Gazi University Journal of Science
GU J Sci
26(2):173-179 (2013)

Fixed Point Theorems for Weakly Compatible Mappings in Intuitionistic Fuzzy Metric Spaces

Sunny CHAUHAN[1,♠], B. D. PANT[2], Sandeep BHATT[3]

[1]*Near Nehru Training Centre, H. No. 274, NaiBasti B-14, Bijnor-246701, Uttar Pradesh, INDIA*

[2]*Government Degree College, Champawat-262523, Uttarakhand, INDIA*

[3]*Department of Applied Sciences and Humanities, Govind Ballabh Pant Engineering College,*

Pauri Garhwal-246001, Uttarakhand, INDIA

Received: 19.01.2013 Accepted:01.02.2013

ABSTRACT

Sintunavarat and Kumam [Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces, J. Appl. Math. vol. 2011, Article ID 637958, 14 pages, 2011] defined the notion of (CLRg) property which is more general than (E.A) property. The aim of this paper is to prove a common fixed point theorem for a pair of weakly compatible mappings in intuitionistic fuzzy metric spaces employing (CLRg) property. Our results improve and generalize several previously known fixed point theorems of the existing literature.
Keywords: Intuitionistic fuzzy metric space, property (E.A), common limit range property, weakly compatible mappings, fixed point.
2010 Mathematics Subject Classification: Primary 47H10, Secondary 54H25.

1. INTRODUCTION

The concept of fuzzy sets was initially investigated by Zadeh [36]. As a generalization of fuzzy sets, Atanassov [7] introduced the idea of intuitionistic fuzzy set. Further, Çoker [9] introduced the idea of the topology of intuitionistic fuzzy sets. Mondal and Samanta [21] introduced the definition of the intuitionistic gradation of openness. In 2004, Park [23] introduced and discussed a notion of intuitionistic fuzzy metric spaces (briefly, IFM-spaces) which is based both on the idea of intuitionistic fuzzy sets and the concept of a fuzzy metric space given by George and Veeramani [11].

♠Correspondingauthor, e-mail: sun.gkv@gmail.com

Inspired by the idea of intuitionistic fuzzy sets, Alaca et al. [6] defined the notion of IFM-spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michálek [14]. Further, they [5] proved intuitionistic fuzzy Banach and intuitionistic fuzzy Edelstein contraction theorems, with the different definition of Cauchy sequences and completeness than the ones given in [23]. In 2006, Turkoglu et al. [35] extended the notion of compatible mappings to IFM-spaces. Alaca [3] weakened the notion of compatibility by using the notion of weakly compatible maps in IFM-spaces and showed that every pair of compatible mappings is weakly compatible but reverse is not true. Many authors have proved a number of fixed point theorems for different contractions in IFM-spaces (see [2, 4, 8, 10, 12, 22, 24, 16, 18, 26, 28, 29, 30, 34]).

In 2002, Aamri and El-Moutawakil [1] defined the notion of property (E.A) for self mappings which contained the class of non-compatible mappings in metric spaces. It is observed that the property (E.A) requires the completeness (or closedness) of the underlying subspaces for the existence of common fixed point. In an interesting paper, Sintunavarat and Kumam [31] introduced the notion of 'common limit range property' (briefly, (CLRg) property with respect to mapping g) in fuzzy metric spaces. They showed that the notion of (CLRg) property never requires the condition of the closedness of the subspace (also see [32]).

In 2008, Alaca et al. [6] proved a common fixed point theorem for continuous compatible mappings on complete IFM-space. Further, Kumar [15] and Manro [19] improved and generalized the results of Alaca et al. [6] and Kumar and Vats [17] under strict contractive conditions. Most recently Tanveer et al. [33] proved common fixed point theorems for weakly compatible mappings in modified IFM-spaces using common property (E.A).

The object of this paper is prove a common fixed point theorem for a pair of weakly compatible mappings in IFM-space by using the notion of (CLRg) property. We also present a result for two finite families of self mappings by using pairwise commuting due to Imdad et al. [13].

2. PRELIMINARIES

Definition 2.1 [27] A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $*$ satisfies the following conditions:

(1) $*$ is commutative and associative,
(2) $*$ is continuous,
(3) $a * 1 = a, \forall a \in [0,1]$,
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $\forall a, b, c, d \in [0,1]$.

Examples of t-norms are $a * b = \min\{a, b\}$ and $a * b = ab$.

Definition 2.2 [27] A binary operation $\diamond: [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $*$ satisfies the following conditions:

(1) $\diamond$ is commutative and associative,
(2) $\diamond$ is continuous,
(3) $a \diamond 0 = a, \forall a \in [0,1]$,
(4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, $\forall a, b, c, d \in [0,1]$.

Examples of t-norms are $a \diamond b = \max\{a, b\}$ and $a \diamond b = \min\{1, a + b\}$.

Remark 2.1 The concepts of t-norms and t-conorms are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [20] in his study of statistical metric spaces.

Atanassov [7], Kramosil and Michálek [14] and Alaca et al. [5] defined the following definition in framework of IFM-spaces:

Definition 2.3 A 5-tuple $(X, M, N, *, \diamond)$ is said to be an IFM-space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: $\forall x, y, z \in X, t, s > 0$

(1) $M(x, y, t) + N(x, y, t) \leq 1$,
(2) $M(x, y, 0) = 0$,
(3) $M(x, y, t) = 1$ iff $x = y$,
(4) $M(x, y, t) = M(y, x, t)$,
(5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(6) $M(x, y, \cdot): (0, \infty) \to [0,1]$ is left continuous,
(7) $\lim_{n \to \infty} M(x, y, t) = 1$,
(8) $N(x, y, 0) = 1$,
(9) $N(x, y, t) = 0$ iff $x = y$,
(10) $N(x, y, t) = N(y, x, t)$,
(11) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
(12) $N(x, y, \cdot): (0, \infty) \to [0,1]$ is right continuous,
(13) $\lim_{n \to \infty} N(x, y, t) = 0$,

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.2 Every fuzzy metric space $(X, M, *)$ is an IFM-space of the form $(X, M, 1 - M, *\diamond)$ such that t-norm $*$ and t-conorms $\diamond$ are associated, that is, $x \diamond y = 1 - ((1 - x) * (1 - y)), \forall x, y \in X$.

Example 2.1 [23] (Induced intuitionistic fuzzy metric) Let (X, d) be a metric space. Denote $a * b = ab$ and $a \diamond b = \min\{1, a + b\}, \forall a, b \in [0,1]$ and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = \frac{ht^n}{ht^n + md(x, y)}, N_d(x, y, t) = \frac{d(x, y)}{kt^n + md(x, y)},$$

$\forall h, k, m, n \in \mathbb{N}$. Then $(X, M, N, *, \diamond)$ is an IFM-space.

Remark 2.3 [23] Note the above example holds even with the t-norm $a * b = \min\{a, b\}$ and the t-conorm

$a \diamond b = \max\{a, b\}$ and hence (M, N) is an intuitionistic fuzzy metric with respect to any continuous t-norm and continuous t-conorm. In the above example by taking $h = k = m = n = 1$, we get

$$M_d(x, y, t) = \frac{t}{t + md(x,y)}, N_d(x, y, t) = \frac{d(x,y)}{t + md(x,y)}.$$

Then $(X, M, N, *, \diamond)$ is an IFM-space induced by the metric d. It is obvious that $N(x, y, t) = 1 - M(x, y, t)$.

Remark 2.4 In IFM-space $(X, M, N, *, \diamond)$, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing $\forall x, y \in X$.

The following definition is on the lines of Aamri and Moutawakil [1].

Definition 2.4 A pair (f, g) of self mappings defined on an IFM-space $(X, M, N, *, \diamond)$ is said to satisfy the property (E.A), if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z,$$

for some $z \in X$.

Definition 2.5 [3] A pair (f, g) of self mappings defined on a non-empty set X is said to be weakly compatible if they commute at their coincidence points, that is, if $fx = gx$ for some $x \in X$ then $fgx = gfx$.

Remark 2.5 The notions of weak compatibility and property (E.A) are independent to each other (see [25, Example 2.2]).

Inspired by the work of Sintunavarat and Kumam [31], we defined the notion of (CLRg) property in framework of IFM-space.

Definition 2.6 A pair (f, g) of self mappings defined on an IFM-space $(X, M, N, *, \diamond)$ is said to satisfy the (CLRg) property, if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu,$$

for some $u \in X$.

Example 2.2 Let $X = [0, \infty)$ with metric d defined by $d(x, y) = |x - y|$ and define

$$M(x, y, t) = \begin{cases} \frac{t}{t + |x - y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

$$N(x, y, t) = \begin{cases} \frac{|x - y|}{t + |x - y|}, & \text{if } t > 0; \\ 1, & \text{if } t = 0, \end{cases}$$

$\forall x, y \in X$. Clearly $(X, M, N, *, \diamond)$ be an IFM-space where $*$ and $\diamond$ are continuous t-norm and continuous t-conorm defined by $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$, $\forall a, b \in [0,1]$. Define the self mappings f and g on X by $f(x) = x + 3$ and $g(x) = 4x$, $\forall x \in X$.

Consider a sequence $\{x_n\} = \left\{1 + \frac{1}{n}\right\}_{n\in\mathbb{N}}$ in X, we have

$$\lim_{n\to\infty} f\left(1 + \frac{1}{n}\right) = \lim_{n\to\infty}\left(4 + \frac{1}{n}\right) = 4 = g(1) = \lim_{n\to\infty}\left(4 + \frac{4}{n}\right) = \lim_{n\to\infty} g\left(1 + \frac{1}{n}\right),$$

which shows that the pair (f, g) enjoys the (CLRg) property.

Example 2.3 The conclusion of Example 2.2 remains true if we replace the self mappings f and g by $f(x) = \frac{x}{4}$ and $g(x) = \frac{3x}{4}$, $\forall x \in X$. Consider a sequence $\{x_n\} = \left\{\frac{1}{n}\right\}_{n\in\mathbb{N}}$ in X, we have

$$\lim_{n\to\infty} f\left(\frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{1}{4n}\right) = 0 = g(0) = \lim_{n\to\infty}\left(\frac{3}{4n}\right) = \lim_{n\to\infty} g\left(\frac{1}{n}\right),$$

therefore, the mappings f and g satisfy the (CLRg) property.

Lemma 2.1 [29] Let $(X, M, N, *, \diamond)$ be an IFM-space and $\forall x, y \in X, t > 0$ and if for a constant $k \in (0,1)$

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t),$$

then $x = y$.

Definition 2.7 [13] Two families of self mappings $\{f_i\}_{i=1}^{m}$ and $\{g_k\}_{k=1}^{n}$ are said to be pairwise commuting if

(1) $f_i f_j = f_j f_i$, $\forall i, j \in \{1, 2, \ldots, m\}$,
(2) $g_k g_l = g_l g_k$, $\forall k, l \in \{1, 2, \ldots, n\}$,
(3) $f_i g_k = g_k f_i$, $\forall i \in \{1, 2, \ldots, m\}$ and $\forall k \in \{1, 2, \ldots, n\}$.

3. RESULTS

Theorem 3.1 Let $(X, M, N, *, \diamond)$ be an IFM-space with $t * t \geq t$ and $(1 - t) \diamond (1 - t) \leq (1 - t)$, $\forall t \in [0,1]$. Further, let the pair (f, g) of self mappings is weakly compatible satisfying

$$M(fx, fy, kt) \geq \left\{ \begin{array}{c} M(gx, gy, t) * M(fx, gx, t) * M(fy, gy, t) \\ * M(fx, gy, t) * M(fy, gx, t) \end{array} \right\} \tag{3.1}$$

and

$$N(fx, fy, kt) \leq \left\{ \begin{array}{c} N(gx, gy, t) \diamond N(fx, gx, t) \diamond N(fy, gy, t) \\ \diamond N(fx, gy, t) \diamond N(fy, gx, t) \end{array} \right\} \tag{3.2}$$

$\forall x, y \in X$, $k \in (0,1)$ and $t > 0$. If f and g enjoy the (CLRg) property then f and g have a unique common fixed point in X.

Proof. Since the pairs (f, g) satisfies the (CLRg) property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu,$$

for some $u \in X$. Now we assert that $fu = gu$. On using inequalities (3.1) and (3.2) with $x = x_n$, $y = u$, we get

$$M(fx_n, fu, kt) \geq \left\{ \begin{array}{c} M(gx_n, gu, t) * M(fx_n, gx_n, t) * M(fu, gu, t) \\ * M(fx_n, gu, t) * M(fu, gx_n, t) \end{array} \right\},$$

and

$$N(fx_n, fu, kt) \leq \left\{ \begin{matrix} N(gx_n, gu, t) \diamond N(fx_n, gx_n, t) \diamond N(fu, gu, t) \\ \diamond N(fx_n, gu, t) \diamond N(fu, gx_n, t) \end{matrix} \right\},$$

Taking limit as $n \to \infty$, we have

$$M(gu, fu, kt) \geq \left\{ \begin{matrix} M(gu, gu, t) * M(gu, gu, t) * M(fu, gu, t) \\ * M(gu, gu, t) * M(fu, gx_n, t) \end{matrix} \right\},$$

and

$$N(gu, fu, kt) \leq \left\{ \begin{matrix} N(gu, gu, t) \diamond N(gu, gu, t) \diamond N(fu, gu, t) \\ \diamond N(gu, gu, t) \diamond N(fu, gu, t) \end{matrix} \right\}.$$

It implies

$$M(fu, gu, kt) \geq \{1 * 1 * M(fu, gu, t) * 1 * M(fu, gx_n, t)\} = M(fu, gu, t),$$

and

$$N(fu, gu, kt) \leq \{0 \diamond 0 \diamond N(fu, gu, t) \diamond 0 \diamond N(fu, gu, t)\} = N(fu, gu, t).$$

In view of Lemma 2.1, we have $fu = gu$. Now we let $z = fu = gu$. Since the pair (f, g) is weakly compatible, we get $fz = fgu = gfu = gz$. Now we show that z is a common fixed point of the mappings f and g. To prove this, using inequalities (3.1) and (3.2) with $x = z$, $y = u$, we have

$$M(fz, fu, kt) \geq \left\{ \begin{matrix} M(gz, gu, t) * M(fz, gz, t) * M(fu, gu, t) \\ * M(fz, gu, t) * M(fu, gz, t) \end{matrix} \right\},$$

and

$$N(fz, fu, kt) \leq \left\{ \begin{matrix} N(gz, gu, t) \diamond N(fz, gz, t) \diamond N(fu, gu, t) \\ \diamond N(fz, gu, t) \diamond N(fu, gz, t) \end{matrix} \right\}.$$

It implies

$$M(fz, z, kt) \geq \left\{ \begin{matrix} M(fz, z, t) * 1 * 1 \\ * M(fz, z, t) * M(z, fz, t) \end{matrix} \right\} = M(fz, z, t),$$

and

$$N(fz, z, kt) \leq \left\{ \begin{matrix} N(fz, z, t) \diamond 0 \diamond 0 \\ \diamond N(fz, z, t) \diamond N(z, fz, t) \end{matrix} \right\} = N(fz, z, t).$$

On employing Lemma 2.1, we get $z = fz = gz$ which shows that z is a common fixed point of the mappings f and g.

Uniqueness: Let w be another common fixed point of the mappings f and g. On using inequalities (3.1) and (3.2) with $x = z$, $y = w$, we have

$$M(fz, fw, kt) \geq \left\{ \begin{matrix} M(gz, gw, t) * M(fz, gz, t) * M(fw, gw, t) \\ * M(fz, gw, t) * M(fw, gz, t) \end{matrix} \right\},$$

and

$$N(fz, fw, kt) \leq \left\{ \begin{matrix} N(gz, gw, t) \diamond N(fz, gz, t) \diamond N(fw, gw, t) \\ \diamond N(fz, gw, t) \diamond N(fw, gz, t) \end{matrix} \right\},$$

or, equivalently,

$$M(z, w, kt) \geq \left\{ \begin{matrix} M(z, w, t) * M(z, z, t) * M(w, w, t) \\ * M(z, w, t) * M(w, z, t) \end{matrix} \right\} = M(z, w, t)$$

and

$$N(z, w, kt) \leq \left\{ \begin{matrix} N(z, w, t) \diamond N(z, z, t) \diamond N(w, w, t) \\ \diamond N(z, w, t) \diamond N(w, z, t) \end{matrix} \right\} = N(z, w, t).$$

Appealing to Lemma 2.1, we have $z = w$. Therefore the mappings f and g have a unique common fixed point in X.

Remark 3.1 From the proof of Theorem 3.1, it is asserted that (CLRg) property never requires the completeness (or closedness) of the underlying subspace and containment of ranges amongst involved mappings.

Example 3.1 Let $X = [1, 15)$ with metric d defined by $d(x, y) = |x - y|$ and define

$$M(x, y, t) = \begin{cases} \frac{t}{t + |x - y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

$$N(x, y, t) = \begin{cases} \frac{|x - y|}{t + |x - y|}, & \text{if } t > 0; \\ 1, & \text{if } t = 0, \end{cases}$$

$\forall x, y \in X$. Then $(X, M, N, *, \diamond)$ is an IFM-space where $*$ and $\diamond$ are continuous t-norm and continuous t-conorm defined by $a * b = \min\{a, b\}$and $a \diamond b = \max\{a, b\}$, $\forall a, b \in [0, 1]$. Now we define the self mappings$f$ and g on X by

$$f(x) = \begin{cases} 1, & \text{if } x \in \{1\} \cup (3, 15); \\ 8, & \text{if } x \in (1, 3]. \end{cases}$$

$$g(x) = \begin{cases} 1, & \text{if } x = 1; \\ 7, & \text{if } x \in (1, 3]; \\ \frac{x+1}{4}, & \text{if } x \in (3, 15). \end{cases}$$

Consider a sequence $\{x_n\} = \left\{3 + \frac{1}{n}\right\}_{n \in \mathbb{N}}$or $\{x_n\} = 1$. Then we have

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = 1 = g(1) \in X.$$

Hence the pair (f, g) enjoys the (CLRg) property. It is noticed that $f(X) = \{1, 8\} \not\subset [1, 4) \cup \{7\} = g(X)$. Here $g(X)$ is not a closed subspace of X. Thus all the conditions of Theorem 3.1 are satisfied for some $k \in (0, 1)$ and 1 is a unique common fixed point of the mappings f and g. Also all the involved mappings are discontinuous at their unique common fixed point.

Our next theorem is proved for a pair of weakly compatible mappings by using the notion of property (E.A) under additional assumption of closedness of the underlying subspace.

Theorem 3.2 Let $(X, M, N, *, \diamond)$ be an IFM-space with $t * t \geq t$and $(1 - t) \diamond (1 - t) \leq (1 - t)$, $\forall t \in [0, 1]$. Further, let the pair (f, g) of self mappings is weakly compatible satisfying inequalities (3.1) and (3.2) of Theorem 3.1. If f and g enjoy the property (E.A) and $g(X)$ is a closed subspace of X then f and g have a unique common fixed point in X.

Proof. If the pairs(f, g) satisfies the property (E.A), then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = z,$$

for some $z \in X$. Since $g(X)$ is a closed subspace of X, there exists a point $u \in X$ such that $gu = z$ which shows that the pair (f,g) satisfies the (CLRg) property. The rest of the proof can be completed on the lines of the proof of Theorem 3.1. Therefore the mappings f and g have a unique common fixed point in X.

Example 3.2 In the setting of Example 3.1, replace the mapping g by the following: besides retaining the rest

$$g(x) = \begin{cases} 1, & \text{if } x = 1; \\ 4, & \text{if } x \in (1,3]; \\ \frac{x+1}{4}, & \text{if } x \in (3,15). \end{cases}$$

Choose a sequence $\{x_n\} = \left\{3 + \frac{1}{n}\right\}_{n \in \mathbb{N}}$ or $\{x_n\} = 1$. Then we have

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = 1 \in X.$$

It implies the pair (f,g) satisfies the property (E.A). Also $f(X) = \{1,8\} \not\subset [1,4] = g(X)$. Here $g(X)$ is a closed subspace of X. Thus all the conditions of Theorem 3.2 are satisfied for some $k \in (0,1)$ and 1 is a unique common fixed point of the mappings f and g.

Since the pair of non-compatible mappings implies to the pair satisfying the property (E.A), we get the following results:

Corollary 3.1 Let $(X,M,N,*,\diamond)$ be an IFM-space with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$, $\forall t \in [0,1]$. Further, let the pair (f,g) of self mappings is weakly compatible satisfying inequalities (3.1) and (3.2) of Theorem 3.1. If f and g are non-compatible and $g(X)$ is a closed subspace of X then f and g have a unique common fixed point in X.

Now we untilize Definition 2.7 (which is a natural extension of commutativity condition to two finite families) to define a new result in IFM-space.

Theorem 3.3 Let $\{f_i\}_{i=1}^{m}$ and $\{g_j\}_{j=1}^{n}$ be two finite families of self mappings in IFM-space $(X,M,N,*,\diamond)$ with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$, $\forall t \in [0,1]$ such that $f = f_1 f_2 \cdots f_m$ and $g = g_1 g_2 \cdots g_n$ which satisfy the inequalities (3.1) and (3.2). Suppose that the pair (f,g) enjoys the (CLRg) property.

Moreover $\{f_i\}_{i=1}^{m}$ and $\{g_j\}_{j=1}^{n}$ have a unique common fixed point provided the pair of families (f_i, g_j) commutes pairwise where $i = \{1,2,\ldots,m\}$ and $j = \{1,2,\ldots,n\}$.

Proof. The proof of this theorem is similar to Theorem 4.1 contained in [12], hence we did not include the details.

Putting $f_1 = f_2 = \cdots = f_m = f$ and $g_1 = g_2 = \cdots = g_n = g$ in Theorem 4.1, we get the following natural result:

Corollary 3.2 Let f and g be two self mappings in IFM-space $(X,M,N,*,\diamond)$ with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$, $\forall t \in [0,1]$. Further, let the pair (f^m, g^n) of self mappings enjoys the (CLRg^n) property. If there exists a constant $k \in (0,1)$ such that

$$M(fx,fy,kt) \geq \left\{ \begin{matrix} M(gx,gy,t) * M(fx,gx,t) * M(fy,gy,t) \\ * M(fx,gy,t) * M(fy,gx,t) \end{matrix} \right\} \quad (3.3)$$

and

$$N(fx,fy,kt) \leq \left\{ \begin{matrix} N(gx,gy,t) \diamond N(fx,gx,t) \diamond N(fy,gy,t) \\ \diamond N(fx,gy,t) \diamond N(fy,gx,t) \end{matrix} \right\} \quad (3.4)$$

$\forall x,y \in X$, $t > 0$ and m,n are fixed positive integers, then f and g have a unique common fixed point in X provided the pair (f^m, g^n) commutes pairwise.

Remark 3.2 The results similar to Theorem 3.3 and Corollary 3.2 can be outlined in view of Theorem 3.2 and Corollary 3.1. The details of possible corollaries are not included here.

CONCLUSION

Theorem 3.1 is proved for a pair of weakly compatible mappings using the (CLRg) property in IFM-space without any requirement on containment of ranges amongst the involved mappings and completeness of the whole space (or closedness of any subspace). Theorem 3.1 improves the results of Kumar [15, Theorem 3.3] and Tanveer et al. [33, Corollary 3.2]. Theorem 3.3 and Corollary 3.2 also generalize the main results of Alaca et al. [6, Theorem 18] and Huang et al. [12] as the conditions of completeness (or closedness) of the underlying subspace, continuity of one or more mappings and containment of ranges amongst involved mappings are completely relaxed.

ACKNOWLEDGEMENT

The authors are thankful to Professor CihangirAlaca for the reprints of his valuable papers [3, 5] and the editorial board of "Gazi University Journal of Science" for supporting this research work.

REFERENCES

[1] Aamri,M.,Moutawakil, D.El, "Some new common fixed point theorems under strict contractive conditions", ***J. Math. Anal. Appl.,*** 270(1):181-188(2002).

[2] Abu-Donia,H.M.,Nase,A. A., "Common fixed point theorems in intuitionistic fuzzy metric spaces", ***MohuXitongyuShuxue,*** 22(2):100-106(2008). MR2428179

[3] Alaca, C., "A common fixed point theorem for weak compatible mappings in intuitionistic fuzzy metric spaces", ***Int. J. Pure Appl. Math.,*** 32(4):537-548(2006). MR2275086

[4] Alaca,C., "On fixed point theorems in intuitionistic fuzzy metric spaces", ***Commun. Korean Math. Soc.,*** 24(4):565-579(2009).

[5] Alaca,C.,Turkoglu,D.,Yildiz,C., "Fixed points in intuitionistic fuzzy metric spaces", ***Chaos Solit. Fract.,*** 29(5):1073-1078(2006).

[6] Alaca, C.,Turkoglu,D.,Yildiz,C., "Common fixed points of compatible maps in intuitionistic fuzzy metric spaces", ***Southeast Asian Bull. Math.,*** 32(1):21-33(2008).

[7] Atanassov, K.,"Intuitionistic fuzzy sets", ***Fuzzy Sets Syst.,*** 20(1):87-96(1986).

[8] Ćiŕic,LjB.,Ješić,S.N.,Ume, J.S., "The existence theorems for fixed and periodic points of nonexpansive mappings in intuitionistic fuzzy metric spaces", ***Chaos Solit. Fract.,*** 37(3):781-791(2008).

[9] Çoker,D., "An introduction to intuitionistic fuzzy topological spaces", ***Fuzzy Sets Syst.,*** 88(1):81-89(1997).

[10] Dimri,R.C.,Gariya,N.S., "Coincidences and common fixed points in intuitionistic fuzzy metric spaces", ***Indian J. Math.,*** 52(3):479-490(2010).

[11] George,A.,Veeramani,P., "On some results in fuzzy metric spaces", ***Fuzzy Sets Syst.,*** 64(3):395-399(1994).

[12] Huang,X., Zhu,C.,Wen, Xi, "Common fixed point theorems for families of compatible mappings in intuitionistic fuzzy metric spaces", ***Ann. Univ. Ferrara Sez. VII Sci. Mat.,*** 56(2):305-326(2010).

[13] Imdad, M.,Ali,J.,Tanveer,M., "Coincidence and common fixed point theorems for nonlinear contractions in Menger PM spaces", ***Chaos Solit. Fract.,*** 42(5):3121-3129(2009).

[14] Kramosil, I.,Michálek, J., "Fuzzy metrics and statistical metric spaces", ***Kybernetika (Prague),*** 11(5):336-344(1975).

[15] Kumar, S., "Common fixed point theorems in intuitionistic fuzzy metric spaces using property (E.A)", ***J. Indian Math. Soc. (N.S.),*** 76(1-4):93-104(2009).

[16] Kumar,S.,Kutukcu, S., "Fixed points of expansion maps in intuitionistic fuzzy metric spaces", ***Sci. Stud. Res. Ser. Math. Inform.,*** 20(1):119-132(2010).

[17] Kumar,S.,Vats, R.K., "Common fixed points for weakly compatible maps in intuitionistic fuzzy metric spaces", ***Adv. Fuzzy Math.,*** 4(1):9-22(2009).

[18] Kutukcu, S.,Yildiz,C.,Turkoglu, D., "Fixed points of contractive mappings in intuitionistic fuzzy metric spaces", ***J. Comput. Anal. Appl.,*** 9(2):181-193(2007).

[19] Manro,S.,Bhatia,S.S.,Kumar, S., "Common fixed point theorem for weakly compatible maps satisfying (E.A.) property in intuitionistic fuzzy metric spaces", ***Punjab Univ. J. Math. (Lahore),*** 42:51-56(2010).

[20] Menger,K., "Statistical metrics", ***Proc. Nat. Acad. Sci. U.S.A.,*** 28:535-537(1942).

[21] Mondal,T.K.,Samanta,S. K., "On intuitionistic gradation of openness", ***Fuzzy Sets Syst.,*** 131(2):323-336(2002).

[22] Pant, B.D.,Kumar,S.,Chauhan, S., "Common fixed point of weakly compatible maps on intuitionistic fuzzy metric spaces", ***J. Adv. Stud. Topol.,*** 1(1):41-49(2010).

[23] Park, J. H., "Intuitionistic fuzzy metric spaces", ***Chaos Solit. Fract.,*** 22(5):1039-1046(2004).

[24] Park,J. S., "Some results on intuitionistic fuzzy metric spaces", ***Far East J. Math. Sci. (FJMS),*** 33(1):65-75(2009).

[25] Pathak, H. K.,López,R.R.,Verma, R.K., "A common fixed point theorem using implicit relation and property (E.A) in metric spaces", ***Filomat,*** 21(2):211-234(2007).

[26] Saadati, R.,Park,J. H., "On the intuitionistic fuzzy topological spaces", ***Chaos, Solit. Fract.,*** 27(2):331-344(2006).

[27] Schweizer,B.,Sklar, A., "Probabilistic Metric Spaces", North-Holland Series in Probability and Applied Mathematics",***North-Holland Publishing Co., New York,*** ISBN: 0-444-00666-4(1983).

[28] Sedghi,S.,Shobe,N.,Aliouche, A., "Common fixed point theorems in intuitionistic fuzzy metric spaces through conditions of integral type", ***Appl. Math. Inf. Sci.,*** 2(1):61-82(2008).

[29] Sharma,S.,Deshpande,B., "Common fixed point theorems for finite number of mappings without continuity and compatibility on intuitionistic fuzzy metric spaces", ***Chaos Solit. Fract.,*** 40(5):2242-2256(2009).

[30] Sharma,S.,Kutukcu,S.,Rathore, R.S., "Common fixed point for multivalued mappings in intuitionistic fuzzy metric spaces", ***Commun. Korean Math. Soc.,*** 22(3):391-399(2007).

[31] Sintunavarat,W.,Kumam,P., "Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces", ***J. Appl. Math.,*** Vol. 2011, Article ID 637958, 14 pages, (2011).

[32] Sintunavarat,W., Kumam, P., "Common fixed points for R-weakly commuting in fuzzy metric spaces", ***Ann. Univ. Ferrara Sez. VII Sci. Mat.,*** 58(2):389-406(2012).

[33] Tanveer,M.,Imdad,M.,Gopal,D., Kumar, D., "Common fixed point theorems in modified intuitionistic fuzzy metric spaces with common property (E.A)", ***Fixed Point Theory Appl.,*** 2012, 2012:36, 17 pages.

[34] Turkoglu,D.,Alaca, C., Cho, Y.J.,Yildiz,C., "Common fixed point theorems in intuitionistic fuzzy metric spaces", ***J. Appl. Math. Comput.,*** 22(1-2):411-424(2006).

[35] Turkoglu,D.,Alaca,C.,Yildiz,C., "Compatible maps and compatible maps of type (α) and (β) in intuitionistic fuzzy metric spaces", ***Demonstratio Math.,*** 39(3):671-684(2006).

[36] Zadeh, L.A., "Fuzzy sets", ***Inform. Control,*** 8:338-353(1965).