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Pressure Corrections in the Potential Flow Analysis of Electrohydrodynamics Kelvin-Helmholtz Instability of Cylindrical Interface through Porous Media

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ABSTRACT

The effect of pressure correction on the linear analysis of Kelvin-Helmholtz instability of cylindrical interface in presence of saturated porous bed structure has been carried out, considering viscous potential flow theory. In the viscous potential flow theory, viscosity enters through normal stress balance and tangential stresses are not considered. The assume fluids in the system are considered to be viscous and incompressible with different kinematic viscosities. The fluids are subjected to be uniform electric field which is acting in the axial direction. A dispersion relation that accounts for the axisymmetric waves has been obtained and stability criterion has been given in terms of relative velocity. The viscous pressure is derived by mechanical energy equation and this pressure correction applied to compute the growth rate of Kelvin-Helmholtz instability. The difference graphs have been drawn, to show the effect of various physical parameters such as porosity and permeability of medium, viscosity ratio, upper fluid fraction on the stability of the system. By the observation of the graphs, that axial electric field has stabilizing effect while porous media has destabilizing effect on the stability of the system.

Keywords: Viscous potential flow; Pressure corrections; Kelvin-Helmholtz instability; Porous media; Cylindrical interface; Axial Electric field.

1. INTRODUCTION

If in a system, when the two different physical properties fluids of are superposed one over another and moving with a relative velocity, the instability occurs at the interface (contact plane of both fluids with each other). This type of instability is called Kelvin-Helmholtz instability [1,2]. The Kelvin-Helmholtz instability occurs in various kinds of situations in our real life like when meteor entering on the earth's atmosphere, wind flowing over the clouds, liquid jets and cooling of fuel rods by liquid coolants in the nuclear reactor etc.

The cylindrical geometry models play very important roles while investigating the stability problems related to liquid jets and cooling of nuclear reactor fuel rods by liquid coolants. Nayak and Chakrborty [3] considered the Kelvin-Helmholtz instability of the cylindrical interface with heat and mass transfer and conclude that plane geometry configuration is more stable than cylindrical one. The Kelvin-Helmholtz instability in a cylindrical flow with a shear layer has been considered by Wu and Wang [4]. The Instabilities (Kelvin-Helmholtz and capillary) were discussed and it is observed that capillary instability can occur in a

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vacuum, but Kelvin-Helmholtz instability cannot occur in a vacuum.

In viscous potential flow theory viscosity enters through normal stress balance and Tangential stresses are not considered. No-slip condition at the boundary is not enforced so that two dimensional solutions satisfy the three dimensional solutions in this theory. In viscous potential flow, a viscous term in the Navier-Stokes equation is identically zero when the vorticity is zero but the viscous stresses are not zero. The viscous potential flow of Kelvin-Helmholtz instability is studied by Joseph and Funada [5]. They concluded that the stability criterion for viscous potential flow is given by the critical value of relative velocity. The viscous potential flow analysis of Capillary instability has been studied by Funada and Joseph [6]. They observed that viscous potential flow is a better approximation of the exact solution instead of inviscid model. Funada and Joseph [7] extended their study of capillary instability viscoelastic fluids of Maxwell types and observed that the growth rates are larger for viscoelastic fluids than the equivalent Newtonian fluids.

Various vorticity and circulation theorems of inviscid potential flow also hold good in viscous potential flow. The viscous potential flow analysis of Kelvin-Helmholtz instability with heat and mass transfer in plane geometry model has been investigated by Asthana and Agrawal [5]. They observed that when the lower fluid viscosity is high, heat and mass transfer has a stabilizing effect and destabilizing effect when lower fluid viscosity is low. Awasthi and Agrawal [6] have studied the viscous potential flow analysis of Kelvin-Helmholtz instability in cylindrical interface and originate that the viscous potential flow solution is more stable instead to inviscid potential flow solution. Funada and Joseph [8] have done the viscous potential flow analysis of Kelvin-Helmholtz instability in a channel and originate that the stability criterion for viscous potential flow is given by the critical value of the relative velocity. From this study Funada and Joseph concluded that the critical value is maximized when the viscosity ratio equals the density ratio.

The study of the relation between the electric field and fluid mechanics is called electrohydrodynamics. The electrohydrodynamic effects and flow are important in many problems of colloidal hydrodynamics such as separation of charged particles during electrophoresis of colloids, proteins etc. Elcoot [9] considered nonlinear analysis of capillary instability in the presence of axial electric field. He observed that, the viscosity plays a dual role in the nonlinear stability criterion. El-Sayed and Callebaut [10] considered nonlinear electrohydrodynamic stability of the interfacial waves of two superposed dielectric fluids. They observed that the normal electric field usually has a destabilizing effect. This effect is relatively stronger when the dielectric constant and the depth of the upper fluid is the greater at the same time and depth of the low fluid is low. Nonlinear electrohydrodynamic Kelvin-Helmholtz instability conditions of a cylindrical interface under the influence of an axial electric field have been studied by Othman [11] and Elhefnawy et al. [12].

Dhiman at el. [13] investigated viscoelastic potential flow analysis of Kelvin-Helmholtz instability in the presence of tangential magnetic field and conclude that the elasticity of the fluid enhances the growth of disturbance wave, at low viscosity fluid is more unstable as compared to high viscosity and inclusion of irrotational shear stresses have stabilizing effects on the stability of the system.

Awasthi et al. [14] considered Pressure corrections for the potential flow analysis of Kelvin-Helmholtz instability with heat and mass transfer. A comparison between viscous potential flow (VPF) solution and viscous contribution to the pressure for potential flow (VCVPF) solution has been made and it is found that the effect of irrotational shearing stresses stabilizes the system. Recently, Dhiman et al. [15] have applied the viscous potential flow analysis of Kelvin-Helmholtz instability of cylindrical interface in the presence of axial electric field in viscous media and observed that the axial electric field has stabilized effect. Wang et al. [16] considered the viscous contribution to the pressure for potential flow analysis of capillary instability of two viscous fluids; and they found that the viscous correction of the pressure leads to an outstanding approximation of the exact solution, uniform in the wave number, when one of the fluids is gas.

In the present article, the analysis of pressure correction on Kelvin-Helmholtz instability of cylindrical interface in the presence of saturated porous bed structure has been carried out by using viscous potential flow theory. In viscous potential flow theory, viscosity enters through normal stress balance and tangential stresses are not considered. The assume fluids are considered to be viscous and incompressible with different kinematic viscosities. The assumed fluids are subjected to be a uniform electric field which is acting in the axial direction. A dispersion relation that accounts for the axisymmetric waves has been obtained and stability criterion has been given in terms of relative velocity. The viscous pressure is derived from the mechanical energy equation and this pressure correction applied to compute the growth rate of Kelvin-Helmholtz instability. The results of the assumed model is discourse and shown by graphically. Finally, a comparison has been made between the results of the present study with the results obtained by Awasthi and Agrawal [15].

2. PROBLEM FORMULATION

We consider a system of two incompressible, irrotational, dielectric and viscous fluid layers separated by a cylindrical interface in an annular configuration as shown in figure-1 (a), figure-1 (b) of constant porosity and permeability ε, k_1 respectively. The system is assumed in cylindrical coordinates (r, ϑ, z) and in the equilibrium state z-axis is the axis of symmetry of the system. The cylindrical interface of both fluids is supposed at radius R. In the undisturbed state, the viscosity of inner fluid is μ_1, density ρ_1 and permittivity β_1 occupies the region $r_1 < r < R$ and second fluid which is outer fluid viscosity of fluid is

μ_2, density ρ_2 and permittivity β_2 occupies the region $R < r < r_2$. Both the fluids are flowing with uniform velocities V_1 and V_2, along the z-axis in the system. The bounding surfaces $r = r_1$ and $r = r_2$ are considered as rigid surface. Surface tension at the interface of the fluids is σ. Let the uniform applied electric field constant E_0 is applied in the axial-direction.

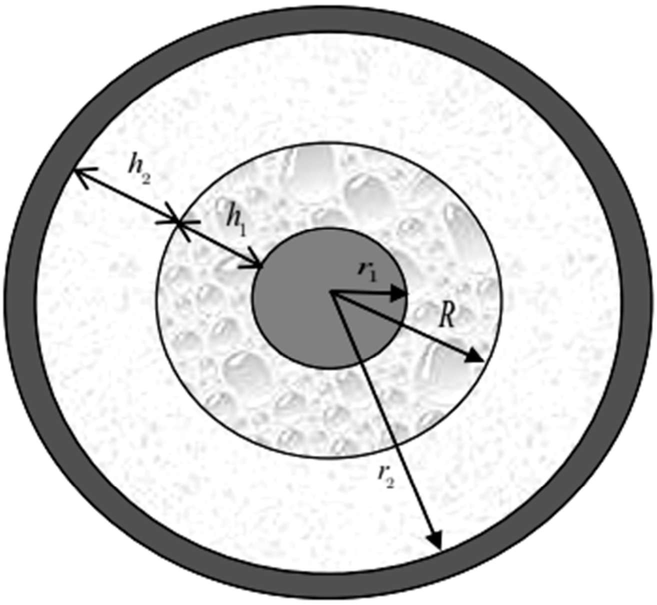

Figure 1 (a): Equilibrium configuration of the system.

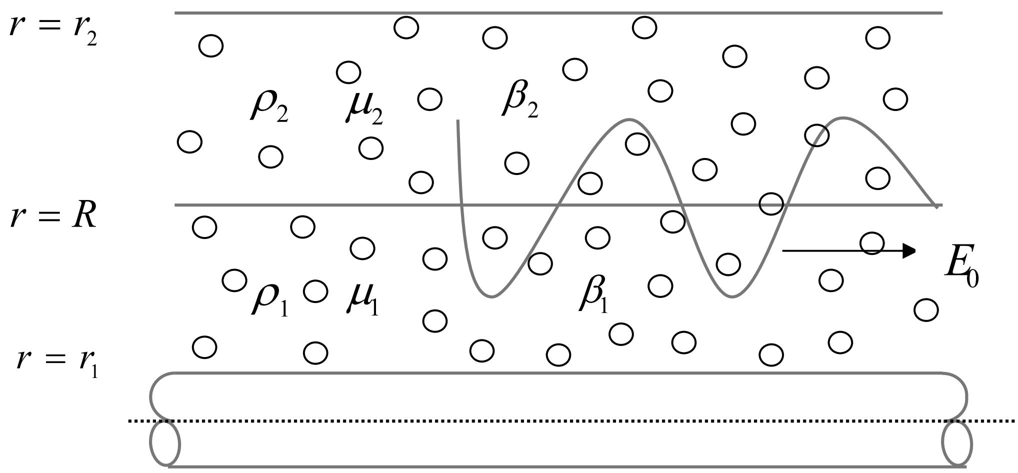

Figure 1 (b): Equilibrium configuration of the system.

The equation of interface after disturbance in the fluid, is

$$F(r,z,t) = r - R - \chi(z,t) = 0 \tag{1}$$

where χ is the perturbation in the radius of the interface the equilibrium value R,
For this unit outward normal to the first order term, is given by

$$n=\frac{\nabla F}{|\nabla F|}=\left\{1+\left(\frac{\partial\chi}{\partial z}\right)^2\right\}^{-\frac{1}{2}}\left(e_r-\frac{\partial\chi}{\partial z}e_z\right) \tag{2}$$

Where e_r and e_z are unit vectors along the *r* and *z* direction respectively. Only axisymmetric disturbances are considered in this analysis.

Assuming that Darcy's law is valid, we obtain the following momentum and continuity equations.

$$\rho\frac{dv}{dt}=-\nabla p-\frac{\mu}{k_1}v \tag{3}$$

$$\nabla\cdot v=0 \tag{4}$$

Here p represents pressure, μ denotes to the fluid viscosity, k_1 is the medium permeability and v is the Darcian (filter) velocity. Taking an average of the fluid velocity over a volume V we get the intrinsic average velocity v, which is related to v by the Dupuit-Forchheimer relationship $v = \varepsilon$ v .Where ε represents the porosity of the medium which is defined as the fraction of the total volume of the medium that is occupied by void space. Small axisymmetric disturbances are superimposed to the basic state. After disturbance, the interface is given by

In both fluid layer velocity is given by the potential function $\phi\,(r,z,t)$ and potential function satisfies Laplace equation.

$$\nabla^2\phi_{(j)}(r,z,t)=0\,,\ (j=1,2) \tag{5}$$

Where $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$

Here we used Quasi static approximation in this the transverse component of the electromagnetic field is dominating and the longitudinal component is neglect able, so the wave equation reduced to Passion's equation

In each fluid electric potential function is denoted by $\psi_{(j)}(r,z,t)$, *j=1,2* is for inner and outer fluid in the assumed system. And fluid electric potential function satisfied the Laplace equation

$$\nabla^2\psi_{(j)}(r,z,t)=0 \tag{6}$$

The electric field is given by $E_j=E_0e_z-\nabla\,\psi_{(j)}$

Condition on the boundary walls are given by

$$\frac{\partial\phi_{(j)}}{\partial r}=0\ ,\ \frac{\partial\psi_{(j)}}{\partial r}=0\quad \text{For } (\,r=r_j,\ j=1,2) \tag{7}$$

Kinematic conditions are given as ($r = R$)

$$\frac{\partial\chi}{\partial t}+V_{(j)}\frac{\partial\chi}{\partial z}=\frac{\partial\phi_{(j)}}{\partial r} \tag{8}$$

Tangential component of electric field should be continuous across the interface *i.e*

$$[\![E_t]\!]=0 \quad \text{Or} \tag{9}$$

$$\left[\!\left[\frac{\partial\psi}{\partial z}\right]\!\right]+\frac{\partial\chi}{\partial z}\left[\!\left[\frac{\partial\psi}{\partial r}\right]\!\right]=0$$

Where $E_t = |n \times E|$ is the tangential component of the electric field and $[\![x]\!] = x_{(2)} - x_{(1)}$ in which superscripts refers to outside and inside fluid, respectively.

The boundary condition corresponding to normal component of electric field, at the interface is given by

$$[\![\beta E_n]\!] = 0 \qquad \text{Or} \tag{10}$$

$$\left[\!\left[\beta \frac{\partial \psi}{\partial r}\right]\!\right] + \frac{\partial \chi}{\partial z}\left[\!\left[\beta\left(E_0 - \frac{\partial \psi}{\partial z}\right)\right]\!\right] = 0$$

Where $E_n = |n.E|$ Is the normal component of the electric field.
The interfacial condition for conservation of momentum is:

$$p_2 - p_1 - 2\mu_{(2)} n.\nabla \otimes \nabla \phi_{(2)}.n + 2\mu_{(1)} n.\nabla \otimes \nabla \phi_{(1)}.n - \frac{1}{2}\left[\!\left[\beta\left(E_n^2 - E_t^2\right)\right]\!\right] = -\sigma \nabla \cdot n \tag{11}$$

where p represent the pressure in conservation of momentum equation

3. PRESSURE CORRECTIONS FOR POTENTIAL FLOW

The pressure correction for the viscous potential flow analysis is another irrotational theory in which the shear stresses do not vanish.

Let $\boldsymbol{n_1} = \boldsymbol{e_r}$ be the unit outward normal at the interface for the inner fluid and $\boldsymbol{n_2} = -\boldsymbol{n_1}$ is the unit outward normal for the outer fluid; and $\boldsymbol{t} = \boldsymbol{e_z}$ be the unit tangent vector. We use the superscripts 'v' for viscous and 'i' for irrotational, respectively. The normal and shear parts of the viscous stress are represented by τ^n and τ^s, respectively.
The mechanical energy equations for outside and inside fluids are, respectively.

$$\begin{aligned}\frac{d}{dt}\int_{V_2} \frac{\rho^{(2)}}{2}|\boldsymbol{u_2}|^2 dV &= \int_A \left(\boldsymbol{u_2} \cdot \boldsymbol{T} \cdot \boldsymbol{n_2}\right) dA - \int_{V_2} 2\mu^{(2)} \boldsymbol{D_2} : \boldsymbol{D_2}\, dV \\ &= -\int_A \left(\boldsymbol{u_2} \cdot \boldsymbol{n_1}(-p_2^i + \tau_2^n) + \boldsymbol{u_2} \cdot \boldsymbol{t}\ \tau_2^s\right) dV - \int_{V_2} 2\mu^{(2)} \boldsymbol{D_2} : \boldsymbol{D_2} dV\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{d}{dt}\int_{V_1} \frac{\rho^{(1)}}{2}|\boldsymbol{u_1}|^2 dV &= \int_A \left(\boldsymbol{u_1} \cdot \boldsymbol{T} \cdot \boldsymbol{n_1}\right) dA - \int_{V_1} 2\mu^{(1)} \boldsymbol{D_1} : \boldsymbol{D_1}\, dV \\ &= \int_A \left(\boldsymbol{u_1} \cdot \boldsymbol{n_1}(-p_1^i + \tau_1^n) + \boldsymbol{u_1} \cdot \boldsymbol{t}\ \tau_1^s\right) dV - \int_{V_1} 2\mu^{(1)} \boldsymbol{D_1} : \boldsymbol{D_1} dV\end{aligned} \tag{13}$$

where $\boldsymbol{D_j}(j = 1, 2)$ is the symmetric part of the rate of strain tensor for inside and outside fluids, respectively.

As the normal velocities are continuous at the interface, we have

$$\boldsymbol{u_2} \cdot \boldsymbol{n_1} = \boldsymbol{u_1} \cdot \boldsymbol{n_1} = u_n \,,$$

Summing equations (12) and (13), we have

$$\begin{aligned}\frac{d}{dt}\int_{V_2} \frac{\rho^{(2)}}{2}|\boldsymbol{u_2}|^2 dV + \frac{d}{dt}\int_{V_1} \frac{\rho^{(1)}}{2}|\boldsymbol{u_1}|^2 dV &= \int_A \left(u_n(-p_1^i + \tau_1^n + p_2^i - \tau_2^n) + \boldsymbol{u_2} \cdot \boldsymbol{t}\ \tau_2^s - \boldsymbol{u_1} \cdot \boldsymbol{t}\ \tau_1^s\right) dA \\ &\quad - \int_{V_2} 2\mu^{(2)} \boldsymbol{D_2} : \boldsymbol{D_2} dV - \int_{V_1} 2\mu^{(1)} \boldsymbol{D_1} : \boldsymbol{D_1} dV\end{aligned} \tag{14}$$

Now two viscous pressure correction terms p_1^v and p_2^v have been introduced for the inner and outer side of the flow region, we can resolve the discontinuity of the shear stress and tangential velocity at the interface, so

$\tau_1^s = \tau_2^s = \tau^s$ and $\boldsymbol{u_2} \cdot \boldsymbol{t} = \boldsymbol{u_1} \cdot \boldsymbol{t} = u_s$

Hence, the equation (14) becomes

$$\frac{d}{dt}\int_{V_2} \frac{\rho^{(2)}}{2}|\boldsymbol{u_2}|^2 dV + \frac{d}{dt}\int_{V_1} \frac{\rho^{(1)}}{2}|\boldsymbol{u_1}|^2 dV = \int_A \left(u_n(-p_1^i - p_1^v + \tau_1^n + p_2^i + p_2^v - \tau_2^n)\right) dA - \int_{V_2} 2\mu^{(2)}\boldsymbol{D_2} : \boldsymbol{D_2} dV - \int_{V_1} 2\mu^{(1)}\boldsymbol{D_1} : \boldsymbol{D_1} dV \tag{15}$$

We can obtain an equation which relates the pressure corrections to the uncompensated irrotational shear stresses by comparing equations (14) and (15)

$$\int_A \left(u_n(-p_1^v + p_2^v)\right) dA = \int_A \left(\boldsymbol{u_1} \cdot \boldsymbol{t}\tau_1^s - \boldsymbol{u_2} \cdot \boldsymbol{t}\tau_2^s\right) dA \tag{16}$$

The governing equation for the pressure corrections is given by

$$\nabla^2 p_j^v = 0 \qquad \text{for } (j = 1, 2) \tag{17}$$

Using the normal mode method, the solution of equation (17) can be written as

$$p_1^v = -\left(C_k i I_n(kr) + E_k i K_n(kr)\right)\exp\left(i\left(kz - \omega t\right)\right) \tag{18}$$

$$p_2^v = -\left(D_k i I_n(kr) + F_k i K_n(kr)\right)\exp\left(i\left(kz - \omega t\right)\right) \tag{19}$$

Here $I_n(kr)$ and $K_n(kr)$ denote the modified Bessel functions of the first kind and second kind of order n, respectively, $i = \sqrt{-1}$ and C_k, E_k, D_k, F_k all denote complex constants.

At the interface $r = R$ the difference in the viscous pressure is expressed as

$$-p_1^v + p_2^v = \left\{(C_k - D_k) i I_n(kR) + (E_k - F_k) i K_n(kR)\right\}\exp\left(i\left(kz - \omega t\right)\right) \tag{20}$$

The equation of conservation of momentum (11) on including the viscous pressure can be written as:

$$p_2^i + p_2^v - p_1^i - p_1^v - 2\mu_{(2)} n.\nabla \otimes \nabla\phi_{(2)}.n + 2\mu_{(1)} n.\nabla \otimes \nabla\phi_{(1)}.n - \frac{1}{2}\left[\beta\left(E_n^2 - E_t^2\right)\right] = -\sigma\nabla \cdot n \tag{21}$$

Let the interface elevation is given by

$$\chi = C\,\exp[i\left(kz - wt\right)] + c.c \tag{22}$$

where *k*, *w* and *C* denote the wave number, complex growth rate and complex constant respectively.

$$\phi_{(1)} = \frac{1}{k}\left(ikV_1 - iw\right) C E_{(1)}\left(\mathrm{kr}\right)\,\exp[i\left(kz - wt\right)] + c.c \tag{23}$$

$$\phi_{(2)} = \frac{1}{k}\left(ikV_2 - iw\right) C E_{(2)}\left(\mathrm{kr}\right)\,\exp[i\left(kz - wt\right)] + c.c \tag{24}$$

$$\psi_{(1)} = \frac{iE_0 k\left(\beta_2-\beta_1\right) g_2\left(k\right)}{\beta_1 g_2\left(k\right) G_1\left(k\right) - \beta_2 g_1\left(k\right) G_2\left(k\right)} \left[I_0\left(kr\right) K_0\left(kr_1\right) - I_0\left(kr_1\right) K_0\left(kr\right)\right] A \exp\left(ikz - iwt\right) + c.c. \quad (25)$$

$$\psi_{(2)} = \frac{iE_0 k\left(\beta_2-\beta_1\right) g_1\left(k\right)}{\beta_1 g_2\left(k\right) G_1\left(k\right) - \beta_2 g_1\left(k\right) G_2\left(k\right)} \left[I_0\left(kr\right) K_0\left(kr_2\right) - I_0\left(kr_2\right) K_0\left(kr\right)\right] A \exp\left(ikz - iwt\right) + c.c. \quad (26)$$

Where *c.c.* stands for complex conjugate of proceeding term $E_{(1)}\left(kr\right)$ and $E_{(2)}\left(kr\right)$ is given by:

$$E_{(1)}\left(kr\right) = \frac{I_0\left(kr\right) K_0'\left(kr_1\right) - K_0\left(kr\right) I_0'\left(kr_1\right)}{I_0'\left(kR\right) K_0'\left(kr_1\right) - K_0'\left(kR\right) I_0'\left(kr_1\right)} \quad (27)$$

$$E_{(2)}\left(kr\right) = \frac{I_0\left(kr\right) K_0'\left(kr_2\right) - K_0\left(kr\right) I_0'\left(kr_2\right)}{I_0'\left(kR\right) K_0'\left(kr_2\right) - K_0'\left(kR\right) I_0'\left(kr_2\right)} \quad (28)$$

$$g_j\left(k\right) = I_0\left(kr_j\right) K_0\left(kR\right) - K_0\left(kr_j\right) I_0\left(kR\right) \quad (29)$$

$$G_j\left(k\right) = I_1\left(kR\right) K_0\left(kr_j\right) + K_1\left(kRr_j\right) I_0\left(kr_j\right) \quad (30)$$

The contribution of irrotational shearing stresses will be obtained by solving equation (16) along with equation (20). So we have

$$\begin{aligned}&\left[(C_k - D_k) i I_n(kR) + (E_k - F_k) i K_n(kR)\right] \\ &\qquad = 2kC\left[\mu^{(1)}\left(ikU_1 - i\omega\right) E^{(1)}(kR) - \mu^{(2)}\left(ikU_2 - i\omega\right) E^{(2)}(kR)\right]\end{aligned} \quad (31)$$

4. DISPERSION RELATION

Using Bernoulli's equation for irrotational pressure and linearizing the equation (11) (Awasthi and Asthana [16]), we get

$$\left[\frac{\rho}{\varepsilon}\frac{\partial\phi}{\partial t} + \frac{\rho V}{\varepsilon}\frac{\partial\phi}{\partial z} + \frac{\mu}{k_1}\phi - p^v + 2\frac{\mu}{\varepsilon}\frac{\partial^2\phi}{\partial r^2} + \beta E_0 \frac{\partial\psi}{\partial z}\right] = \sigma\left(\frac{\partial^2\chi}{\partial z^2} + \frac{\chi}{R^2}\right) \quad (32)$$

Substituting the values of χ, $\phi_{(1)}, \phi_{(2)}, \psi_{(1)}, \psi_{(2)}$ in equation (21), the dispersion relation is obtained as:

$$D(\omega, k) = a_0\omega^2 + \left(a_1 + ib_1\right)\omega + \left(a_2 + ib_2\right) = 0 \quad (33)$$

Where
$$a_0 = \frac{\rho_1 E_1\left(kr\right)}{\varepsilon} - \frac{\rho_2 E_2\left(kr\right)}{\varepsilon}$$

$$a_1 = -2k\left(\frac{\rho_1 V_1 E_1\left(kr\right)}{\varepsilon} - \frac{\rho_2 V_2 E_2\left(kr\right)}{\varepsilon}\right)$$

$$b_1 = \left(\frac{\mu_1 E_1\left(kr\right)}{k_1} - \frac{\mu_2 E_2\left(kr\right)}{k_1} + \frac{2\mu_1 k\left(F_1\left(kr\right) + E_1\left(kr\right)\right)}{\varepsilon} - \frac{2\mu_2 k\left(F_2\left(kr\right) + E_2\left(kr\right)\right)}{\varepsilon}\right)$$

$$a_2 = \frac{k^2}{\varepsilon}\left(\rho_1 V_1^2 E_1\left(kr\right) - \rho_2 V_2^2 E_2\left(kr\right)\right) - \sigma k(k^2 - \frac{1}{R^2}) - \frac{k^3 E_0^2\left(\beta_2 - \beta_1\right)^2 g_1 g_2}{\beta_1 g_2\left(k\right) G_1\left(k\right) - \beta_2 g_1\left(k\right) G_2\left(k\right)}$$

$$b_2 = \frac{k}{k_1}\left(\mu_2 V_2 E_2\left(kr\right) - \mu_1 V_1 E_1\left(kr\right)\right) + 2k^2\left(\frac{\mu_2 V_2\left(F_2\left(kr\right) + E_2\left(kr\right)\right)}{\varepsilon} - \frac{\mu_1 V_1\left(F_1\left(kr\right) + E_1\left(kr\right)\right)}{\varepsilon}\right)$$

$F_1(kr) = E_1(kr) - \dfrac{1}{kR}$, $\qquad F_2(kr) = E_2(kr) - \dfrac{1}{kR}$

Let $\omega = \omega_R + i\omega_I$, and equating the real and imaginary parts of equation (33) will reduces to

$$a_0(\omega_R^2 - \omega_I^2) + (a_1\omega_R - b_1\omega_I) + a_2 = 0 \tag{23}$$

And $2a_0\omega_R\omega_I + a_1\omega_I + b_1\omega_R + b_2 = 0$ (24)

So $\omega_R = -\dfrac{a_1\omega_I + b_2}{2a_0\omega_I + b_1}$

Putting this value in equation (23), we obtained a quadratic equation in ω_I as

$$A_4\omega_I^4 + A_3\omega_I^3 + A_2\omega_I^2 + A_1\omega_I + A_0 = 0 \tag{25}$$

Where

$$\begin{aligned}
&A_4 = -4a_0^3, \qquad A_3 = -8a_0^2 b_1, \\
&A_2 = 4a_0^2 a_2 - 5a_0 b_1^2 - a_0 a_1^2 \\
&A_1 = 4a_0 a_2 b_1 - b_1^3 - a_1^2 b_1 \\
&A_0 = a_0 b_2^2 - a_1 b_1 b_2 + a_2 b_1^2
\end{aligned}$$

$$V^2\left(k^2\left(\begin{aligned}
&\frac{\rho_1\mu_2^2E_1E_2^2}{\varepsilon k_1^2} + \frac{4k^2\rho_1\mu_2^2E_1F_2^2}{\varepsilon^3} + \frac{4k^2\rho_1\mu_2^2E_1E_2^2}{\varepsilon^3} + \frac{8k^2\rho_1\mu_2^2E_1E_2F_2}{\varepsilon^3} + \frac{4k\rho_1\mu_1^2E_1^2F_1}{\varepsilon^3} \\
&+\frac{4k\rho_1\mu_2^2E_1E_2F_2}{\varepsilon^2k_1} + \frac{4k\rho_1\mu_2^2E_1E_2^2}{\varepsilon^2k_1} - \frac{4k\rho_1\mu_1^2E_1^2F_1}{\varepsilon^2k_1} - \frac{\rho_2\mu_1^2E_1^2E_2}{k_1^2\varepsilon} - \frac{4k^2\rho_2\mu_1^2E_2F_1^2}{\varepsilon^3} \\
&-\frac{4k^2\rho_2\mu_1^2E_1^2E_2}{\varepsilon^3} - \frac{8k^2\rho_2\mu_1^2E_1E_2F_1}{\varepsilon^3} - \frac{4k\rho_2\mu_1^2E_1E_2F_1}{\varepsilon^2k_1} - \frac{4k\rho_2\mu_1^2E_1^2E_2}{\varepsilon^2k_1}
\end{aligned}\right)\right)$$

$$= \left(\frac{\sigma k}{R^2} - \sigma k^3 + \frac{k^3E_0^2(\beta_2-\beta_1)^2 g_1g_2}{\beta_2g_1G_2 - \beta_1g_2G_1}\right)\times b_1^2 \text{ m} \tag{26}$$

Here V is the relative velocity which is $V_2 - V_1$.We can obtain the lowest point on the neutral curve $V^2(k) = 0$ as

$$V_c^2(k) = \min\left[V^2(k)\right] = V^2(k_c) \tag{27}$$

Critical wave length is given by $\lambda_c = \dfrac{2\pi}{k_c}$. The flow is unstable when

$$V^2 = \left(-V^2\right) \geq V_c^2 \tag{28}$$

This criterion is symmetric with respect to V and -V, depending only on the absolute value of the difference.

5. RESULT AND DISCUSSIONS

In this section, the numerical computation has been carried out by using the equation (26). In the assumed system two different fluids has been considered. Water and air have been taken for system of interest containing water in the inner region and air in the outer region. The following parametric values have been taken for both fluids.

$\rho_2 = 0.0012\ \mathrm{gm/cm^3}$, $\rho_1 = 1.0\ \mathrm{gm/cm^3}$

$\mu_2 = 0.00018$ poise, $\mu_1 = 0.01$ poise $\sigma = 72.3$ dyne/cm

$\beta_2 = 1.0$ $\beta_1 = 80.37$ $\varepsilon = 0.5$

$r_2 = 2$ cm $r_1 = 1$ cm $R = 1.05$ cm

$E = 5, 10, 15, 20$ Volt

The neutral curve of the relative velocity divides the plane into the stable region (below the curve) and unstable region (above the curve). To observed the effect of porosity on the instability of the interface in the term of relative velocity, we have plotted Fig. 2, which show the variation of neutral curve for relative velocity for various values of porosity. It is observed that increases in porosity unstable region decreases, growth rate also increases so it has shown destabilizing effect on the stability of the system.

In Fig. 3 the graph for relative velocity and wave number is plotted for the different Electric Field 05, 10, 15 and 20 Volt. Studying the curve for different Electric Field, it has been observed increases in Electric Field growth rate of the system also increases, and unstable region is decreasing with respect to Electric Field. The Fig. 4 is Growth rate verses wave number for observing the effect of permeability of the system. As it is observed medium permeability increases, the stable region also increases and therefore, medium permeability has stabilizing effect on the stability of the system. The threshold permeability of the system is 0.45.

The effect of electric field has been studied in the Fig. 5 for the different values of electric fields. As the curves shows, increases the electric fields behaviour of the system is more stable. The effects of permittivity ration of viscous and viscoelastic fluids $\hat{B}\left(\beta_2/\beta_1\right)$ over the critical value of relative velocity have been shows in the Fig. 6. Increases in $\hat{B}$, the critical value of relative velocity, first decreases and thereafter increases. This concludes that permittivity ratio of two fluids plays dual role in the stability criterion of the system. The effect of electric field is null and voided when $\hat{B}\left(\beta_2/\beta_1\right) = 1,\ \beta_1 = \beta_2$. (both fluids permittivity is same)

In Fig. 7 a comparison has been shown between the neutral curve of Electrohydrodynamic Kelvin – Helmholtz Instability of Cylindrical Interface through Porous Media (Dhiman and Awasthi [15]) with obtained result from the present analysis.

6. RESULT FIGURES

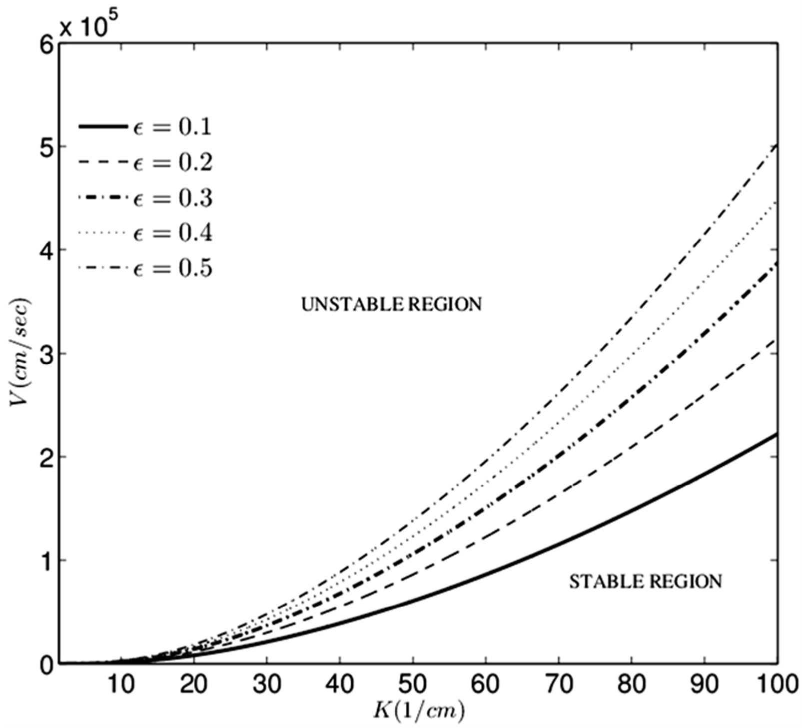

Figure 2: Velocity Vs wave number for different Porosity

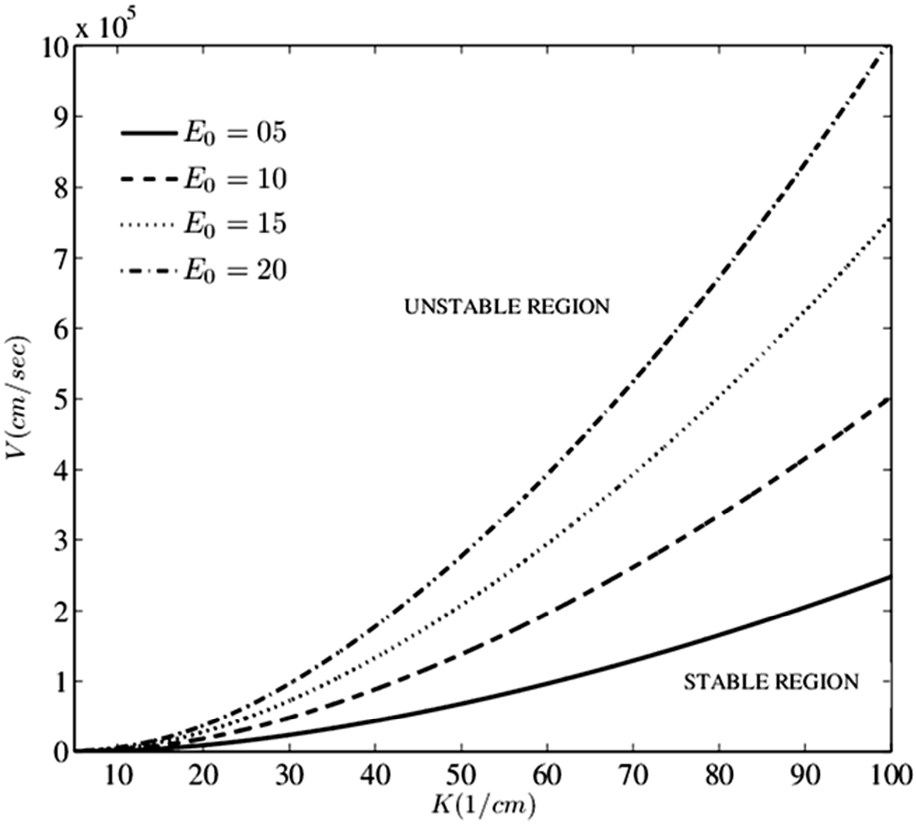

Figure 3: Velocity Vs wave number for different values of Electric Field

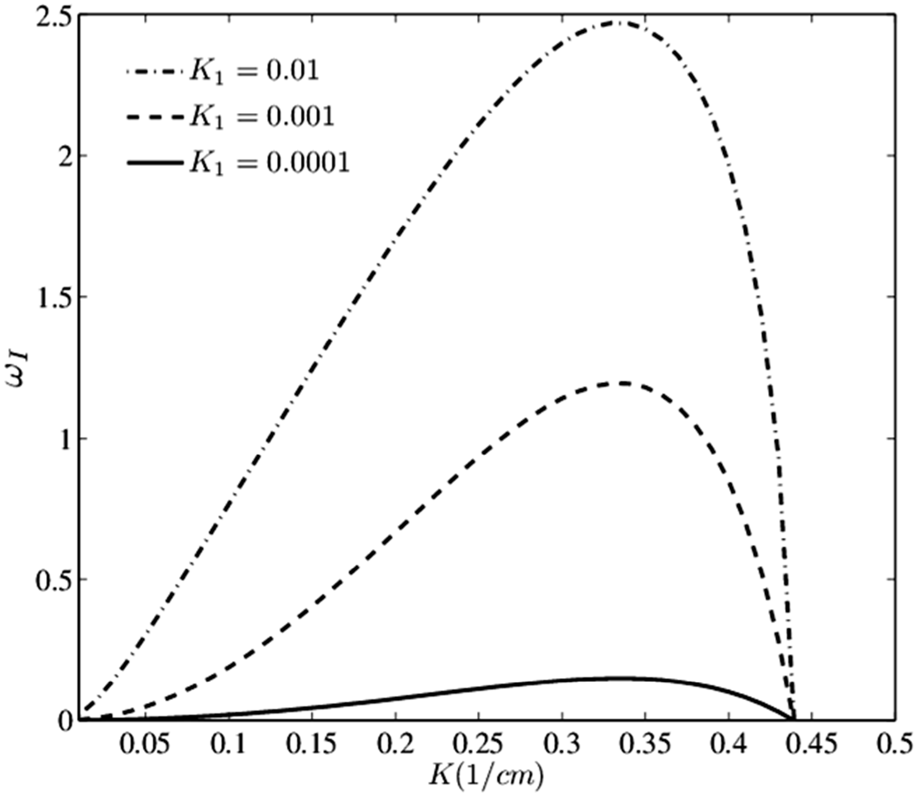

Figure 4: Growth rate Vs wave number curves for different values of Permeability

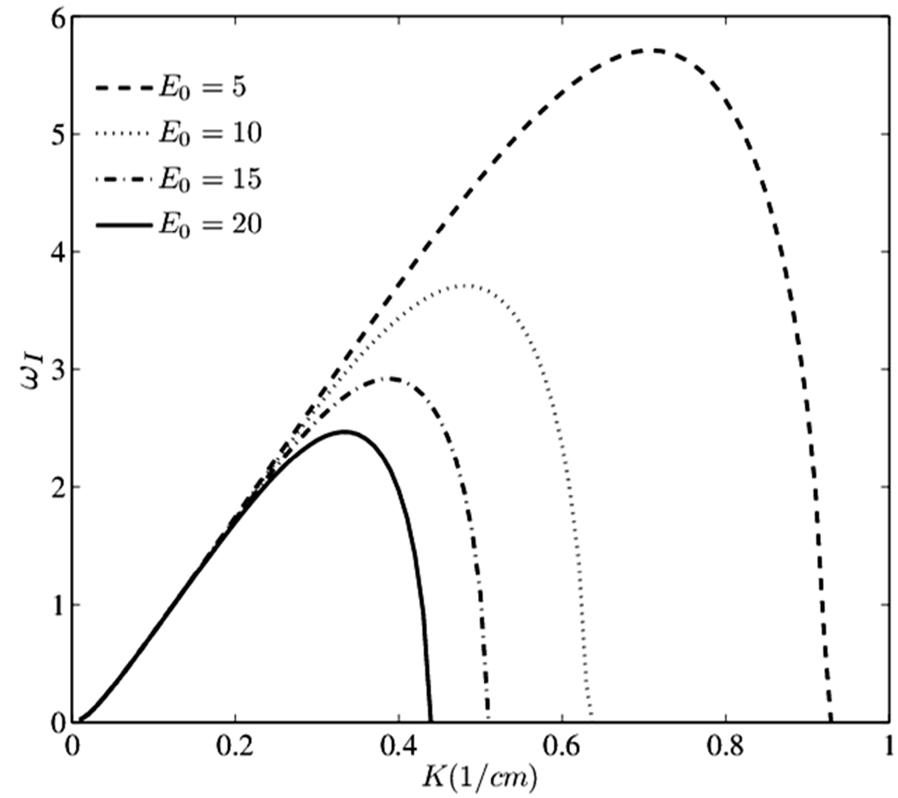

Figure 5: Growth rate Vs wave number curves for different values of Electric Fields

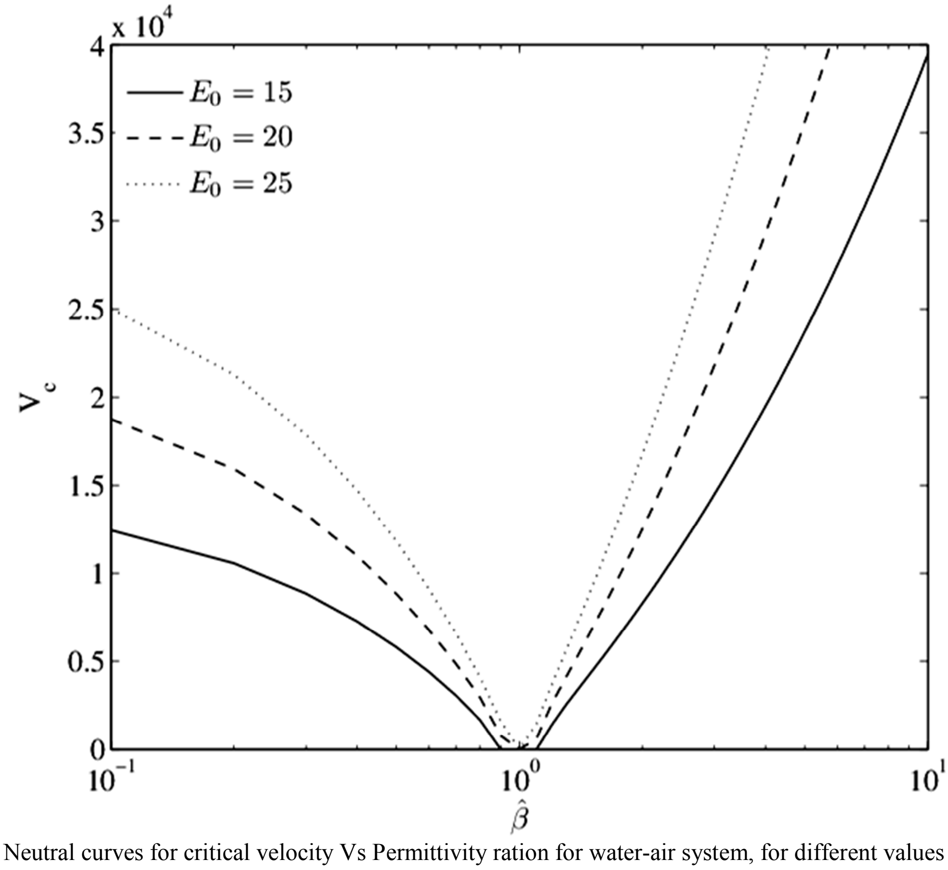

Figure 6: Neutral curves for critical velocity Vs Permittivity ration for water-air system, for different values of Electric Fields

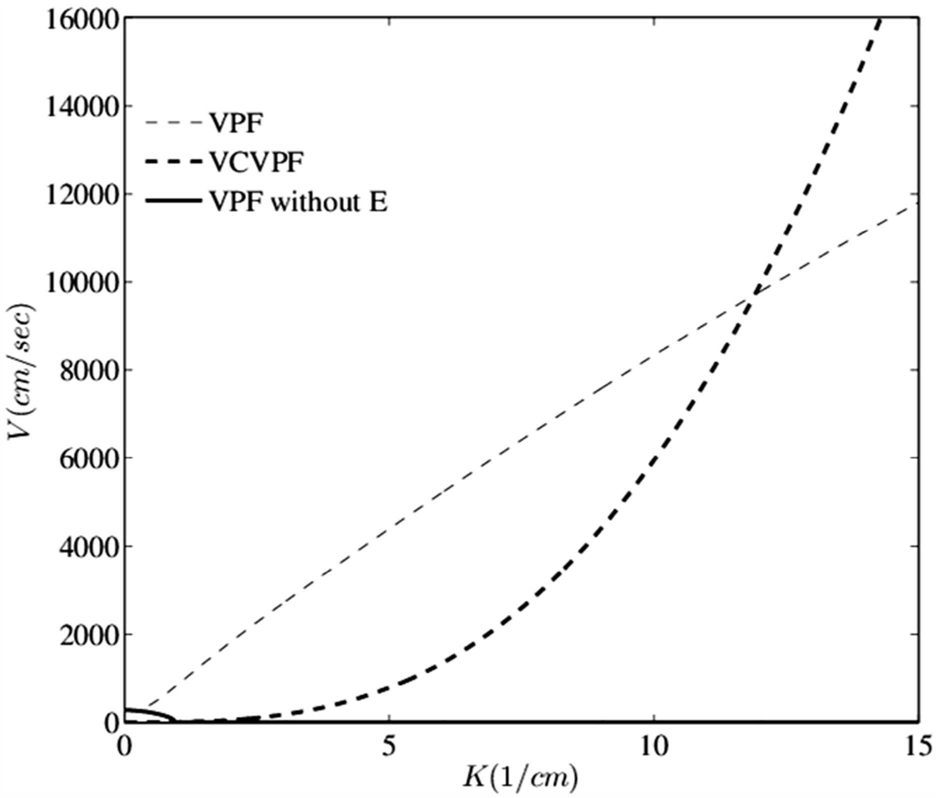

Figure 5: Comparison between Velocity for VPF, VCVPF and VPF in absence of Electric Field, solution.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this paper.

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