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A General Fixed Point Theorem for Pairs of Mappings Satisfying a ϕ - Implicit Relation in G - Metric Spaces

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ABSTRACT

In this paper a general fixed point theorem for two pairs of weakly compatible mappings satisfying ϕ - implicit relations in G - metric spaces, which generalize and improve Theorem 2.1 [9], is proved.

Key Words: G - metric spaces, common fixed point, weakly compatible mappings, ϕ - implicit relation.

1. INTRODUCTION

Let (X,d) be a metric space and $S,T:(X,d)\to(X,d)$ be two mappings. In 1994, Pant [17] introduced the notion of pointwise R - weakly commuting mappings. It is proved in [18] that pointwise R - weakly commutativity is equivalent to commutativity in coincidence points. Jungck [8] defined S and T to be weakly compatible if $Sx=Tx$ implies $STx=TSx$. Thus, S and T are weakly compatible if and only if S and T are pointwise R - weakly commuting.

In [5], [6], Dhage introduced a new class of generalized metric space, named D - metric spaces. Mustafa and Sims [11], [12] proved that most of the claims concerning the fundamental topological structures on D - metric spaces are incorrect and introduced an appropriate notion of generalized metric space, named G - metric space. In fact, Mustafa, Sims and other authors studied many fixed point results for self mappings in G - metric spaces under certain conditions [13], [14], [15], [16], [28] and other papers.

Several classical fixed point theorems and common fixed point theorems have been recently unified by considering a general condition by an implicit relation in [19], [20] and other papers. Actually, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, ultra - metric spaces, convex metric spaces, reflexive spaces, compact metric spaces, paracompact metric spaces, in two or three metric spaces, for single valued mappings, hybrid pairs of mappings and set valued mappings. Quite recently, this method is used in the study of fixed points for mappings satisfying an contractive condition of integral type, in fuzzy metric spaces, probabilistic metric spaces and intuitionistic metric spaces.

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The notion of ϕ - implicit relation is introduced in [4]. The study of fixed points for mappings satisfying implicit relations in G - metric spaces is initiated in [21], [22], [24], [26] and in other papers.

The study of fixed points for mappings satisfying a ϕ - contractive implicit relation in G - metric spaces is initiated in [23], [25]. In [23], the authors proved a fixed point theorem for a mapping satisfying a ϕ - implicit relation. Quite recently, in [27] the authors extended this results for a pair of mappings. In [9], a common fixed point theorem for two pairs of weakly compatible mappings in G - metric spaces is proved.

In this paper a general fixed point theorem for two pairs of weakly compatible mappings in G - metric spaces, satisfying ϕ - implicit relations which generalizes and improves the results from Theorem 2.1 [9].

2. PRELIMINARIES

Definition 2.1 [12] Let X be a nonempty set and $G: X^3 \to \mathrm{P}_+$ be a function satisfying the following properties:

$(G_1): G(x,y,z)=0$ if $x=y=z$,

$(G_2): 0<G(x,x,y)$ for all $x,y\in X$ with $x\neq y$,

$(G_3): G(x,x,y)\leq G(x,y,z)$ for all $x,y,z\in X$ with $z\neq y$,

$(G_4): G(x,y,z)=G(y,z,x)=\ldots$ (symmetry in all three variables),

$(G_5): G(x,y,z)\leq G(x,a,a)+G(a,y,z)$ for all $x,y,z,a\in X$ (rectangle inequality).

The function G is called a G - metric on X and the pair (X,G) is called a G - metric space.

Note that if $G(x,y,z)=0$ then $x=y=z$.

Definition 2.2 [12] Let (X,G) be a G - metric space. A sequence (x_n) in (X,G) is said to be:

a) G - convergent if for $\varepsilon>0$, there is an $x\in X$ and $k\in \mathrm{N}$ such that for all $n,m\in\mathrm{N}, n,m\geq k$, $G(x,x_n,x_m)<\varepsilon$.

b) G - Cauchy if for $\varepsilon>0$, there is $k\in\mathrm{N}$ such that for all $n,m,p\in\mathrm{N}$, with $n,m,p\geq k$, $G(x_n,x_m,x_p)<\varepsilon$, that is $G(x_n,x_m,x_p)\to 0$ as $n,m,p\to\infty$.

A G - metric space (X,G) is said to be G - complete if every G - Cauchy sequence is G - convergent.

Lemma 2.1 [12] Let (X,G) be a G - metric space. Then, the following properties are equivalent:

1) (x_n) is G - convergent to x;

2) $G(x_n,x_n,x)\to 0$ as $n\to\infty$;

3) $G(x_n,x,x)\to 0$ as $n\to\infty$;

4) $G(x_n,x_m,x)\to 0$ as $n,m\to\infty$.

Lemma 2.2 [12] If (X,G) is a G - metric space, the following properties are equivalent:

1) (x_n) is G - Cauchy;

2) For $\varepsilon>0$, there exists $k\in\mathrm{N}$ such that $G(x_n,x_m,x_m)<\varepsilon$ for all $m,n\in\mathrm{N}$, $m,n\geq k$.

Lemma 2.3 [12] Let (X,G) be a G - metric space. Then, the function $G(x,y,z)$ is jointly continuous in all three of its variables.

Note that each G - metric generates a topology τ_G on X [12], whose base is a family of open G - balls $\{B_G(x,\varepsilon): x\in X, \varepsilon>0\}$, where $B_G(x,\varepsilon)=\{y\in X: G(x,y,y)<\varepsilon\}$ for all $x,y\in X$ and $\varepsilon>0$.

A nonempty set $A\subset X$ is G - closed if $A=\overline{A}$.

Lemma 2.4 [10] Let (X,G) be a G - metric space and A a subset of X. A is G - closed if for any G - convergent sequence in A with $\lim_{n\to\infty}x_n=x$, then $x\in A$.

In [2], [22], [23] and other papers some fixed point theorems for weakly compatible mappings in G - metric spaces are proved. Quite recently, in [9] the following theorem is proved.

Theorem 2.1 (Theorem 2.1 [9]) Let (X,G) be a complete G - metric space. Suppose that (f,S) and (g,T) are two pairs weakly compatible self - mappings on X satisfying

$$G(fx,fx,gy) \le h\max\{G(Sx,Sx,Ty),G(fx,fx,Sx),G(gy,gy,Ty),\\ \frac{1}{2}[G(fx,fx,Ty)+G(gy,gy,Sx)]\} \tag{2.1}$$

and

$$G(fx,gy,gy) \le h\max\{G(Sx,Ty,Ty),G(fx,Sx,Sx),G(gy,Ty,Ty),\\ \frac{1}{2}[G(fx,Ty,Ty)+G(gy,Sx,Sx)]\} \tag{2.2}$$

for all $x,y \in X$, where $h \in \left[0,\frac{1}{2}\right)$. Suppose that $f(X) \subset T(X)$ and $g(X) \subset S(X)$. If one of $T(X)$ or $S(X)$ is a G - closed subspace of X, then f,g,S and T have an unique common fixed point.

The purpose of this paper is to prove a general fixed point theorem for two pairs of weakly compatible self - mappings of (X,G) satisfying ϕ - implicit relations in G - metric spaces which generalize and improve Theorem 2.1.

3. IMPLICIT RELATIONS

Definition 3.1 [23] A function $f:[0,\infty) \to 0,\infty)$ is a ϕ - function, $f \in \phi$, if f is nondecreasing function such that $\sum_{n=1}^{\infty} f^n(t) < +\infty$, for all $t>0$ and $f(0)=0$.

Definition 3.2 [23] Let F_ϕ be the set of all continuous functions $F(t_1,...,t_6): \mathrm{P}_+^6 \to \mathrm{P}$ such that

(F_1): F is nonincreasing in variable t_5,

(F_2): there exists a function $\phi_1 \in \phi$ such that for all $u,v \ge 0$ with $F(u,v,v,u,u+v,0) \le 0$ implies $u \le \phi_1(v)$,

(F_3): there exists a function $\phi_2 \in \phi$ such that for all $t,t' > 0$, $F(t,t,0,0,t,t') \le 0$ implies $t \le \phi_2(t')$.

The following examples are presented in [23].

Example 3.1.

$$F(t_1,...,t_6)=t_1-k\max\left\{t_2,t_3,t_4,\frac{t_5+t_6}{2}\right\},$$

where $k \in 0,1)$.

Example 3.2.

$F(t_1,...,t_6)=t_1-at_2-bt_3-ct_4-dt_5-et_6$, where $a>0,b,c,d,e \ge 0$ and $a+b+c+2d+e<1$.

Example 3.3.

$F(t_1,...,t_6)=t_1-k\max\{t_2,t_3,t_4,t_5,t_6\}$, where $k \in \left[0,\frac{1}{2}\right)$.

Example 3.4.

$F(t_1,...,t_6)=t_1^2-t_1(at_2+bt_3+ct_4)-dt_5t_6$, where $a>0,b,c,d \ge 0$ and $a+b+c+d<1$.

Example 3.5.

$$F(t_1,...,t_6)=t_1-k\max\left\{t_2,\frac{t_3+t_4}{2},\frac{t_5+t_6}{2}\right\},$$

where $k \in 0,1)$.

Example 3.6.

$$F(t_1,...,t_6)=t_1^3-c\frac{t_3^2t_4^2+t_5^2t_6^2}{1+t_2+t_3+t_4},$$

where $c \in [0,1)$.

Example 3.7.

$$F(t_1,...,t_6)=t_1^2-at_2^2-c\frac{t_5t_6}{1+t_3+t_4},$$

where $a \ge 0$ and $a+c<1$.

Example 3.8.

$F(t_1,...,t_6)=t_1-at_2-bt_3-c\max\{2t_4,t_5+t_6\}$, where $a>0,b,c \ge 0$ and $a+b+2c<1$.

Example 3.9.

$F(t_1,...,t_6)=t_1-at_2-bt_3-c\max\{t_4+t_5,2t_6\}$, where $a>0,b,c \ge 0$ and $a+b+3c<1$.

Example 3.10.

$F(t_1,...,t_6) = t_1 - c\max\{t_2, t_3, \sqrt{t_4 t_6}, \sqrt{t_5 t_6}\}$,
where $c \in (0,1)$.

4. MAIN RESULTS

Theorem 4.1 [1] Let f, g be weakly compatible self mappings on a nonempty set X. If f and g have an unique point of coincidence $w = fx = gx$, then w is the unique common fixed point of f and g.

Theorem 4.2 Let f, g, S and T be self mappings of a G - metric space such that

$$H_1(G(Sx,Ty,Ty), G(fx,gy,gy), G(fx,Sx,Sx), G(gy,Ty,Ty), G(fx,Ty,Ty), G(gy,Sx,Sx)) \le 0, \tag{4.1}$$

$$H_2(G(Tx,Sy,Sy), G(gx,fy,fy), G(gx,Tx,Tx), G(fy,Sy,Sy), G(gx,Sy,Sy), G(fy,Tx,Tx)) \le 0, \tag{4.2}$$

for all $x, y \in X$, where H_1, H_2 satisfy property (F_3).

If there exist $u, v \in X$ such that $fu = Su$ and $gv = Tv$, then there exists $t \in X$ such that t is the unique point of coincidence of f and S, as well t is the unique point of coincidence of g and T.

Proof. First we prove that $fu = gv$. Suppose that $fu \neq gv$. Then, by (4.1) we have successively

$$H_1(G(Su,Tv,Tv), G(fu,gv,gv), G(fu,Su,Su), G(gv,Tv,Tv), G(fu,Tv,Tv), G(gv,Su,Su)) \le 0,$$

$$H_1(G(fu,gv,gv), G(fu,gv,gv), 0, 0, G(fu,gv,gv), G(gv,fu,fu)) \le 0,$$

By (F_3) we obtain

$$G(fu,gv,gv) \le \phi_2(G(gv,fu,fu)) < G(gv,fu,fu).$$

Similarly, by (4.2) we have successively

$$H_2(G(Tv,Su,Su), G(gv,fu,fu), G(gv,Tv,Tv), G(fv,Su,Su), G(gv,Su,Su), G(fu,Tv,Tv)) \le 0,$$

$$H_2(G(gv,fu,fu), G(gv,fu,fu), 0, 0, G(gv,fu,fu), G(fu,gv,gv)) \le 0,$$

By (F_3) we obtain

$$G(gv,fu,fu) \le \phi_2(G(fu,gv,gv)) < G(fu,gv,gv).$$

Hence,

$$G(fu,gv,gv) \le G(gv,fu,fu) < G(fu,gv,gv),$$

a contradiction. Hence, $fu = gv$ which implies $t = fu = gv = Su = Tv$ for some $t \in X$. Therefore t is a common point of coincidence of (f,S) and (g,T). We prove that t is the unique point of coincidence of (f,S). Suppose that there is other point of coincidence $z = fw = Sw$, $z \neq t$.

By (4.1) we have successively

$$H_1(G(Sw,Tv,Tv), G(fw,gv,gv), G(fw,Sw,Sw), G(gv,Tv,Tv), G(fw,Tv,Tv), G(gv,Sw,Sw)) \le 0,$$

$$H_1(G(fw,gv,gv), G(fw,gv,gv), 0, 0, G(fw,gv,gv), G(gv,fw,fw)) \le 0.$$

By (F_3) we have

$$G(fv,gv,gv) \le \phi_2(G(gv,fw,fw)) < G(gv,fw,fw).$$

Similarly, by (4.2) and (F_3) we obtain

$$G(gv,fw,fw) < G(fw,gv,gv).$$

Hence,

$$G(fw,gv,gv) < G(gv,fw,fw) < G(fw,gv,gv),$$

a contradiction. Hence, $fw = gv = fu = t$. Therefore,

$$fw = Sw = t = z.$$

Similarly, we prove that t is the unique point of coincidence of g and T.

Theorem 4.3. Let (X,G) be a G - metric space and $\{f,S\}$ and $\{g,T\}$ be two pairs of self mappings of X satisfying the inequalities (4.1), (4.2) for all $x, y \in X$, $H_1, H_2 \in \mathsf{F}_\phi$ and

$$S(X) \subset g(X) \text{ and} T(X) \subset f(X). \tag{4.3}$$

If one of $f(X)$ or $g(X)$ is a G - closed subset of X, then

a) (f,S) have a coincidence point,

b) (g,T) have a coincidence point.

Moreover, if (f,S) and (g,T) are weakly compatible, then f,g,S and T have an unique common fixed point.

Proof. Let $x_0 \in X$ be an arbitrary point of X. Since $S(X) \subset g(X)$ and $T(X) \subset f(X)$, there exist $x_1, x_2 \in X$ such that $Sx_0 = gx_1$ and $Tx_1 = fx_2$.

By continuing this process, for $n = 0,1,2,...$ we choose $x_n, y_n \in X$ such that

$$y_{2n} = Sx_{2n} = gx_{2n+1}, \quad y_{2n+1} = Tx_{2n+1} = fx_{2n+2}.$$

By (4.1) we have successively

$$H_1(G(Sx_{2n}, Tx_{2n+1}, Tx_{2n+1}), G(fx_{2n}, gx_{2n+1}, gx_{2n+1}), G(fx_{2n}, Sx_{2n}, Sx_{2n}),$$
$$G(gx_{2n+1}, Tx_{2n+1}, Tx_{2n+1}), G(fx_{2n}, Tx_{2n+1}, Tx_{2n+1}), G(gx_{2n+1}, Sx_{2n}, Sx_{2n})) \leq 0,$$

$$H_1(G(y_{2n}, y_{2n+1}, y_{2n+1}), G(yx_{2n-1}, y_{2n}, y_{2n}), G(y_{2n-1}, y_{2n}, y_{2n}),$$
$$G(y_{2n}, y_{2n+1}, y_{2n+1}), G(y_{2n-1}, y_{2n+1}, y_{2n+1}), 0) \leq 0.$$

By (F_1) and (G_5) we obtain

$$H_1(G(y_{2n}, y_{2n+1}, y_{2n+1}), G(y_{2n-1}, y_{2n}, y_{2n}), G(y_{2n-1}, y_{2n}, y_{2n}),$$
$$G(y_{2n}, y_{2n+1}, y_{2n+1}), G(y_{2n-1}, y_{2n}, y_{2n}) + G(y_{2n}, y_{2n+1}, y_{2n+1}), 0) \leq 0.$$

By (F_2) we obtain

$$G(y_{2n}, y_{2n+1}, y_{2n+1}) \leq \phi_1(G(y_{2n-1}, y_{2n}, y_{2n})).$$

Again, by (4.2) we have successively

$$H_2(G(Tx_{2n+1}, Sx_{2n+2}, Sx_{2n+2}), G(gx_{2n+1}, fx_{2n+2}, fx_{2n+2}), G(gx_{2n+1}, Tx_{2n+1}, Tx_{2n+1}),$$
$$G(fx_{2n+1}, Sx_{2n+2}, Sx_{2n+2}), G(gx_{2n+1}, Sx_{2n+2}, Sx_{2n+2}), G(fx_{2n+2}, Tx_{2n+1}, Tx_{2n+1})) \leq 0,$$

$$H_2(G(y_{2n+1}, y_{2n+2}, y_{2n+2}), G(yx_{2n}, y_{2n+1}, y_{2n+1}), G(y_{2n}, y_{2n+1}, y_{2n+1}),$$
$$G(y_{2n+1}, y_{2n+2}, y_{2n+2}), G(y_{2n}, y_{2n+2}, y_{2n+2}), 0) \leq 0.$$

By (F_1) and (G_5) we have that

$$H_2(G(y_{2n+1}, y_{2n+2}, y_{2n+2}), G(y_{2n}, y_{2n+1}, y_{2n+1}), G(y_{2n}, y_{2n+1}, y_{2n+1}),$$
$$G(y_{2n+1}, y_{2n+2}, y_{2n+2}), G(y_{2n}, y_{2n+1}, y_{2n+1}) + G(y_{2n}, y_{2n+2}, y_{2n+2}), 0) \leq 0.$$

By (F_2) we have

$$G(y_{2n+1}, y_{2n+2}, y_{2n+2}) \leq \phi_1(G(y_{2n}, y_{2n+1}, y_{2n+1})),$$

which implies

$$G(y_n, y_{n+1}, y_{n+1}) \leq \phi_1(G(y_{n-1}, y_n, y_n)), n = 1,2,...$$

Then

$$G(y_n, y_{n+1}, y_{n+1}) \le \phi_1^n(G(y_0, y_1, y_1)).$$

We prove that (y_n) is a G - Cauchy sequence in X.

For $n, m \in \mathrm{N}$ with $m > n$, we have repeating (G_5) that

$$\begin{aligned} G(y_n, y_m, y_m) &\le G(y_n, y_{n+1}, y_{n+1}) + G(y_{n+1}, y_{n+2}, y_{n+2}) + ... + G(y_{m-1}, y_m, y_m) \\ &\le \sum_{k=n}^{m-1} \phi_1^k(G(y_0, y_1, y_1)). \end{aligned}$$

Since $\sum_{k=1}^{\infty} \phi_1^k(G(y_0, y_1, y_1)) < +\infty$, then for any $\varepsilon > 0$, there exists $k \in \mathrm{N}$ such that for $m, n \ge k$, $\sum_{k=n}^{m-1} \phi_1^k(G(y_0, y_1, y_1)) < \varepsilon$. Hence, by Lemma 2.2, y_n is a G - Cauchy sequence. Since (X, G) is G - complete, there exists $z \in X$ such that $y_n \to z$ as $n \to \infty$. This implies that $\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} y_{2n+1} = z$.

Suppose that $g(X)$ is closed. It follows that $z = gu$, for some $u \in X$. Using (4.1) we have successively

$$\begin{gathered} H_1(G(Sx_{2n}, Tu, Tu), G(fx_{2n}, gu, gu), G(fx_{2n}, Sx_{2n}, Sx_{2n}), \\ G(gu, Tu, Tu), G(fx_{2n}, Tu, Tu), G(gu, Sx_{2n}, Sx_{2n})) \le 0, \end{gathered}$$

$$\begin{gathered} H_1(G(y_{2n}, Tu, Tu), G(y_{2n-1}, gu, gu), G(y_{2n-1}, y_{2n}, y_{2n}), \\ G(gu, Tu, Tu), G(y_{2n-1}, Tu, Tu), G(gu, y_{2n}, y_{2n})) \le 0. \end{gathered}$$

Letting n tend to infinity we obtain

$$H_1(G(z, Tu, Tu), 0, 0, G(z, Tu, Tu), G(z, Tu, Tu), 0) \le 0,$$

which implies by (F_2) that $G(z, Tu, Tu) \le \phi(0) = 0$. Hence, $z = Tu = gu$ and u is a coincidence point of g and T.

Since $T(X) \subset f(X)$, there exists $v \in X$ such that $z = Tu = fv = gu$. Then by (4.2) we have successively

$$\begin{gathered} H_2(G(Tu, Sv, Sv), G(gu, fv, fv), G(gu, Tu, Tu), \\ G(fv, Sv, Sv), G(gu, Sv, Sv), G(fv, Tu, Tu)) \le 0, \end{gathered}$$

$$H_2(G(gu, Sv, Sv), 0, 0, G(gu, Sv, Sv), G(gu, Sv, Sv), 0) \le 0$$

which implies by (F_2) that

$$G(gu, Sv, Sv) \le \phi(0) = 0,$$

i.e. $z = gu = Sv = fv$. Hence, v is a coincidence point of S and f.

Therefore, z is the common point of coincidence of (f, S) and (g, T). By Theorem 4.1, z is the unique point of coincidence of (f, S) and (g, T). By Theorem 4.1, z is the unique common fixed point of f, g, S and T.

Corollary 4.1 Let (X,G) be a complete G - metric space. Suppose that (f,S) and (g,T) are weakly compatible pairs of self mappings of X satisfying

$$G(Sx,Ty,Ty) \le h\max\{G(fx,gy,gy),G(fx,Sx,Sx),\\ G(gy,Ty,Ty),\frac{1}{2}[G(fx,Ty,Ty)+G(gy,Sx,Sx)]\}, \tag{4.4}$$

$$G(Tx,Sy,Sy) \le h\max\{G(gx,fy,fy),G(gx,Tx,Tx),\\ G(fy,Sy,Sy),\frac{1}{2}[G(gx,Sy,Sy)+G(fy,Tx,Tx)]\} \le 0, \tag{4.5}$$

for all $x,y \in X$ and $h \in 0,1)$. Suppose that $S(X) \subset g(X)$ and $T(X) \subset f(X)$. If one of $g(X)$ or $f(X)$ is a G - closed subspace of X, then f,g,S and T have an unique common fixed point.

Proof. The proof it follows from Theorem 4.3 and Example 3.1 with $\phi_1(t)=\phi_2(t)=kt$.

Remark 4.1

1. In the proof of Theorem 2.1 [3], page 4, lines 10 and 11 from the bottom, there exists some written mistakes and hence the proof of the fact that the sequence (y_n) is a G - Cauchy sequence is not correct. Similarly, in the proof of Theorem 2.1 [9]. For a correct form of Theorem 2.1, we suggest the inequality

$$G(fx,gy,gy) \quad \le \quad h\max\{G(Sx,Ty,Ty),G(Sx,fx,fx),G(Ty,gy,gy),\\ \frac{1}{2}[G(Sx,gy,gy)+G(Ty,fx,fx)]\}$$

instead inequality (2.2).

2. Corollary 4.1 is a generalization and the correct form of Theorem 2.1 because $h \in 0,1)$ instead $h \in \left[0,\frac{1}{2}\right)$ and the fact that (y_n) is a G - Cauchy sequence is correct.
3. By Examples 3.2 – 3.10 we obtain new particular results.

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