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Research Article

On the Reversibility Problem

Yasemin ÇENGELLENMİŞ[1], Abdullah DERTLİ[2*]

[1]Trakya University, Faculty of Sciences, Mathematics Department, Edirne, Turkey

[2] Ondokuz Mayıs University, Faculty of Arts and Sciences, Mathematics Department, Samsun, Turkey

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Abstract

By defining a non-trivial automorphism over $R = Z_4 + uZ_4 + vZ_4 + uvZ_4$, where $u^2 = u, v^2 = v, uv = vu$, and $Z_4 = \{0,1,2,3\}$, the skew cyclic codes over the finite ring R are introduced. By using the skew cyclic codes over R, it is solved the reversibility problem for DNA 4-bases. Thanks to this, the reversible DNA codes are obtained.

Keywords: Skew cyclic codes, Reversibility problem.

Ters Sıralama Problemi Hakkında

Öz

Bu çalışmada, $u^2 = u, v^2 = v, uv = vu$ ve $Z_4 = \{0,1,2,3\}$ olmak üzere $R = Z_4 + uZ_4 + vZ_4 + uvZ_4$ sonlu halkası üzerinde aşikar olmayan bir otomorfizma tanımlanarak R halkası üzerinde çarpık devirli kodlar verilmiştir. R halkası üzerinde çarpık devirli kodlar kullanılarak DNA 4-bazlar için ters sıralama problemi çözülmüştür. Bu sayede ters sıralı DNA kodlar elde edilmiştir.

Anahtar Kelimeler: Çarpık devirli kodlar, Ters sıralama problemi.

1. Introduction

The transmission and storage of information take place in digital platform and the coding theory is necessary in order to correct and detect errors in the platform. There is another platform. In the platform, the correcting and detecting errors are necessary but it does not take place in digital. It is DNA.

The idea of computing with DNA was given by T. Head (Head, 1987). L.Adleman performed the computation using DNA strands (Adleman, 1994). In fact, DNA sequence has four bases which are (A) Adenine, (G) Guanine, (T) Thymine and (C) Cytosine. DNA has also two strands. They are related by the Watson Crick Complement rule $A^c = T, T^c = A, C^c = G, G^c = C$.

To perform computation using DNA strands, a spesific set of DNA sequences are required with particular properties. The aim of this paper is to obtain the set of DNA strands satisfying various constraints, by using the special error correcting codes over the some finite rings which enjoy DNA properties. One of the constraints is reverse constaint. This leads to reversible codes.

To explain the reversibility problem, let's take a codeword as (δ_1, δ_2). It is corresponding $GTTAGGCA$. The reverse of (δ_1, δ_2) is (δ_2, δ_1). The vector (δ_2, δ_1) is corresponding to $GGCAGTTA$. It is not the reverse of $GTTAGGCA$. The reverse of $GTTAGGCA$ is $ACGGATTG$.

There are many methods in order to solve this problem (Bayram et al., 2015; Dertli and Cengellenmis, 2017; Kumar and Singh, 2018).

One of the methods is to use skew cyclic codes. By using the skew cyclic codes over the finite rings $F_4 + uF_4 + vF_4 + uvF_4$, $F_{16} + uF_{16} + vF_{16} + uvF_{16}$, $u^2 = u, v^2 = v, uv = vu$ and $F_{4^{2k}}[u_1, ..., u_s] / \langle u_1^2 - u_1, ..., u_s^2 - u_s \rangle$, where $k, s \geq 1$, $u_i^2 = u_i, u_i u_j = u_j u_i$, it is solved the reversibility problem for DNA 4-bases, DNA 8-bases and DNA $2^{s+1}k$ -bases, respectively (Cengellenmis and Dertli, 2019; Gursoy et al., 2017; Gursoy et al., 2017).

In this paper, motivated by the previous work (Cengellenmis and Dertli, 2019; Gursoy et al., 2017; Gursoy et al., 2017), we study the reversibility problem for DNA 4-bases , by using the skew cyclic codes over the finite ring $R = Z_4 + uZ_4 + vZ_4 + uvZ_4$, where $u^2 = u, v^2 = v, uv = vu.$ In section 2, some knowledges are given and a new Gray map is introduced. In section 3, by defining a non trivial authomorphism on R, the skew cyclic codes over R are introduced. Thanks to them, the reversible DNA codes are obtained.

2. Material and Methods

The finite ring

$R = Z_4 + uZ_4 + vZ_4 + uvZ_4 =$
$\{a_0 + ua_1 + va_2 + uva_3 \ : \ a_i \in Z_4, i = 0,1,2,3\}$
with $u^2 = u, v^2 = v, uv = vu$ is commutative with characteristic 4 and elements 256.

$$\begin{aligned} R &= Z_4 + uZ_4 + v(Z_4 + uZ_4), \qquad v^2 = v, u^2 = u \\ &= R_1 + vR_1, \qquad v^2 = v \end{aligned}$$

(Kumar and Singh, 2018), the finite ring $R_1 = Z_4 + uZ_4$, where $u^2 = u$ is studied. The reversible DNA codes are obtained with different method. By using the matching the elements of Z_4 and $S_{D_4} = \{A, T, C, G\}$ which is given as $A \to 0, T \to 1, C \to 3, G \to 2$ and by using the Gray map from $R_1 = Z_4 + uZ_4, u^2 = u$ to Z_4^2, $a + ub$ goes to $(a, a+b)$. They define a ξ correspondence between the elements of the finite ring $Z_4 + uZ_4$ and a set of DNA double pairs $S_{D_{16}} = \{AA, TT, CC, GG, ..., AG\}$ as follows

elements α	DNA double pairs $\xi(\alpha)$
0	AA
1	TT
2	GG
3	CC
u	AT
$1+u$	TG
$2+u$	GC
$3+u$	CA
$2u$	AG
$1+2u$	TC
$2+2u$	GA
$3+2u$	CT
$3u$	CA
$1+3u$	TA
$2+3u$	GT
$3+3u$	CG

We define the Gray map as follows,

$$\varphi : R \to R_1^2$$
$$a+ub+v(c+ud) \mapsto (a+ub,(a+c)+u(b+d))$$

By using two matchings and the new Gray map above, we get a matching between the elements of R and a set of DNA 4-bases $S_{D_{256}} = \{AAAA, TTTT, ...\}$ as follows. We are called the matching as Ψ.

$$\Psi : R \to S_{D_{256}}$$
$$a+ub+v(c+ud) \mapsto (\xi(a+ub), \xi(a+c+u(b+d)))$$

where $\Psi = \gamma \circ \varphi$ and

$$\gamma : R_1^2 \to S_{D_{256}}$$
$$(s,t) \mapsto (\xi(s), \xi(t))$$

where $s,t \in R_1$.

3. Research Findings

3.1. Skew cyclic codes over R

Definition 3.1.1: *Let B be a finite ring and θ be a non trivial automorphism on B. A subset C of B^n is called a skew cyclic code of length n if C satisfies the following conditions,*

1) *C is a submodule of B^n*

2) *If $c=(c_0,c_1,...,c_{n-1}) \in C$, then $\sigma_\theta(c) = (\theta(c_{n-1}), \theta(c_0), ..., \theta(c_{n-2})) \in C$, where σ_θ is the skew cyclic shift operator.*

By defining a non trivial automorphism θ on R as follows, we can define skew cyclic codes over R

$$\theta : R \to R$$
$$a+vb \mapsto \theta(a+vb) = \theta^{'}(a+b) - v\theta^{'}(b)$$

where $a,b \in R_1 = Z_4 + uZ_4, u^2 = u$ and $\theta^{'}$ is a non trivial authomorphism on $R_1 = Z_4 + uZ_4$ as follows

$$\theta^{'} : R_1 \to R_1$$
$$p+uz \mapsto (p+z) - uz$$

where $p,z \in Z_4$. The orders of θ and $\theta^{'}$ are 2.

The set of polynomials

$$R[x,\theta] = \{a_0 + a_1x + ... + a_{n-1}x^{n-1} \ : \ a_i \in R, n \in N\}$$

is the skew polynomial ring over R with the usual addition of polynomials and the non-commutative multiplication given by

$$(ax^i)(bx^j) = a\theta^i(b)x^{i+j}.$$

In polynomial representation, a skew cyclic code of length n over R is defined as a left ideal of the quotient ring $R_{\theta,n} = R[x,\theta]/\langle x^n - 1\rangle$, if the order of θ divides n, that is n is even. If the order of θ does not divide n, a skew cyclic code of length n over R is defined as a left $R[x,\theta]$-submodule of $R_{\theta,n}$, since the set

$$R_{\theta,n} = R[x,\theta]/\langle x^n - 1\rangle = \{ f(x) + \langle x^n - 1\rangle \ : \ f(x) \in R[x,\theta]\}$$

is a left $R[x,\theta]$-module with the multiplication from left defined by

$$r(x)(f(x) + \langle x^n - 1\rangle) = r(x)f(x) + \langle x^n - 1\rangle$$

where for any $r(x) \in R[x,\theta]$.

In both case, the following is hold.

Theorem 3.1.2: *Let C be a skew cyclic code over R and let $f(x)$ be a polynomial in C of minimal degree. If the leading coefficient of $f(x)$ is a unit in R, then $C = \langle f(x)\rangle$, where $f(x)$ is a right divisor of $x^n - 1$.*

3.2. Reversible DNA codes from skew cyclic codes over R

Definition 3.2.1: *For $x = (x_0, x_1, ..., x_{n-1}) \in R^n$, the vector $(x_{n-1}, x_{n-2}, ..., x_1, x_0)$ is called the reverse of x and is denoted by x^r. A linear code C of length n over R is called reversible if $x^r \in C$ for every $x \in C$.*

Each element α of R_1 and $\theta'(\alpha)$ are mapped to DNA pairs, which are reverse of each other. For example, $\xi(u) = AT$, while $\xi(\theta'(u)) = TA$.

Similarly, by using a map $\Psi = \gamma \circ \varphi$, we can explain a relationship between skew cyclic codes and DNA codes. $\Psi(\beta)$ and $\Psi(\theta(\beta))$ are DNA reverse of each other, for every $\beta \in R$.

For $\beta = a + vb \in R$, where $a, b \in R_1$, we have

$$\begin{aligned}\Psi(\beta) &= \gamma(\varphi(a+vb)) = \gamma(a, a+b) \\ &= (\xi(a))\xi(a+b))\end{aligned}$$

On the other hand,

$$\begin{aligned}\Psi(\theta(\beta)) &= \Psi(\theta'(a+b) - v\theta'(b)) \\ &= (\xi(\theta'(a+b)), \xi(\theta'(a)))\end{aligned}$$

Example3.2.2: *If $\beta = 1 + u + v(2+3u) \in R$, then we have*

$$\begin{aligned}\Psi(\beta) &= \gamma(\varphi(\beta)) = \gamma(1+u, 1+u+2+3u) \\ &= (\xi(1+u), \xi(3)) = (TG, CC)\end{aligned}$$

On the other hand,

$$\begin{aligned}\Psi(\theta(\beta)) &= \Psi(\theta'(3) - v\theta'(2+3u)) \\ &= \gamma(\theta'(3), \theta'(1-3u)) \\ &= (\xi(\theta'(3)), \xi(\theta'(1-3u))) \\ &= (CC, GT)\end{aligned}$$

This map can be extended as follows. For any $s = (s_0, ..., s_{n-1}) \in R^n$,

$$\Psi(s)^r = (\Psi(s_0), \Psi(s_1), ..., \Psi(s_{n-1}))^r = (\Psi(\theta(s_{n-1})), ..., \Psi(\theta(s_1)), \Psi(\theta(s_0)))$$

Definition 3.2.3: *Let C be a code of length n over R. If $\Psi(c)^r \in \Psi(C)$, for all $c \in C$, then C or equivalently $\Psi(C)$ is called a* *reversible DNA code.*

Definition 3.2.4: *Let* $f(x) = a_0 + a_1 x + ... + a_s x^s$ *be a polynomial of degree* s *over* R. $f(x)$ *is called a palindromic polynomial if* $a_i = a_{s-i}$ *for all* $i \in \{0,1,...,s\}$. $f(x)$ *is called a* θ*-palindromic polynomial if* $a_i = \theta(a_{s-i})$ *for all* $i \in \{0,1,...,s\}$.

The skew cyclic code of odd length over R with respect to θ is a cyclic code, as the order of θ is 2. So we will take the length n to be even.

Theorem 3.2.5: *Let* $C = < f(x) >$ *be a skew cyclic code of length* n *over* R, *where* $f(x)$ *is a right divisor of* $x^n - 1$ *and* $\deg(f(x))$ *is odd. If* $f(x)$ *is a* θ*-palindromic polynomial, then* $\Psi(C)$ *is a reversible DNA code.*

Proof: Let $f(x)$ be a θ-palindromic polynomial and $f(x) = a_0 + a_1 x + ... + a_{2s-1} x^{2s-1}$. So $a_i = \theta(a_{2s-1-i})$, for all $i = 0,1,...,s-1$. Let $h(x) = h_0 + h_1 x + ... h_{2k-1} x^{2k-1}$. Let b_l be the coefficient of x^l in $h(x) f(x)$, where $l = 0,1,..,n-1$. For any $t < n/2$, the coefficient of x^t in $h(x) f(x)$ is

$$b_t = \sum_{j=0}^{t} h_j \theta^j (a_{t-j})$$

and the coefficient of x^{n-t} is $b_{n-t} = \sum_{j=0}^{t} h_{2k-1-j} \theta^{2k-1-j} (a_{2s-1-(t-j)})$.

The polynomial $h(x) f(x) = \sum_{d=0}^{2k-1} h_d x^d f(x)$ corresponds a vector $b = (b_0, b_1, ..., b_{n-1}) \in C$.

The vector $\Psi(b)^r = ((\Psi(b_0), ..., \Psi(b_{n-1})))^r$ is equal to the vector $\Psi(z)$, where the vector z corresponds the polynomial $\sum_{d=0}^{2k-1} \theta(h_d) x^{2k-1-d} f(x)$.

So, $\Psi(C)$ is a reversible DNA code.

Theorem 3.2.6: *Let* $C = < f(x) >$ *be a skew cyclic code of length* n *over* R, *where* $f(x)$ *is a right divisor of* $x^n - 1$ *and* $\deg(f(x))$ *is even. If* $f(x)$ *is a palindromic polynomial then* $\Psi(C)$ *is a reversible DNA code.*

Proof: Let $f(x)$ be a palindromic polynomial with even degree. $f(x) = a_0 + a_1 x + ... + a_{2s} x^{2s}$ and $a_i = a_{2s-i}$, for all $i = 0,1,...,s$. Let $h(x) = h_0 + h_1 x + ... + h_{2k} x^{2k}$. Let b_l be the coefficient of x^l in $h(x) f(x)$, where $l = 0,1,..,n-1$. For any $t < n/2$, the coefficient of x^t in $h(x) f(x)$ is

$$b_t = \sum_{j=0}^{t} h_j \theta^j (a_{t-j})$$

and the coefficient of x^{n-t} is $b_{n-t} = \sum_{j=0}^{t} h_{2k-j} \theta^{2k-j} (a_{2s-(t-j)})$.

The polynomial $h(x) f(x) = \sum_{d=0}^{2k} h_d x^d f(x)$ corresponds a vector $b = (b_0, b_1, ..., b_{n-1}) \in C$.

The vector $\Psi(b)^r = ((\Psi(b_0), ..., \Psi(b_{n-1})))^r$ is equal to the vector $\Psi(z)$, where the vector z corresponds the polynomial $\sum_{d=0}^{2k} \theta(h_d) x^{2k-d} f(x)$.

So, $\Psi(C)$ is a reversible DNA code.

4. Conclusion

By using the skew cyclic codes over R, the reversibility problem for DNA 4-bases is solved. By this means, the reversible DNA codes are obtained.

5. References

Adleman L. 1994. “Molecular computation of the solutions to combinatorial problems”, *Science*, 266,1021-1024.

Bayram A., Oztas E., Siap I. 2015. “Codes over $F_4 + vF_4$ and some DNA applications”, *Designs, Codes and Cryptography*, DOI: 10.107/s10623-015-01008.

Cengellenmis Y., Dertli A. 2020. “On the skew cyclic codes and the reversibility problem for DNA 4-bases”, *Mathematics in Computer Science*, 14, 431-435.

Dertli A., Cengellenmis Y. 2017. “On cyclic DNA codes over the rings $Z_4 + wZ_4$ and $Z_4 + wZ_4 + vZ_4 + wvZ_4$”, *Biomath*, 6, 1712167.

Gursoy F., Oztas S. E., Siap I. 2017. “Reversible DNA codes over $F_{16} + uF_{16} + vF_{16} + uvF_{16}$”, *American Institude of Mathematical Sciences*, 11, 307-312.

Gursoy F., Oztas S. E.,Ozkan A. 2017. “Reversible DNA codes over a family of non-chain ring”, *arXiv:1711.02385v1*.

Head, T. 1987. “Formal language theory and DNA: an analysis of the generative capacity of specific recombinant behaviors”, *Bulletin of mathematical biology*, 49, 737-759.

Kumar, N., Singh A. K. 2018. “DNA computing over the ring $Z_4 + vZ_4$”, *International Journal of Biomathematics* , 11, 1850090.