots, I_{t_n}\left(\left(0_T, a\left(t_i\right)\right), t_n\right)\right) \\ &= f\left(1_T, 1_T, \ldots, 1_T\right)=\widetilde{1} \end{aligned}$$

b. Since $I\left(\left(a\left(t_i\right), 1_T\right), t\right)=\widetilde{1}$ for all $a\left(t_i\right) \in TIFP_t^*$ and every time moment $t \in T$,The equation is satisfied

$$\begin{aligned} I\left(a\left(t_i\right), 1_T\right) &= f\left(I_{t_1}\left(\left(a\left(t_i\right), 1_T\right), t_1\right), I_{t_2}\left(\left(a\left(t_i\right), 1_T\right), t_2\right), \ldots, I_{t_n}\left(\left(a\left(t_i\right), 1_T\right), t_n\right)\right) \\ &= f\left(1_T, 1_T, \ldots, 1_T\right)=\widetilde{1} \end{aligned}$$

c. Since $I_t\left(\left(1_T, 0_T\right), t\right)=\widetilde{0}$ for every time moment $t \in T$,The equation is satisfied

$$\begin{aligned} I\left(1_T, 0_T\right) &= f\left(I_{t_1}\left(\left(1_T, 0_T\right), t_1\right), I_{t_2}\left(\left(1_T, 0_T\right), t_2\right), \ldots, I_{t_n}\left(\left(1_T, 0_T\right), t_n\right)\right) \\ &= f\left(0_T, 0_T, \ldots, 0_T\right)=\widetilde{0} \end{aligned}$$

I-2: Since I_t is decreasing in first variable, the inequality $I_t\left(y\left(t_i\right), z\left(t_i\right), t\right) \leq I_t\left(x\left(t_i\right), z\left(t_i\right), t\right)$ is satisfied at every time moment t and each $x=\left(x_1\left(t\right), x_2\left(t\right)\right)$, $y=\left(y_1\left(t\right), y_2\left(t\right)\right)$, $z=\left(z_1\left(t\right), z_2\left(t\right)\right) \in TIFP_t^*$ such that $x \leq y$. As the definition of f, the following inequality is obtained such that:

$$\begin{aligned} & I\left(\left(y\left(t_i\right), z\left(t_i\right)\right), t\right) \\ = \; & f\left(I_{t_1}\left(\left(y\left(t_i\right), z\left(t_i\right)\right), t_1\right), I_{t_2}\left(\left(y\left(t_i\right), z\left(t_i\right)\right), t_2\right), \ldots, I_{t_n}\left(\left(y\left(t_i\right), z\left(t_i\right)\right), t_n\right)\right) \\ \leq \; & f\left(I_{t_1}\left(\left(x\left(t_i\right), z\left(t_i\right)\right), t_1\right), I_{t_2}\left(\left(x\left(t_i\right), z\left(t_i\right)\right), t_2\right), \ldots, I_{t_n}\left(\left(x\left(t_i\right), z\left(t_i\right)\right), t_n\right)\right) \\ = \; & I\left(\left(x\left(t_i\right), z\left(t_i\right)\right), t\right) \end{aligned}$$

I-3: Since I_t is increasing in second variable, the inequality $I_t\left(y\left(t_i\right), x\left(t_i\right), t\right) \leq I_t\left(z\left(t_i\right), x\left(t_i\right), t\right)$ is satisfied at every time moment t and each $x=\left(x_1\left(t\right), x_2\left(t\right)\right)$, $y=\left(y_1\left(t\right), y_2\left(t\right)\right)$, $z=\left(z_1\left(t\right), z_2\left(t\right)\right) \in TIFP_t^*$ such that $y \leq z$. As the definition of f, the following inequality is obtained such that:

$$\begin{aligned} & I\left(\left(y\left(t_i\right), x\left(t_i\right)\right), t\right) \\ = \; & f\left(I_{t_1}\left(\left(y\left(t_i\right), x\left(t_i\right)\right), t_1\right), I_{t_2}\left(\left(y\left(t_i\right), x\left(t_i\right)\right), t_2\right), \ldots, I_{t_n}\left(\left(y\left(t_i\right), x\left(t_i\right)\right), t_n\right)\right) \\ \leq \; & f\left(I_{t_1}\left(\left(z\left(t_i\right), x\left(t_i\right)\right), t_1\right), I_{t_2}\left(\left(z\left(t_i\right), x\left(t_i\right)\right), t_2\right), \ldots, I_{t_n}\left(\left(z\left(t_i\right), x\left(t_i\right)\right), t_n\right)\right) \\ = \; & I\left(\left(z\left(t_i\right), x\left(t_i\right)\right), t\right) \end{aligned}$$

□

Theorem 7. *Let $T=\{t_1, t_2, \ldots t_n\}$ be a finite time set which has $n \geq 2$ elements, $I_{t_i}^c$ be an overall intuitionistic fuzzy coimplication and $f:\left(TIFP_T^*\right)^n \rightarrow IFP^*$ $(n \geq 2)$ be a function satisfied following conditions:*

(1) *$f\left(a\left(t_i\right), a\left(t_i\right), \ldots, a\left(t_i\right)\right)=a\left(t_i\right)$ for $a\left(t_i\right) \in TIFP_T^*$,*
(2) *$f\left(a\left(t_1\right), a\left(t_2\right), \ldots, a\left(t_n\right)\right) \leq f\left(b\left(t_1\right), b\left(t_2\right), \ldots, b\left(t_n\right)\right)$ for any pair $\left(a\left(t_1\right),\right.$ $\left.a\left(t_2\right), \ldots, a\left(t_n\right)\right)$ and $\left(b\left(t_1\right), b\left(t_2\right), \ldots, b\left(t_n\right)\right)$ of $n-$tuples in $\left(TIFP_T^*\right)^n$ such that $a\left(t_i\right) \leq b\left(t_i\right)$ $(i \in\{1,2, \ldots, n\})$,*

(3) f *is a continuous function.*

Then the mapping $I^C : TIFP_T^* \to IFP^*$ *defined as*

$$\begin{aligned} & I^C\left(x\left(t_i\right), y\left(t_i\right)\right) \\ = \ & f\left(I_{t_1}^c\left(\left(x\left(t_i\right), y\left(t_i\right)\right), t_1\right), I_{t_2}^c\left(\left(x\left(t_i\right), y\left(t_i\right)\right), t_2\right), ..., I_{t_n}^c\left(\left(x\left(t_i\right), y\left(t_i\right)\right), t_n\right)\right) \end{aligned}$$

$(i \in \{1, 2, ..., n\})$ *is an intuitionistic fuzzy coimplication on* $TIFP_T^*$*.*

5. Conclusion

It is understood from the definitions and theorems given in the whole article, from the judgments obtained from a temporal system, that a conclusion judgment could be obtained by aggregation functions. This provides a way for crisp outlets to be obtained from temporal intuitionistic fuzzy systems. In this study; temporal intuitionistic fuzzy negation, temporal intuitionistic fuzzy triangular norm and temporal intuitionistic fuzzy triangular conorm have been researched. The aim of this study is to define negator, t-norm and t-conorms, which is the generalization of negation, conjunctions and disconjunctions in the temporal intuitionistic fuzzy sets and to examine the De Morgan relations between these concepts. The thing to note here is that conjunctions generalized with $t-$norm and $t-$conorm is changed depending on time. we will carry concept of implication and coimplication to temporal intuitionistic fuzzy sets. With the new implication definitions, a causal structure will be established which will match the variable structure of the systems depending on the position and time variables. It is evident that successful results will be achieved in this type of system, which is being dealt with by this new structure.

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