Gazi University Journal of Science
GU J Sci
27(3):923-932 (2014)

Soft $I-$Sets and Soft $I-$Continuity of Functions

Metin AKDAĞ[1], Fethullah EROL[1,♠]

[1]Cumhuriyet University, Science Faculty, Department Of Mathematics 58140 Sivas, TURKEY

Received:04/04/2014 Accepted:07/08/2014

Abstract

In this paper, we introduce the notion of soft $I-open$ sets and investigate some properties of soft $I-openess$. Additionally, we study soft $I-continuous$ and soft $I-open$ functions. Some characterizations and several properties concerning these functions are obtained.

Keywords. Soft sets, soft ideal, soft continuity, soft $I-continuouity$

1. INTRODUCTION

Molodtsov [1] introduced soft set theory in 1999. Then Shabir and Naz [2] applied this theory to topological structure in 2011. They introduced and studied some concepts such as soft topological space, soft interior, soft closure and soft subspace etc. Kharal and Ahmad [3] defined the notion of soft mappings on soft classes. Then Aygünoğlu and Aygün [4] introduced soft continuity of soft mappings, soft product topology and studied soft compactness. Nazmul and Samanta [5] studied the neighbourhood properties in a soft topological space.

The topic of ideals in general topological spaces is treated in the classic text by Kuratowski [6]. This topic has an excellent potential for applications in other branches of mathematics. The Cantor-Bendixson Theorem examplifies this potential. This subject was continued to study by general topologists in recent years [6, 11, 15, 17]. In 1990, Jankovic and Hamlett [7] introduced another topology $\tau^*(I)$ by using a given topology τ, which satisfies $\tau \subset \tau^*(I)$. In 1952, Hashimoto [9,10] introduced the notion of ideal continuity. Then Jankovic and Hamlett [11] defined I-open set in ideal topological spaces. Later, Abd El-Monsef [12] studied I-continuity for functions. Then Hatır and Noiri [13] introduced the semi-I-open set and semi-I-continuity in 2002. Kale and Guler [8] gave the definition of soft ideal and studied the properties the soft ideal topological space. Moreover they introduced the notion of soft $I-regularity$ and soft $I-normality$.

The purpose of this paper is to define soft $I-open$ set and soft $I-continuity$ of functions and investigate their basic properties.

♠Corresponding author, e-mail: feerol@cumhuriyet.edu.tr

2. PRELIMINARIES

Throughout this paper, X will be a nonempty initial universal set and A will be a set of parameters. Let $P(X)$ denote the power set of X and $S(X)$ denote the set of all soft sets over X.

Definition 1. [1] A pair (F,A) is called a soft set over X, where F is a mapping from A to $P(X)$.

Definition 2. [19] Let (F_1,A) and (F_2,A) be soft sets over a common universe X. Then (F_1,A) is said to be a soft subset of (F_2,A) if $F_1(\alpha)\subset F_2(\alpha)$, for all $\alpha\in A$ and this relation is denoted by $(F_1,A)\tilde{\subset}(F_2,A)$. Also, (F_1,A) is said to be a soft equal to (F_2,A) if $F_1(\alpha)=F_2(\alpha)$, for all $\alpha\in A$ and this relation is denoted by $(F_1,A)=(F_2,A)$.

Definition 3. [20] A soft set (F,A) over X is said to be a null soft set if $F(\alpha)=\emptyset$ for all $\alpha\in A$ and this denoted by $\tilde{\emptyset}$. Also, (F,A) is said to be an absolute soft set if $F(\alpha)=X$, for all $\alpha\in A$ and this denoted by $\tilde{X}$.

Definition 4. [21] The complement of a soft set (F,A) is defined as $(F,A)^c=(F^c,A)$, where $F^c(\alpha)=(F(\alpha))^c=X-F(\alpha)$, for all $\alpha\in A$. Clearly, we have $(\tilde{\emptyset})^c=\tilde{X}$ and $(\tilde{X})^c=\tilde{\emptyset}$.

Definition 5. [2] The difference of two soft sets (F_1,A) and (F_2,A) is defined by $(F_1,A)-(F_2,A)=(F_1-F_2,A)$, where $(F_1-F_2)(\alpha)=F_1(\alpha)-F_2(\alpha)$, for all $\alpha\in A$.

Definition 6. [22] Let $\{(F_j,A): j\in J\}$ be a nonempty family of soft sets over a common universe X. The intersection of these soft sets denoted by $\cap_{j\in J}$, is defined by $\cap_{j\in J}(F_j,A)=\cap_{j\in J}(F_j,A)$, where $(\cap_{j\in J}F_j)(\alpha)=\cap_{j\in J}(F_j(\alpha))$, for all $\alpha\in A$. The union of these soft sets denoted by $\cup_{j\in J}$, is defined by $\cup_{j\in J}(F_j,A)=\cup_{j\in J}(F_j,A)$, where $(\cup_{j\in J}F_j)(\alpha)=\cup_{j\in J}(F_j(\alpha))$ for all $\alpha\in A$.

Definition 7. [5] A soft set (E,A) over X is said to be a soft element if $\exists\alpha\in A$, $\beta\neq\alpha$ such that $E(\alpha)=\{x\}$ and $E(\beta)=\emptyset$, for all $\beta\in A$. Such a soft element is denoted by E_α^x. The soft element E_α^x is said to be in the soft set (G,A) if $x\in(G,A)$, and denoted by $E_\alpha^x\tilde{\in}(G,A)$.

Definition 8. [2] Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if,

(i) $\tilde{\emptyset},\tilde{X}\tilde{\in}\tau$

(ii) the intersection of any two soft sets in τ belongs to τ

(iii) the union of any number of soft sets in τ belongs to τ.

The triple (X,τ,A) is called a soft topological space over X. The members of τ are said to be τ-soft open sets (simply, soft open set in X). A soft set over X is said to be soft closed in X if its complement belongs to τ.

Definition 9. [2] Let Y be a nonempty subset of X, then $\tilde{Y}$ denotes the soft set (Y,A) over X for which $Y(\alpha)=Y$, for all $\alpha\in A$.

Definition 10. [2] Let (F,A) be a soft set over X and Y be a nonempty subset of X. Then the sub soft set of (F,A) over Y denoted by $({}^YF,A)$ is defined as ${}^YF(\alpha)=Y\cap F(\alpha)$, for each $\alpha\in A$. In other word $({}^YF,A)=\tilde{Y}\tilde{\cap}(F,A)$.

Definition 11. [2] Let (X,τ,A) be a soft topological space over X and Y be a nonempty subset of X. Then $\tau_Y=\{({}^YF,A):(F,A)\in\tau)\}$ is said to be the soft relative topology on Y and (Y,τ_Y,A) is called a sof subspace of (X,τ,A).

Definition 12. [2] Let (X,τ,A) be a soft topological space over X and (F,A) be a soft set over X. The soft closure of (F,A) denoted by $Cl(F,A)$ is the intersection of all closed soft super sets of (F,A). The soft interior of (F,A) denoted by $Int(F,A)$ is the union of all open soft subsets of (F,A).

Definition 13. [5] Let (X,τ) be a soft topological space over X. A soft set (F,A) is said to be a neighbourhood of the soft set (H,A) if there exist a soft set $(G,A)\in\tau$ such that $(H,A)\tilde{\subset}(G,A)\tilde{\subset}(F,A)$. If $(H,A)=E_\alpha^x$, then (F,A) is said to be a soft neighbourhood of the soft element E_α^x. The soft neighbourhood system of soft element E_α^x, denoted by $N(E_\alpha^x)$, is the family of all its soft neighbourhood. The soft open neighbourhood system of soft element E_α^x, denoted by $V(E_\alpha^x)$, is the family of all its soft open neighbourhood.

Lemma 1. [5] A soft element $E_\alpha^x\tilde{\in}Cl(F,A)$ if and only if each soft neighbourhood of E_α^x intersects (F,A).

Definition 14. [23] Let (X,τ,A) be a soft topological space over X, (G,A) be a soft closed set in X and E_α^x be a soft point such that $E_\alpha^x\tilde{\notin}(G,A)$. If there exist soft open sets (F_1,A) and (F_2,A) such that $E_\alpha^x\tilde{\in}(F_1,A)$, $(G,A)\tilde{\subset}(F_2,A)$ and $(F_1,A)\tilde{\cap}(F_2,A)=\tilde{\emptyset}$, then (X,τ) is called a soft regular space.

Definition 15. [23] Let (X,τ,A) be a soft topological space over X, and let (G_1,A) and (G_2,A) be two disjoint soft closed sets. If there exist two soft open sets (F_1,A) and (F_2,A) such that $(G_1,A)\tilde{\subseteq}(F_1,A)$, $(G_2,A)\tilde{\subseteq}(F_2,A)$ and $(F_1,A)\tilde{\cap}(F_2,A)=\tilde{\emptyset}$, then (X,τ) is called a soft normal space.

Definition 16. [18] A soft ideal I is a nonempty collection of soft sets over X if

(i) $(F,A)\tilde{\in} I$, $(G,A)\tilde{\subseteq}(F,A)$ implies $(G,A)\tilde{\in} I$

(ii) $(F,A)\tilde{\in} I$, $(G,A)\tilde{\in} I$ implies $(F,A)\tilde{\cup}(G,A)\tilde{\in} I$.

A soft topological space (X,τ,A) with a soft ideal I called soft ideal topological space and denoted by (X,τ,A,I).

Definition 17. [18] Let (F,A) be a soft set in a soft ideal topological space (X,τ,A,I) and $(.)^*$ be a soft operator from $S(X)$ to $S(X)$. Then the soft local mapping of (F,A) defined by $(F,A)^*(I,\tau)=\{E_\alpha^x:(U,A)\tilde{\cap}(F,A)\tilde{\notin} I$ for every $(U,A)\tilde{\in} V(E_\alpha^x)$ denoted by $(F,A)^*$ simply. Also, the soft set operator Cl^* is called a soft $^*-closure$ and is defined as $Cl^*(F,A)$=$(F,A)\tilde{\cup}(F,A)^*$ for a soft subset (F,A).

Lemma 2. [18] Let (X,τ,A,I) be a soft ideal topological space and (F,A),(G,A) be two soft sets. Then;

(i) $(F,A)\tilde{\subseteq}(G,A)$ implies $(F,A)^*\tilde{\subseteq}(G,A)^*$ and $(F,A)\tilde{\cup}(G,A)^*=(F,A)^*\tilde{\cup}(G,A)^*$.

(ii) $(F,A)^*\tilde{\subseteq}Cl(F,A)$ and $((F,A)^*)^*\tilde{\subseteq}(F,A)^*$.

(iii) (F,A) is soft open and $(F,A)\tilde{\cap}(G,A)\tilde{\in} I$ implies $(F,A)\tilde{\cap}(G,A)^*=\tilde{\emptyset}$

(iv) $(F,A)^*$is soft closed.

(v) If (F,A) is soft closed then$(F,A)^*\tilde{\subseteq}(F,A)$.

Proposition 1. [18] Let (X,τ,A,I) be a soft ideal topological space and (F,A),(G,A) be two soft sets. Then;

(i) $Cl^*(\tilde{\emptyset})=\tilde{\emptyset}$ and $Cl^*(\tilde{X})=\tilde{X}$.

(ii) $(F,A)\tilde{\subseteq}Cl^*(F,A)$ and $Cl^*(Cl^*(F,A))$= $Cl^*(F,A)$.

(iii) If $(F,A)\tilde{\subseteq}(G,A)$ then $Cl^*(F,A)\tilde{\subseteq}Cl^*(G,A)$

(iv) $Cl^*(F,A)\tilde{\cup}\ Cl^*(G,A)$=$Cl^*((F,A)\tilde{\cup}(G,A))$.

Definition 18. Let (X,τ,A,I) be a soft ideal topological space, (Y,ϑ,B) be a soft topological space, $P:A\to B$ and $U:X\to Y$ be mappings. Then the mapping $f_{PU}:(X,\tau,A,I)\to(Y,\vartheta,B)$ is defined as follows:

(i) The image of (F,A) a soft set of (X,τ,A) underf_{PU} written as $f_{PU}(F,A)=(f_{PU}(F),P(A))$, is a soft set of (Y,ϑ,B) such that

$$f_{PU}(F)(y)=\begin{cases}\bigcup_{x\in P^{-1}(Y)\cap A}U(F(x)), & \text{if } P^{-1}(y)\cap A\neq\emptyset\\ \emptyset\quad , & \text{otherwise}\end{cases}$$

for all $y\in B$.

(ii) The inverse image of (G,B) a soft set under f_{PU} written as $\overset{-1}{f_{PU}}(G,B)=(\overset{-1}{f_{PU}}(G),P^{-1}(B))$, is a soft set of (X,τ,A) such that

$$\overset{-1}{f_{PU}}(G)(x)=\begin{cases}U^{-1}\left(G(P(x))\right), & P(x)\in B\\ \emptyset & ,\text{otherwise}\end{cases}$$

for all $x\in A$.

3. SOFT *I*-OPEN SETS AND SOFT *I*-CLOSED SETS

Definition 19. A subset S of an ideal topological space (X, τ, A, I) is said to be

(i) [15] $I - open$ if $S \subset Int(S^*)$

(ii) [16] $\alpha - I - open$ if $S \subset Int(Cl^*(Int(S)))$

(iii) [17] $pre - I - open$ if $S \subset Int(Cl^*(S))$.

(iv) [16] $semi - I - open$ if $S \subset Cl^*(Int(S))$.

Definition 20. A soft subset (F, A) of an soft ideal topological space (X, τ, A, I) is said soft $I - open$ if $(F, A) \tilde{\subseteq} Int(F, A)^*$.

We denote by $SIO(X, \tau, A, I) = \{(F, A): (F, A) \tilde{\subseteq} Int(F, A)^*\}$ the family of all soft $I - open$ sets of a soft topological space (X, τ, A, I).

Remark 1. It is clear that soft $I - openness$ and soft openness are independent concepts.

Example 1. Let a soft ideal topological space (X, τ, A, I) given as follows:

$X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$,

$\tau = \{\tilde{\emptyset}, \tilde{X}, \{(e_1, \{h_1\})\}, \{(e_2, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\}$,

$I = \{\tilde{\emptyset}, \{(e_2, \{h_1\})\}\}$. Then $(F, A) = \{(e_1, \{h_2\})\}$ is soft $I - open$ set but is not soft $open\ set$.

Example 2. Let a soft ideal topological space (X, τ, A, I) given as follows:

$X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $\tau = \{\tilde{\emptyset}, \tilde{X}, \{(e_1, \{h_1\})\}, \{(e_2, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\}$,

$I = \{\tilde{\emptyset}, \{(e_1, \{h_1\})\}\}$. Then $(F, A) = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}$ is soft $open\ set$ but is not soft $I - open$ set.

Definition 21. [25] A soft subset (F, A) of a soft topological space (X, τ, A) is said soft $pre - open$ if $(F, A) \tilde{\subseteq} Int(Cl(F, A))$.

Remark 2. Every soft $I - open$ set is soft $pre - open$ set but the converse is not true in general as shown by the following example.

Example 3. Let a soft ideal topological space (X, τ, A, I) given as follows:

$X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $\tau = \{\tilde{\emptyset}, \tilde{X}, \{(e_1, \{h_1\})\}, \{(e_2, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\}$, $I = \{\tilde{\emptyset}\}$.

Then $(H, A) = \{(e_1, \{h_2\})\}$ is soft $pre - open$ set but is not soft $I - open$ set.

Remark 3. The intersection of two soft $I - open$ sets need not be soft $I - open$ as shown by the following example.

Example 4. Let a soft ideal topological space (X, τ, A, I) given as follows:

$X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$, $\tau = \{\tilde{\emptyset}, \tilde{X}, \{(e_1, \{h_1\})\}, \{(e_2, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\}$, $I = \{\tilde{\emptyset}, \{(e_1, \{h_1\})\}\}$. Then $(F, A) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$ and $(G, A) = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$ are two soft $I - open$ sets but $(F, A) \tilde{\cap} (G, A) = \{(e_1, \{h_1\})\}$ is not soft $I - open$ set.

Theorem 1. For any soft $I - open$ set (F, A) of a space (X, τ, A, I), we have $(F, A)^* = (Int(F, A)^*)^*$.

Proof Obvious.

Definition 22. A soft subset (F, A) of a soft ideal topological space (X, τ, A, I) is said to be soft $I - closed$ if its complement is soft $I - open$.

By $SIC(X, \tau, A, I)$ we denote the family of all soft $I - closed$ sets of a soft topological space (X, τ, A, I).

Remark 4. For soft subset (F, A) of a soft ideal topological space (X, τ, A, I),we have $\tilde{X} - (Int(F, A))^* \neq Int(\tilde{X} - (F, A))^*$.

Example 5. Let a soft ideal topological space (X,τ,A,I) given as follows:

$X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$, $\tau=\{\widetilde{\emptyset},\tilde{X},\{(e_1,\{h_2\})\},\{(e_2,\{h_1\})\},\{(e_1,\{h_2\}),(e_2,\{h_1\})\}\}$,
$I=\{\widetilde{\emptyset},\{(e_1,\{h_1\})\},\{(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$. For a soft subset $(F,A)=\{(e_1,\{h_2\})\}$, we have $\tilde{X}-(Int(F,A))^*=\tilde{X}$ but $Int(\tilde{X}-(F,A))^*=\{(e_1,\{h_1\}),(e_2,\{h_2\})\}$.

Theorem 2. Let (F,A) be a soft subset of a soft ideal topological space (X,τ,A,I). If (F,A) is soft $I-closed$ set then $(F,A)\,\widetilde{\supseteq}\,(Int(F,A))^*$.

Proof The proof is obvious from the definition of soft $I-closed$ set and $(F,A)^*$.

Theorem 3. Let (F,A) be a soft subset of a soft ideal topological space (X,τ,A,I) and $\tilde{X}-(Int(F,A))^*=Int(\tilde{X}-(F,A))^*$. Then (F,A) is soft $I-closed$. Then (F,A) is soft $I-closed$ set iff $(F,A)\,\widetilde{\supseteq}\,(Int(F,A))^*$.

Proof Obvious.

Theorem 4. Let (X,τ,A,I) be a soft ideal topological space and (F,A), (G,A) be two soft set in X. Then,

(i) If $\{(F_\alpha,A):\alpha\in\Delta\}$ is soft-I-open sets, then $\sqcup\{(F_\alpha,A):\alpha\in\Delta\}$ is soft $I-open$ set.

(ii) If $(F,A)\in SIO(X,\tau,A,I)$ and $(G,A)\in\tau$, then $(F,A)\,\widetilde{\sqcap}\,(G,A)\in SIO(X,\tau,A,I)$.

(iii) If $(F,A)\in SIO(X,\tau,A,I)$ and (G,A) is α-open set then $(F,A)\,\widetilde{\sqcap}\,(G,A)\in SPO(X,\tau,A)$.

Proof (i) Since $\{(F_\alpha,A):\alpha\in\Delta\}$ is soft $I-open$ sets, then $(F_\alpha,A)\,\widetilde{\sqsubseteq}\,Int(F_\alpha,A)^*$ for every $\alpha\in\Delta$.Thus $\sqcup\,(F\alpha,A)\,\widetilde{\sqsubseteq}\sqcup\,Int(F_\alpha,A)^*\,\widetilde{\sqsubseteq}\,Int(\sqcup\,(F_\alpha,A)^*)\,\widetilde{\sqsubseteq}\,Int(\sqcup\,(F_\alpha,A))^*$ for every $\alpha\in\Delta$.

(ii)$(F,A)\,\widetilde{\sqcap}\,(G,A)\,\widetilde{\sqsubseteq}\,Int(F_\alpha,A)^*\,\widetilde{\sqcap}\,(G,A)=Int(F_\alpha,A)^*\,\widetilde{\sqcap}\,(G,A))$. Thus $(F,A)\,\widetilde{\sqcap}\,(G,A)\,\widetilde{\sqsubseteq}\,Int((F_\alpha,A)\,\widetilde{\sqcap}\,(G,A))^*$.

(iii) Since $(F,A)^*(I)$ is soft closed and $(F,A)^*(I)\,\widetilde{\sqsubseteq}\,Cl(F,A)$.

Corollary 1. The union of soft $I-closed$ set and soft closed set is soft $I-closed$.

Corollary 2. The union of soft $I-closed$ set and soft α-closed set is soft $pre-closed$.

Theorem 5. If (F,A) is soft $I-open$ and soft $semi-closed$ set in soft ideal topological space (X,τ,A,I), then $(F,A)=Int((F,A)^*)$.

Proof Obvious.

Theorem 6. Let (F,A) is soft $I-open$ set in X and (G,B) is soft $I-open$ set in Y then $(F,A)\times(G,B)$ is soft $I-open$ set in $X\times Y$ if $(F,A)^*\times(G,B)^*=((F,A)\times(G,B))^*$, where $X\times Y$ is the product space.

Proof $(F,A)\times(G,B)\,\widetilde{\sqsubseteq}\,Int((F,A)^*)\times Int((G,B)^*)=Int((F,A)^*\times(G,B)^*)=Int(((F,A)\times(G,B))^*)$. Thus $(F,A)\times(G,B)$ is soft $I-open$ set in $X\times Y$.

Theorem 7. If $(F,A)\,\widetilde{\sqsubseteq}\,(G,A)\,\widetilde{\sqsubseteq}\,Cl(F,A)$ and (F,A) is soft $I-open$ in X, then (G,A) is soft $\beta-open$.

Proof Obvious.

Theorem 8. Let (G,A) be a soft $I-open$ set in a soft ideal topological space (X,τ,A,I), then $Cl(F,A)\,\widetilde{\sqcap}\,(G,A)\,\widetilde{\sqsubseteq}\,((F,A)\,\widetilde{\sqcap}\,(G,A))^*$, for every $(F,A)\in SO(X,\tau,A)$.

Proof Let $(F,A)\in SO(X,\tau,A)$, then $Cl(F,A)=Cl\big(Int(F,A)\big)$. Since $(G,A)\in SIO(X,\tau,A,I)$ then $Cl(F,A)\,\widetilde{\sqcap}\,(G,A)\,\widetilde{\sqsubseteq}\,Cl(Int(F,A))\,\widetilde{\sqcap}\,(Int(G,A)))^*\,\widetilde{\sqsubseteq}\,Cl(Int\big((F,A)\,\widetilde{\sqcap}\,(G,A)\big)^*)\,\widetilde{\sqsubseteq}\,Cl((F,A)\,\widetilde{\sqcap}\,(G,A)^*)=((F,A)\,\widetilde{\sqcap}\,(G,A))^*$.

Theorem 9. If (X,τ,A,I) be a soft ideal topological space, $(F,A)\in\tau$ and $(G,A)\in SIO(X,\tau,A,I)$, then there exists an soft open set (H,A) of X such that $(F,A)\,\widetilde{\sqcap}\,(H,A)=\widetilde{\emptyset}$, implies $(F,A)\,\widetilde{\sqcap}\,(G,A)=\widetilde{\emptyset}$.

Proof Since $(G,A)\in SIO(X,\tau,A,I)$, then $(G,A)\,\widetilde{\sqsubseteq}\,Int(G,A)^*$. By taking $(H,A)=Int(G,A)^*$ to be an soft open set such that $(G,A)\,\widetilde{\sqsubseteq}\,(H,A)$, but $(F,A)\,\widetilde{\sqcap}\,(H,A)=\widetilde{\emptyset}$, then $(H,A)\,\widetilde{\sqsubseteq}\,\tilde{X}-(F,A)$ implies that $Cl(H,A)\,\widetilde{\sqsubseteq}\,\tilde{X}-(F,A)$. Hence $(G,A)\,\widetilde{\sqsubseteq}\,\tilde{X}-(F,A)$ and $(F,A)\,\widetilde{\sqcap}\,(G,A)=\widetilde{\emptyset}$.

4. SOFT I −CONTINUOUS FUNCTIONS

Definition 23. A function $f:(X,\tau,A,I)\rightarrow(Y,\sigma,B)$ is said to be soft $I-continuous$ if $f^{-1}(F,B)$ is soft $I-open$ set in (X,τ,A,I) for each soft open set (F,B) of (Y,σ,B).

Definition 24. [25] A function $f:(X,\tau,A)\rightarrow(Y,\sigma,B)$ is said to be soft $pre\ continuous$ if $f^{-1}(F,B)$ is soft pre open set in (X,τ,A) for each soft open set (F,B) of (Y,σ,B).

Remark 5. It is obvious that soft $I-continuity$ implies soft $pre\ continuity$. But these converse is not true in general.

Example 6. Let a soft ideal topological space (X,τ,A,I) and a soft topological space (Y,ϑ,B) given as follows: $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$, $\tau=\{\widetilde{\emptyset},\tilde{X},\{(e_1,\{h_1\}),(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $I=\{\widetilde{\emptyset},\{(e_1,\{h_1\})\},\{(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $Y=\{y_1,y_2\}$, $B=\{k_1,k_2\}$, $\vartheta=\{\widetilde{\emptyset},\tilde{Y},\{(k_1,\{y_2\})\}\}$. Also, let $U:X\rightarrow Y$, $U(h_1)=y_2$, $U(h_2)=y_1$, $P:A\rightarrow B$, $P(e_1)=k_1$, $P(e_2)=k_2$. Then the soft function $f_{UP}:(X,\tau,A,I)\rightarrow(Y,\vartheta,B)$ is soft $pre-continuous$ but is not soft $I-continuous$.

Definition 25. [25] A function $f:(X,\tau,A)\rightarrow(Y,\sigma,B)$ is said to be soft continuous if $f^{-1}(F,B)$ is soft open set in (X,τ,A) for each soft open set (F,B) of (Y,σ,B).

Remark 6. The following two examples show that the concept of soft continuity and soft $I-continuity$ are independent.

Example 7. Let a soft ideal topological space (X,τ,A,I) and a soft topological space (Y,ϑ,B) given as follows: $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$, $\tau=\{\widetilde{\emptyset},\tilde{X},\{(e_1,\{h_1\}),(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $I=\{\widetilde{\emptyset},\{(e_1,\{h_1\})\},\{(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $Y=\{y_1,y_2\}$, $B=\{k_1,k_2\}$, $\vartheta=\{\widetilde{\emptyset},\tilde{Y},\{(k_1,\{y_2\})\}\}$. Also, let $U:X\rightarrow Y$, $U(h_1)=y_1$, $U(h_2)=y_2$, $P:A\rightarrow B$, $P(e_1)=k_1$, $P(e_2)=k_2$. Then the soft function $f_{UP}:(X,\tau,A,I)\rightarrow(Y,\vartheta,B)$ is soft $I-continuous$ but is not soft continuous.

Example 8. Let a soft ideal topological space (X,τ,A,I) and a soft topological space (Y,ϑ,B) given as follows: $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$, $\tau=\{\widetilde{\emptyset},\tilde{X},\{(e_1,\{h_1\}),(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $I=\{\widetilde{\emptyset},\{(e_1,\{h_1\})\},\{(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $Y=\{y_1,y_2\}$, $B=\{k_1,k_2\}$, $\vartheta=\{\widetilde{\emptyset},\tilde{Y},\{(k_1,\{y_1\}),(k_2,\{y_2\})\}\}$. Also, let $U:X\rightarrow Y$, $U(h_1)=y_1$, $U(h_2)=y_2$, $P:A\rightarrow B$, $P(e_1)=k_1$, $P(e_2)=k_2$. Then the soft function $f_{UP}:(X,\tau,A,I)\rightarrow(Y,\vartheta,B)$ is soft $continuous$ but is not soft $I-continuous$.

Theorem 10. For a function $f:(X,\tau,A,I)\rightarrow(Y,\sigma,B)$ the following are equivalent:

(i) f is soft $I-continuous$,

(ii) for each $E_\alpha^x\tilde{\in}(X,A)$ and each $(V,B)\in\sigma$ containing $f(E_\alpha^x)$, there exists $(U,A)\in SIO(X,\tau,A,I)$ containing E_α^x such that $f(U,A)\tilde{\subset}(V,B)$,

(iii) For each $E_\alpha^x\tilde{\in}(X,A)$ and $(V,B)\in\sigma$ containing $f(E_\alpha^x)$, $(f^{-1}(V,B))^*$ is a neighbourhood of E_α^x.

Proof (i)⇔(ii) Since $(V,B)\in\sigma$ containing $f(E_\alpha^x)$, then by (i) $f^{-1}(V,B)$ is soft $I-open$ in X. By taking $(U,A)=f^{-1}(V,B)$ which containing $f(E_\alpha^x)$, thus $f(U,A)\tilde{\subset}(V,B)$.

(ii)⇒(iii) Since $(V,B)\in\sigma$ containing $f(E_\alpha^x)$, then by (ii) there exists $(G,A)\in SIO(X,\tau,A,I)$ containing $f(E_\alpha^x)$, such that $f(G,A)\tilde{\subset}(V,B)$. So, $E_\alpha^x\tilde{\in}(G,A)\tilde{\subset}Int((G,A)^*)\tilde{\subset}Int\big(f^{-1}(V,B)\big)^*\tilde{\subset}(f^{-1}(V,B))^*$. Hence $(f^{-1}(V,B))^*$ is a neighbourhood of E_α^x.

(iii)⇒(i) Obvious.

Theorem 11. For a function $f:(X,\tau,A,I)\rightarrow(Y,\sigma,B)$ the following are equivalent:

(i) f is soft $I-continuous$,

(ii) The inverse image of each soft closed set in Y is soft $I-close$d,

(iii) $(Int(f^{-1}(G,B)))^*\tilde{\subset}f^{-1}((G,B)^*)$ for each *-dense-in-itself soft subset (G,B) of Y,

(iv) $f(Int((F,A)))^*\tilde{\subset}(f(F,A))^*$, for each subset (F,A) of X, and for each *-perfect soft subset of Y.

Proof (i)⇒(ii) Let (F,B) be a soft closed set of Y, then $\tilde{X}-(F,B)$ is soft open set, and by (i), $f^{-1}(\tilde{Y}-(F,B))=\tilde{X}-f^{-1}(F,B)$ is soft $I-open$. Thus $f^{-1}(F,B)$ is soft $I-closed$.

(ii)⇒(iii) (G,B) be a soft subset of Y, since $(G,B)^*$ is soft closed, then by (ii), $f^{-1}((G,B)^*)$ is $soft\ I-closed$. Thus $(Int(f^{-1}((G,B)^*)))^* \tilde{\subseteq} f^{-1}((G,B)^*)$. Since (G,B) is *-dense-in-itself soft subset, then $(Int(f^{-1}(G,B)))^* \tilde{\subseteq} (Int(f^{-1}((G,B)^*)))^* \tilde{\subseteq} f^{-1}((G,B)^*)$.

(iii)⇒(iv) Let (F,A) be a soft subset of X, and $(G,B)=f(F,A)$, then by (iii), $(Int(F,A))^* \tilde{\subseteq} (Int(f^{-1}(G,B)))^* \tilde{\subseteq} f^{-1}((G,B)^*)$. Hence, $f((Int(F,A))^*) \tilde{\subseteq} (G,B)^* = (f(F,A))^*$.

(iv)⇒(i) Let $(G,B)\in\sigma$, $(H,B)=\tilde{Y}-(G,B)$ and $(F,A)=f^{-1}(H,B)$, then $f(F,A)\tilde{\subseteq}(H,B)$ and by (iv), $f(Int((F,A)))^* \tilde{\subseteq} (f(F,A))^* \tilde{\subseteq} (H,B)^* = (H,B)$ (because (H,B) is *-perfect). Thus, $(Int(f^{-1}(H,B)))^* \tilde{\subseteq} (Int(F,A))^* \tilde{\subseteq} f^{-1}(H,B)$, and therefore $f^{-1}(H,B)=f^{-1}(\tilde{Y}-(G,B))=\tilde{X}-f^{-1}(G,B)$ is soft $I-closed$. Hence $f^{-1}(G,B)$ is soft $I-open$. Thus f is soft $I-continuous$.

Theorem 12. A soft function $f:(X,\tau,A,I)\to(Y,\sigma,B)$ is soft $I-continuous$ if and only if the graph soft function $g: X\to X\times Y$, defined by $g(E_\alpha^x)=(E_\alpha^x, f(E_\alpha^x))$, for each $E_\alpha^x \tilde{\in} X$, is soft $I-continuous$.

Proof (⇒) Suppose that f is soft $I-continuous$. Let $E_\alpha^x \tilde{\in} X$ and $(W, A\times B)$ be any soft open set of $X\times Y$ containing $g(E_\alpha^x)=(E_\alpha^x, f(E_\alpha^x))$, . Then there exists a basic soft open set $(U,A)\times(V,B)$ such that $g(E_\alpha^x)=(E_\alpha^x, f(E_\alpha^x))\in(U,A)\times(V,B)\tilde{\subseteq}(W,A\times B)$. Since f is soft $I-continuous$, there exists a soft $I-open$ set (U_0,A_0) of X containing E_α^x such that $f\big((U_0,A_0)\big)\tilde{\subseteq}(V,B)$. Since $(U_0,A_0)\tilde{\cap}(U,A)\in SIO(X,\tau,A,I)$ and $(U_0,A_0)\tilde{\cap}(U,A)\tilde{\subseteq}(U,A)$, then $g\big((U_0,A_0)\tilde{\cap}(U,A)\big)\tilde{\subseteq}(U,A)\times(V,B)\tilde{\subseteq}(W,A\times B)$. This shows that g is soft $I-continuous$.

(⇐) Suppose that g is soft $I-continuous$. Let $E_\alpha^x \tilde{\in} X$ and (V,B) be any soft open set of Y containing $f(E_\alpha^x)$. Then $X\times(V,B)$ is open in $X\times Y$. Since g is soft $I-continuous$, there exists $(U,A)\in SIO(X,\tau,A,I)$ containing E_α^x such that $g(U,A)\tilde{\subseteq}X\times(V,B)$. Therefore, we obtain $f(U,A)\tilde{\subseteq}(V,B)$.This shows that f is soft $I-continuous$.

Theorem 13. Let $f:(X,\tau,A,I)\to(Y,\sigma,B)$ be a soft $I-continuous$ function and $(U,A)\in\tau$. Then the restriction $f\mid_{(U,A)}: ((U,A),\tau\mid_{(U,A)}, I\mid_{(U,A)})\to(Y,\sigma,B)$ is soft $I-continuous$.

Proof Let (V,B) be any soft open set of (Y,σ).Then $f^{-1}(V,B)\tilde{\subseteq} Int\big(f^{-1}(V,B)\big)^*$ and so,
$(U,A)\tilde{\cap} f^{-1}(V,B)\tilde{\subseteq}(U,A)\tilde{\cap} Int\big(f^{-1}(V,B)\big)^*$. Thus $\big(f\mid_{(U,A)}\big)^{-1}(V,B)=(U,A)\tilde{\cap} f^{-1}(V,B)$
$\tilde{\subseteq}(U,A)\tilde{\cap} Int(f^{-1}(V,B))^*$, since $(U,A)\in\tau$, then $(f\mid_{(U,A)})^{-1}(V,B)=Int((U,A)\tilde{\cap}((f^{-1}(V,B)))^*)$
$\tilde{\subseteq} Int((U,A)\tilde{\cap}(f^{-1}(V,B)))^* = Int((f\mid_{(U,A)})^{-1}(V,B)^*$. Thus we have that $(f\mid_{(U,A)})^{-1}(V,B)\in SIO((U,A),\tau\mid_{(U,A)})$. This shows that $f\mid_{(U,A)}$ is soft $I-continuous$.

Theorem 14. Let $f:(X,\tau,A,I)\to(Y,\sigma,B)$ be a soft $I-continuous$ function and $\{(U_\alpha,A):\alpha\in\Delta\}$ be an soft open cover of X. If the restriction function $f\mid_{(U,A)}: ((U,A),\tau\mid_{(U,A)}, I\mid_{(U,A)})\to(Y,\sigma,B)$ is soft $I-continuous$ for each $\alpha\in\Delta$, then f is soft $I-continuous$.

Proof The proof is similar to previous theorem.

Theorem 15. Let $f:(X,\tau,A,I)\to(Y,\sigma,B)$ be a soft $I-continuous$ function and soft open function, then the inverse image of each soft $I-open$ set in Y is soft $preopen$ in X.

Proof Obvious.

Theorem 16. Let $f:(X,\tau,A,I)\to(Y,\sigma,B)$ be a soft $I-continuous$ function and $(f^{-1}(V,B)^*)\ \tilde{\subseteq}(f^{-1}(V,B))^*$ for each soft subset (V,B) of Y. Then the inverse image of each $I-open$ set is soft $I-open$.

Proof Obvious.

Remark 7. The composition of two soft $I-continuous$ function need not to be soft $I-continuous$, in general, as shown by the following example.

Example 9. $X=Y=Z=\{h_1,h_2\}$, $A=B=C=\{e_1,e_2\}$, $\sigma=\{\tilde{\emptyset},\tilde{X},\{(e_1,\{h_2\})\}\}$, $\vartheta=\{\tilde{\emptyset},\tilde{Y},\{(e_1,\{h_1\})\},\{(e_2,\{h_2\})\}\}$, $\tau=\{\tilde{\emptyset},\tilde{X}\}$ $J=\{\tilde{\emptyset},\{(e_1,\{h_1\})\},\{(e_2,\{h_2\})\},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $I=\{\tilde{\emptyset},\{(e_1,\{h_2\})\}\}$. Let U be a identify function from X to X and let P be a identify function from A to A. Then the soft functions $f_{UP}:(X,\tau,A,I)\to(Y,\vartheta,B)$ and $g_{UP}:(Y,\vartheta,B,J)\to(Z,\sigma,C)$ are soft $I-continuous$ but $gof:(X,\tau,A,I)\to(Z,\sigma,C)$ which the composition of f and g is not soft $I-continuous$.

Theorem 17. For soft functions $f:(X,\tau,A,I)\to(Y,\sigma,B,J)$ and $g:(Y,\sigma,B,J)\to(Z,\eta,C)$ the following are hold:

(i) if f is soft $I-continuous$ and g is soft $continuous$ then gof is soft $I-continuous$.

(ii) if f is soft $I-irresolute$ and g is soft $I-continuous$ then gof is soft $I-continuous$.

Proof (i) Let (H,C) be a soft open subset of Z. Since g is soft continuous then $g^{-1}(H,C)$ is soft open in Y. Since f is soft $I-continuous$ then $f^{-1}(g^{-1}(H,C))=(gof)^{-1}$ is soft $I-open$ in X. Thus gof is soft $I-continuous$.

(ii) Let (H,C) be a soft open subset of Z. Since g is soft $I-continuous$ then $g^{-1}(H,C)$ is soft $I-open$ set in Y. Since f is soft $I-irresolute$ then $f^{-1}(g^{-1}(H,C))=(gof)^{-1}$ is soft $I-open$ in X. Thus gof is soft $I-continuous$.

Lemma 3. For any soft function $f:(X,\tau,A,I)\rightarrow(Y,\sigma,B)$, $f(I)$ is an soft ideal on Y.

Proof i. Let $f(F,A)\in f(I)$ and $f(G,A)\tilde{\subseteq} f(F,A)$. Then $(F,A)\in I$ and $(G,A)\tilde{\subseteq}(F,A)$. Since I is soft ideal then $(G,A)\in I$. Thus $f(G,A)\in f(I)$.

ii. Let $f(F,A)\in f(I)$ and $f(G,A)\in f(I)$.Then $(F,A)\in I$ and $(G,A)\in I$. Since I is soft ideal then $(F,A)\tilde{\cup}(G,A)\in I$. Thus $f((F,A)\tilde{\cup}(G,A))=f(F,A)\tilde{\cup} f(G,A)\in f(I)$. Therefore $f(I)$ is $soft\ ideal$.

Definition 26. An soft ideal topological space (X,τ,A,I) is said to be soft $I-compact$ if for every soft $open$ cover $\{(W_i,Ai): i\in\Delta\}$ of X, there exists a finite subset Δ_0 of Δ such that $\tilde{X}-\sqcup\ \{(W_i,A_i): i\in\Delta_0\}\in I$.

Theorem 18 The image of a soft $I-compact$ space under a $I-continuous$ surjective function is $f(I)-compact$.

Proof Let $f:(X,\tau,A,I)\rightarrow(Y,\sigma,B)$ be a soft $I-continuous$ surjection function and $\{(W_i,A_i): i\in\Delta\}$ an open cover of Y. Then $f^{-1}\{(W_i,A_i): i\in\Delta\}$ is $I-open$ cover of X. By the hypothesis, there exists a finite subset Δ_0 of Δ such that $\tilde{X}-\sqcup$ $\{f^{-1}(W_i,A_i): i\in\Delta_0\}\in I$. Therefore, $f(\tilde{X}-\sqcup\{f^{-1}(W_i,A_i): i\in\Delta_0\})=\tilde{Y}-\sqcup\ \{(W_i,A_i): i\in\Delta_0\}\in f(I)$ which shows that is $f(I)$-compact.

5. SOFT $I-Open$ AND SOFT $I-Closed$ FUNCTIONS

Definition 27. A soft function $f:(X,\tau,A)\rightarrow(Y,\sigma,B,J)$ is called soft $I-open$ if $f(U,A)\in SIO(Y,\sigma,J)$ for each soft open set (U,A) in X.

Definition 28 A soft function $f:(X,\tau,A)\rightarrow(Y,\sigma,B,J)$ is called soft $I-closed$ if $f(U,A)\in SIC(Y,\sigma,J)$ for each soft closed set (U,A) in X.

Definition 29 [24] A soft function $f:(X,\tau,A)\rightarrow(Y,\sigma,B)$ is called soft $open$(resp. $closed$) if $f(U,A)\in SO(Y,\sigma,B)$ (resp. $f(U,A)\in SC(Y,\sigma,B)$) for each soft $open(resp. soft\ closed)$ set (U,A) in X.

Remark 8. (1) Every soft $I-open$ (resp. $I-closed$) function is soft $pre-open$ (resp. $pre-closed$) and the converse is not true in general as shown by the following examples.

(2) Soft $I-open$ function and soft $open$ function are independent as shown by the following example.

Example 10. Let a soft topological space (X,τ,A) and a soft ideal topological space (Y,ϑ,B,J) given as follows: $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$, $\tau=\{\tilde{\emptyset},\tilde{X},\{(e_1,\{h_1\})\}\}$, $Y=\{y_1,y_2\}$, $B=\{k_1,k_2\}$, $\vartheta=\{\tilde{\emptyset},\tilde{Y},\{(k_1,\{y_1\}),(k_2,\{y_2\})\},\{(k_1,\{y_1\}),(k_2,\{y_2\})\}\}$, $J=\{\tilde{\emptyset},\{(k_1,\{y_1\})\},\{(k_2,\{y_2\})\},\{(k_1,\{y_1\}),(k_2,\{y_2\})\}\}$. Also, let $U:X\rightarrow Y$, $U(h_1)=y_1$, $U(h_2)=y_2$, $P:A\rightarrow B$, $P(e_1)=k_1$, $P(e_2)=k_2$. Then the soft function $f_{UP}:(X,\tau,A)\rightarrow(Y,\vartheta,B,J)$ is soft $pre-open$ but is not soft $I-open$.

Example 11. Let two soft topological space (X,τ,A) and a soft ideal topological space (Y,ϑ,B,J) given as follows: $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$, $\tau=\{\tilde{\emptyset},\tilde{X},\{(e_1,\{h_2\})\}\}$, $Y=\{y_1,y_2\}$, $B=\{k_1,k_2\}$, $\vartheta=\{\tilde{\emptyset},\tilde{Y},\{(k_1,\{y_1\}),(k_2,\{y_2\})\},\{(k_1,\{y_1\}),(k_2,\{y_2\})\}\}$, $J=\{\tilde{\emptyset},\{(k_1,\{y_1\})\},\{(k_2,\{y_2\})\},\{(k_1,\{y_1\}),(k_2,\{y_2\})\}\}$. Also, let $U:X\rightarrow Y$, $U(h_1)=y_1$, $U(h_2)=y_2$, $P:A\rightarrow B$, $P(e_1)=k_1$, $P(e_2)=k_2$. Then the soft function $f_{UP}:(X,\tau,A)\rightarrow(Y,\vartheta,B,J))$ is soft $I-open$ but is not soft $open$.

Example 12. Let a soft topological space (X,τ,A) and a soft ideal topological space (Y,ϑ,B,J) given as follows: $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$, $\tau=\{\tilde{\emptyset},\tilde{X},\{(e_1,\{h_1\}),(e_2,\{h_2\})\}\}$, $Y=\{y_1,y_2\}$, $B=\{k_1,k_2\}$, $\vartheta=\{\tilde{\emptyset},\tilde{Y},\{(k_1,\{y_1\}),(k_2,\{y_2\})\},\{(k_1,\{y_1\}),(k_2,\{y_2\})\}\}$, $J=\{\tilde{\emptyset},\{(k_1,\{y_1\})\},\{(k_2,\{y_2\})\},\{(k_1,\{y_1\}),(k_2,\{y_2\})\}\}$. Also, let $U:X\rightarrow Y$, $U(h_1)=y_1$, $U(h_2)=y_2$, $P:A\rightarrow B$, $P(e_1)=k_1$, $P(e_2)=k_2$. Then the soft function $f_{UP}:(X,\tau,A)\rightarrow(Y,\vartheta,B,J)$ is soft $open$ but is not soft $I-open$.

Theorem 19. A soft function $f:(X,\tau,A)\rightarrow(Y,\sigma,B,J)$ is $I-open$ function if and only if for each $E_\alpha^x\tilde{\in} X$ and each soft neighborhood (U,A) of E_α^x, there exists $(V,B)\in SIO(Y,\sigma,B)$ containing $f(E_\alpha^x)$ such that $(V,B)\tilde{\subseteq} f(U,A)$.

Proof Suppose that f is a soft $I-open$ function. For each $E_\alpha^x \tilde{\in} X$ and each soft neighborhood (U,A) of E_α^x, there exists $(U_0,A_0) \in \tau$ such that $E_\alpha^x \tilde{\in} (U_0,A_0) \tilde{\subset} (U,A)$. Since f is soft $I-open$, $(V,B) = f((U_0,A_0)) \in SIO(Y,\sigma,B)$ and $f(E_\alpha^x) \in (V,B) \tilde{\subset} f(U,A)$.

Conversely, let (U,A) be an soft open set of (X,τ,A,I). For each $E_\alpha^x \tilde{\in} (U,A)$, there exists $(V,B) \in SIO(Y,\sigma,B)$ such that $f(E_\alpha^x) \tilde{\in} (V,B) \tilde{\subset} f(U,A)$. Therefore, we obtain $f(U,A) = \sqcup \{(V,B): E_\alpha^x \tilde{\in} (U,A)\}$ and $f(U,A) \in SIO(Y,\sigma,B)$. This shows that f is soft $I-open$ function.

Theorem 20. Let $f:(X,\tau,A) \to (Y,\sigma,B,J)$ be a soft $I-open$ (resp. soft $I-closed$) function, (W,A) any soft subset of Y and (F,A) a soft $closed$ (resp. soft $open$) subset of X containing $f^{-1}(W,A)$, then there exists a soft $I-closed$ (soft $I-open$) subset (H,A) of Y containing (W,A) such that $f^{-1}(H,A) \tilde{\subset} (F,A)$.

Proof Suppose that f is a soft $I-open$ function. Let (W,A) be any soft subset of Y and (F,A) a soft closed subset of X containing $f^{-1}(W,A)$. Then $\tilde{X}-(F,A)$ is soft open and since f is soft $I-open$, $f(\tilde{X}-(F,A))$ is soft $I-open$. Hence $(H,A) = \tilde{Y} - f(\tilde{X}-(F,A))$ is soft $I-closed$. It follows from $f^{-1}(W,A) \tilde{\subset} (F,A)$ that $(W,A) \tilde{\subset} (H,A)$. Moreover, we obtain $f^{-1}(H,A) \tilde{\subset} (F,A)$. For a soft $I-closed$ function, we can prove similarly.

Theorem 21. For any soft bijective function $f:(X,\tau,A) \to (Y,\sigma,B,J)$ the following are equivalent:

(i) $f^{-1}:(Y,\sigma,B,J) \to (X,\tau,A)$ is soft $I-continuous$,

(ii) f is soft $I-open$,

(iii) f is soft $I-closed$.

Proof (i)⇒(ii) Let (F,A) be a soft open subset in X. Since f^{-1} is soft $I-continuous$, then $(f^{-1})^{-1}(F,A) = f(F,A)$ is soft $I-open$ in Y. Then f is soft $I-open$.

(ii)⇒(iii) Let (F,A) be a soft closed subset in X, then $X-(F,A)$ is soft open set and since f is soft $I-open$ function, then $f(\tilde{X}-(F,A)) = \tilde{X} - f(F,A)$ is soft closed set, then $f(F,A)$ is soft open set. Thus f is soft $I-closed$.

(iii)⇒(i) Let (F,A) be s soft open subset in X. Then $\tilde{X} - f(F,A)$ is soft closed set, and since f is soft $I-closed$, then $f(\tilde{X}-(F,A)) = \tilde{X} - f(F,A)$ soft $I-closed$. Thus $f(F,A) = (f^{-1})^{-1}(F,A)$ is soft I-open. Therefore f^{-1} is soft $I-continuous$.

Theorem 22. If $f:(X,\tau,A,I) \to (Y,\sigma,B,J)$ is soft open function and $g:(Y,\sigma,B,J) \to (Z,\eta,C,K)$ is soft $I-open$ function then gof is soft $I-open$ function.

Proof Obvious.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] D. Moldtsov, Soft set theory-First results, Computers and Mathematics with Applications, 37 (1999) 19-31.

[2] M. Shabir and M. Naz, On soft topological spaces, Computers and Mathematics with Applications, 61 (2011) 1786-1799.

[3] A. Kharal and B. Ahmad, Mappings on soft classes, New Mathematics and Natural Computation, Vol.7 No.3 (2011) 471-481.

[4] A. Aygunoglu and H. A. Aygun, Some notes on soft topological spaces, Natural Computer Application, DOI 10.1007/s 00521-011-0722-3 (2011).

[5] S.K. Nazmul and S.K. Samanta, Neighbourhood properties of soft topological spaces, Annals of Fuzzy Mathematics and Informatics.

[6] K. Kuratowski, Topology, Vol. I, Academic Press, New York, 1966.

[7] D. Jankovic, T.R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly 97 (4) (1990) 295-310.

[8] G. Kale and A.C. Guler, On Soft Ideal Topological Spaces.

[9] H. Hashimoto, On some local properties on spaces, Math. Japonica, (1952) II, 127-134.

[10] H. Hashimoto, On the *topology and its applications, Fund. Math., (1976) 91, 5-10.

[11] T.R. Hamlett and D. Jankovic, Ideals in topological spaces and the set operator Ψ, Bollettino U.M.I., (1990) vol. 7, 863-874.

[12] M.E. Abd El-Monsef, E.F. Lashien and A.A. Nasef, On I-open sets and I-continuous functions, Kyungpook Math. J., (1992) vol. 32 (1), 21-30

[13] E. Hatır and T. Noiri, On semi-I-continuous sets and semi-I-continuous functions, Acta. Math. Hungar., (2005), 107 (4), 345-353.

[14] D. Jankovic, T.R. Hamlett, Compatible extensions of ideals, Boll. Un. Mat. Ital. (7), 6-B (1992) 453-465.

[15] E. Hatır and T. Noiri, On decompositions o fcontinuity via idealization, Acta. Math. Hungar., 96 (2002), 341-349.

[16] J. Dontchev, On pre-I-open sets and a decomposition of I-continuity, Banyan Math. J., 2 (1996).

[17] G. Kale and A.Caksu Guler, On Soft Ideal Topological Spaces, Neural Computing and Applications, 1-14.

[18] F. Feng, Y.B. Zhao, Soft semirings, Computer and Mathematics with Applications, 56 (2008), 2621-2628.

[19] P.K. Maji, B. Biswas and A.R. Roy, Soft set theory, Computer and Mathematics with Applications, 45 (2003), 555-562.

[20] M.I. Ali, F. Feng, X. Liu, W.K. Min and M. Shabir, On some new operations in soft set theory, Computer and Mathematics with Applications, 57 (2009), 1547-1553.

[21] I. Zorlutuna, M. Akdag, W.K. Min and S. Atmaca, Remarks on soft topological spaces, to appear in Annals of Fuzzy Mathematics and Informatics.

[22] W. Rong, The countabilities of soft topological spaces, International Journal of Computational and Mathematical Sciences, 6 (2012).

[23] J. Mahanta, P.K. Das, On soft topological space via semiopen and semiclosed soft sets.

[24] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-Latif, γoperation and decomposition of some forms of soft continuity in soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 2013.