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A Common Tripled Fixed Point Theorem For Jungck Type Mappings In G-Metric Spaces

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ABSTRACT

In this paper, we prove a common tripled fixed point theorem for Jungck type mappings in G-metric spaces. We give an example to illustrate our result.

Keywords: G-metric space, Jungck type maps, tripled fixed point, W*-compatible maps..
MSC: 54H25, 47H10.

1. INTRODUCTION

Dhage [1-4] introduced the notion of D-metric space as a generalization of usual metric space. Recently, Mustafa and Sims [29,30] and Naidu, Rao and Rao [23-25] have shown that most of the results concerning Dhage's D-metric spaces are invalid. In [30], they introduced an improved version of the generalized metric space structure which they called G-metric spaces. For more results on G-metric spaces, one can refer to the papers [6-9,12,18,19,21,22,28,31- 36].

Very recently, Berinde and Borcut [26] proved some tripled fixed and coincidence point theorems for contractive type mappings in partially ordered metric spaces. Later several authors obtained coincidence and common tripled fixed point theorems in various spaces, for example, refer [5,8-11,13-17,20,26,27].

In this paper, we establish a unique common tripled fixed point theorem for Jungck type mappings in G-metric spaces. Also we give an example to illustrate our result.

Now, we give some basic definitions and results which are used throughout the paper.

Definition 1.1 ([30]). Let X be a nonempty set and let $G : X \times X \times X \rightarrow R^+$ be a function satisfying the following properties :
(G_1): $G(x, y, z) = 0$ if $x = y = z$,
(G_2): $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,
(G_3): $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,
(G_4): $G(x, y, z) = G(x, z, y) = G(y, z, x) = ...$, symmetry in all three variables,
(G_5): $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.
Then the function G is called a generalized metric or a G-metric on X and the pair (X, G) is called a G-metric space.

Proposition 1.2 ([30]). Let (X, G) be a G-metric space. Then for any $x, y, z, a \in X$, it follows that

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(i) if $G(x, y, z) = 0$ then $x = y = z$,
(ii) $G(x, y, z) \le G(x, x, y) + G(x, x, z)$,
(iii) $G(x, y, y) \le 2G(x, x, y)$,
(iv) $G(x, y, z) \le G(x, a, z) + G(a, y, z)$,
(v) $G(x, y, z) \le \frac{2}{3}\,[G(x, a, a) + G(y, a, a) + G(z, a, a)]$.

Definition 1.3 ([30]). Let (X, G) be a G-metric space and $\{x_n\}$ be a sequence in X. A point $x \in X$ is said to be limit of $\{x_n\}$ if $\lim_{n, m\to\infty} G(x, x_n, x_m) = 0$. In this case, the sequence $\{x_n\}$ is said to be G-convergent to x.

Definition 1.4 ([30]). Let (X, G) be a G-metric space and $\{x_n\}$ be a sequence in X. $\{x_n\}$ is called G-Cauchy iff $\lim_{l,n, m\to\infty} G(x_l, x_n, x_m) = 0$ or equivalently $\lim_{n, m\to\infty} G(x_n, x_n, x_m) = 0$.
(X, G) is called G-complete if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).

Proposition 1.5 ([30]). Let (X, G) be a G-metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.

Proposition 1.6 ([30]). Let (X, G) be a G-metric space. Then for a sequence $\{x_n\} \subseteq X$ and a point $x \in X$, the following are equivalent
(i) $\{x_n\}$ is G-convergent to x,
(ii) $G(x_n, x_n, x) \to 0$ as $n \to \infty$,
(iii) $G(x_n, x, x) \to 0$ as $n \to \infty$,
(iv) $G(x_m, x_n, x) \to 0$ as $m, n \to \infty$.

Definition 1.7 ([30]). A G-metric space (X, G) is said to be symmetric if $G(x, y, y) = G(x, x, y)$ for all $x, y \in X$.

Definition 1.8 ([30]). Let (X, G) and (X', G') be two G-metric spaces and let $f : (X, G) \to (X', G')$ be a function then f is said to be G-continuous at a point $a \in X$ if and only if, given $\varepsilon > 0$, there exists $\delta > 0$ such that $x, y \in X$ and $G(a, x, y) < \delta$ implies $G'(f(a), f(x), f(y)) < \varepsilon$. A function f is G-continuous at X if and only if it is G-continuous at all $a \in X$.

Now we give the following definitions.

Definition 1.9. An element $(x, y, z) \in X \times X \times X$ is called

(i) a tripled coincident point of $S, T : X \times X \times X \to X$ if $S(x, y, z) = T(x, y, z)$, $S(y, z, x) = T(y, z, x)$ and $S(z, x, y) = T(z, x, y)$ and $(S(x, y, z), S(y, z, x), S(z, x, y))$ is called a tripled point of coincidence of S and T.
(ii) a common tripled fixed point of $S, T : X \times X \times X \to X$ if $x = S(x, y, z) = T(x, y, z)$, $y = S(y, z, x) = T(y, z, x)$ and $z = S(z, x, y) = T(z, x, y)$.

Definition 1.10. The mappings $S, T : X \times X \times X \to X$ are called W*-compatible if $S(T(x, y, z), T(y, z, x), T(z, x, y)) = T(S(x, y, z), S(y, z, x), S(z, x, y))$ whenever $S(x, y, z) = T(x, y, z)$, $S(y, z, x) = T(y, z, x)$ and $S(z, x, y) = T(z, x, y)$.

Let R and R^+ denote the set of all real numbers and set of all non-negative real numbers respectively.

Example 1.11. Let $X = R$ and $S, T : X \times X \times X \to X$ be defined by $S(x, y, z) = 3x + 2y + 4z - 10$ and $T(x, y, z) = 4x + 3y + 5z - 14$ for all $x, y, z \in X$. Then (1, 2, 1) is a tripled coincidence point of S and T. Clearly (1, 2, 3) is a tripled point of coincidence of S and T. In this example S and T are not W*-compatible.

Example 1.12. Let $X = R$ and $S, T : X \times X \times X \to X$ be defined by $S(x, y, z) = 3x + 2y + 2z - 12$ and $T(x, y, z) = 4x + 3y + 3z - 18$ for all $x, y, z \in X$. Clearly, S and T are W*-compatible and (1, 2, 3) is a common tripled fixed point of S and T.

2. MAIN RESULT

Let Φ be the set of all continuous and non-decreasing functions $\varphi : R^+ \to R^+$ such that $\lim_{n\to\infty} \varphi^n(t) = 0$ for all $t > 0$. Then $\varphi(t) < t$ for all $t > 0$ and $\varphi(0) = 0$.

Theorem 2.1. Let (X, G) be a G-metric space and $S, T : X \times X \times X \to X$ be mappings satisfying

(2.1.1) $$G(S(x, y, z), S(x, y, z), S(u, v, w)) \le \varphi\left(\max\begin{Bmatrix} G(T(x, y, z), T(x, y, z), T(u, v, w)), \\ G(T(y, z, x), T(y, z, x), T(v, w, u)), \\ G(T(z, x, y), T(z, x, y), T(w, u, v)) \end{Bmatrix}\right)$$

for all $x, y, z, u, v, w \in X$, where $\varphi \in \Phi$,

(2.1.2) $S(X \times X \times X) \subseteq T(X \times X \times X)$,

(2.1.3) $\{(T(x, y, z), T(y, z, x), T(z, x, y))/x, y, z \in X\}$ is a complete subspace of $X \times X \times X$ and

(2.1.4) the pair (S, T) is W*-compatible.

Then S and T have a unique common tripled fixed point in $X \times X \times X$ of the form (t, t, t).

Proof. Let x_0, y_0, z_0 be arbitrary points in X. From (2.1.2), there exist sequences $\{x_n\}$, $\{y_n\}$, $\{z_n\}$, $\{u_n\}$, $\{v_n\}$ and $\{w_n\}$ in X such that

$$u_n = S(x_n, y_n, z_n) = T(x_{n+1}, y_{n+1}, z_{n+1}),$$
$$v_n = S(y_n, z_n, x_n) = T(y_{n+1}, z_{n+1}, x_{n+1}), \quad (1)$$
$$w_n = S(z_n, x_n, y_n) = T(z_{n+1}, x_{n+1}, y_{n+1}), \quad n = 0, 1, 2, \cdots.$$

Case(I): Assume that $u_n \ne u_{n+1}$ or $v_n \ne v_{n+1}$ or $w_n \ne w_{n+1}$ for all n.

Now

$$G(u_n,u_n,u_{n+1})=G(S(x_n,y_n,z_n),S(x_n,y_n,z_n),S(x_{n+1},y_{n+1},z_{n+1}))$$
$$\le \varphi\left(\max\left\{\begin{array}{c}G(u_{n-1},u_{n-1},u_n),\ G(v_{n-1},v_{n-1},v_n)\\ G(w_{n-1},w_{n-1},w_n)\end{array}\right\}\right).$$

Similarly, we have

$$G(v_n,v_n,v_{n+1})$$
$$\le \varphi\left(\max\left\{\begin{array}{c}G(u_{n-1},u_{n-1},u_n),\ G(v_{n-1},v_{n-1},v_n)\\ G(w_{n-1},w_{n-1},w_n)\end{array}\right\}\right),$$

$$G(w_n,w_n,w_{n+1})$$
$$\le \varphi\left(\max\left\{\begin{array}{c}G(u_{n-1},u_{n-1},u_n),\ G(v_{n-1},v_{n-1},v_n)\\ G(w_{n-1},w_{n-1},w_n)\end{array}\right\}\right)$$

Thus

$$\max\left\{\begin{array}{c}G(u_n,u_n,u_{n+1}),\ G(v_n,v_n,v_{n+1})\\ G(w_n,w_n,w_{n+1})\end{array}\right\}$$
$$\le \varphi\left(\max\left\{\begin{array}{c}G(u_{n-1},u_{n-1},u_n),\ G(v_{n-1},v_{n-1},v_n)\\ G(w_{n-1},w_{n-1},w_n)\end{array}\right\}\right)$$

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$$\le \varphi^n\left(\max\left\{\begin{array}{c}G(u_0,u_0,u_1),\ G(v_0,v_0,v_1)\\ G(w_0,w_0,w_1)\end{array}\right\}\right)$$

Since $\lim_{n\to\infty}\varphi^n(t)=0$ for all $t>0$, we have for given $\varepsilon>0$ there exists a positive integer n_0 such that

$$\varphi^n(\max\{G(u_0,u_0,u_1),G(v_0,v_0,v_1),G(w_0,w_0,w_1)\})<\varepsilon-\varphi(\varepsilon),$$
$\forall n\ge n_0$.

Hence

$$\max\{G(u_n,u_n,u_{n+1}),G(v_n,v_n,v_{n+1}),G(w_n,w_n,w_{n+1})\}$$
$$<\varepsilon-\varphi(\varepsilon),\ \forall n\ge n_0. \qquad (2)$$

Now for $m, n\in N$, we prove by induction on m that

$$\max\{G(u_n,u_n,u_m),G(v_n,v_n,v_m),G(w_n,w_n,w_m)\}$$
$$<\varepsilon\ \forall n\ge n_0. \qquad (3)$$

Since $\varepsilon-\varphi(\varepsilon)<\varepsilon$, then by (2) we conclude that (3) holds when $m=n+1$.

Now suppose that (3) holds for $m=k$.

$$G(u_n,u_n,u_{k+1})\le G(u_n,u_n,u_{n+1})+G(u_{n+1},u_{n+1},u_{k+1})$$
$$<\varepsilon-\varphi(\varepsilon)+G(S(x_{n+1},y_{n+1},z_{n+1}),S(x_{n+1},y_{n+1},z_{n+1}),S(x_{k+1},y_{k+1},z_{k+1}))$$
$$\le\varepsilon-\varphi(\varepsilon)+\varphi(\max\{G(u_n,u_n,u_k),G(v_n,v_n,v_k),G(w_n,w_n,w_k)\})$$
$$<\varepsilon-\varphi(\varepsilon)+\varphi(\varepsilon)=\varepsilon.$$

Similarly, we can show that

$$G(v_n,v_n,v_{k+1})<\varepsilon \text{ and } G(w_n,w_n,w_{k+1})<\varepsilon.$$

Hence we have

$$\max\{G(u_n,u_n,u_{k+1}),G(v_n,v_n,v_{k+1}),G(w_n,w_n,w_{k+1})\}<\varepsilon.$$

Thus (3) holds for all $m\ge n\ge n_0$.

Hence $\{u_n\}$, $\{v_n\}$ and $\{w_n\}$ are G-Cauchy sequences in X.
From (2.1.3), there exists
$(p,q,r)\in\{(T(x,y,z),T(y,z,x),T(z,x,y))/x,y,z\in X\}$
such that $u_n\to p=T(x,y,z)$, $v_n\to q=T(y,z,x)$ and $w_n\to r=T(z,x,y)$ for some $x,y,z\in X$.
Now

$$G(u_n,u_n,S(x,y,z))=G(S(x_n,y_n,z_n),S(x_n,y_n,z_n),S(x,y,z))$$

$$\le\varphi\left(\max\left\{\begin{array}{c}G(u_{n-1},u_{n-1},T(x,y,z)),G(v_{n-1},v_{n-1},T(y,z,x)),\\ G(w_{n-1},w_{n-1},T(z,x,y))\end{array}\right\}\right)$$

Letting $n\to\infty$, we get

$$G(p,p,S(x,y,z))$$
$$\le\varphi\left(\max\left\{\begin{array}{l}G(p,p,T(x,y,z)),G(q,q,T(y,z,x)),\\ G(r,r,T(z,x,y))\end{array}\right\}\right)$$
$$=\varphi(0)=0.$$

This implies that $S(x,y,z)=p$. Similarly, we have $S(y,z,x)=q$ and $S(z,x,y)=r$. Thus $S(x,y,z)=p=T(x,y,z)$, $S(y,z,x)=q=T(y,z,x)$ and $S(z,x,y)=r=T(z,x,y)$. (4)
Thus (x,y,z) is a tripled coincidence point of S and T.

Claim (a): If (x^*,y^*,z^*) is another tripled coincidence point of S and T then $S(x,y,z)=S(x^*,y^*,z^*)$, $S(y,z,x)=S(y^*,z^*,x^*)$ and $S(z,x,y)=S(z^*,x^*,y^*)$.
Consider

$$G(S(x,y,z),S(x,y,z),S(x^*,y^*,z^*))$$
$$\le\varphi\left(\max\left\{\begin{array}{l}G(T(x,y,z),T(x,y,z),T(x^*,y^*,z^*))\\ G(T(y,z,x),T(y,z,x),T(y^*,z^*,x^*))\\ G(T(z,x,y),T(z,x,y),T(z^*,x^*,y^*))\end{array}\right\}\right)$$
$$=\varphi\left(\max\left\{\begin{array}{l}G(S(x,y,z),S(x,y,z),S(x^*,y^*,z^*))\\ G(S(y,z,x),S(y,z,x),S(y^*,z^*,x^*))\\ G(S(z,x,y),S(z,x,y),S(z^*,x^*,y^*))\end{array}\right\}\right)$$

Similarly we have

$$G(S(y, z, x), S(y, z, x), S(y^*, z^*, x^*))$$

$$\leq \varphi\left(\max\begin{Bmatrix} G(S(x, y, z), S(x, y, z), S(x^*, y^*, z^*)) \\ G(S(y, z, x), S(y, z, x), S(y^*, z^*, x^*)) \\ G(S(z, x, y), S(z, x, y), S(z^*, x^*, y^*)) \end{Bmatrix}\right),$$

$$G(S(z, x, y), S(z, x, y), S(z^*, x^*, y^*))$$

$$\leq \varphi\left(\max\begin{Bmatrix} G(S(x, y, z), S(x, y, z), S(x^*, y^*, z^*)) \\ G(S(y, z, x), S(y, z, x), S(y^*, z^*, x^*)) \\ G(S(z, x, y), S(z, x, y), S(z^*, x^*, y^*)) \end{Bmatrix}\right).$$

Thus

$$\max\begin{Bmatrix} G(S(x, y, z), S(x, y, z), S(x^*, y^*, z^*)) \\ G(S(y, z, x), S(y, z, x), S(y^*, z^*, x^*)) \\ G(S(z, x, y), S(z, x, y), S(z^*, x^*, y^*)) \end{Bmatrix}$$

$$\leq \varphi\left(\max\begin{Bmatrix} G(S(x, y, z), S(x, y, z), S(x^*, y^*, z^*)) \\ G(S(y, z, x), S(y, z, x), S(y^*, z^*, x^*)) \\ G(S(z, x, y), S(z, x, y), S(z^*, x^*, y^*)) \end{Bmatrix}\right)$$

which implies that $S(x, y, z) = S(x^*, y^*, z^*)$, $S(y, z, x) = S(y^*, z^*, x^*)$ and $S(z, x, y) = S(z^*, x^*, y^*)$.
Hence the claim (a). So $(S(x, y, z), S(y, z, x), S(z, x, y))$ is a unique tripled point of coincidence of S and T.

Claim(b): $S(x, y, z) = S(y, z, x) = S(z, x, y)$
Consider

$$G(S(x, y, z), S(x, y, z), S(y, z, x))$$

$$\leq \varphi\left(\max\begin{Bmatrix} G(T(x, y, z), T(x, y, z), T(y, z, x)) \\ G(T(y, z, x), T(y, z, x), T(z, x, y)) \\ G(T(z, x, y), T(z, x, y), T(x, y, z)) \end{Bmatrix}\right)$$

$$= \left(\max\begin{Bmatrix} G(S(x, y, z), S(x, y, z), S(y, z, x)) \\ G(S(y, z, x), S(y, z, x), S(z, x, y)) \\ G(S(z, x, y), S(z, x, y), S(x, y, z)) \end{Bmatrix}\right)$$

Similarly we have

$$G(S(y, z, x), S(y, z, x), S(z, x, y))$$

$$\leq \varphi\left(\max\begin{Bmatrix} G(S(x, y, z), S(x, y, z), S(y, z, x)) \\ G(S(y, z, x), S(y, z, x), S(z, x, y)) \\ G(S(z, x, y), S(z, x, y), S(x, y, z)) \end{Bmatrix}\right),$$

$$G(S(z, x, y), S(z, x, y), S(x, y, z))$$

$$\leq \varphi\left(\max\begin{Bmatrix} G(S(x, y, z), S(x, y, z), S(y, z, x)) \\ G(S(y, z, x), S(y, z, x), S(z, x, y)) \\ G(S(z, x, y), S(z, x, y), S(x, y, z)) \end{Bmatrix}\right).$$

Thus

$$\max\begin{Bmatrix} G(S(x, y, z), S(x, y, z), S(y, z, x)) \\ G(S(y, z, x), S(y, z, x), S(z, x, y)) \\ G(S(z, x, y), S(z, x, y), S(x, y, z)) \end{Bmatrix}$$

$$\leq \varphi\left(\max\begin{Bmatrix} G(S(x, y, z), S(x, y, z), S(y, z, x)) \\ G(S(y, z, x), S(y, z, x), S(z, x, y)) \\ G(S(z, x, y), S(z, x, y), S(x, y, z)) \end{Bmatrix}\right)$$

Hence $S(x, y, z) = S(y, z, x) = S(z, x, y)$.
Hence the claim (b).
Now from claim (a), the point $(S(x, y, z), S(y, z, x), S(z, x, y))$ is a unique tripled point of coincidence of S and T. (5)
Let $t = S(x, y, z)$.
Then from (4) and the claim (b), we have
$S(x, y, z) = S(y, z, x) = S(z, x, y) = T(x, y, z) = T(y, z, x) = T(z, x, y) = t$.
Since S and T are w^*-compatible, we have
$S(t, t, t) = T(t, t, t)$.
Thus (t, t, t) is a tripled coincidence point of S and T and hence $(S(t, t, t), S(t, t, t), S(t, t, t))$ is a tripled point of coincidence of S and T.
From (5), we have $S(t, t, t) = S(x, y, z) = t$.
Thus (t, t, t) is a common tripled fixed point of S and T.
Suppose (t^*, t^*, t^*) is another common tripled fixed point of S and T.
Then

$$\begin{aligned} G(t, t, t^*) &= G(S(t, t, t), S(t, t, t), S(t^*, t^*, t^*)) \\ &\leq \varphi\left(\max\left\{G(t, t, t^*), G(t, t, t^*), G(t, t, t^*)\right\}\right) \\ &= \varphi\left(G(t, t, t^*)\right). \end{aligned}$$

Thus $t = t^*$.
Hence (t, t, t) is the unique common tripled fixed point of S and T.

Case (II): If $u_n = u_{n+1}$, $v_n = v_{n+1}$ and $w_n = w_{n+1}$ for some n, then $(x_{n+1}, y_{n+1}, z_{n+1})$ is a tripled coincidence point of S and T.
The rest of the proof follows as in Case (I) from claim (a) onwards with $x = x_{n+1}$, $y = y_{n+1}$ and $z = z_{n+1}$.

The following example illustrates our main theorem.

Example 2.2. Let $X = [0,1)$ and $G(x, y, z) = |x - y| + |y - z| + |z - x|$ for all $x, y, z \in X$.
Let $S, T : X \times X \times X \to X$ be defined by

$$S(x, y, z) = \sin\left(\frac{x+y+z}{8}\right) \text{ and } T(x, y, z) = \frac{x+y+z}{2}.$$

Let $\varphi : R^+ \to R^+$ be defined by $\varphi(t) = \frac{t}{4}$ Then

$$G(S(x,\, y,\, z),\; S(x,\, y,\, z),\; S(u,\, v, w))$$
$$= 2\,|\,S(x,\, y,\, z) - S(u,\, v, w)\,|$$
$$=2\,\left|\sin\left(\frac{x+y+z}{8}\right) - \sin\left(\frac{u+v+w}{8}\right)\right|$$
$$= 4\left|\cos\left(\frac{x+y+z+u+v+w}{16}\right)\sin\left(\frac{x+y+z-u-v-w}{16}\right)\right|$$
$$\le \frac{1}{4}\,|\,x+y+z-u-v-w\,| = \frac{1}{2}\,|\,T(x,\, y,\, z) - T(u,\, v, w)\,|$$
$$= \frac{1}{4}\,G(T(x,\, y,\, z),\; T(x,\, y,\, z),\; T(u,\, v, w))$$
$$\le \varphi\left(\max\begin{Bmatrix} G(T(x,\, y,\, z),\; T(x,\, y,\, z),\; T(u,\, v, w)), \\ G(T(y,\, z,\, x),\; T(y,\, z,\, x),\; T(v, w,\, u)), \\ G(T(z,\, x,\, y),\; T(z,\, x,\, y),\; T(w,\, u,\, v)) \end{Bmatrix}\right).$$

Thus all the conditions of Theorem 2.1 are satisfied and (0,0,0) is the unique common tripled fixed point of S and T.

Now, we give the following two corollaries which follow immediately from Theorem 2.1.

Corollary 2.3. Let (X, G) be a G-metric space and $S : X \times X \times X \to X$ and $f : X \to X$ be mappings satisfying

(2.3.1) $$G(S(x,\, y,\, z),\; S(x,\, y,\, z),\; S(u,\, v, w)) \le \varphi\left(\max\begin{Bmatrix} G(fx,\, fx,\, fu), \\ G(fy,\, fy,\, fv), \\ G(fz,\, fz,\, fw) \end{Bmatrix}\right)$$

$\forall\; x, y, z, u, v, w \in X$ where $\varphi \in \Phi$,

(2.3.2) $S(X \times X \times X) \subseteq f(X)$ and $f(X)$ is complete,

(2.3.3) the pair (S, T) is W^*-compatible.

Then S and f have a unique common tripled fixed point in $X \times X \times X$.

Corollary 2.4. Let (X, G) be a complete G-metric space and $S : X \times X \times X \to X$ be a mapping satisfying

(2.4.1) $$G(S(x,\, y,\, z),\; S(x,\, y,\, z),\; S(u,\, v, w)) \le \varphi\left(\max\begin{Bmatrix} G(x,\, x,\, u), \\ G(y,\, y,\, v), \\ G(z,\, z,\, w) \end{Bmatrix}\right)$$

$\forall\; x, y, z, u, v, w \in X$ where $\varphi \in \Phi$.

Then S has a unique common tripled fixed point in $X \times X \times X$.

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