



Saturation Operation on the Direct Current Motor That is Used for Manipulator Control

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ABSTRACT

In this study, the saturation operation on the motor armature current has been carried out with a manipulator control algorithm. Manipulator system is controlled by state feedback plus integral error control strategy. Controller design tool in the system is pole placement method. Linearization process was performed by means of Jacobian matrices that expresses small refractions around nominal trajectory of manipulator system. The conclusions for two degree of freedom manipulator were shown with simulations by means of a computer program that makes saturation operation.

Keywords: Manipulator control, direct current motor, saturation operation, current limitation.

1. INTRODUCTION

Most desired specialties for the manipulator controller design is to obtain those the precise response, robustness, non-interaction between joints. To achieve these specialties with minimum cost is another important objective.

In most standard manipulator design researches it has seen that the cost measure is not taken into consideration, in fact the direct current motor dynamic is disregarded [1,2,3]. But, when direct current motor dynamics is taking into consideration then the saturation operation is disregarded [4]. However, today in rapidly developing market competition environment, cheapness of the product increasingly makes point of importance as well as the performance of the product while a common application that is used to reduce the cost is to avoid from expensive measurement devices by designing open circuit control structures [5]. This way of approach is effective when the industrial task to be used by the robot is fairly defined, and is failed to the corruptive inputs. A manipulator system cost can be

drawn in by reducing its response speed and sensitivity. However, what in this study we want to do is to have advanced system performance with minimum cost.

If a motor armature current can be saturated at certain value, high torques can be achieved by using smaller, light and cheap motors [6]. The objective here is to provide same high torques by stating at certain value of the motor instead of having high torques by deriving unexpected and high currents which motor requires. Saturation operation makes the possible to use light and cheaper motors in manipulator design. Light motor affects the system response positively. When the motor are heavy then system will be sensible to vibrations in low frequency. Because, if a heavy manipulator arm stopped after a speedy motion then vibrations will be occurred. It causes the longer system response time and precision problem. System is projected with state feedback plus integral error control strategy for two degree of freedom manipulator system [7]. In this study, firstly, the dynamic model of manipulator system without DC motor characteristics is determined as a second order differential equation by using Lagrange

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equations. This equation was linearized by Jacobian matrices that explain small deviation about nominal trajectory of manipulator. Therefore, the linearization coefficients are updated during trajectory in small sampling periods. Coupling terms between joints are treated as disturbance torque in control algorithm to separate the joints. As the controller is robust against disturbances, this approach does not effect the system response adversely. It makes the possible to calculate transfer functions of each joint independently for pole placement. The dynamic model of each joint is obtained by making unification between the linear dynamic model of a joint and dynamic equation of DC motor on this joint. There are researches on motor current limitation [8,9,10]. But, main contribution of this study is that saturation operation has been achieved with a great performance such as robustness, dead beat response, independent joint control and non-interaction between joints in two degrees of freedom manipulator system.

2. MANIPULATOR KINEMATICS

Figure 1 shows a two degrees of freedom manipulator. The point O_2 is assumed to be fixed. There are DC motors at points O and O_1 . Manipulator has two revolute joints and two serial links. Due to kinematics analysis, firstly, base and local coordinate systems are placed on the manipulator. Base coordinate system is fixed at point O . There are local coordinate systems at point O_1 and O_2 .

Position and orientation of end effectors of robot arm can be determined according to base coordinate system. Transformation matrices which gives the relation between adjacent joints is used for this operation. In the same way, the joint variables can be determined by means of inverse of transformation matrices. Kinematic parameters and transformation matrices were determined by using the “Denavit Hartenberk” representation.

Eq. 1 shows numeric values of structural parameter of manipulator and gravity acceleration. Here, M_1 and M_2 represent masses of link 1 and link 2. L_1 and L_2 represent lengths of the links.

$$M_1=M_2=5 \text{ kg}, L_1=L_2=L=0,5 \text{ m}, g=9,81 \text{ m/s}^2 \quad (1)$$

$$D_{11} = (1/3)L^2(M_1 + 4M_2 + 3M_2 \cos(\theta_2)) \quad (4)$$

$$D_{12} = (1/3)M_2L^2 + (1/2)M_2L^2 \cos(\theta_2) \quad (5)$$

$$D_{21} = (1/3)M_2L^2 + (1/2)M_2L^2 \cos(\theta_2) \quad (6)$$

$$D_{22} = (1/3)M_2L^2 \quad (7)$$

$$h_2 = (1/2)L^2 \sin(\theta_2)M_2\dot{\theta}_1\dot{\theta}_1 \quad (8)$$

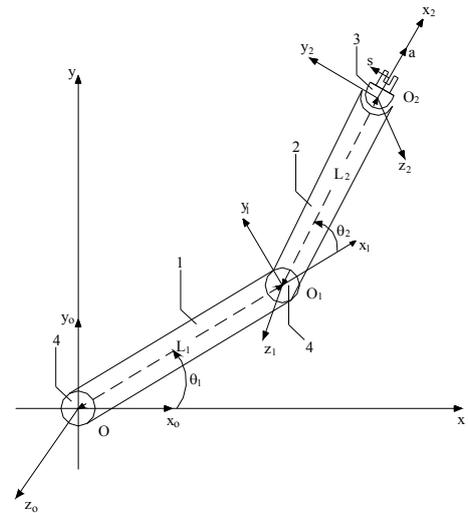


Figure 1. Two degrees of freedom manipulator: 1. First link 2. Second link 3. End effectors 4. Motor

3. MANIPULATOR DYNAMICS

A manipulator system can be represented as a second order differential equation by means of Lagrange equations as described below.

$$T = D(q)\ddot{q} + h(q, \dot{q})\dot{q} + c(q) \quad (2)$$

Here, $n \times 1$ dimension T matrices represents generalized torques of joints. D is $n \times n$ matrix and gives inertia effects of masses. h is $n \times 1$ matrix and represents centrifugal and Coriolis effects. c is $n \times 1$ matrix and shows gravity forces that creates torques to joints. q is $n \times 1$ generalized coordinates and represents angular displacement of joints. Therefore, \ddot{q} is angular acceleration ($\ddot{\theta}$), and \dot{q} is angular velocity ($\dot{\theta}$).

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad (3)$$

Eq. 3 shows dynamic model of manipulator.

$$h_1 = -(1/2)L^2 \sin(\theta_2)M_2\dot{\theta}_2(\dot{\theta}_2 + 2\dot{\theta}_1) \tag{9}$$

$$c_1 = (1/2)gL(M_1 \cos(\theta_1) + M_2 \cos(\theta_1 + \theta_2) + 2M_2 \cos(\theta_1)) \tag{10}$$

$$c_2 = (1/2)gLM_2 \cos(\theta_1 + \theta_2) \tag{11}$$

4. LINEARIZATION OF DYNAMIC MODEL

Dynamic model of manipulator can be written as a set of first-order differential equations.

$$\dot{x} = f[x(t), u(t)] \tag{12}$$

In this equation, nx2 dimensional vector x is state variable which represents angular displacement and its

$$\dot{x}_n = f[x_n(t), u_n(t)] \tag{13} \quad \delta x(t) = x(t) - x_n(t)$$

$$\delta u(t) = u(t) - u_n(t) \tag{15}$$

$$d / dt(x_n + \delta x) = f[x_n(t) + \delta x(t), u_n(t) + \delta u(t)] \tag{16}$$

The right hand side of Eq. 16 may be expanded by using Taylor Series about nominal path (x_n(t), u_n(t)). (∂f/∂x) and (∂f/∂u) represents Jacobian matrices.

$$f[x_n(t) + \delta x(t), u_n(t) + \delta u(t)] = f[x_n(t), u_n(t)] + [(∂f / ∂x)_{x_n, u_n}] \delta x + [(∂f / ∂u)_{x_n, u_n}] \delta u + ... \tag{17}$$

$$\delta \dot{x} = [(∂f / ∂x)_{x_n, u_n}] \delta x + [(∂f / ∂u)_{x_n, u_n}] \delta u \tag{18}$$

$$\delta \ddot{x} = A(x_n, u_n) \delta x + B(x_n, u_n) \delta u \tag{19}$$

The linearized model of two degrees of freedom manipulator was obtained with equations below. In this equation, coefficient matrices A(x_n, u_n) and B(x_n, u_n) represents elements of Jacobian matrices. Therefore, the

derivative, n dimensional vector u represents torques which effects to joints by motion of manipulator. System input without motor dynamics can be considered as to find appropriate torques (u) for joints so as manipulator can track its trajectory. Nominal trajectory (x_n) can be obtained by applying nominal torques (u_n) to joints. For this situation, dynamic model of system is written below. δx(t) and δu(t) represents deviations from nominal trajectory.

elements of these matrices are updated during manipulator motion. By using the state variables:

$$x_1 = \theta_1, x_2 = \theta_2, x_3 = \dot{\theta}_1, x_4 = \dot{\theta}_2, T_1 = u_1, T_2 = u_2 \tag{20}$$

Eq. 19 can be written as follows:

$$\delta \ddot{x}_1 = \delta \ddot{x}_3 \tag{21}$$

$$\delta \ddot{x}_2 = \delta \ddot{x}_4 \tag{22}$$

$$\delta \ddot{x}_3 = a_{31} \delta x_1 + a_{32} \delta x_2 + a_{33} \delta x_3 + a_{34} \delta x_4 + b_3 \delta u_1 \tag{23}$$

$$\delta \ddot{x}_4 = a_{41} \delta x_1 + a_{42} \delta x_2 + a_{43} \delta x_3 + a_{44} \delta x_4 + b_4 \delta u_2 \tag{24}$$

$$\delta \ddot{\theta}_1 = a_{31} \delta \theta_1 + a_{32} \delta \theta_2 + a_{33} \delta \dot{\theta}_1 + a_{34} \delta \dot{\theta}_2 + b_3 \delta T_1 \tag{25}$$

$$\delta \ddot{\theta}_2 = a_{41} \delta \theta_1 + a_{42} \delta \theta_2 + a_{43} \delta \dot{\theta}_1 + a_{44} \delta \dot{\theta}_2 + b_4 \delta T_2 \tag{26}$$

$$\delta x = x - x_n \tag{27}$$

$$\ddot{\theta}_1 = a_{31} \theta_1 + a_{32} \theta_2 + a_{33} \dot{\theta}_1 + a_{34} \dot{\theta}_2 + b_3 u_1 - a_{31} \theta_{n1} + a_{32} \theta_{n2} + a_{33} \dot{\theta}_{n3} + a_{34} \dot{\theta}_{n4} + b_3 u_{n1} + \ddot{\theta}_{n1} \tag{28}$$

$$c = -a_{31}\theta_{n1} - a_{32}\theta_{n2} - a_{33}\dot{\theta}_{n3} - a_{34}\dot{\theta}_{n4} - b_3u_{n1} + \ddot{\theta}_{n1} \quad (29)$$

$$\ddot{\theta}_1 = a_{31}\theta_1 + a_{32}\theta_2 + a_{33}\dot{\theta}_1 + a_{34}\dot{\theta}_2 + b_3u_1 + c \quad (30)$$

$$\ddot{\theta}_2 = a_{41}\theta_1 + a_{42}\theta_2 + a_{43}\dot{\theta}_1 + a_{44}\dot{\theta}_2 + b_4u_2 - a_{41}\theta_{n1} - a_{42}\theta_{n2} - a_{43}\dot{\theta}_{n3} - a_{44}\dot{\theta}_{n4} - b_4u_{n2} + \ddot{\theta}_{n2} \quad (31)$$

$$d = -a_{41}\theta_{n1} - a_{42}\theta_{n2} - a_{43}\dot{\theta}_{n3} - a_{44}\dot{\theta}_{n4} - b_4u_{n2} + \ddot{\theta}_{n2} \quad (32)$$

$$\ddot{\theta}_2 = a_{41}\theta_1 + a_{42}\theta_2 + a_{43}\dot{\theta}_1 + a_{44}\dot{\theta}_2 + b_4u_2 + d \quad (33)$$

5. MODELLING OF DC MOTOR

Equations below describes DC motor dynamics.

$$L_a(di_a/dt) = -R_a i_a + e_a - e_b \quad (34)$$

$$d\theta/dt = \omega \quad (35)$$

$$e_b = K_b \omega \quad (36)$$

$$T_m = K_i i_a \quad (37)$$

L_a = Armature inductance = 0,005 Henry

i_a = Armature current (amp)

R_a = Armature resistance = 1 ohm

e_a = Armature voltage (V)

e_b = Feedback voltage (V)

K_b = Coefficient of feedback voltage = 0,1 Vs/rad

K_i = Current-torque coefficient = 10 Nm/amp

T_m = Motor torque (Nm)

θ = Angular displacement of motor shaft (rad)

$$\left[\ddot{\theta}_1 - (a_{31}\theta_1 + a_{32}\theta_2 + a_{33}\dot{\theta}_1 + a_{34}\dot{\theta}_2 + c) \right] / b_3 = u_1 = T_1 \quad (39)$$

Coefficient c in Eq. 39 represents torque calculated by inverse dynamics of both joints. This function represents first part of controller mentioned in section 1. These torques obtained by inverse dynamics and linearization coefficients are updated during manipulator operation. But, because of the second link of manipulator which create corrective torques to prevent deviation from trajectory is coupling with linearized dynamic model. This condition can be seen in Eq. 39. This interaction is treated as a disturbance in controller algorithm. With this approach independent joint control has become possible and pole placement can be done easily by calculating the characteristic function of each joint.

ω = Angular velocity of motor shaft (rad/s)

Eq. 38 that shows total torque acting on joints can be written by making combination between dynamic equation of DC motor and dynamic model of manipulator.

$$T_m + T_D = J\ddot{\theta} + B_t\dot{\theta} + T \quad (38)$$

where,

J = 0,1 Nms²/rad

B_t = 0,01 Nms/rad

Eq. 38 gives total torque equilibrium for any joint in manipulator system. T_D denotes disturbance torques acting on joints, J is polar inertia moment of motor. B_t is total friction coefficient, T is torque determined by linear dynamic model of manipulator. θ is angular displacement of motor and joint, so gear ratio between motor and joint is 1. T contains linearization coefficients, dynamics of other joints and torques calculated by inverse dynamics. By using linearized dynamic equation determined in Eq. 30 for first joint of two degree of freedom manipulator, the torque acting on joint 1 as a result of manipulator motion can be written as follows.

Poles are placed on the negative axis of s plane and repeated poles are used. That is, each joint has 6 repeated poles on the negative real axis. Fig. 2 shows controller block diagram of two degrees of freedom manipulator.

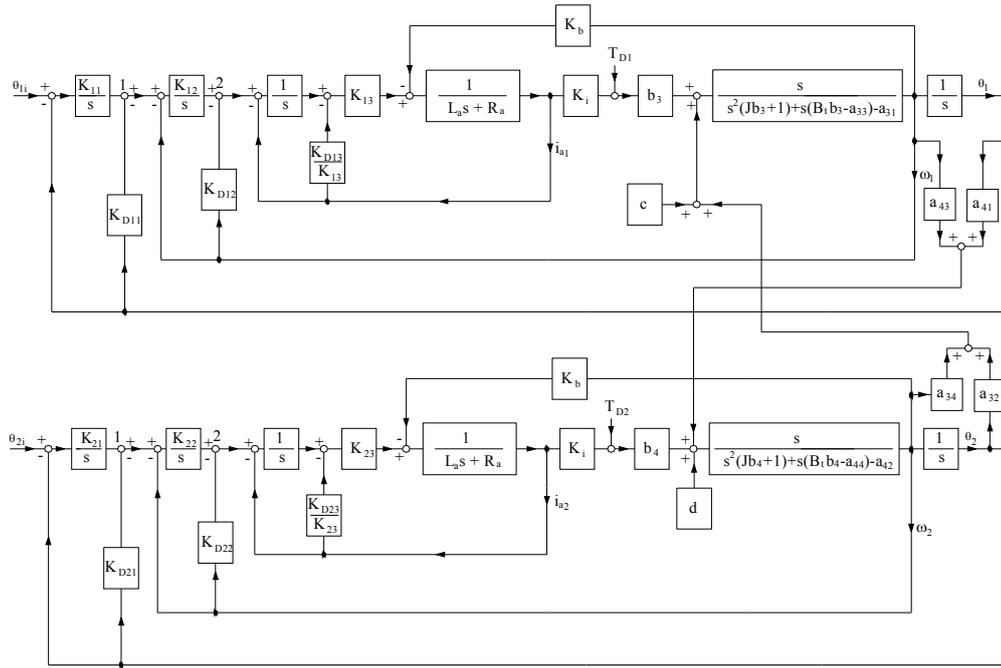


Figure 2. Controller block diagram of manipulator.

6. SATURATION OPERATION

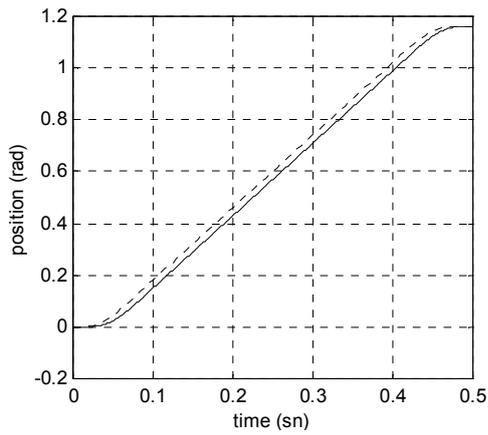
When point 2 input in Figure 1 is saturated, there is an accumulation at second integrator input. This nonlinear reaction causes big and repeated excitations and unsteadiness at system response. The integrator input will also be saturated in order to stop this. However, it is not only enough to terminate the second integrator for the saturation. Because when this process is realized, there is an accumulation at the first integrator. Therefore, the speed should also be limited in order to saturate. Even if the first integrator output is saturated for the speed limitation, there is an accumulation at its input again. In short, the input and outputs of the two integrators are terminated in order to make the saturation operation.

System response time can be made shorter by moving the poles towards the left side of negative real axis of s plane. But, in this case, more powerful amplifier and bigger motors will be needed to supply more torque to joints. If the motor weights increase then system will be more sensitive against vibrations in low frequency. This causes the delay in time response of system. Steel construction of manipulator can be strengthened by using more rigid materials to prevent vibrations. But, more rigid materials and bigger motors will increase the cost, dimension and weight of manipulator. When we consider above mentioned factors, the outstanding advantages of saturation operation can be seen. Light and less expensive motors can be used for manipulator

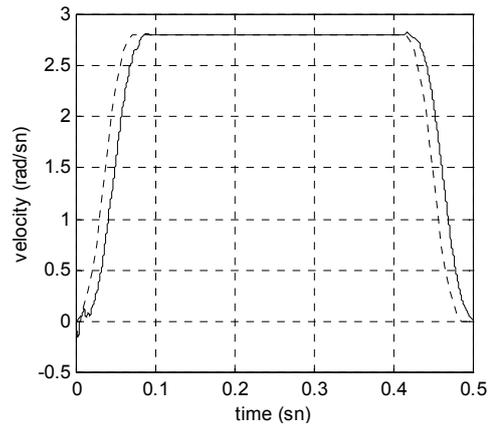
design by limiting at a value of motor currents by saturation operation.

7. SIMULATIONS

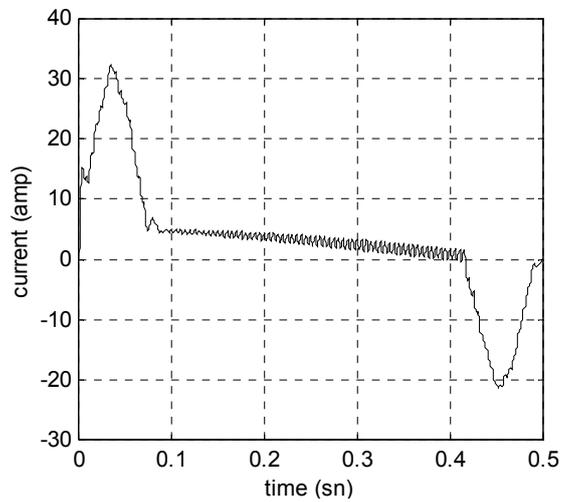
Obtainment of the optimum termination values to prevent the accumulations, occur at the integrator inputs, can be done by means of a computer program. This program written in MATLAB updates the linearization coefficients and controller gains in 5 ms during manipulator trajectory. The maximum acceleration determined in nominal trajectory is 70 rad/s² for the first joint and 40 rad/s² for the second joint. As the first joint of the manipulator derives more current, the saturation operation was done for the first joint. During the movement the second joint has a trajectory similar to the first joint. The saturation values are found out with trial and error method by making simulations. The dotted line at the simulations expresses the desired trajectory with system input, while the straight line expresses the actual trajectory. In these performed simulations the poles were placed at 500 point on the real axis of s plane for both joints, and 6 repeated poles were used. Fig. 3 shows the time response of joint 1 without saturation operation. Fig. 4 shows the time response of joint 1 with saturation operation at 25 current of amp value and velocity of 3,1 rad/s. When the saturation is wanted to be at the values less than 25 amp. for this defined trajectory input, there are excitations at system response.



a) Position response of joint 1 without saturation

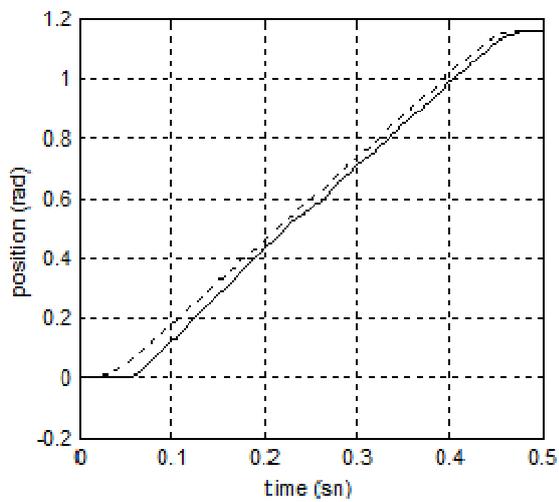


b) Velocity response of joint 1 without saturation

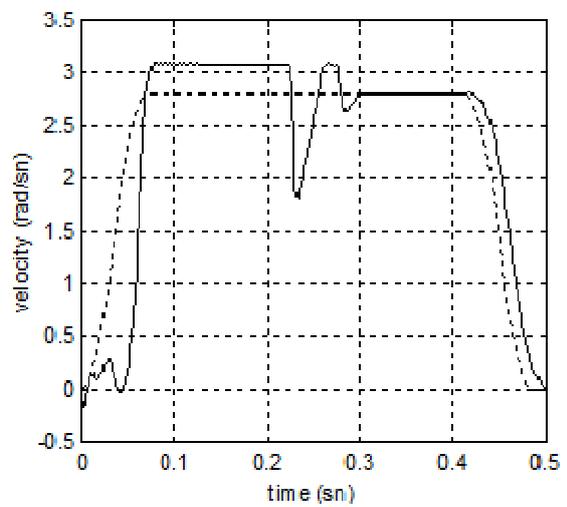


c) Current response of joint 1 without saturation

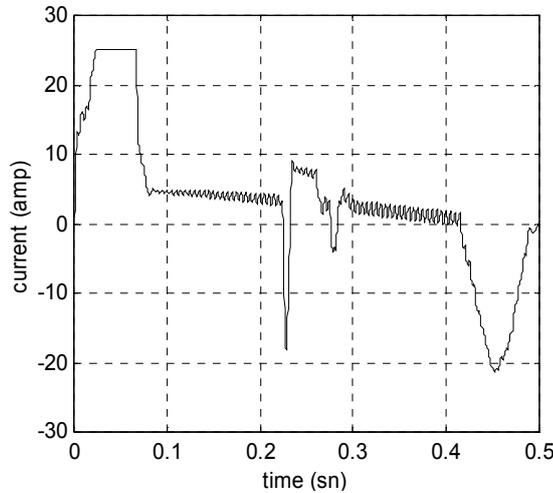
Figure 3. Time response of joint 1 without saturation



a) Position response of joint 1 with saturation



(a) Velocity response of joint 1 with saturation



c) Current response of joint 1 with saturation

Figure 4. Time response of joint 1 with saturation

8. CONCLUSIONS

In simulations for two degrees of freedom manipulator system, saturation operation was performed by achieving desired system performances such as dead beat response, robustness and non-interaction between joints. It is known that the used control is resistant to corruptive inputs and reduces the interaction between joints [7]. It has been found out in simulations for the designed system that speed limitation has an important role in saturation. By proposed saturation procedure, light and cheaper motors can be used for manipulator design.

9. SYMBOLS AND ABBREVIATIONS

Symbol Definition

L_a	Armature inductance (Henry)
i_a	Armature current (amp)
R_a	Armature resistance (ohm)
e_a	Armature voltage (V)
e_b	Feedback voltage (V)
K_b	Feedback voltage coefficient (Vs/rad)
K_i	Current-torque coefficient (Nm/amp)
θ	Angular exchange of motor axle (rad)
ω	Angular speed of motor axle (rad/s)

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