



# Double Folding Model Analysis of Elastic Scattering Data of $^{19}\text{F}+^{159}\text{Tb}$ Reaction

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## ABSTRACT

In this study, the elastic scattering data of  $^{19}\text{F}+^{159}\text{Tb}$  reaction at 98 MeV is investigated. In order to obtain the theoretical results, the double folding model within the framework optical model is used. The results are compared with the related literature as well as the experimental data. This comparison provides information about the similarities and differences of the models used in calculations.

**Key Words:** *Optical model, Double folding model, Elastic scattering.*

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## 1. INTRODUCTION

Angular distributions on projectile-like fragments (PFLs) are important in obtaining information about incomplete fusion reactions. For this purpose, projectile-like fragments for different reactions have been measured in the last years. Sodaye et al. [1] have been measured energy spectra and angular distribution of projectile-like fragments in  $^{12}\text{C}+^{169}\text{Tm}$  reaction at 84 MeV. Momota et al. [2] have also investigated the momentum distributions of projectile-like fragments obtained for  $^{40}\text{Ar}+^9\text{Be}$  reaction at  $E/A=90$  MeV and for  $^{16}\text{O}+^{12}\text{C}$  reaction at  $E/A=290$  MeV. Tripathi et al. [3] have reported angular distributions of projectile-like fragments in  $^{19}\text{F}+^{89}\text{Y}$  reaction and have investigated beam energy dependence of the related cross-section. Tripathi et al. [4] have measured cross-sections of projectile-like fragments in  $^{19}\text{F}+^{66}\text{Zn}$  reaction for the beam energy range of 3-6 MeV/nucleon.

In addition to all these, Amit Kumar et al. [5] have recently shown the angular distributions and cross-sections of projectile-like fragments in the  $^{19}\text{F}+^{159}\text{Tb}$

reaction. In the present study, I aim to make similar calculations regarding the elastic scattering data of  $^{19}\text{F}+^{159}\text{Tb}$  reaction, but using the double folding model within the framework of the optical model. I will then compare the results with those in [5] as well as the experimental data. This comparison justifies the reliability of the present model for the data of interest and clarifies the inter-relation between the two different approaches for the elastic scattering reaction considered.

In the section 2, the theoretical model appeared in the calculations and the results obtained through the present analysis are given. Section 3 is devoted to some concluding words.

## 2. DOUBLE FOLDING MODEL ANALYSIS

In the optical model approach, the total effective potential can be written as the sum of nuclear, Coulomb and centrifugal potentials

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$$V_{total}(r) = V_{Nuclear}(r) + V_{Coulomb}(r) + V_{Centrifugal}(r) \tag{1}$$

The  $V_{Coulomb}(r)$  potential [6], owing to a charge  $Z_p e$  of projectile interacting with a charge  $Z_T e$  of target nucleus distributed uniformly over a sphere of radius  $R_C$ , is

$$V_{Coulomb}(r) = \frac{1}{4\pi\epsilon_0} \frac{Z_p Z_T e^2}{r}, \quad r \geq R_C$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Z_p Z_T e^2}{2R_C} \left(3 - \frac{r^2}{R_C^2}\right), \quad r \leq R_C \tag{2}$$

where  $R_C$  is the Coulomb radius, taken as  $1.25(A_p^{1/3} + A_T^{1/3})$  fm in the calculations.

The  $V_{Centrifugal}(r)$  potential is

$$V_{Centrifugal}(r) = \frac{l(l+1)\hbar^2}{2\mu r^2} \tag{3}$$

where  $\mu$  is the reduced mass of the interacting pair.

The nuclear potential consists of two parts, which are the real part and the imaginary part. The nuclear matter distributions of projectile and target nuclei via an effective nucleon-nucleon interaction potential ( $v_{NN}$ ) are used in obtaining the real part of the  $V_{Nuclear}(r)$  potential. In this context, the double folding potential is given as

$$V_{DF}(r) = \int dr_1 \int dr_2 \rho_P(r_1) \rho_T(r_2) v_{NN}(r_{12}) \tag{4}$$

where  $\rho_P(r_1)$  and  $\rho_T(r_2)$  are the nuclear matter density of projectile ( $^{19}\text{F}$ ) and target ( $^{159}\text{Tb}$ ) nuclei, respectively. The density distributions of  $^{19}\text{F}$  and  $^{159}\text{Tb}$  nuclei have been taken from RIPL-3 [7]. The proton and neutron densities of both  $^{19}\text{F}$  and  $^{159}\text{Tb}$  nuclei have been shown in figure 1.

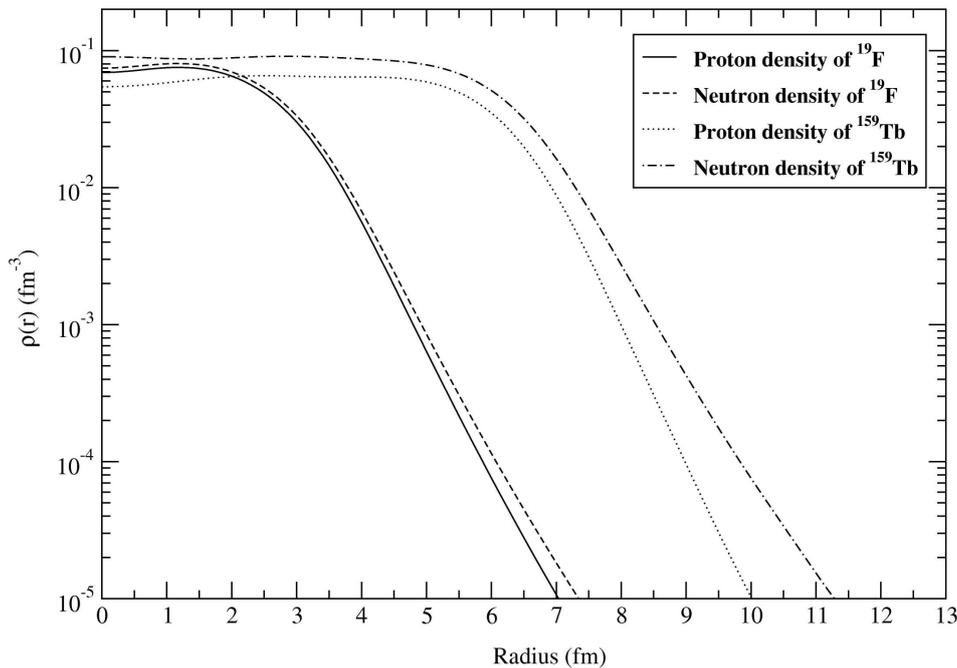


Figure 1. The proton and neutron densities of  $^{19}\text{F}$  and  $^{159}\text{Tb}$  nuclei.

The  $v_{NN}$  is integrated over both density distributions. In literature, some nucleon-nucleon interaction expressions can be found for the folding model potentials. I have taken the M3Y nucleon-nucleon (Michigan 3 Yukawa) realistic interaction which is the most common one. The  $v_{NN}$  is given by [8]

$$v_{NN}(r) = 7999 \frac{\exp(-4r)}{4r} - 2134 \frac{\exp(-2.5r)}{2.5r} + J_{00}(E)\delta(r) \text{ MeV} \tag{5}$$

where  $J_{00}(E)$  is the exchange term, since nucleon exchange is possible between the projectile and the

target.  $J_{00}(E)$  which has a linear energy-dependence can be taken as following form

$$J_{00}(E) = 276 [1 - 0.005 E / A_p] \text{ MeV fm}^3 \quad (6)$$

However, the imaginary part of the  $V_{Nuclear}(r)$  potential has been taken as in the following the Woods-Saxon form

$$W(r) = -\frac{W_0}{1 + \exp((r - R_w) / a_w)} \quad (7)$$

where  $R_w = r_w (A_p^{1/3} + A_T^{1/3})$ .  $A_p$  and  $A_T$  denote mass numbers of projectile and target nuclei respectively. The code FRESKO [9] has been used for all the calculations.

In calculations of the present study, the parameters of the imaginary potential which give convenient results with the experimental data have been searched. All the parameters obtained are shown in table 1. The normalization factor ( $N$ ) is a parameter used in double

folding model calculations, which shows the achievement of the model. The most suitable value of the normalization factor is 1.0. In order to see the validity of our calculations, I have not changed this value and have taken as 1.0. Also, the results obtained are given in comparison with the results of the [5] as well as the experimental data in figure 2. As can be seen from figure 2, the double folding model analysing results are in a remarkable agreement with the data in the whole domain, which obviously clarifies that double folding model explains the elastic scattering data of  $^{19}\text{F}+^{159}\text{Tb}$  reaction in a perfect manner. Also, it generally shows similar behavior with the results of [5].

Table 1: The optical model parameters for  $^{19}\text{F}+^{159}\text{Tb}$  reaction investigated using the double folding model.

System	$N$	$W(\text{MeV})$	$r_w(\text{fm})$	$a_w(\text{fm})$
$^{19}\text{F}+^{159}\text{Tb}$	1.00	14.70	1.19	0.71

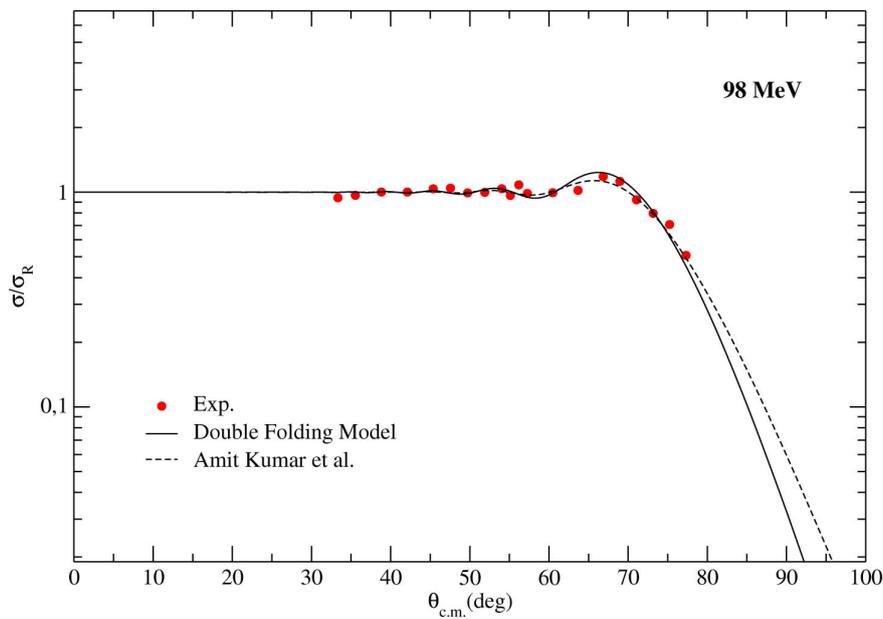


Figure 2. The elastic scattering angular distribution obtained by the double folding model in comparison with the optical model results as well as the experimental data which have been taken from Ref. [5].

Finally, I have found as 1041 mb the cross-section of the  $^{19}\text{F}+^{159}\text{Tb}$  reaction at 98 MeV. Amit Kumar et al. [5] have reported as 1066 mb the magnitude of the cross-section for the same reaction. If one tries to understand the considerable consistency shown in figure 2 between the results obtained and the data, one can conclude that good fits between the different theoretical analysing results and the corresponding elastic scattering angular distribution, as in [10], can be found. The physics behind this, of course, is due to the interaction potentials used in these different theoretical treatments are “phase equivalent”

[11,12] though the structures of these potential functions appear as different, when I remind ourselves the expression of the elastic scattering cross-section in terms of the “phase shift”.

### 3. CONCLUDING REMARKS

In this study, the elastic scattering data of projectile-like fragments of  $^{19}\text{F}+^{159}\text{Tb}$  reaction has been investigated. Within this context, the double folding model with reasonable potential parameters has been used. The calculation results obtained in the present work have been

compared with the experimental data, together with the related results in the previous report [5], and a remarkable agreement has been observed between them. This justifies the reliability of the present technique used in this article.

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#### CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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