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Erratum to the Paper "Some Classes of Kenmotsu Manifolds with Respect to Semi-Symmetric Metric Connection"

D.G. PRAKASHA[1], A. TURGUT VANLI [2,♠], C.S. BAGEWADI[3], D.A. PATIL[4]

[1]*Department of Mathematics, Faculty of Sciences, Karnatak University, Dharwad - 580 003, India.*
[2]*Department of Mathematics, Faculty of Sciences, Gazi University, 06500 Ankara, Turkey.*
[3]*Department of Mathematics, Faculty of Science and Technology, Kuvempu University, Shankaraghatta -577 451, India.*
[4]*Department of Mathematics, Faculty of Basic Sciences, B.V.B. College of Engineering & Technology, Hubli-580 002, India.*

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ABSTRACT

In this paper, we correct the example in the paper "Some classes of Kenmotsu manifolds with respect to semi-symmetric metric connection" Acta Mathematica Sinica, English Series, Vol.29 ,No.7, 1311-1322, July 2013.

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1. INTRODUCTION

Let $\tilde{\nabla}$ be a linear connection in an n--dimensional differentiable manifold M. The torsion tensor $\tilde{T}$ is given by

$$\tilde{T}(X,Y)=\tilde{\nabla}_X Y-\tilde{\nabla}_Y X-[X,Y].$$

The connection $\tilde{\nabla}$ is symmetric if its torsion tensor vanishes, otherwise it is non-symmetric. If there is a Riemannian metric g in M such that $\tilde{\nabla}$ g=0, then the connection $\tilde{\nabla}$ is a metric connection, otherwise it is non-metric. It is known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection. In a Kenmotsu manifold $M(\phi,\xi,\eta,g)$, a semi-symmetric metric connection is defined by

$$\tilde{T}(X,Y)=\eta(Y)X-\eta(X)Y$$

with ξ is the associated vector field (that is, $g(X,\xi)=\eta(X)$).

A relation between the semi-symmetric metric connection $\tilde{\nabla}$ and the Levi-Civita connection ∇ of M is given by

$$\tilde{\nabla}_X Y=\nabla_X Y+\eta(Y)X-g(X,Y)\xi.$$

In a Kenmotsu manifold M of dimension $n\geq 3$, the conharmonic curvature tensor $\tilde{K}$ with respect to semi-symmetric metric connection $\tilde{\nabla}$ is given by

$$\tilde{K}(X,Y)Z=\tilde{R}(X,Y)Z-\frac{1}{n-2}\Big\{\tilde{S}(Y,Z)X-\tilde{S}(X,Z)Y+g(Y,Z)\tilde{Q}X-g(X,Z)\tilde{Q}Y\Big\}$$

♠Corresponding author, e-mail:ayselvanli@gmail.com

for $X,Y,Z \in \Gamma(TM)$ where $\tilde{R}$, $\tilde{S}$ and $\tilde{Q}$ are the Riemannian curvature tensor, Ricci tensor and the Ricci operator with respect to the connection $\tilde{\nabla}$, respectively.

Theorem. A conharmonically flat Kenmotsu manifold with respect to semi-symmetric metric connection is an $\eta-$Einstein manifold with respect to semi-symmetric metric connection.

We give an example which is not true opposite of the Theorem; that is, $M(\phi,\xi,\eta,g)$ is an $\eta-$ Einstein manifold but isn't a conharmonically flat Kenmotsu manifold with respect to semi-symmetric metric connection.

Example. We consider 5-dimensional manifold $\mathrm{M}=\{(\mathrm{x}_1,\mathrm{x}_2,y_1,y_2,z)\in\mathbb{R}^5 : z\neq 0\}$,

where $(\mathrm{x}_1,\mathrm{x}_2,y_1,y_2,z)$ are the standard coordinates in $\mathbb{R}^5$. We choose the vector fields

$$e_1=-e^{-z}\frac{\partial}{\partial \mathrm{x}_1},\quad e_2=-e^{-z}\frac{\partial}{\partial \mathrm{x}_2},$$

$$e_3=e^{-z}\frac{\partial}{\partial y_1}\quad e_4=e^{-z}\frac{\partial}{\partial y_2}\quad e_5=\frac{\partial}{\partial z}$$

which are linearly indepent at each point of M. Let g be the Riemannian metric defined by

$$\mathrm{g}=\sum_{\mathrm{i}=1}^{2}e^{2z}(\mathrm{dx_i}\otimes\mathrm{dx_i}+\mathrm{dy_i}\otimes\mathrm{dy_i})+\eta\otimes\eta$$

where η is the 1-form defined by η (X)=g(X,e_5) for any vector field X on M. Hence, {e_1,e_2,e_3,e_4,e_5} is an orthonormal basis of M. We defined the (1,1) tensor field ϕ as

$$\phi\left(\sum_{i=1}^{2}\left(X_i\frac{\partial}{\partial x_i}+Y_i\frac{\partial}{\partial y_i}\right)+Z_i\frac{\partial}{\partial z}\right)$$

$$=\sum_{i=1}^{2}\left(Y_i\frac{\partial}{\partial x_i}-X_i\frac{\partial}{\partial y_i}\right)$$

Thus, we have

$$\phi(\mathrm{e}_1)=\mathrm{e}_3,\ \phi(\mathrm{e}_2)=\mathrm{e}_4,\ \phi(\mathrm{e}_3)=-\mathrm{e}_1,\ \phi(\mathrm{e}_4)=-\mathrm{e}_2 \text{ and } \phi(\mathrm{e}_5)=0.$$

The linearity property of ϕ and g yields that

$$\eta(e_5)=1,\quad \phi^2X=-X+\eta(X)e_5,$$
$$g(\phi X,\phi Y)=g(X,Y)-\eta(X)\eta(Y)$$

for any vector fields X, Y on M. Thus for $e_5=\xi$, $M(\phi,\xi,\eta,g)$ defines an almost contact metric manifold. The 1-forms η is closed. In addition, we have

$$\Phi=-\sum_{\mathrm{i}=1}^{2}e^{2z}dx_i\Lambda dy_i.$$

Hence, $d\Phi=-\sum_{\mathrm{i}=1}^{2}2e^{2z}dz\Lambda dx_i\Lambda dy_i=2\eta\Lambda\Phi.$

Therefore $M(\phi,\xi,\eta,g)$ is an almost Kenmotsu manifold. It can be seen that $M(\phi,\xi,\eta,g)$ is normal. So, it is Kenmotsu manifold. Moreover, we get

$$[e_i,\xi]=e_i,\quad [e_i,e_j]=0,\quad i,j=1,2,3,4.$$

The Riemannian connection ∇ of the metric g is given

$$2g(\nabla_X Y,Z)=Xg(Y,Z)+Yg(Z,X)-Zg(X,Y)$$
$$+g([X,Y],Z)-g([Y,Z],X)+g([Z,X],Y).$$

Using the Koszul's formula, we obtain

$$\nabla_{e_i}e_i=-\xi,\quad \nabla_{e_i}e_j=0,\quad \nabla_{e_i}\xi=0$$
$$\nabla_{\xi}e_i=-e_i\quad i=1,2,3,4.$$

Therefore, the semi-symmetric metric connection on M is given

$$\tilde{\nabla}_{e_i}e_i=-2\xi,\quad \tilde{\nabla}_{e_i}e_j=0,\quad \tilde{\nabla}_{e_i}\xi=e_i$$
$$\tilde{\nabla}_{\xi}e_i=-e_i\quad i=1,2,3,4.$$

With the help of the above results. It can be easily verified that

$$\tilde{R}(e_i,e_j)e_k=0\quad \tilde{R}(e_i,e_j)e_i=2e_j$$
$$\tilde{R}(e_i,e_j)e_j=-2e_i\quad \tilde{R}(e_i,\xi)e_j=0$$
$$\tilde{R}(\xi,e_j)\xi=2e_i\quad \tilde{R}(e_i,\xi)e_i=2\xi$$
$$\tilde{R}(\xi,e_j)e_j=-4\xi\quad \tilde{R}(e_i,\xi)\xi=0$$
$$i,j=1,2,3,4.$$

From the above expressions of the curvature tensor we obtain

$$\tilde{S}(X,Y)=10g(X,Y)-2\eta(X)\eta(Y)$$

for any vector fields X and Y. Therefore, $M(\phi,\xi,\eta,g)$ is an $\eta-$ Einstein manifold with respect to semi-symmetric metric connection. In addition, we have $\tilde{K}(\xi,e_i)\xi\neq 0.$ Thus, $M(\phi,\xi,\eta,g)$ isn't a conharmonically flat Kenmotsu manifold with respect to semi-symmetric metric connection.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

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