Gazi University Journal of Science
GU J Sci
26(1):97-106 (2013)

A Bi-Criteria Single Machine Scheduling with Rate-Modifying-Activity

Yücel ÖZTÜRKOĞLU[1,♠]

Yaşar University, Faculty of Economics and Administrative Sciences,
International Logistics Management,
Bornova, İzmir, 35100

Received:23/03/2012 Revised:12/12/2012 Accepted:12/12/2012

ABSTRACT

In this paper, we consider a single machine scheduling problem with two criteria: minimizing both total flow time with total tardiness and minimize maximum tardiness with number of tardy jobs. Unlike the classical scheduling problems, we use a job position deterioration, which means that the job processing time increases as a function of the job position. Besides deteriorated jobs, we also consider rate-modifying-activities which alter the efficiency of the deteriorating processor. This is the first paper, to combine both time dependent processing times and problems with rate-modifying-activity in the bi-criteria objectives. To solve the new type of problem, we introduce a new scheduling mathematical model which is based on one developed Ozturkoglu and Bulfin [1]. To analyze the efficiency of the mathematical model, we use three different approaches. According to computational results, up to 50 jobs can be solved in less than one minute.

Keywords: Single-Machine Scheduling, Bi-criteria, Deteriorated Jobs, Rate-Modifying- Activity

1. INTRODUCTION

Competition between companies in the same industry is vital because of the recession in the national and the international economy. Nowadays, the cost of products and response to customer's requirements are important for companies to be successful in the market. So, managers should consider more than one measure when they try to find the best schedule for their production process. Therefore, bi-criteria scheduling is more attractive. For example, to satisfy both customers and production effectiveness, minimizing both total time and number of tardy jobs are appropriate.

Research on bi-criteria scheduling is rare compared to research in single criterion scheduling. Almost all bi-criteria research assumes job processing times are constant. But in a real life situation, job processing times may deteriorate while jobs are waiting to be processed. Both machine and operator may cause this deterioration. For instance, a machine or tools may wear and processing time and quality of the jobs will change. Or the operator's physical condition may cause the processing speed to

♠Corresponding author, e-mail: yucel.ozturkoglu@yasar.edu.tr

change over time. Browne and Yechiali introduced deteriorated processing times in the scheduling literature [2]. They assumed that the processing time of a job grows linearly depending on the start time of the job.

To prevent job deterioration due to the machine, repair or maintenance, called rate-modifying activities (RMA), are needed. The RMA, an activity which affects and changes the production rate of the machines, was first introduced by Lee and Leon [3]

This research addresses the bi-criteria scheduling problems involving single machine with deteriorating jobs and rate-modifying-activities. We study four criteria; total flow time, total tardiness, maximum tardiness and number of tardy jobs. Minimizing total flow time and number of tardy jobs reflect manufacturer satisfaction, while minimizing maximum tardiness and total tardiness are measures of customer satisfaction. To satisfy both sides by using those performance measures, we propose mathematical models and present experimental results for both $1/p_{ij} = (1+\alpha_j)^{i-1} p_j, rm/T_{\max} \mid \sum U$ and $1/p_{ij} = (1+\alpha_j)^{i-1} p_j, rm/\sum F \mid \sum T$.

2. LITERATURE REVIEW

In the bi-criteria literature, some criteria are used very often. Therefore, we divide literature review based on performance measures. We classify the most common criteria studied in bi-criteria single machine scheduling problems by researchers. But before giving information about previous researchers, we want to explain major approaches to solve bi-criteria problems. There are three approaches in scheduling problems.

1) Bi-criteria Approach: Generate Pareto curve for all non-dominated schedules. Using Graham et al. three field notation [4], we denote single machine bi-criteria problems under basic assumptions as $1//\gamma_1, \gamma_2$.
2) Secondary Objective Approach: First try to optimize the primary criterion and then try to solve secondary criterion subject to the optimal value of the primary criterion. Denote the single machine bi-criteria problems under basic assumptions as $1//\gamma_2 \mid \gamma_1$.
3) The weighting method in which both objectives are optimized at the same time by assigning weights. Mathematically, the weighting method can be stated as follows:

$$\min z(x) = w_1 z_1(x) + (1-w_1) z_2(x) \qquad (2.1)$$

In this study, we use three approaches to analyze our mathematical model. Various combinations of the criteria are considered and analyzed as primary and secondary criteria in the literature. We give brief literature review of most common used performance measures.

Total Completion Time and Maximum Tardiness

Smith developed a polynomial time algorithm to use secondary objective approach scheduling problems with these two objectives [5]. Afterwards, Heck and Roberts [6], Emmons [7], Van Wassenhove and Gelders [8] extended Smith's study and developed some algorithms to consider secondary approach. Chen and Bulfin [9] proved that flow time with maximum tardiness bi-criteria problems are NP-hard.

Later, Kondakci et al. presented an algorithm to produce all efficient schedules for any given non decreasing function of the total completion tine and maximum tardiness objectives. [10]. Chen [11] developed a heuristic to find the Pareto optimal schedules of periodic maintenance of the machines.

Weighted Completion Time and Maximum Tardiness

Burns [12] presented an algorithm that provide to a local optimum for both the weighted and unweighted problems in bi-criteria objectives. For the secondary approach, Bansal [13] extended Burns [12] algorithm and applied a branch and bound algorithm to find a globally optimal solution. In his algorithm, he found a locally optimal solution for the problem of minimizing weighted sum of completion times subject to the condition that every job be completed by its due date. Miyazaki [14] solved bi-criteria problem with different approach which one of the criteria as objective and the other as a constraint. He developed a necessary condition under which the local and global solutions are different and developed an algorithm to obtain an improved schedule based on the locally optimal schedule. Shanthikumar and Buzacott [15], Potts and Van Wassenhove [16] developed some heuristics for problem $1//T_{\max} \mid F$. Posner [17] and Bacghi and Ahmadi [18] considered with these two performance measures with deadlines and they found tightest bound for same objectives. Chen and Bulfin [9] proved that weighted flow time with maximum tardiness bi-criteria problems $1//w_T \mid T_{\max}$ and $1//w_u \mid T_{\max}$ are NP-hard.

Total Completion Time and Number of Tardy Job

Emmons [7] was the first study which is presented a branch and bound algorithm based on these criteria for the secondary approach. Then, Nelson et al. [19] presented branching procedures on the bi-criteria approach. Chen and Bulfin [9] proved that these objectives are NP-hard.

Maximum Tardiness and Number of Tardy Job

Firstly, Shanthikumar [20] developed a branch and bound algorithm for the problem $1 \| T_{\max} \mid \sum U_j$. Later on, Nelson et al. [19] and Chen and Bulfin [21] proposed both heuristic and branch and bound algorithms for problem $1 \| \sum U_j \mid T_{\max}$. Huo et al. [21] considered complexity relationship of single machine problems $1 \| \sum U_j \mid \max\{w_j T_j\}$ and $1 \| \max\{w_j T_j\} \mid \sum U_j$ with weighted tardiness. Also they proposed several fast heuristics.

Most papers mentioned above assume processing time is constant. To the best of our knowledge, there is no study which combines bi-criteria scheduling problems with deteriorating jobs. So this paper is the first study which combines deteriorating jobs with rate-modifying-activity in bi-criteria objectives.

3. PROBLEM DESCRIPTION

As stated earlier, this study focuses on the single machine bi-criteria scheduling problem. There are n jobs to be processed on a single machine. The jobs are available at time zero and are independent of each other. Preemption is not allowed. The machine can handle one job at a time. Each job has an initial processing time p_j before deterioration, a due date d_j and an actual processing time p_{ji} which is the processing time of job j if done *i* positions after an RMA or the initial position. We calculate p_{ji} by;

$$p_{ji} = (1+\alpha_i)^{i-1} p_j \tag{3.1}$$

where α_i is the deterioration rate of jobs for $0 < \alpha_i \leq 1$ when delayed by one position. This is non-linear deterioration based on position rather than start time. And, q is the fixed period of time to perform the rma.

Decision Variables

$$x_{ijk} = \begin{cases} 1 & \text{if job j is in the ith position after an rma which is done before position k + 1,} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if an rma is done before position i} \\ 0 & \text{otherwise} \end{cases}$$

C_i = Completion time of the job in position *i*.

Our model is based on by Öztürkoğlu and Bulfin [1]. The constraints for our model are;

$$C_1 = \sum_{j=1}^{n} p_{j1} x_{1j0} \tag{3.2}$$

$$C_i = C_{i-1} + \sum_{k=1}^{i} \sum_{j=1}^{n} p_{jk} x_{i,j,i-k} + q\, y_i \qquad i = 2,.....,n \tag{3.3}$$

$$\sum_{i=1}^{n} \sum_{k=0}^{i-1} x_{ijk} = 1 \qquad j = 1,.....,n \tag{3.4}$$

$$\sum_{k=0}^{i-1} \sum_{j=1}^{n} x_{ijk} = 1 \qquad i = 1,.....,n \tag{3.5}$$

$$x_{kji} \leq y_{i+1} \qquad k = 2.....,n; \; j = 1,.....,n; \; i = 1,.....,k-1 \tag{3.6}$$

$$x_{ijk} \in \{0,1\} \qquad i, j = 1,.....,n; \quad k = 0,.....,n \tag{3.7}$$

$$y_k \in \{0,1\} \qquad k = 2,.....,n \tag{3.8}$$

$$C_i \geq 0 \qquad i = 1,.....,n \tag{3.9}$$

In constraint (3.2), the completion time of the job in position one is equal to the processing time of the job assigned to position one. Before the first position, there is no rma ($y_1 = 0$). In constraint (3.3), the completion time of the job in position i is equal to the completion time of the job in position $i-1$ plus the processing time of the job assigned to position i plus the rma time if assigned. In constraint (3.4), each job is assigned to exactly one position. In constraint (3.5), each position is scheduled for only one job. Constraint (3.6) requires an rma to be done in the related position if jobs are scheduled after rma and to control the sequence of the rma. Lastly, constraints (3.7), (3.8) and (3.9) indicate the decision variables are binary and all other variables are non-negative.

4. CRITERION

In this paper we focus on four different objectives which are to minimize total flow time, total tardiness, maximum tardiness and number of tardy jobs. Different combinations of two of these criteria are studied. The mathematical formulation for each criterion is given below.

4.1. Total Completion Time

Minimizing flow time is to keep the work in process inventory at a low level. Also, it tries to minimize completion times, lateness and job waiting times. It is defined as;

$$\min z = \sum_{i=1}^{n} C_i \qquad (4.1)$$

4
5

Subject to:

Constraint sets (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8) and (3.9) respectively.

4.2. Total Tardiness

Minimizing total tardiness is to reduce penalties cause by late jobs. Let T_j be the tardiness of job j .

$$\min z = \sum_{j=1}^{n} T_j \qquad (4.2)$$

Constraint sets (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9) and

$$T_j \ge C_j - d_j \qquad j = 1,..,n \qquad (4.3)$$

4.3. Maximum Tardiness

Minimizing maximum tardiness is a measure of customer satisfaction based on due dates.

$$\min z = T_{\max} \qquad (4.4)$$

Constraint sets (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9) and

$$T_{\max} \ge C_i - d_i \qquad i = 1,..,n$$

(4.5)

4.4. Number of Tardy Jobs

Often used in real applications, we try to finish as many jobs as possible on time because of the penalty costs.

$$\min z = \sum_{i=1}^{n} N_i \qquad (4.6)$$

Constraint sets (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9) and

$$N_i \ge MT_i \qquad i = 1,..,n$$

(4.7)

$$N_i \in \{0,1\} \qquad i = 1,..,n$$

(4.8)

M is a very big number.

5. COMPUTATIONAL EXPERIMENTS

To understand the behavior of the mathematical model, three approaches are used. The proposed mathematical model is coded using AMPL and solved by CPLEX 9.1 on a computer with Pentium IV 2.8 GHz processor and 1GB of RAM. We perform an empirical study of the three bi-criteria approach. In the next subsection, we describe how we generate the data. And then, we give results and analysis of experiments.

5.1. Data Generations

In our experiments, we consider 25 and 50 jobs. Job processing times are generated from a uniform distribution on the interval [1-50]. To generate the due dates, we use τ and R based on Huo et al. [19] which are denote the due date range and tardiness factor respectively. To generate each jobs due date, we use a discrete uniform distribution with intervals between $\sum_{j=1}^{n} p_j (1-\tau - R/2)\ and \sum_{j=1}^{n} p_j (1-\tau + R/2)$. Table 1 gives problem parameters.

Table 1. The parameters of the Problem

Parameter	Values
p_j	U~ [1-50]
$\#of\ jobs$	25 and 50
α	0.025, 0.05, 0.075
q	2, 5, 8 (min.)
τ	0.25, 075
R	0.25, 0.50
ψ	0.25, 0.50, 0.75

Ten replications of each of the possibilities were run for each combination of performance measure. Totally, 540 instances were generated. The results and analysis of experiments are given in the below.

5.2. Bi-criteria Approach

One of the commonly used methods in bi-criteria is Pareto curve. In this method, s_1 is dominated by solution s_2. In that approach s_2 is not worse than s_1 among objectives and s_2 can be strictly better than s_1 for at least one of the objectives. This solution is called non-dominated solutions and the set of non-dominated solutions in the feasible problem space is the Pareto optimal set.

To try to find efficient Pareto curve we have plotted the 25 points which are our objective functions. These points lie on the objective function line. To obtain these points, we use 25 jobs with 0.025 deterioration rate. All other parameters are the same.

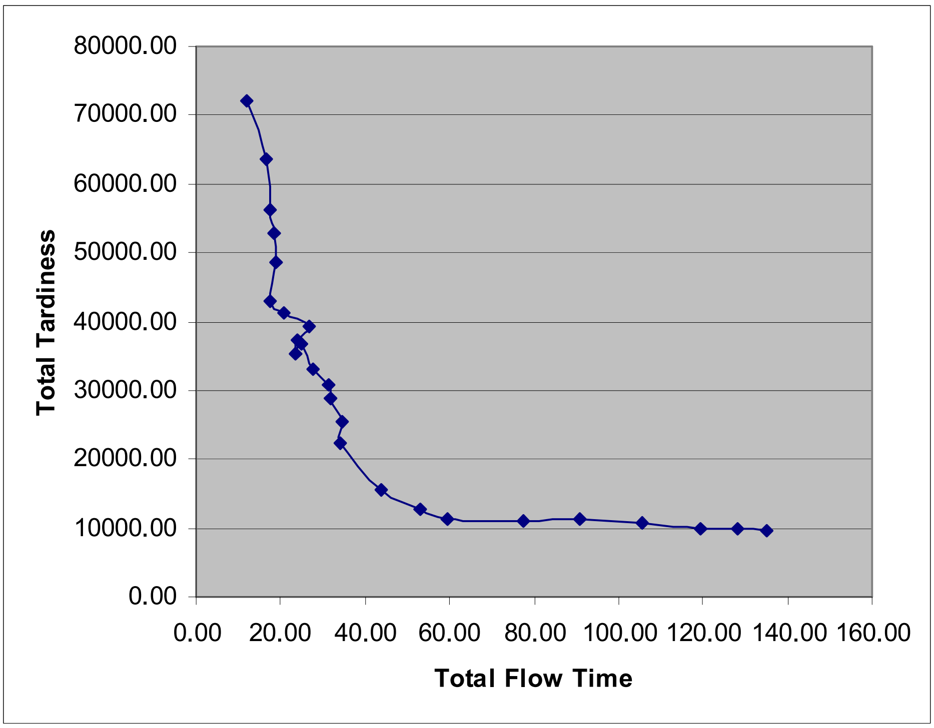

Figure 1. Pareto Curve for objective $1/_{ij} = (1+\alpha_j)^{i-1} p_j, rm / \sum F, \sum T$

Non-dominated means that there is no other solution in which one objective function can be improved without a simultaneous detriment to the other objective. In Figure 1, each of these points determines the extreme points of the dominated set in the decision space. All points are equally acceptable as the solution to the bi-criteria optimization problem. But, decision maker should be select only one solution set for practical reasons. Therefore, the decision maker may choose a schedule that provides a more balanced performance on the two criteria employed.

5.3. Secondary Objective Approach:

In the secondary criterion approach, problem can be solved in two separate stages. Firstly, solve the problem for the primary criterion, called S_1, in that stage ignore the secondary criterion, called S_2. In the next stage, we solve the problem for the secondary criterion, subject to the constraint that the optimal value S_1 does not change. In our model, first we solve our model only with primary objective. After getting the optimal solution, then solve the same model with secondary objective and add first optimal solution value likes a constraint. So, model can be solved by secondary objective method.

Tables 2 and Table 3 show the computational time and given an RMA of the $1/_{ij}=(1+\alpha_j)^{i-1}p_j,rm/T_{\max}\mid\sum U$ and $1/_{ij}=(1+\alpha_j)^{i-1}p_j,rm/\sum T\mid\sum F$ respectively.

Table 2. Average Run Time (sec.) for $1/_{ij}=(1+\alpha_j)^{i-1}p_j,rm/T_{\max}\mid\sum U$

Jobs numb.	Det. Rate	Rma time	Ave. Comp. Time (sec.)	Num. of given RMA
25	0.025	2	5.7	9
	0.025	5	5.6	9
	0.025	8	5.9	9
	0.05	2	6.2	10
	0.05	5	6.3	10
	0.05	8	6.1	10
	0.075	2	8.6	11
	0.075	5	8.5	12
	0.075	8	8.8	12
50	0.025	2	18.7	20
	0.025	5	19.0	21
	0.025	8	19.4	21
	0.05	2	22.2	22
	0.05	5	24.3	22
	0.05	8	24.1	23
	0.075	2	28.5	24
	0.075	5	28.7	24
	0.075	8	29.4	24

Table 3. Average Run Time (sec.) for $1/_{ij} = (1+\alpha_j)^{i-1} p_j, rm / \sum T \mid \sum F$

Jobs numb.	Det. Rate	Rma time	Ave. Comp. Time (sec.)	Num. of given RMA
	0.025	2	19.1	8
	0.025	5	19.8	8
	0.025	8	20.2	8
	0.05	2	22.9	8
25	0.05	5	23.3	8
	0.05	8	23.4	9
	0.075	2	26.6	9
	0.075	5	27.1	10
	0.075	8	28.5	10
	0.025	2	32.9	17
	0.025	5	32.7	17
	0.025	8	34.3	16
	0.05	2	36.8	17
50	0.05	5	37.8	18
	0.05	8	39.1	18
	0.075	2	38.4	19
	0.075	5	39.9	19
	0.075	8	40.9	19

The computational results are summarized in Table 2 and Table 3. Based on the tables, the number of rmas is based on both deterioration rate and rma time. A larger deterioration rate results in more rmas. It is obvious that for larger rma times, larger computation time is needed.

5.4. Weighted Method

The two objectives can be optimized at the same time by assigning the proper weights in the weighting method. Mathematically, the weighting method can be stated as follows:

$$\min z(x) = w_1 z_1(x) + (1-w_1) z_2(x)$$

We use three different value of w_i (0.25, 0.50, 0.75) in our calculations. The extension of weight ranges is important for the stability of solution. Before using the Equation 5.1, we normalize our objective function values to obtain reliable results.

Normalization

In practice, multiple objectives have different dimensions and difficult to compare different objective types. The individual preferences of the objectives are described by weights. These weights are assigned by the decision maker. But assigning proper weights are very difficult and causes problems unless assigns convenient weights. To prevent problems, the normalization of objectives is necessary to get reliable solutions. The normalization of different objectives permits the compare various dimensions and the find out the relationship between indifferent objectives.

In our data set, normalization is necessary. We transform the data into a range between 0 and 1. The normalization of the objective function values are found by using below equation;

$$t_i = (f_i(x) - z_i^L) / (z_i^U - z_i^L)$$

(5.2)(5.1)

This value provides the best normalization results as we normalize the objective functions by the true intervals of their variation over the Pareto Optimal set. The parameters of the equation (5.2) is given in the below;

$f_i(x) =$ original value ,

$z_i^U \quad = \max(f_i(x))$,

$z_i^L \quad = \min(f_i(x))$,

$t_i \quad =$ transformed value.

After normalization, we run our new data set and obtain Table 4 and Table 5.

Table 4. Weighted Method for objective of $1/_{ij} = (1+\alpha_j)^{i-1} p_j, rm / f(T,F)$

Jobs numb.	Det. Rate	Rma time	$w_1 = 0.25$	$w_1 = 0.5$	$w_1 = 0.75$	Ave.Flow.T (sec.)	Ave.Num. of given RMA
25	0.025	2	6007.1	4257.2	2507.3	2.2	9
	0.025	5	6192.9	4388.5	2584.1	3.1	5
	0.025	8	6322.3	4480.5	2638.6	3.3	4
	0.05	2	6295.4	4446.4	2597.3	3.5	10
	0.05	5	6575.01	4643.1	2711.2	3.6	7
	0.05	8	6770.1	4780.6	2790.8	3.6	4
	0.075	2	6467.4	4573.8	2680.1	3.9	19
	0.075	5	6808.1	4811.8	2818.5	4.1	13
	0.075	8	7034.2	4973.48	2912.2	4.0	9
50	0.025	2	24789.9	17060.5	9331	18.3	16
	0.025	5	25499.6	17547.8	9595.9	21.5	14
	0.025	8	25943.7	17851.7	9759.6	19.5	10
	0.05	2	16395.8	11315.5	6235.2	18.2	20
	0.05	5	17197.9	11866.8	6532.7	23.6	13
	0.05	8	17717.2	12261.3	6751.2	21.1	10
	0.075	2	14993.8	9674.1	4949.4	21.6	28
	0.075	5	15108.7	11716.9	5432.7	22.7	22
	0.075	8	15473.9	11920.3	5748.1	24.1	16

Table 5. Weighted Method for objective of $1/_{ij} = (1+\alpha_j)^{i-1} p_j, rm / f(T_{\max}, U)$

Jobs numb.	Det. Rate	Rma time	$w_1 = 0.25$	$w_1 = 0.5$	$w_1 = 0.75$	Ave. Comp. Time (sec.)	Num. of given RMA
25	0.025	2	8857.2	6675.3	4964.2	6.2	8
	0.025	5	8909.1	6818.1	4997.6	6.1	6
	0.025	8	9121.7	6908.6	5271.9	9.7	5
	0.05	2	7995.3	5881.3	4495.4	14.5	12
	0.05	5	8093.1	5934.2	4214.6	20.1	10
	0.05	8	8380.6	5534.9	3984.3	20.2	9
	0.075	2	7307.8	4983.4	3439.5	9.9	15
	0.075	5	7495.5	4811.8	3692.4	20.2	13
	0.075	8	7693.9	4731.5	3934.5	21.2	10
50	0.025	2	25822.2	16346.7	13488.7	58.3	20
	0.025	5	26981.7	16984.6	14989.1	89.5	18
	0.025	8	27349.4	17964.2	15349.1	89.5	15
	0.05	2	22901.6	11594.7	8320.9	28.2	26
	0.05	5	23714.5	13688.7	9341.3	56.6	25
	0.05	8	26341.8	14143.8	9918.6	61.1	22
	0.075	2	19737.0	9985.2	8374.9	11.6	30
	0.075	5	23813.7	10671.1	8964.1	32.7	26
	0.075	8	25749.1	13462.7	10438.6	45.1	25

Table 4 and Table 5 show a summary of the computational results of both objectives. Problems with $w = 0.75$ have the smallest objective function. As rma time and deterioration rate get bigger, the objective function gets bigger for the same weights. If we fix rma time, the objective function increases as the deterioration rate increases. The decision manager can make his/her decisions quickly and control their manufacturing systems by choosing proper weights.

6. CONCLUSIONS

This is the first study on bi-criteria scheduling with deteriorating jobs and rate-modifying-activity. We address a real-life scheduling problem with periodic maintenance activity. In reality, scheduling maintenance will result in some jobs being tardy and a larger flow time. Thus, the bi-criteria used in this study are minimize total flow time with total tardiness, and minimize maximum tardiness with number of tardy jobs. Generally, mathematical models have not been used extensively for scheduling problems. In this study, all combinations are studied with the proposed mathematical programming model.

This is the first study which uses all three approaches to analyze the efficiency of the mathematical model with bi-criteria objectives. First, we use the model to find the Pareto Curve for both objectives, so a manager can make his decision from points on the curve. Then we use secondary objective method. Computational results show that the solution of the problem is dependent on the number of jobs and the other parameters of the problem. Although, 50 job problems are solved in around one minute, exponential growth in solution times makes larger problems much harder to solve. Lastly, we use weighted method to analyze model.

REFERENCES

[1] Öztürkoğlu, Y. and Bulfin, R., A unique integer mathematical model for scheduling deteriorating jobs with rate-modifying activities on a single machine. ***International Journal Advanced Manufacturing Technology***, 57: 753-762, (2011).

[2] Browne, S., Yechiali U., Scheduling deteriorating jobs on a single processor. ***Operations Research***, 38: 495-498, (1990).

[3] Lee, C.Y. and Leon, V.J., Machine scheduling with a rate-modifying activity. ***European Journal of Operational Research***, 128: 119-128, (2001).

[4] Graham, R.L., Lawler, E.L., Lenstra, J.K. and Rinnooy, K. A.H.G., Optimization and approximation in deterministic sequencing and scheduling: A Survey. ***Annual Discrete Mathematics***, 5: 287–326, (1979).

[5] Smith, W.E., Various optimizers for single state production. ***Naval Research Logistics Quarterly***, 3: 59-66, (1956).

[6] Heck, H., Roberts S., A note on the extension of a result on scheduling with secondary criteria. ***Naval Research Logistics Quarterly***, 19: 403-405, (1972).

[7] Emmons, H., A note on a scheduling problem with dual criteria. ***Naval Research Logistics Quarterly***, 22: 615-616, (1975).

[8] Van-Wassenhove, L. N., Gelders, L.F., Solving a bicriterion scheduling problem. ***European Journal of Operational Research***, 4: 42-8, (1980).

[9] Chen, C. L. and Bulfin, R. L., Complexity of single machine, multi-criteria scheduling problems. ***European Journal of Operational Research***, 70: 115-125, (1993).

[10] Kondakci, S., Azizoglu, M., Koksalan, M., Note: Bicriteria scheduling for minimizing flow time and maximum tardiness. ***Naval Research Logistics***, 43: 929-36, (1996).

[11] Chen, W.-J., Minimizing total flow time and maximum tardiness with periodic maintenance. ***Journal of Quality in Maintenance Engineering***, 13-3: 293-303, (2007).

[12] Burns, R. N., Scheduling to minimize weighted sum of completion times with secondary criteria. ***Naval Research Logistics Quarterly***, 23-1:125-129, (1976).

[13] Bansal, S. P., Single machine scheduling to minimize the weighted sum of completion times with secondary criterion: A branch and bound approach. ***European Journal of Operations Research***, 5: 177-181, (1980).

[14] Miyazaki, S., One machine scheduling problem with dual criteria. ***Journal of the Operations Research Society of Japan,*** 24-1: 37-50, (1982).

[15] Shanthikumar, J. G., Buzacott, J. A., On the use of decomposition approaches in a single machine scheduling problem. ***Journal of the Operations Research Society of Japan,*** 25: 29-47 (1982).

[16] Potts, C. N., Van Wassenhove,L. N., An algorithm for single machine sequencing with deadlines to minimize total weighted completion time. ***European Journal of Operations Research***, 12: 379-387, (1983).

[17] Posner, M. E., Minimizing weighted completion times with deadlines. ***Operations Research***, 33: 562-574, (1985).

[18] Bacghi,U. and Ahmadi, R. H., An improved lower bound for minimizing weighted completion times with deadlines. ***Operations Research***, 35: 311-313, (1987).

[19] Nelson, R.T., Sarin, R.K., Daniels, R.L., Scheduling with multiple performance measures: The one-machine case. ***Management Science***, 32: 464-480, (1986).

[20] Shanthikumar, J. G., Scheduling n jobs on one machine to minimize the maximum tardiness with minimum number tardy. ***Computers and Operations Research*****,** 10: 255-266, (1983).

[21] Chen, C.L., Bulfin, R. L., Scheduling a single machine to minimize two criteria: Maximum tardiness and number of tardy jobs. ***IIE Transactions***, 26: 76-84, (1994).

[22] Huo Y., Leung, J.Y.T., Zhao, H., Bi-criteria scheduling problems: Number of tardy jobs and maximum weighted tardiness. ***European Journal of Operational Research*****,** 177: 116-134, (2007).