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ORIGINAL ARTICLE

Using Seismic Isolation Elements to Protect Cylindrical Steel Liquid Storage Tanks from Destructive Forces of Earthquakes

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ABSTRACT
This study compares various seismic isolation methods used to reduce the damage caused by impulsive and oscillating impacts on liquid storage tanks during earthquakes. In the research seven models were created, for the reference model no seismic isolation element was used, three models used different rubber bearings of varying hardness as the seismic isolation element and three models used dampers together with rubber bearings of differing hardness. Implementing a system of seismic isolation is very important for both the economy of a country and the environment of countries that are located in zones of high seismic risk. The Nonlinear time history analyses of the models were performed in accordance with the data from the, El Centro (USA), Northridge (USA), and Izmit (Turkey) earthquakes. The way in which the models reacted to the impact of seismic resonance differed according to the elements used. The comparison of the results show that using rubber bearings and dampers can protect the tank systems from the destructive effects of earthquakes.

Key words: Cylindrical steel liquid storage tank; Seismic isolation; Dynamic analysis; Time history analysis, Rubber bearings, Dampers.

1. INTRODUCTION

There has always been a need for human beings to store essential commodities such as water and food, however over the centuries, as social life and industry have developed so have the storage needs of society. During the First World War, steel and reinforced concrete were used and later prestressed concrete was added to the available structural technologies for constructing storage containers. Then, steel tanks began to be used since they were easy to dismount and reassemble [1]. More recently, mixed systems have started to be employed. These tanks are used, for example, for to store dangerous liquids which could cause serious injury to people and livestock or damage the natural environment if the tank wall was breached resulting in spillage. After the major earthquake in 1999 in Turkey, the resulting fire at the Tupras refinery in Izmit clearly demonstrates the need to ensure that liquid storage tanks, in particular those containing fuel, are sufficiently isolated from the damaging effects of seismic activity [2]. This can be achieved by removing the direct contact between the tank superstructure and the ground [3].

In this study, the oscillating and impulsive impacts on the liquid storage tanks during earthquakes were replicated on seven models. In the research, for the reference model no seismic isolation element was used, three models used different rubber bearings of varying hardness as the seismic isolation element and three models used dampers together with rubber bearings of

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differing hardness. The nonlinear time history analyses of the models were performed in accordance with the data from the El Centro (USA), Northridge (USA), and Izmit (Turkey) earthquakes. The impulsive and oscillation effects, acceleration records, recorded in all three earthquakes were used in the nonlinear analyses. Ground–structure interaction was ignored but the liquid–structure interaction was taken into account in the analyses of the models.

As a result of the analyses of the models using different seismic isolation elements, a comparison was made between the shear force, overturning moments of models; accelerations, vibrations, and the relative displacement of the roof of the models. This comparison should indicate which seismic isolation element(s) can prevent the vibrations, which create the resonance effect, from reaching a liquid fuel tank.

2. CYLINDRICAL LIQUID STORAGE TANKS

Considering the construction of liquid storage tanks both structural and hydraulic issues need to be taken into account. The main research topics for cylindrical liquid storage tanks are the tank- liquid, tank- ground interactions and the oscillating and impulsive effects.

Cylindrical tanks are used to store a wide range of harmless liquids however, they are also used to store dangerous liquids such as petrol, liquified petroleum gas (LPG) and liquified natural gas (LNG). Controlling a fire in these structures is difficult and they can also damage other buildings around them. In the Izmit earthquake, the fire that broke out in one of the fuel tanks at the Tupras refinery spread to the other tanks and resulted in serious financial losses [4]. The damage to the fuel tanks at the Tupras refinery is shown in Figure 1.

Figure 1. Naptha fuel storage tanks damaged by fire resulting from the earthquake at theTüpraş Refinery, Izmit

In the literature, there are many studies related to the calculation and construction techniques of the liquid storage tanks. The first studies were undertaken in the second half of the 19th century followed by work in the 20th century with the main researchers being Guthrie, Rayleigh and Nikolai [5, 6]. The first study about the hydrodynamic compressions caused by oscillating and impulsive effects was performed by Westergaard and published in 1931 [7]. After this many more studies were performed to detect hydrodynamic pressure by the determination of various acceptance differences [8]. In addition to the impulsive effects Housner took the oscillating effects into consideration [9].

Veletsos performed a study which analyzed both impulse and oscillation effects related to hydrodynamic pressures [10]. Westergard took an approach concerning the structure–liquid and structure–ground interactions [7].

In this study; the radius of the model is 10 m with a height of 9.6 m. The tank is anchored at 10 points in a radial direction with horizontal viscous–elastic damper element. In the selection of the 10 points, the experimental points of the measurement devices used by Housner [9] in his vibration tests were taken into consideration. The roof and base of the tank was made of sheet steel material therefore, there will be a friction effect [11]. In order to clearly see the deformations to the model, the tank height was divided into 10 sections.

3. SEISMIC ISOLATION

Since earthquakes cannot be prevented, methods must be developed to reduce the damage they cause [12].

This can be achieved by ensuring that vulnerable structures have increased damping or that the displacement caused by earthquakes is reduced.

3.1. Rubber Bearings

The vibration of a structure constructed using base isolation is different from the vibration in the ground to which it is fixed. The vibration of the structure moves away from the natural vibration. In this way, the forces affecting the structure are reduced. The most commonly used seismic isolation method for base isolation is rubber bearings which were used for the first time in Skopje in the construction of a primary school and new approaches were brought to base isolation technology [13, 18]. In 1973 a system was developed in the USA at the National Research Center and a roll of rubber of a few millimeters was cut into circles and steel plates of a few millimeters thickness were placed between them [19, 20]. The lead seed rubber system also resembles the layered rubber system. There are additional advantages in that the lead seed inside the rubber has a high absorption capacity thus provides additional damping and the lead seeds reduce the material fatigue due to the recurrent loadings that result from earthquake movements. In the structures isolated by using rubber bearings, some problems are encountered in returning the structure to the initial location after many displacements. In order to overcome this problem, damping apparatus is used with the rubber bearings.

3.2. Damping apparatus

The aim of the damping apparatus is to reduce the destructive energy affecting the structure due to dynamic loads such as wind and earthquake. There are various systems designed to increase the damping of the structure and reduce relative displacements. The best known examples are viscous–liquid dampers based on the principle of liquid with high viscosity passing through a pipe which was proposed in 1992 by Constantinou [15]. According to the working principles, adjusted mass dampers that provide the horizontal control of the structure and adjusted liquid dampers are commonly used especially in high structures. The main principle of the adjusted liquid damper is based on the absorption of the kinetic energy of the structure in the same way as the adjusted mass dampers.

3.3. Dynamic Analysis Approaches for Seismic Isolation Applications

In liquid storage tanks, rubber bearings and dampers are among the most important methods of achieving seismic isolation, these systems extend the vibration of the tank and therefore, the resonance interval is reduced. Another advantage of this system is that it provides a mechanism for energy damping.

The structure model considered for seismic isolation of single freedom degree systems is given in Figure 2.

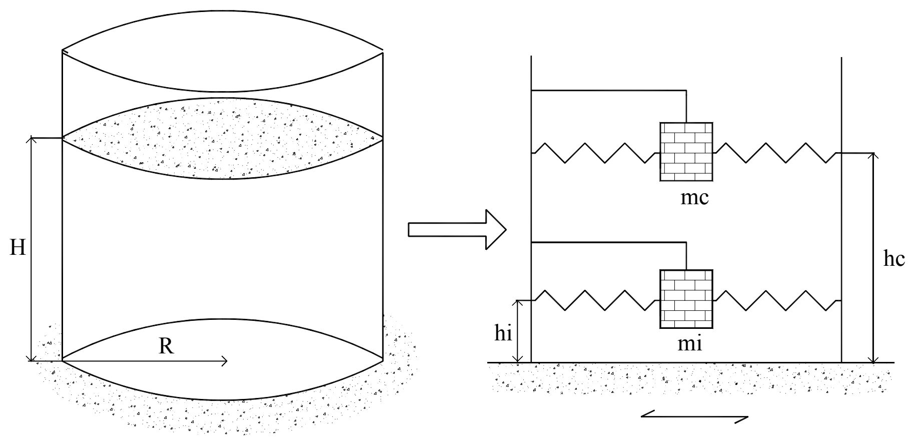

Figure 2. Equivalent of full of liquid storage tank as modeled single freedom degree.

The mode number of the models in which seismic isolation has been applied is one more than the mode number of models to which an isolator has not been applied. Since a displacement at the isolator placed under the structure occurs in addition to the displacements of the structure, the displacements should be removed from the displacements of the structure [21-26].

In this study, rubber bearings and dampers were used. The rubber bearings used in the system under analysis, showed nonlinear behavior. For the dynamic analysis of the systems showing nonlinear behavior, some dynamic modeling approaches were used. In the structures such as liquid storage tanks, using nonlinear dynamic analysis is inevitable. In the analysis the impulsive and oscillation effects were considered in these tanks and the models were analyzed using the SAP 2000 Nonlinear 9.0.3 software [27]. The program also allows the viscous damper to be defined as a link element. In this study shell and NLlink elements were used to model the liquid storage tanks with SAP 2000 program.

In the SAP 2000 program, a nonlinear dynamic analysis application is performed using “Rapid Nonlinear Analysis Method” developed by Wilson [28]. The force–deformation relationship of the rubber bearings are given in Equations 8 and 9. Here, k_2 and k_3: elastic spring constants, $yield_2$ and $yield_3$: yielding forces, $ratio_2$ and $ratio_3$: proportion of the last yielding stiffness to elastic stiffness, z_2 and z_3: hysteric variables.

$$f_{u2} = ratio_1 k_2 d_{u2} + (1 - ratio_2) yield_2 z_2 \qquad (8)$$

$$f_{u3} = ratio_3 k_3 d_{u3} + (1 - ratio_3) yield_3 z_3 \qquad (9)$$

3.4. Formulation of Models

In this study an investigation into the impulsive and oscillating effects on liquid storage tanks was undertaken using three different rigidity of rubber bearings; soft, medium-hard and hard, in addition, dampers in a radial direction were used together with each rubber bearing. The results obtained from the nonlinear time history analysis of these tank models were compared to the results obtained from the reference tank model fixed to the ground. The El Centro (USA), Northridge (USA), and Izmit (Turkey) earthquake data were separately applied to the tank systems. The characteristic features of the rubber bearings and dampers are given in Tables 1 and 2 [12]. The north–south components of acceleration values of the El Centro (USA), Northridge (USA), and Izmit (Turkey) earthquakes were applied in the x direction and the East–West acceleration values were applied in the y direction. The Izmit earthquake acceleration values were analyzed in 40 second time – definition interval with 0.005 seconds breaks in 8000 steps.
The El Centro and Northridge earthquake values were analyzed in 30 second time–definition intervals with 0.02 second breaks in 1500 steps..

Table 1. Properties of rubber bearings

Material Property	Vertical stiffness (t/m)	Initial stiffness (t/m)	Effect stiffness (t/m)	Yield force (t)	Stiffness ratio	Thickness (m)	Mass (t)
Soft	178.55	178.60	26.78	2.27	0.20	0.18	0.017
Medium hard	140.00	794.00	110.00	7.94	0.04	0.18	0.017
Hard	280.00	1270.00	190.00	12.70	0.055	0.18	0.017

The vertical components of the earthquakes in the z direction were not taken into consideration in the analysis as they were minimal values. In the analyses the ground–structure interaction were not taken into consideration in the model.

Table 2. Properties of Dampers

Linear properties		Nonlinear properties	
Effective stiffness	0 t/m	Stiffness	$1{,}786.\ 10^7$ t/m
Effective damping	0 k-sec/in	Damping	837,300 ton-s/m
---	----	Damping exponent	0.5

While forming a single freedom degree system model for the tank, impulsive and oscillating masses were connected to the structure as a rigid mass. The modeled tank with a fixed support together with the application of the rubber bearings and dampers is shown in Figure 3.

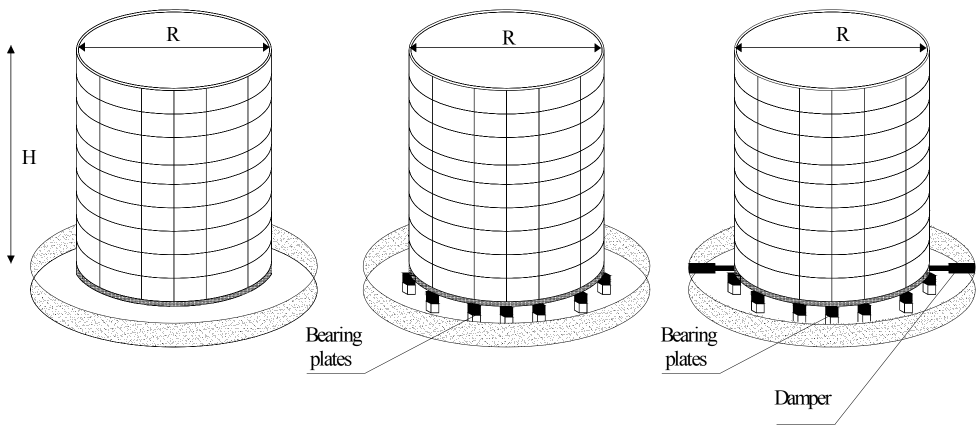

Figure 3. Reference and seismic isolated liquid storage tank models

Generally, the tanks are freely sitting on the ground, however, in order to investigate the effects of rubber bearings and dampers, the first tank model was considered to be fixed to the ground. Later, analysis was performed using high damping rubber bearings placed under the supports.

In the models viscous dampers were used in order to prevent buckles at places close to the ground and lessen the horizontal displacements of the rubber bearings. The analysis was performed using dampers at the level of 0.05xh of the tank height at 10 points in a radial direction. In the time history analysis of the model with fixed support, a usage of 85% - 98% of the modal mass was proposed. Considering the impulsive and oscillating masses, the system mode number was taken to be 5.

4. ANALYSIS RESULTS

The tank was considered to be a model fixed in the ground and the ground–structure interaction was not observed. Furthermore, impulsive and oscillation masses were placed as a concentrated mass to the tank axle. The nonlinear analyses of tanks according to the three different earthquake acceleration records, the ratio of shear force occurring at the base of tanks, the ratio of the overturning moment at the base of tanks, the ratio of acceleration and the vibration of tank models are given in Figures 4, 5, 6, and 7, respectively.

4.1. Shearing Forces Formed at the Base of the Tank

When the El Centro earthquake acceleration records were used; the shearing forces formed at the base of the models using soft, medium-hard, and hard rubber bearings were 3.5%, 6.5%, and 9% respectively of the shearing force formed at the base of the reference tank model.

For the El Centro earthquake acceleration records; the shearing forces formed at the base of the model using a soft rubber bearing and damper, a medium-hard rubber and damper, a hard rubber bearing and damper were 141%, 89.5%, and 79.5% of the shearing force of the reference tank model, respectively (Figure 4.).

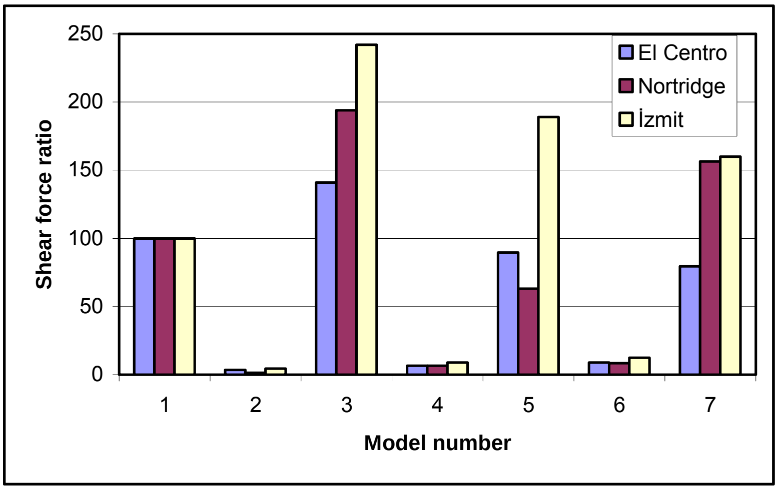

Figure 4. Ratio of shearing force occurred at base of tank models

When the Northridge earthquake acceleration records were used; the shearing forces formed at the base of the model using soft rubber, medium-hard rubber and, hard rubber were 1.5%, 6.5%, and 8.5% of the shearing force of the reference tank model, respectively.

When Northridge earthquake acceleration records were used; the shearing forces formed at the base of the model using a soft rubber bearing and damper, a medium-hard rubber and damper, a hard rubber bearing and damper were 194%, 63%, and 156.5% of the shearing force of the reference tank model, respectively (Figure 4.).

When the Izmit earthquake acceleration records were used; the shearing forces formed at the base of the model using soft rubber, medium-hard rubber, and hard rubber were 4.5%, 9%, and 12.5% of the shearing force of the reference tank model, respectively.

When the Izmit earthquake acceleration records were used; the shearing forces formed at the base of the model using a soft rubber bearing and damper, a medium-hard rubber and damper, and a hard rubber bearing and damper were 242%, 189%, and 160% of the shearing force of the reference model, respectively (Figure 4).

4.2. Overturning Moments Formed at the Base of the Tank

For the El Centro earthquake acceleration records; the overturning moments formed at the base of the model using soft, medium-hard rubber, and hard rubber were 4%, 7%, and 9% of the overturning moment of the reference tank model, respectively (Figure 5).

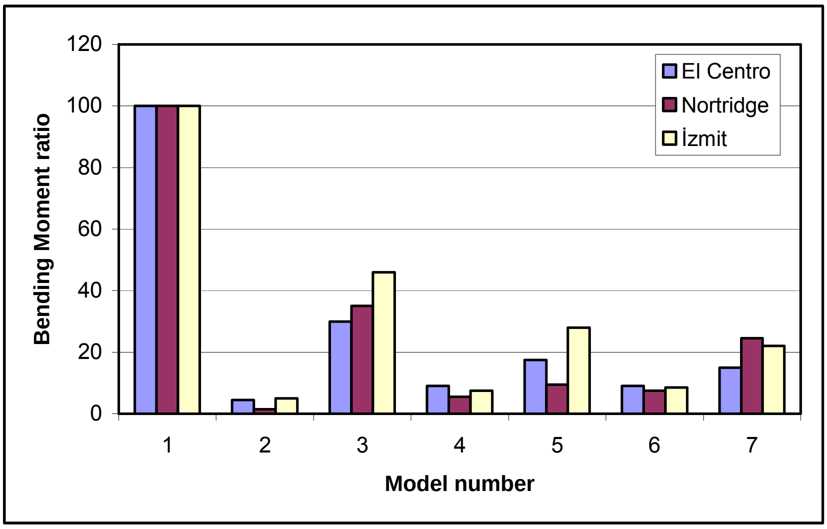

Figure 5. Ratio of moment occurred at base of tank models

When the El Centro earthquake acceleration records were used; the overturning moments formed at the base of the model using a soft rubber bearing and damper, a medium-hard rubber bearing and damper, and a hard rubber bearing and damper were 25%, 9.5%, and 16% of the overturning moment of the reference tank model, respectively.

When the Northridge earthquake acceleration records were used; the overturning moments formed at the base of the model used soft, medium-hard rubber, and hard rubber were 1.5%, 5.5%, and 7.5% of the overturning moment of the reference tank model, respectively.

When the Northridge earthquake acceleration records were used; the overturning moment formed at the base of the model used a soft rubber bearing and damper, a medium-hard rubber bearing and damper, and a hard rubber bearing and damper were 35%, 9.5%, and 24.5% of the overturning moment of the reference tank model, respectively.

When the Izmit Earthquake acceleration records were used; the overturning moments formed at the base of the model used a soft rubber, a medium-hard rubber, and a hard rubber were 5%, 7.5%, and 8.5% of the overturning moment of the reference tank model, respectively.

When the Izmit earthquake acceleration records were used; the overturning moments formed at the base of the model used a soft rubber bearing and damper, a medium-hard rubber bearing and damper, and a hard rubber bearing and damper were 46%, 28%, and 22% of the overturning moment of the reference tank model, respectively (Figure 5.).

4.3. Acceleration Formed at the Roof of the Tank

When the El Centro earthquake acceleration records were used; the accelerations formed at the roof of the model used a soft rubber bearing, a medium-hard rubber bearing, and a hard rubber bearing were 42%, 44%, and 45% of the acceleration of the reference tank model, respectively.

When the El Centro earthquake acceleration records were used; the accelerations formed at the roof of the model used a soft rubber bearing and damper, a medium-hard rubber bearing and damper, and a hard rubber bearing and damper were 153%, 86%, and 83% of the overturning moment of the reference tank model, respectively (Figure 6.)

When the Northridge earthquake acceleration records were used; the accelerations formed at the roof of the model used a soft rubber bearing, a medium-hard rubber bearing, and a hard rubber bearing were 43%, 48%, and 52% of the overturning moment of the reference tank model, respectively.

When the Northridge earthquake acceleration records were used; the accelerations formed at the roof of the model used a soft rubber bearing and damper, a medium-hard rubber bearing and damper, and a hard rubber bearing and damper were 109%, 65%, and 104% of the overturning moment of the reference tank model, respectively (Figure 6.)

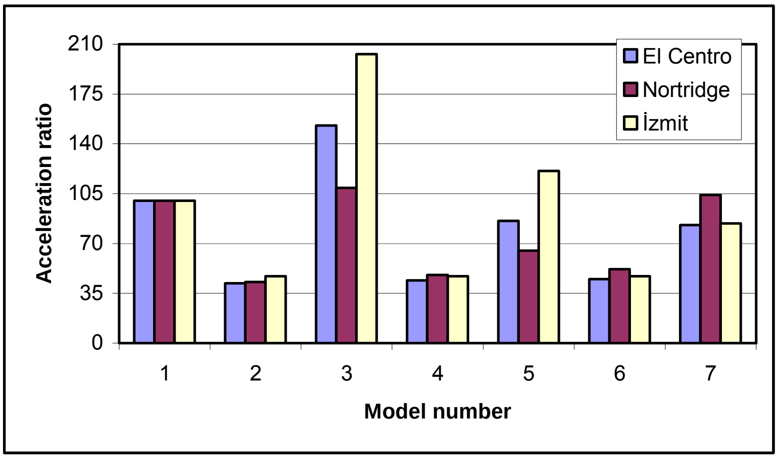

Figure 6. Ratio of acceleration occurred at tankmodels

When the Izmit earthquake acceleration records were used; the accelerations of the roof of the model used a soft rubber bearing and damper, medium-hard rubber bearing and damper, and hard rubber bearing and damper were 47%, 47%, and 47% of the overturning moment of the reference tank model, respectively.
When the Izmit earthquake acceleration records were used; the accelerations of the roof of the model used a soft rubber bearing, medium-hard rubber bearing, and hard rubber bearing were 203%, 121%, and 84% of the overturning moment of the reference tank model, respectively (Figure 6.)

4.4. Vibrations of Tank Models

In the analyses of the tank models with over 90% mass participation rate according to the data from the El Centro (USA), Northridge (USA), and Izmit (Turkey) earthquakes, the vibration of the models isolated with a soft rubber bearing, a medium-hard rubber bearing, a medium-hard rubber bearing and damper, a hard rubber bearing were 8.7, 3.9, 4.5, and 3.62 seconds, respectively. As seen from the results of the analysis, the damper had no effect on the models that used soft and hard rubber bearings. For the model that used the medium-hard rubber bearing together with a damper the vibration of the model was extended by 0.56 seconds (Figure 7).

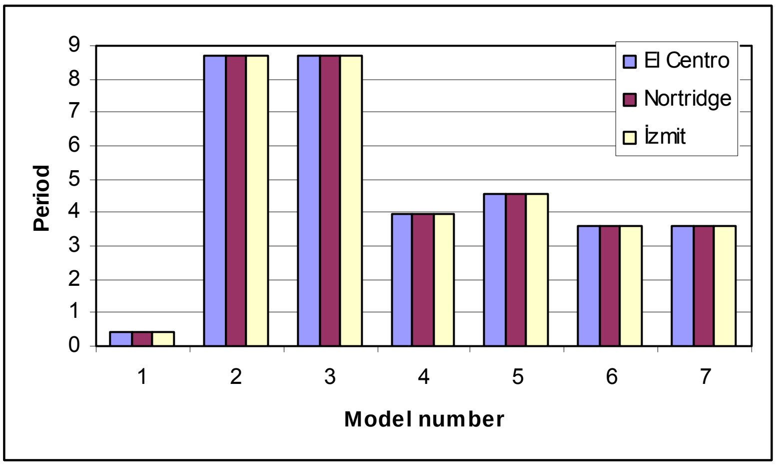

Figure 7. Vibrations of liquid storage tanks

4.5. Maximum Displacement of Tank Models

When the tanks modeled using rubber bearings of different stiffness were analyzed according to the data from the El Centro (USA), Northridge (USA), and Izmit (Turkey) earthquakes, the relative displacements formed at the base of the model that used a soft rubber bearing, a medium-hard rubber bearing, a hard rubber bearing were 0.0071m, 0.0111m, and 0.0166m, respectively.

When the analyses of the models in which dampers were used together with soft, medium-hard and hard rubber bearings were performed according to the data of three different earthquakes, the relative displacements formed at the base of the model that used a soft rubber bearing and damper, a medium-hard rubber bearing and damper, a hard rubber bearing damper were -0.0180 m, -0.0089 m, and -0.0069 m, respectively.

5. CONCLUSIONS

In the analyses of the models according to the acceleration records of the Izmit, El Centro and Northridge earthquakes, it was seen that ratio of shear force that occurred at the base of the model using a soft rubber bearing to the shear force that occurred at the base of reference element was at a minimum whereas, the ratio of shear force that occurred at the base of model using the same rubber bearing with a damper to the shear force that occurred at the base of reference element was at a maximum. It was seen that models with fixed dampers were subjected to more shear force than those using a rubber bearing.

The ratio of overturning moments that occurred at the base of the models using seismic isolation elements was compared with the reference element, the maximum and minimum values were obtained from the model that used a rubber bearing with a damper, and the soft rubber bearing, respectively.

The ratio of accelerations recorded in the analyses of models using different types of seismic isolation material and reference element were compared. The maximum and minimum values were obtained from the model using a rubber bearing with a damper, and only a soft rubber bearing, respectively.

The vibration of models using a seismic isolation element and the reference element were compared, models having maximum vibration were those that used only a soft rubber bearing and also those using a soft rubber bearing with a damper. It was seen that seismic isolation element(s) prevented the vibrations, which create the resonance effect, from reaching a liquid fuel tank.

The models using a rubber bearing were considered, it was seen that the amount of energy consumed by the models using the soft rubber was less than when hard rubber was used. When the base displacement values were observed, it was seen that displacements were more limited in the models using a dampers. The transmission of kinetic energy in a vertical direction to the tank was prevented by means of seismic isolation elements.

The results show that using rubber bearings and dampers can protect tank models from the destructive effects of earthquakes.

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