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A New Approach To Cross Efficiency In Data Envelopment Analysis and Performance Evaluation of Turkey Cities

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ABSTRACT

The cross-efficiency evaluation (CEE) method, which was developed as a contribution to classical Data Envelopment Analysis (DEA), has been successively used to solve problems involving unrealistic weight distribution as well as problems that do not require prior information for ranking the decision making units (DMUs). Originally, the CEE method included the efficiency evaluations that were obtained for a DMU by the classical DEA for the reuse of optimal weights in the other DMUs. As the optimal weights in the classical DEA solutions usually have multiple solutions, this reduces the usefulness of the CEE method. Hence, this study suggests a new technique that could be used in the second stage of the CEE method for removing the problem of multiple optimal weights and for determining the reasonable ranks of DMUs. The performance of the proposed model is examined on real data set relative to the efficiencies of Turkey cities.

Key Words: *Data envelopment analysis, cross efficiency, ranking.*

1. INTRODUCTION

Data Envelopment Analysis (DEA) is a nonparametric efficiency method that was first developed by Charnes et al. [7] with the purpose of measuring the relative efficiencies of similar economical DMUs for the goods and services produced. This method is characterized by the following properties: it describes the inefficiency amount and sources for each DMU; it does not put forward any functional assumption on the variables; and it has relative evaluated efficiencies (since the efficiency for each DMU is computed with respect to the other DMUs). Thus far, the DEA method has been widely applied in various areas and is being developed further through interaction with various techniques. In fact, DEA seems to be the most popular approximation that has gained rapid development and widespread applications, particularly in the areas of operations research and management science. However, the improvements made to the DEA technique have resulted in several new problems [1, 15]. These problems are interdependent and have been known for a long time, for example, the issue

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of unrealistic weights distribution, the weak discrimination power, and having multiple optimal solutions to weights for efficient DMUs.

The problem of unrealistic weight distribution for DEA occurs when some DMUs are rated as efficient owing to input and output weights having extreme or zero values [4, 5, 6, 12, 17]. Moreover, having multiple optimal solutions to weights affects to a great extent the consistency of operations related to weights cross efficiency method is the most frequently studied topic in DEA literature. The problem of unrealistic weight distribution is dealt with using techniques of weight restriction. Besides, the case having multiple solutions to weights has an effect on the other two problems (unrealistic weights distribution and the weak discrimination power). In this way, the issues are interdependent. The problem of having multiple solutions to optimal weights occurs specifically in the cross efficiency method wherein the efficiencies of DMUs are evaluated by means of optimal weights of any given DMU.

On the other hand, the problem of weak discrimination power or the lack of discrimination power appears in the case where the number of DMUs under evaluation is insufficient in comparison with the total number of inputs and outputs. For the classical DEA models, this situation often results in the solutions determining several DMUs as efficient. To arrive at a solution for this problem of DEA, the models of super efficiency and cross efficiency approaches have been developed [2, 6, 11]. For all efficient DMUs, the super efficiency concept has been proposed when the number of efficient DMUs is more than one. Andersen and Petersen [2] introduced one of the super efficiency models for ranking efficient DMUs in DEA. Their method enables an extreme efficient unit o to achieve an efficiency score greater than one by removing the p^{th} constraint in the envelopment linear programming formulation.

Sexton et al. [16] developed the cross efficiency method to rate the DMUs. Their technique made use of the cross evaluation scores computed as related to all DMUs and hence identified the best DMUs [3]. The basic idea of cross evaluation is to use DEA as machinery in peer evaluation rather than self evaluation. Peer evaluation refers to the assigned score for each DMU that is obtained using the optimal weights of other DMUs. The advantages of the cross efficiency method include the ability to rate DMUs and the capability of being a useful tool without the need of any expert opinion or prerequisites to solve unenviable cases such as multiple solutions and solutions with extreme or zero values for the weights in DEA.

Despite the extensive use of the cross efficiency method, it has some limitations arising from the classical DEA. Doyle and Green [11] stated that the non-uniqueness, i.e., having multiple solutions to optimal weights in DEA, decreases the usefulness of the cross efficiency method. Sexton et al. [16] and Doyle and Green [11] recommended the use of a secondary objective (model) for the cross efficiency evaluation related to the non-uniqueness of optimal weights in DEA. They proposed the aggressive and benevolent models for achieving the secondary objective. The basic idea in the benevolent approach is to obtain the set of optimal weights maximizing not only the efficiency of a DMU under evaluation but also the average efficiency of other DMUs. On the contrary, the aggressive models are focused on searching for the set of optimal weights minimizing the average efficiency of other DMUs. Recently, Liang et al. [14] made some new suggestions for the second stage in cross efficiency evaluation. Their suggestions are comprised of three different models: the minimization of deviations from the ideal point (the minsum efficiency); the minimization of maximum inefficiency amount (the minmax efficiency); and the minimization of absolute deviations from the average efficiency. The first two models are used in the multicriteria DEA approach proposed by Li and Reeves [13] for dealing with the problems of ranking the units as well as unrealistic weight distribution.

This study is aimed at presenting an alternative suggestion for the approaches used in cross efficiency evaluation and determining the reasonable ranks of DMUs. In the proposed model the input and output components are respectively approximated to the weighted input and weighted output sums in order to provide reasonable ranking. The performance of the proposed model is examined on real data set relative to the efficiencies of Turkey cities.

This article is organized as follows. In Section 2, the basic DEA model and the related concepts are introduced. In Section 3, the cross efficiency method is presented and its aggressive formulation is explained. In Section 4, the model in which the input and output components are respectively approximated to the weighted input and weighted sums is presented for the second stage of cross efficiency evaluation. In Section 5, the basic CCR model, super efficiency method, the aggressive cross efficiency method, and the proposed model for cross efficiency evaluation are applied to real data set relative to the efficiencies of Turkey cities and their solutions are compared. Lastly, in Section 6, a summary of the research and the results are provided.

2. DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis (DEA) is a mathematical programming approach that utilizes multiple inputs and outputs to measure the relative efficiencies within a group of decision making units (DMUs). In DEA, it is assumed that there are n DMUs to be evaluated in terms of m inputs and s outputs. Let x_{ij} ($i = 1, \ldots, m$) and y_{rj} ($r = 1, \ldots, s$) represent the input and output values of DMU_j ($j = 1, \ldots, n$), respectively. Subsequently, the efficiency of DMU_p can be calculated as

$$\theta_p = \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \qquad (1)$$

where, v_i ($i = 1, \ldots, m$) and u_r ($r = 1, \ldots, s$) are the input and output weights assigned to i^{th} input and r^{th} output, respectively. Charnes et al. [7] established a model as

$$\max \ \theta_p = \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad j = 1, 2, \ldots, n \tag{2}$$

$$u_r \ge 0, \qquad r = 1, \ldots, s$$

$$v_i \ge 0, \qquad i = 1, \ldots, m$$

where, DMU_p refers to the DMU under evaluation. This fractional program, well known as the CCR model, can be converted into a linear programming problem wherein the optimal value of the objective function indicates the relative efficiency of DMU_p. Hence, the reformulated linear programming problem can be defined as follows:

$$\max \ \theta_p = \sum_{r=1}^{s} u_r y_{rp}$$

s.t.

$$\sum_{i=1}^{m} v_i x_{ip} = 1 \tag{3}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1, 2, \ldots, n$$

$$u_r \ge 0, \quad r = 1, \ldots, s$$

$$v_i \ge 0, \qquad i = 1, \ldots, m$$

In the above-mentioned models, DMU_p is considered to be efficient if and only if $\theta_p^* = 1$; otherwise, it is referred to as non-efficient.

3. CROSS EFFICIENCY EVALUATION

The cross efficiency method was developed as a DEA extension tool to be utilized for identifying the best performing DMUs and for ranking DMUs using cross efficiency scores that are linked to all DMUs [3]. The basic idea of the cross efficiency method that alleviates the weak discrimination of the classical DEA model can be explained in two stages: In the first stage, the classical DEA analysis is performed and the optimal weights of inputs and outputs are calculated for each DMU. However, the optimal weights computed by classical DEA have multiple solutions especially for the efficient DMUs and these solutions provide unrealistic weights, i.e., weights with extreme or zero values. In the second stage, these drawbacks are reduced and a suitable set of weights preserving the efficiency values obtained by DEA is selected for each DMU.

In the first stage, the optimal weights of inputs and outputs are calculated for each DMU using the classical DEA formulation. Given the results of the first stage, the weights used by the DMU can be utilized for calculating the peer rated efficiency for each of the other DMUs. The peer evaluation score, $\theta_{p,j}$, indicates the efficiency score for DMU_j using the weights obtained by DMU_p [16].

$$\theta_{p,j} = \frac{\sum_{r=1}^{s} u_{r,p} y_{rj}}{\sum_{i=1}^{m} v_{i,p} x_{ij}} \tag{4}$$

In general, the optimal weights obtained using classical DEA in the first stage are multiple solutions. Therefore, the values $\theta_{p,j}$ will change depending on these values in the second stage. To reduce this undesirable case, there are some model suggestions for preserving the self efficiency scores, $\theta_{p,p}$, obtained for each DMU. The one so-called aggressive efficiency model developed by Sexton et al. [16] and extended by Doyle and Green [11] is given in (5) below. In this approach, an attempt is made to minimize the efficiencies of other DMUs while preserving the efficiency of the DMU under evaluation. Contrary to this, in the benevolent approach, an attempt is made to maximize the efficiencies of other DMUs. Since the discrimination of DMUs is an important problem in DEA, the aggressive model seems more beneficial when compared to the benevolent model in respect of the discrimination problem.

$$\min \ \sum_{r=1}^{s} \left(u_{rp} \sum_{j=1; j \ne p}^{n} y_{rj} \right)$$

s.t.

$$\sum_{i=1}^{m}\left(v_{ip}\sum_{j=1;\,j\neq p}^{n}x_{ij}\right)=1 \qquad (5)$$

$$\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}\leq 0,\; j=1,\,\ldots,\,n;\; j\neq p$$

$$\sum_{r=1}^{s}u_r y_{rp}-\theta_{p,p}\sum_{i=1}^{m}v_i x_{ip}=0$$

$$u_r\geq 0,\; r=1,\,\ldots,\,s$$

$$v_i\geq 0,\; i=1,\,\ldots,\,m$$

The second stage is repeated for each DMU_p; $p=1,\,\ldots,\,n$. The weights $u_{r,p}$ and $v_{i,p}$ obtained from *model (5)* are used in computing the score $\theta_{p,j}$ for DMU_j through equality (4). Following the computation of all the cross evaluation scores, the cross efficiency score for DMU_k is derived using the method proposed by Anderson et al. [3] as follows:

$$CE_k=\frac{\sum_{j=1}^{n}\theta_{j,k}}{n}. \qquad (6)$$

4. APPROXIMATING THE INPUT AND OUTPUT COMPONENTS TO WEIGHTED INPUT AND OUTPUT SUM

It is known that the optimal weights in the classical DEA, especially the optimal weights obtained for the efficient units, are multiple optimal. In addition, in the case wherein the values of an output (input) variable are greater than the values of other output (input) variables, the weight assigned to this output (input) generally becomes zero or very near zero. In this way, a variable that is capable of affecting the performance of a DMU makes no (ability of) contribution to the efficiency of the DMU under evaluation. This drawback can be eliminated by giving importance to each input and each output in proportion to their respective greatness.

The proposed approach is aimed at approximating each weighted output (input) component to weighted output (input) sum in order to contribute to the efficiency account of each output component in proportion to the output (input) values, i.e., to the extent of their greatness or smallness. It is also aimed at obtaining weights that are more appropriate when compared to those obtained by the classical DEA model. In the second stage of cross evaluation, a model is presented (8) in which the classical DEA efficiency scores for each unit are preserved and more appropriate optimal weight values are selected for the units for which the optimal weights obtained by classical DEA in the first stage possibly have multiple and inappropriate solutions.

$$\min\; w_p=z_p$$

s.t.

$$\sum_{r=1}^{s}u_r y_{rp}=\theta_p^*$$

$$\sum_{i=1}^{m}v_i x_{ip}=1 \qquad (8)$$

$$\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}\leq 0,\; j=1,\;2,\;\ldots,\;n$$

$$\sum_{r=1}^{s}u_r y_{rp}-u_r y_{rp}\leq z_p,\; r=1,2,\;\ldots,\;s$$

$$\sum_{i=1}^{m}v_i x_{ip}-v_i x_{ip}\leq z_p,\; i=1,2,\;\ldots,\;m$$

$$u_r\geq 0,\qquad r=1,2,\;\ldots,\;s$$

$$v_i\geq 0,\qquad i=1,2,\;\ldots,\;m$$

$$z_p\geq 0$$

where, θ_p^* is the efficiency value for DMU_p obtained from the classical DEA given in (3). The z_p variable in the model symbolizes the maximum deviation from the weighted output sum and the weighted input sum for the output component and the input component, respectively.

The new approach is similar to the Cooper et al. model [8, 9]. Cooper et al. [8, 9] proposed a two-step procedure that could be used for the selection of the weights from the alternative optimal solution set. The two-step procedure proposed for the selection of weights is based on two general criteria of selection and is implemented by means of two mixed integer linear programming problems. In this procedure, based on the weights chosen in the first step, those weights are selected in the second step which maximize the relative value of the variable with the minimum value for the corresponding "virtual" input or output that is represented by $v_i x_{ip}$ and $u_r y_{rp}$. In the second step of the Cooper et al. model, those weights are determined that have associated programs of performance in which the inputs and outputs globally maximize their relative "importance". For a detailed discussion about the relative importance, consider the following references [8, 9, 10, 18].

5. A REAL-WORLD APPLICATION: *EFFICIENCY EVALUATION OF TURKEY CITIES*

The ranking values of the proposed *model (8)*, super efficiency method and the aggressive cross efficiency were evaluated using a real data set relative to the efficiency of 81 Turkey cities. The set extracted from [19] characterizes each country by four inputs and seventeen outputs as illustrated in *Appendix A*. The output and input variables are provided in Table 1.

The classical DEA (CCR model) efficiency scores, super efficiency scores, aggressive cross efficiency scores, and cross efficiency scores obtained using *model (8)*, as well as the rank values of cities (DMUs) obtained using these models for each DMU are given in Table 2.

Owing to the structure of objective functions for the proposed models, normalized data obtained by dividing each input-output with their highest value are used.

The results are shown in Table 2. In the table, column 3 displays the rank of cities obtained from SPO (State Planning Organization) by means of some multivariate statistical methods. The SPO is headed by an undersecretary in Turkey. It is comprised of eight departments: economic planning, social planning, coordination, and priority regional development, relations with the European Union, credit allocation, foreign capital investment, and evaluation of yearly programs. The main duties of the SPO are to advise the government in determining economic, social, and cultural policies, as well as targets of the country and to prepare development plans and annual programs that conform to the targets determined by the government. From the socio-economic, cultural, industry, tourism, and trade points of view, the most prominent cities in Turkey are İstanbul, Ankara, İzmir, Bursa, Kocaeli, Eskişehir, Yalova, Adana, and Antalya. The SPO calculations have assigned the best ranking values to these cities.

Table 1. The output and input variables.

Outputs:
y_1 : Urbanization rate, 2003
y_2 : The ratio of employment in agriculture sector to total employment, 2003
y_3 : The ratio of employment in industry sector to total employment, 2003
y_4 : The ratio of employment in trade sector to total employment, 2003
y_5 : The ratio of paid workers to total employment, 2003
y_6 : The ratio of paid women workers to total employment, 2003
y_7 : The ratio of employers to total employment, 2003
y_8 : The ratio of literate population, 2003
y_9 : Number of doctors per ten thousand people, 2003
y_{10} : Number of hospital beds per ten thousand people, 2003
y_{11} : Agricultural production value, 2003
y_{12} : The ratio of gross domestic product of country to total gross domestic product, 2003
y_{13} : Gross domestic product in per capita, 2003
y_{14} : Number of banks, 2003
y_{15} : The total population receiving adequate drinking water, 2003
y_{16} : Number of private cars per ten thousand people, 2003
y_{17} : Electricity consumption amount per capita, 2003
Inputs:
x_1 : Infant mortality, 2003
x_2 : Municipal expenditures per capita, 2000
x_3 : Public investment amount per capita, 2000
x_4 : Number of people having free health card, 2003

Table 2. Turkey cities results.

				CCR Super Efficiency			CCR Aggressive Cross Efficiency			*Proposed Model (8)*		
No	Cities (DMUs)	Rank based SPO	CCR Eff.	Eff.	Rank	diff	Eff.	Rank	diff	Eff.	Rank	diff.
1	Adana	**8**	1	1.465	12	4	0.572	39	31	0.701	5	3
2	Adıyaman	**65**	1	1.131	41	24	0.639	21	44	0.248	63	2
3	Afyon	**44**	0.917	0.917	67	23	0.562	43	1	0.345	42	2
4	Ağrı	**80**	1	1.385	16	64	0.560	44	36	0.152	80	0
5	Amasya	**39**	0.814	0.814	78	39	0.507	58	19	0.401	37	2
6	Ankara	**2**	1	1.727	5	3	0.541	48	46	0.766	3	1
7	Antalya	**10**	1	1.534	11	1	0.597	36	26	0.685	6	4
8	Artvin	**43**	1	1.433	14	29	0.626	26	17	0.342	43	0
9	Aydın	**22**	1	1.081	48	26	0.620	30	8	0.518	23	1
10	Balıkesir	**15**	1	1.195	30	15	0.633	25	10	0.618	12	3
11	Bilecik	**18**	0.942	0.942	66	48	0.457	70	52	0.584	16	2
12	Bingöl	**76**	0.903	0.903	69	7	0.291	81	5	0.205	75	1
13	Bitlis	**79**	1	1.031	54	25	0.563	42	37	0.158	79	0
14	Bolu	**14**	1	1.815	4	10	0.764	2	12	0.625	11	3
15	Burdur	**31**	1	1.237	26	5	0.703	4	27	0.457	28	3
16	Bursa	**5**	1	1.177	34	29	0.467	64	59	0.661	8	3
17	Çanakkale	**24**	1	1.002	58	34	0.566	41	17	0.514	24	0
18	Çankırı	**59**	1	1.191	32	27	0.672	13	46	0.284	58	1
19	Çorum	**46**	0.847	0.847	75	29	0.465	67	21	0.334	45	1
20	Denizli	**12**	1	1.083	47	35	0.614	31	19	0.611	13	1
21	Diyarbakır	**63**	0.648	0.648	80	17	0.347	80	17	0.241	64	1
22	Edirne	**16**	1	1.192	31	15	0.635	22	6	0.562	19	3
23	Elazığ	**36**	1	1.306	22	14	0.694	7	29	0.415	34	2
24	Erzincan	**58**	0.996	0.996	62	4	0.472	63	5	0.281	59	1
25	Erzurum	**60**	1	1.136	39	21	0.507	59	1	0.276	60	0
26	Eskişehir	**6**	1	1.019	57	51	0.544	46	40	0.714	4	2
27	Gaziantep	**20**	1	1.673	6	14	0.654	18	2	0.571	17	3
28	Giresun	**50**	0.998	0.998	60	10	0.593	37	13	0.321	49	1
29	Gümüşhane	**71**	1	1.198	29	42	0.550	45	26	0.225	69	2
30	Hakkari	**77**	0.986	0.986	64	13	0.449	71	6	0.201	76	1
31	Hatay	**29**	1	1.092	46	17	0.719	3	26	0.437	31	2
32	Isparta	**28**	1	1.434	13	15	0.634	24	4	0.469	27	1
33	Mersin	**17**	0.830	0.830	77	60	0.474	62	45	0.511	25	8
34	İstanbul	**1**	1	4.383	1	0	0.703	5	4	0.852	1	0
35	İzmir	**3**	1	1.366	17	14	0.678	12	9	0.801	2	1
36	Kars	**67**	0.771	0.771	79	12	0.403	77	10	0.221	70	3
37	Kastamonu	**51**	0.999	0.999	59	8	0.528	55	4	0.318	50	1
38	Kayseri	**19**	1	1.212	27	8	0.666	16	3	0.566	18	1
39	Kırklareli	**11**	1	1.140	38	27	0.684	10	1	0.602	14	3
40	Kırşehir	**42**	1	1.030	55	13	0.568	40	2	0.333	46	4
41	Kocaeli	**4**	1	1.408	15	11	0.383	78	74	0.671	7	3

42	Konya	**26**	1	1.543	10	16	0.681	11	15	0.521	22	4
43	Kütahya	**40**	0.988	0.988	63	23	0.601	34	6	0.352	41	1
44	Malatya	**41**	1	1.021	56	15	0.610	32	9	0.366	40	1
45	Manisa	**25**	1	1.624	7	18	0.813	1	24	0.529	21	4
46	K.Maraş	**48**	1	1.067	51	3	0.535	51	3	0.324	48	0
47	Mardin	**72**	1	2.526	2	70	0.640	20	52	0.217	71	1
48	Muğla	**13**	1	1.053	52	39	0.427	74	61	0.594	15	2
49	Muş	**81**	1	1.164	35	46	0.467	65	16	0.145	81	0
50	Nevşehir	**34**	0.858	0.858	73	39	0.491	60	26	0.429	33	1
51	Niğde	**49**	0.877	0.877	70	21	0.534	52	3	0.291	57	8
52	Ordu	**62**	1	1.073	49	13	0.621	28	34	0.262	62	0
53	Rize	**37**	1	1.124	42	5	0.660	17	20	0.409	35	2
54	Sakarya	**23**	0.916	0.916	68	45	0.510	57	34	0.541	20	3
55	Samsun	**32**	0.852	0.852	74	42	0.478	61	29	0.434	32	0
56	Siirt	**73**	1	1.561	9	64	0.598	35	38	0.209	74	1
57	Sinop	**57**	0.627	0.627	81	24	0.370	79	22	0.294	56	1
58	Sivas	**53**	0.997	0.997	61	8	0.439	73	20	0.301	55	2
59	Tekirdağ	**7**	1	1.351	19	12	0.670	14	7	0.659	9	2
60	Tokat	**61**	1	1.135	40	21	0.649	19	42	0.271	61	0
61	Trabzon	**38**	1	1.184	33	5	0.692	9	29	0.398	38	0
62	Tunceli	**52**	1	1.101	45	7	0.423	75	23	0.302	54	2
63	Şanlıurfa	**68**	1	1.238	25	43	0.539	49	19	0.234	66	2
64	Uşak	**30**	1	1.310	20	10	0.692	8	22	0.404	36	6
65	Van	**75**	1	1.207	28	47	0.538	50	25	0.198	77	2
66	Yozgat	**64**	1	1.117	43	21	0.623	27	37	0.237	65	1
67	Zonguldak	**21**	1	1.037	53	32	0.519	56	35	0.444	29	8
68	Aksaray	**56**	1	2.199	3	53	0.669	15	41	0.307	53	3
69	Bayburt	**66**	0.874	0.874	71	5	0.462	69	3	0.231	67	1
70	Karaman	**35**	0.865	0.865	72	37	0.533	54	19	0.379	39	4
71	Kırıkkale	**33**	1	1.104	44	11	0.462	68	35	0.441	30	3
72	Batman	**70**	1	1.157	36	34	0.586	38	32	0.214	72	2
73	Şırnak	**78**	1	1.359	18	60	0.542	47	31	0.197	78	0
74	Bartın	**55**	0.971	0.971	65	10	0.533	53	2	0.309	52	3
75	Ardahan	**74**	1	1.073	50	24	0.448	72	2	0.211	73	1
76	Iğdır	**69**	1	1.148	37	32	0.413	76	7	0.228	68	1
77	Yalova	**9**	1	1.573	8	1	0.602	33	24	0.643	10	1
78	Karabük	**27**	1	1.287	23	4	0.621	29	2	0.509	26	1
79	Kilis	**54**	1	1.265	24	30	0.634	23	31	0.315	51	3
80	Osmaniye	**47**	1	1.308	21	26	0.695	6	41	0.328	47	0
81	Düzce	**45**	0.833	0.833	76	31	0.465	66	21	0.341	44	1
				Sum difference		***1910***	Sum difference		***1802***	Sum difference		***154***

In Table 2, the efficiency, the DMU_p, and the CCR Eff are displayed in column 4. In columns 5, 6, and 7, the CCR super efficiency, the rank value, and the absolute differences from rank based SPO are respectively reported. In columns 8, 9, and 10, the CCR aggressive cross efficiency, rank value, and the absolute differences from rank based SPO are reported, respectively. In columns 11, 12, and 13, the cross efficiency value obtained from proposed *model (8)* efficiency, the rank value, and the absolute differences from rank based SPO are reported, respectively.

According to the ratings of proposed *model (8)*, İstanbul, İzmir, Ankara, Adana, Antalya, Bursa, Yalova, Eskişehir and Kocaeli are considered to be the best cities of Turkey. However, the super efficiency and aggressive cross efficiency methods do not rate all of these cities as the best ones.

At the bottom of column 7, the *1910* value indicates the sum of difference between super efficiency ranking and SPO ranking. For the aggressive cross efficiency method and proposed *model (8),* this difference is *1802* and *154,* respectively. It was found that the ranking values obtained from proposed *model (8)* is closer to the SPO ranking value when compared with the super efficiency and aggressive cross efficiency ranking values. Thus, it was concluded that the rankings of the new *model (8)* is more compatible with SPO ranking than with the super efficiency and aggressive cross efficiency rankings.

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APPENDIX A: Data of 81 Turkey cities.

No	Cities	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	x_1	x_2	x_3	x_4
1	**Adana**	75.58	43.09	14.41	11.9	48.53	10.16	2.71	86.88	14	24	2.38	3.05	2057	190	91.3	761	1.6	44	144	73	17
2	**Adıyaman**	54.33	73.64	4.8	4.25	23.17	4.31	1.07	79.83	5	11	0.77	0.39	780	22	81.51	216	0.8	42	34	93	36
3	**Afyon**	45.77	70.11	6.41	5.13	24.39	3.21	1.28	88.26	8	24	1.86	0.71	1081	72	95.12	368	0.8	45	69	92	17
4	**Ağrı**	47.72	73.44	1.46	3.48	22.75	1.47	0.7	67.95	2	5	0.88	0.22	515	18	63.11	83	0.4	58	20	55	25
5	**Amasya**	53.83	61.47	5.8	5.64	33	4.5	1.01	87.39	7	13	1.03	0.38	1287	31	98.26	520	0.9	47	68	96	27
6	**Ankara**	88.34	16.21	13.41	13.81	72.06	16.86	3.93	93.26	32	38	2.87	8.33	2588	672	98.54	1614	1.2	36	140	426	6
7	**Antalya**	54.45	49.66	5.5	18.05	41.37	9.67	3.17	91.86	13	16	3.05	2.5	1813	200	86.16	885	1.4	32	83	257	11
8	**Artvin**	43.87	60.85	5.39	4.82	34.03	4.22	1.03	86.8	1	30	0.34	0.27	1776	29	68.61	402	1	43	40	66	19
9	**Aydın**	51.87	61.95	7.54	9.16	38.15	10.71	2.85	87.44	12	17	2.47	1.4	1838	131	92.53	724	1	39	69	184	13
10	**Balıkesir**	53.66	56.75	8.63	8.93	35.82	6.63	2.19	88.34	9	24	3.77	1.52	1765	134	89.99	887	1.4	41	55	249	12
11	**Bilecik**	64.01	46.51	19.29	5.89	47.85	7.18	1.26	91.55	9	15	0.47	0.34	2204	29	97.14	556	0.7	42	58	475	11
12	**Bingöl**	48.66	69.94	1.4	2.84	26.96	1.63	0.85	73.61	5	17	0.38	0.14	673	12	74.16	96	0	60	30	1708	43
13	**Bitlis**	56.48	69.84	2.28	3.37	25.78	2.05	0.78	72.37	4	11	0.47	0.17	552	18	77.11	110	0	52	29	56	28
14	**Bolu**	52.72	56.57	11.11	7.32	37.49	6.07	1.67	89.63	11	44	1.85	0.78	3569	36	97.21	1063	2	38	47	190	10
15	**Burdur**	54.48	60.13	8.31	6.11	31.34	4.82	1.34	89.67	10	28	0.76	0.35	1714	37	96.56	801	1	33	58	99	18
16	**Bursa**	76.75	33.56	28.17	11.82	55.6	13.71	3.52	91.72	12	20	3.19	3.68	2155	238	94.4	822	3	39	125	519	7
17	**Çanakkale**	46.36	56.01	9.19	7.15	37.56	6.68	1.59	89.51	9	20	1.62	0.81	2172	67	95.32	643	2	34	59	322	8
18	**Çankırı**	52.22	66.09	5.23	4.12	28.65	3.45	1.09	88.16	7	18	0.56	0.22	1002	24	81.86	205	0	37	45	46	20
19	**Çorum**	52.24	67.61	7.35	5.69	26.27	3.49	1.37	83.11	7	26	1.27	0.69	1431	47	98.05	418	1	51	50	388	32
20	**Denizli**	48.69	53.54	18.98	7.71	39.19	11.22	2.23	89.57	12	17	1.49	1.19	1743	103	66.34	855	2	39	66	178	9
21	**Diyarbakır**	60	63.86	3.82	5.76	32.21	4.45	1.3	69.57	7	21	1.83	1.15	1056	51	51.49	123	1	57	71	333	23
22	**Edirne**	57.35	49.6	9.01	8.42	43.29	7.61	1.69	88.89	19	31	1.13	0.73	2271	54	89.75	665	1	38	57	193	15
23	**Elazığ**	63.95	58.6	6.01	5.88	35.57	3.77	1.7	82.31	14	41	0.86	0.65	1417	33	83.27	363	2	39	41	163	17
24	**Erzincan**	54.35	62.03	4.17	4.79	33.06	2.91	1	87.16	7	20	0.64	0.24	956	27	93.29	326	0	37	43	447	16
25	**Erzurum**	59.8	62.3	3.71	6.06	31.65	3.11	1.58	83.64	12	32	1.34	0.69	914	54	93.53	231	1	65	62	83	27
26	**Eskişehir**	78.9	35.31	18.93	10.5	56.63	11.62	2.59	92.94	16	41	1.15	1.2	2110	75	95.28	952	1	40	90	269	10
27	**Gaziantep**	78.52	39.13	21.28	11.43	49.35	5.56	3.33	83.78	8	20	1.22	1.36	1318	88	76.74	455	2	44	46	159	14
28	**Giresun**	54.09	70.31	5.07	5.71	22.99	4.36	1.26	83.35	6	22	0.64	0.49	1176	47	80.82	266	1	38	42	174	28
29	**Gümüşhane**	41.49	76.54	2.6	2.91	19.35	2.14	0.59	86.4	7	17	0.3	0.14	933	15	92.53	212	0	32	38	301	30
30	**Hakkari**	58.95	52.44	1.35	2.51	44.39	1.6	0.56	70.69	4	6	0.23	0.13	696	8	86.76	59	0	55	26	132	31
31	**Hatay**	46.37	61.63	8.41	6.7	32.47	5	1.69	86.02	6	12	2.15	1.52	1509	95	91.77	516	2	38	53	73	22
32	**Isparta**	58.71	56.9	8.34	5.73	37.19	5.54	1.52	92.01	14	53	1.11	0.54	1318	52	94.98	550	1	32	61	243	18
33	**Mersin**	60.51	57.64	7.93	9.33	35.7	7.93	2.29	89.16	8	17	3.29	2.75	2074	138	77.09	532	1	45	85	276	12
34	**İstanbul**	90.69	8.13	32.15	18.73	75.95	19.39	5.97	93.39	21	34	0.75	22.11	2750	2214	60	1000	2	39	139	222	5
35	**İzmir**	81.07	28.54	20.58	14.54	61.98	16.28	3.99	91.66	23	29	4.33	7.3	2696	613	94.91	986	3	40	100	272	7
36	**Kars**	43.73	68.14	2.17	3.65	27.75	2.47	1.05	82.94	5	12	0.58	0.19	719	22	82.48	188	1	65	42	174	32
37	**Kastamonu**	46.35	71.14	6.32	4.59	23.32	3.52	1.36	80.8	9	40	0.96	0.46	1525	50	79.49	559	1	44	47	329	30
38	**Kayseri**	69.06	46.98	16.42	8.53	44.09	5.69	2.96	88.89	12	23	1.23	1.22	1430	93	99.57	735	1	42	74	80	19
39	**Kırklareli**	57.6	48.2	17.81	7.46	44.31	9.18	1.78	92.88	9	22	0.84	0.72	2740	49	100	656	3	34	57	192	9

40	**Kırşehir**	58.21	65.08	5.93	5.84	28.67	3.7	1.38	87.52	9	20	0.57	0.25	1212	21	99.16	482	1	35	49	262	32
41	**Kocaeli**	59.94	39.04	20.32	9.21	52.84	8.53	2.16	92.04	11	17	0.88	4.55	4696	129	99.31	637	5	42	157	503	7
42	**Konya**	59.07	62.42	9.05	6.87	29.65	3.48	2.22	90.07	8	15	4.72	2.49	1414	149	91.92	505	1	35	73	119	14
43	**Kütahya**	48.54	66.9	9.89	4.81	27.99	2.89	1.3	89.09	6	18	1.1	0.74	1411	55	98.49	636	1	40	50	161	16
44	**Malatya**	58.54	63.93	6.24	6.25	29.63	4.41	1.43	85.35	11	16	1.21	0.8	1163	46	84.85	326	1	35	45	212	12
45	**Manisa**	56.72	61.54	11.9	7.06	35.91	8.39	1.56	86.27	10	22	3.22	2.09	2062	138	87.18	618	1	41	50	80	13
46	**K.Maraş**	53.47	65.63	9.84	5.35	29.42	3.76	1.28	83.42	6	10	1.44	0.98	1215	48	82.6	303	1	37	40	398	23
47	**Mardin**	55.49	69.92	2.37	4.05	24.81	2.23	0.73	71.2	3	7	0.74	0.41	718	30	84.3	100	1	43	45	17	30
48	**Muğla**	37.51	55.02	5.95	13.73	36.67	7.35	2.82	92.72	11	19	2.03	1.53	2659	134	83.14	952	2	35	91	680	11
49	**Muş**	35.16	83.44	1.54	1.86	14.37	1.33	0.48	69.44	3	8	0.81	0.16	453	14	82.11	71	0	55	23	288	23
50	**Nevşehir**	44.05	70.25	4.97	6.77	27.6	5.6	1.6	88.41	9	18	1.07	0.45	1823	36	100	518	1	43	79	172	16
51	**Niğde**	36.43	73.49	5.27	4.99	23.15	4.45	1.1	86.23	8	18	1.38	0.44	1565	23	99.75	343	1	47	59	132	28
52	**Ordu**	46.93	73.55	4.22	5.26	20.42	3.81	1.23	83.1	6	17	1.35	0.61	862	54	76.44	286	1	37	46	112	32
53	**Rize**	56.09	64.34	9.44	6.03	28.9	4.07	1.66	87.66	7	21	0.53	0.45	1531	37	76.38	375	1	32	43	163	8
54	**Sakarya**	60.81	51.93	12.89	9.06	38.72	5.92	2.62	90.85	7	18	2.05	1.11	1825	61	97.1	618	1	42	100	134	7
55	**Samsun**	52.54	63.37	6.86	7.87	28.94	5.5	1.98	86.21	14	27	2.66	1.41	1452	95	78.41	527	1	48	72	305	32
56	**Siirt**	58.22	56.87	2.58	4.04	39.33	2.52	0.62	68.66	4	11	0.31	0.19	880	12	80.63	119	1	63	21	35	31
57	**Sinop**	44.9	71.05	5.33	4.58	23.59	4.37	1.16	82.72	8	24	0.45	0.22	1189	24	90.33	455	1	57	74	313	37
58	**Sivas**	55.86	66.46	5.43	5.09	28.46	3.31	1.31	85.4	11	30	1.42	0.67	1098	57	96.17	303	0.7	53	42	682	21
59	**Tekirdağ**	63.4	38.77	26.22	8.57	52.76	12.39	2.09	93.01	10	17	1.26	1.07	2134	84	97.4	457	4	39	61	184	6
60	**Tokat**	48.52	74.03	4.91	4.38	20.52	2.62	0.95	85.67	5	16	1.44	0.74	1107	45	99.94	329	0	45	52	40	24
61	**Trabzon**	49.12	64.31	5.33	7.1	27.64	4.81	1.97	88.49	11	26	0.93	0.95	1208	90	92.27	313	1	31	48	154	17
62	**Tunceli**	58.21	42.3	1.81	2.44	54.83	3.59	0.43	82.99	9	19	0.21	0.1	1270	16	76.89	101	1	36	74	115	31
63	**Şanlıurfa**	58.34	72.8	3.47	5.24	24.3	2.75	1.08	67.67	4	9	2.36	0.93	805	50	86.38	206	1	37	37	421	20
64	**Uşak**	56.48	60.15	14.91	6.39	31.67	5.39	2.24	87.54	9	19	0.75	0.33	1282	38	69.21	692	2	42	56	64	10
65	**Van**	50.94	67.17	2.46	4.34	27.85	2.14	0.89	68.05	6	14	1.17	0.49	695	29	71.32	162	0	61	24	104	29
66	**Yozgat**	46.15	77.31	3.7	3.81	19.75	2.54	0.81	86.17	5	13	1.29	0.43	781	47	95.69	192	1	45	36	104	27
67	**Zonguldak**	40.66	59.05	15.36	6.08	35.03	5.49	1.25	87.81	10	28	0.48	1.18	2380	70	55.67	878	4	45	78	175	10
68	**Aksaray**	50.55	69.97	5.67	6.21	23.61	2.89	1.75	86.34	9	14	0.95	0.29	900	26	94.36	392	6	48	51	74	26
69	**Bayburt**	42.48	74.68	2.27	3.37	20.7	1.5	1.48	86.49	7	10	0.17	0.06	825	9	98.08	259	0.4	44	51	196	30
70	**Karaman**	57.53	65.01	11.14	5.39	28.76	5.24	1.45	89.72	9	12	1.21	0.34	1752	18	92.44	544	1	48	53	195	22
71	**Kırıkkale**	74.39	51.22	10.41	7.16	41.84	4.09	1.58	89.12	12	16	0.41	0.66	2140	25	73.8	323	1	34	73	309	14
72	**Batman**	66.6	63.48	5.97	5.46	31.07	3.03	1.38	70.96	4	5	0.41	0.35	949	14	58.06	111	0.7	50	27	65	22
73	**Şırnak**	59.83	46.56	1.85	3.71	48.36	1.88	0.73	65.75	3	6	0.18	0.15	518	18	76.65	51	1	51	21	63	19
74	**Bartın**	26.06	71.27	7.9	4.63	23.7	3.79	1.15	84.03	9	17	0.22	0.13	855	20	77.55	465	0.8	42	45	167	19
75	**Ardahan**	29.7	77.79	1.06	2.48	19.4	1.93	0.66	84.6	6	12	0.44	0.07	671	12	64.97	103	0.3	77	26	77	31
76	**Iğdır**	48.38	68.37	2.2	4.61	26.42	2.97	1.28	75.46	6	6	0.32	0.1	729	13	61.85	141	0	52	25	464	26
77	**Yalova**	58.52	38.5	13.71	10.81	51.27	10.28	2.73	92.93	11	12	0.17	0.39	2910	21	78.74	475	4	43	76	84	12
78	**Karabük**	70.08	41.85	18.71	8.03	49.98	7.32	1.95	86.92	9	26	0.18	0.25	1409	27	78.9	319	2	38	73	77	14
79	**Kilis**	65.36	54.72	7.49	7.07	37.54	3.71	1.55	80.41	10	17	0.31	0.13	1463	6	76.91	175	1	48	35	75	20
80	**Osmaniye**	68	59.99	6.86	6.96	32.59	4.4	1.42	86.02	6	7	0.59	0.36	983	24	83.96	240	0	36	42	46	8
81	**Düzce**	41.57	57.38	12.28	7.17	35.34	6.41	1.92	89.44	6	17	0.56	0.26	1025	23	96.75	154	1	50	58	194	16