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On a Fixed Point Theorem with PPF Dependence in the Razumikhin Class

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ABSTRACT

The aim of this paper is to prove fixed point theorem with PPF dependence for mappings involving (ϕ,ψ)-rational type contraction in Razumikhin class. Our result extends and generalizes the result of Jaggi [9].

Keywords: PPF dependent fixed point, rational type contraction, Razumikhin class.

Mathematics Subject Classification: 47H10; 54H25.

1. INTRODUCTION

Fixed point theory is one of the well-known traditional theories in mathematics that has a broad set of applications. In 1922, Polish mathematician Stephan Banach published his famous contraction principle. Since then, this principle has been extended and generalized in several ways either by using the contractive condition or by imposing some additional conditions on an ambient space. This principle is one of the cornerstones in the development of fixed point theory. From inspiration of this work, several mathematicians heavily studied this field. For example, the work of Kannan [11],Chatterjea [3], Borinde [1], Ciric [4], Geraghty [8], Meir and Keeler [13], Suzuki [15] and so forth. Das and Gupta [5] and Jaggi [9] were the pioneers in proving fixed point theorems using contractive conditions involving rational expressions.

On the other hand, Bernfeld et al. [2] introduced the conceptof Past-Present-Future (for short PPF) dependent fixedpoint or the fixed point with PPF dependence which is onetype of fixed points formappings that have different domainsand ranges. They also established the existence of PPF dependentfixed point theorems in the Razumikhin class for Banach type contraction mappings. These results are usefulfor proving the solutions of nonlinear functional differentialand integral equations which may depend upon the past history, present data, and future consideration. Some papers about fixed point theorems with PPF dependence have appeared in the literature (see e.g., [6, 7, 10, 12, and 14]).

In [14] authors present some fixed point theorems for contraction of rational type with PPF dependence. While in [12] authors introduced the new type of contraction mappings called Ciric-rational type contraction and gave sufficient condition for the existenceof PPF dependent

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fixed point theorems inRazumikhin class. They apply their results to study the existence and uniqueness of solution of a nonlinear integral equation.

In this paper, we prove a PPF dependent fixed point theorem for mappings involving (ϕ,ψ) - rational type contraction in the Razumikhin class.

2. PRELIMINARIES

In this section, we recall some concepts and definitions thatwill be required in the sequel. Throughout this paper, E will denote a Banach space with the norm $\|.\|_E$, and $E_0 = C([a,b],E)$ will denote the space of the continuous E-valued functions defined on $[a,b]$ and equipped with the norm $\|.\|_{E_0}$ given by $\|\phi\|_{E_0} = \sup_{t\in[a,b]}\|\phi(t)\|_E$, for $\phi \in E_0$.

Let $T : E_0 \to E$ be a mapping. A point $\phi \in E_0$ is said to be a PPF dependence fixed point of T or a fixed point with PPF dependence of T if $T\phi = \phi(c)$ for some $c \in [a,b]$.
For a fixed element $c \in [a,b]$, the Razumikhin class $\Re_c$ is defined by

$$\Re_c = \{\phi \in E_0 : \|\phi\|_{E_0} = \|\phi(c)\|_E\}$$

Remark 2.1: Note that, for $x \in E$ fixed, the function ϕ_x defined by $\phi_x(t) = x$ for $t \in [a,b]$ satisfies $\phi_x \in E_0, \|\phi_x\|_{E_0} = \|\phi_x(c)\|_E = \|x\|$ for any $c \in [a,b]$, and therefore $\phi_x \in \Re_c$ for any $c \in [a,b]$. Consequently, $\Re_c \neq \phi$ for any $c \in [a,b]$.

We say that the class $\Re_c$ is topologically closed with respect to difference if for any $\phi,\xi \in \Re_c$, we have $\phi - \xi \in \Re_c$. Similarly, we say that the class $\Re_c$ is topologically closed with respect to the topology on E_0 induced by the norm $\|.\|_{E_0}$.
The Razumikhin class plays an important role in the existence of PPF dependent fixed point.

Definition 2.1(see Bernfeld et al. [2]). The mapping $T : E_0 \to E$ is said to be Banach type contraction if there exists a real number $\alpha \in [0,1)$ such that

$$\|T\phi - T\xi\|_E \leq \alpha\|\phi - \xi\|_{E_0}$$

for all $\phi,\xi \in E_0$.

The following PPF dependent fixed point theorem is proved by Bernfeld et al. [2].

Theorem 2.1([2]). Let $T : E_0 \to E$ be a Banach type contraction. If $\Re_c$ is topologically closed and algebraically closed with respect to difference, then *T*has aunique PPF dependent fixed point in $\Re_c$.

3. MAIN RESULTS

Throughout this section, we will denote Ψ the family of continuous and monotone non-decreasing functions $\psi : [0,\infty) \to [0,\infty)$ such that $\psi(t) = 0$ if and only if $t = 0$ and Φ the family of lower semi-continuous functions $\phi : [0,\infty) \to [0,\infty)$ such that $\phi(t) = 0$ if and only if $t = 0$.

Theorem 3.1. Let $T: E_0 \to E$ be a mapping satisfying

$$\psi(\|T\xi - T\eta\|_E) \le \psi(M(\xi,\eta)) - \phi(N(\xi,\eta)) \quad (1)$$

for all $\xi,\eta \in E_0$, where $\psi \in \Psi$ and $\phi \in \Phi$.

$$M(\xi,\eta) = \max\left\{ \frac{\|\eta(c) - T\eta\|_E \|\xi(c) - T\xi\|_E}{1 + \|\xi - \eta\|_{E_0}}, \frac{\|\xi(c) - T\xi\|_E}{1 + \|\xi - \eta\|_{E_0}}, \|\xi - \eta\|_{E_0} \right\}$$

And

$$N(\xi,\eta) = \max\left\{ \frac{\|\eta(c) - T\eta\|_E \left[1 + \|\xi(c) - T\xi\|_E\right]}{1 + \|\xi - \eta\|_{E_0}}, \|\xi - \eta\|_{E_0} \right\}$$

If $\Re_c$ is topologically closed and algebraicallyclosed with respect to difference, then *T*has a unique PPFdependent fixed point in $\Re_c$

Proof: Let ξ_0 be an arbitrary function in $\Re_c$ (whose existence is guaranteed by Remark 2.1). Since $\xi_0 \in \Re_c \subset E_0$ put $x_1 = T\xi_0 \in E$. Again by Remark 2.1, we can find $\xi_1 \in \Re_c$ such that $T\xi_0 = x_1 = \xi_1(c)$.Since $\xi_1 \in \Re_c \subset E_0$, put $x_2 = T\xi_1 \in E.$

Using the same argument, we can find $\xi_2 \in \Re_c$ such that $T\xi_1 = x_2 = \xi_2(c).$

Repeating this process, we can obtain a sequence $\{\xi_n\}$ in $\Re_c$ such that

$T\xi_{n-1} = \xi_n(c)$ for any $n \in N$ (2)

Since $\Re_c$ is algebraically closed with respect to difference, we have

$$\|\xi_p - \xi_q\|_{E_0} = \|\xi_p(c) - \xi_q(c)\|_E \text{ for all } p,q \in N. \quad (3)$$

First, we will prove that $\lim_{n\to\infty} \|\xi_n - \xi_{n+1}\|_{E_0} = 0$.

In fact, taking into account (2) and (3), we get

$$\|\xi_{n+1} - \xi_n\|_{E_0} = \|\xi_{n+1}(c) - \xi_n(c)\|_E$$

$= \|T\xi_n - T\xi_{n-1}\|_E$ for any $n \in N$,

and therefore from (1) with the help of (2) and (3).

$$\psi\left(\|\xi_n - \xi_{n+1}\|_{E_0}\right) = \psi\left(\|T\xi_{n-1} - T\xi_n\|_{E_0}\right)$$

$$\leq \psi\left(M(\xi_{n-1}, \xi_n)\right) - \phi\left(N(\xi_{n-1}, \xi_n)\right)$$

where

$$M(\xi_{n-1}, \xi_n) = \max\left\{\frac{\|\xi_n(c) - T\xi_n\|_E \|\xi_{n-1}(c) - T\xi_{n-1}\|_E}{1 + \|\xi_{n-1} - \xi_n\|_{E_0}}, \frac{\|\xi_{n-1}(c) - T\xi_{n-1}\|_E}{1 + \|\xi_{n-1} - \xi_n\|_{E_0}}, \|\xi_{n-1} - \xi_n\|_{E_0}\right\}$$

$$= \max\left\{\frac{\|\xi_n(c) - \xi_{n+1}(c)\|_E \|\xi_{n-1}(c) - \xi_n(c)\|_E}{1 + \|\xi_{n-1} - \xi_n\|_{E_0}}, \frac{\|\xi_{n-1}(c) - \xi_n(c)\|_E}{1 + \|\xi_{n-1} - \xi_n\|_{E_0}}, \|\xi_{n-1} - \xi_n\|_{E_0}\right\}$$

$$= \max\left\{\frac{\|\xi_n - \xi_{n+1}\|_{E_0} \|\xi_{n-1} - \xi_n\|_{E_0}}{1 + \|\xi_n - \xi_{n-1}\|_{E_0}}, \frac{1 + \|\xi_{n-1} - \xi_n\|_{E_0}}{1 + \|\xi_n - \xi_{n-1}\|_{E_0}}, \|\xi_n - \xi_{n-1}\|_{E_0}\right\}$$

(Since $\|\xi_{n-1} - \xi_n\|_{E_0} \leq 1 + \|\xi_{n-1} - \xi_n\|_{E_0}$)

$$= \max\left\{\|\xi_n - \xi_{n+1}\|_{E_0}, \|\xi_{n-1} - \xi_n\|_{E_0}\right\}$$

and

$$N(\xi_{n-1}, \xi_n) = \max\left\{\frac{\|\xi_n(c) - T\xi_n\|_E \|\xi_{n-1}(c) - T\xi_{n-1}\|_E}{1 + \|\xi_{n-1} - \xi_n\|_{E_0}}, \|\xi_{n-1} - \xi_n\|_{E_0}\right\}$$

$$= \max\left\{\frac{\|\xi_n(c) - \xi_{n+1}(c)\|_E \|\xi_{n-1}(c) - \xi_n(c)\|_E}{1 + \|\xi_{n-1} - \xi_n\|_{E_0}}, \|\xi_{n-1} - \xi_n\|_{E_0}\right\}$$

$$= \max\left\{\frac{\|\xi_n - \xi_{n+1}\|_{E_0} \|\xi_{n-1} - \xi_n\|_{E_0}}{1 + \|\xi_{n-1} - \xi_n\|_{E_0}}, \|\xi_{n-1} - \xi_n\|_{E_0}\right\}$$

$$= \max\left\{\|\xi_n - \xi_{n+1}\|_{E_0}, \|\xi_{n-1} - \xi_n\|_{E_0}\right\}$$

Hence we obtain

$$\psi\left(\|\xi_n - \xi_{n+1}\|_{E_0}\right) \leq \psi\left(\max\left\{\|\xi_n - \xi_{n+1}\|_{E_0}, \|\xi_{n-1} - \xi_n\|_{E_0}\right\}\right) - \phi\left(\max\left\{\|\xi_n - \xi_{n+1}\|_{E_0}, \|\xi_{n-1} - \xi_n\|_{E_0}\right\}\right) \tag{4}$$

Now, let us suppose that there exists $n_0 \in N$ such that $\left\|\xi_{n_0+1} - \xi_{n_0}\right\|_{E_0} = 0.$

In this case $\xi_{n_0+1} = \xi_{n_0}$.

Consequently, $\xi_{n_0+1}(c) = \xi_{n_0}(c)$.

By (2), we have

$$T\xi_{n_0} = \xi_{n_0+1}(c) = \xi_{n_0}(c) \tag{5}$$

Therefore, ξ_{n_0} would be the PPF dependent fixed point.

In the sequel, we suppose that $\left\|\xi_n - \xi_{n+1}\right\|_{E_0} \neq 0.$ for any $n \in N$.

Now, if

$$\max\left\{\left\|\xi_n - \xi_{n+1}\right\|_{E_0}, \left\|\xi_{n-1} - \xi_n\right\|_{E_0}\right\} = \left\|\xi_n - \xi_{n+1}\right\|_{E_0},$$

From (4), we get

$$\psi\left(\left\|\xi_n - \xi_{n+1}\right\|_{E_0}\right) \le \psi\left(\left\|\xi_n - \xi_{n+1}\right\|_{E_0}\right) - \phi\left(\left\|\xi_n - \xi_{n+1}\right\|_{E_0}\right) < \psi\left(\left\|\xi_n - \xi_{n+1}\right\|_{E_0}\right),$$

which is a contradiction .So we have

$$\left\|\xi_n - \xi_{n+1}\right\|_{E_0} \le \left\|\xi_{n-1} - \xi_n\right\|_{E_0} \text{ for any } n \in N \tag{6}$$

Thus, the sequence $\left\{\left\|\xi_n - \xi_{n+1}\right\|_{E_0}\right\}$ is a decreasing sequence of non-negative real numbers.

Put

$$r = \lim_{n\to\infty}\left\|\xi_n - \xi_{n+1}\right\|_{E_0} = 0, \tag{7}$$

where $r \ge 0$.

Suppose that $r > 0$. Taking the upper limit in equation (4) as $n \to \infty$ and using (7) with the properties of ψ, ϕ , we have

$$\psi(r) \le \psi(r) - \lim\inf \phi\left(\left\|\xi_{n-1} - \xi_n\right\|_{E_0}\right)$$

$$\le \psi(r) - \phi(r) < \psi(r),$$

which is a contradiction .

Therefore,

$$\lim_{n\to\infty}\left\|\xi_n - \xi_{n+1}\right\|_{E_0} = 0 \tag{8}$$

Next, we will show that $\{\xi_n\}$is a Cauchy sequence in E_0. In contrary caselet $\lim_{n\to\infty}\|\xi_n-\xi_{n+1}\|_{E_0}=0$,byLemma 2.1 of [8], we can find $\varepsilon>0$ and subsequences $\{\xi_{n(k)}\}$ and $\{\xi_{m(k)}\}$ of $\{\xi_n\}$ satisfying

(i) $n(k)>m(k)\geq k$ for $k>0$;

(ii) $\varepsilon\leq\|\xi_{n(k)}-\xi_{m(k)}\|_{E_0}$, $\|\xi_{n(k)-1}-\xi_{m(k)}\|_{E_0}<\varepsilon$ for $k>0$;

(ii) $\lim_{k\to\infty}\|\xi_{n(k)-1}-\xi_{m(k)-1}\|_{E_0}=\lim_{k\to\infty}\|\xi_{n(k)-1}-\xi_{m(k)}\|_{E_0}$

$$=\lim_{k\to\infty}\|\xi_{n(k)}-\xi_{m(k)-1}\|_{E_0}$$
$$=\lim_{k\to\infty}\|\xi_{n(k)}-\xi_{m(k)}\|_{E_0}=\varepsilon \tag{9}$$

Since $\xi_{n(k)},\xi_{m(k)+1}\in\Re_c$ for any $k\in N$. From (2) and (3), we have

$$\|\xi_{n(k)}-\xi_{m(k)}\|_{E_0}=\|\xi_{n(k)}(c)-\xi_{m(k)}(c)\|_E$$
$$=\|T\xi_{n(k)-1}-T\xi_{m(k)-1}\|_E \text{ for any } k\in N. \tag{10}$$

Now using (1), (2) and (3), we obtain

$$\psi\left(\|\xi_{m(k)}-\xi_{n(k)}\|_{E_0}\right)=\psi\left(\|T\xi_{m(k)-1}-T\xi_{n(k)-1}\|_{E_0}\right)$$
$$=\psi\left(M(\xi_{m(k)-1}-\xi_{n(k)-1})\right)-\phi\left(N(\xi_{m(k)-1}-\xi_{n(k)-1})\right) \tag{11}$$

where

$$M(\xi_{m(k)-1},\xi_{n(k)-1})=\max\left\{\frac{\|\xi_{n(k)-1}(c)-T\xi_{n(k)-1}\|_E\|\xi_{m(k)-1}(c)-T\xi_{m(k)-1}\|_E}{1+\|\xi_{m(k)-1}-\xi_{n(k)-1}\|_{E_0}},\right.$$

$$\left.\frac{\|\xi_{m(k)-1}(c)-T\xi_{m(k)-1}\|_E}{1+\|\xi_{m(k)-1}-\xi_{n(k)-1}\|_{E_0}},\left\|\|\xi_{m(k)-1}-\xi_{n(k)-1}\|_{E_0}\right\|\right\}$$

$$=\max\left\{\frac{\|\xi_{n(k)-1}(c)-\xi_{n(k)}(c)\|_E\|\xi_{m(k)-1}(c)-\xi_{m(k)}(c)\|_E}{1+\|\xi_{m(k)-1}-\xi_{n(k)-1}\|_{E_0}},\right.$$

$$\left.\frac{1+\|\xi_{m(k)-1}(c)-\xi_{m(k)}(c)\|_E}{1+\|\xi_{m(k)-1}-\xi_{n(k)-1}\|_{E_0}},\left\|\|\xi_{m(k)-1}-\xi_{n(k)-1}\|_{E_0}\right\|\right\}$$

$$= \max\left\{ \frac{\left\|\xi_{n(k)-1} - \xi_{n(k)}\right\|_{E_0} \left\|\xi_{m(k)-1} - \xi_{m(k)}\right\|_{E_0}}{1+\left\|\xi_{m(k)-1} - \xi_{n(k)-1}\right\|_{E_0}}, \frac{1+\left\|\xi_{m(k)-1} - \xi_{m(k)}\right\|_{E_0}}{1+\left\|\xi_{m(k)-1} - \xi_{n(k)-1}\right\|_{E_0}}, \left\|\left\|\xi_{m(k)-1} - \xi_{n(k)-1}\right\|_{E_0}\right\| \right\}$$

Taking limit as $n \to \infty$, we obtain

$$\lim_{k\to\infty} M(\xi_{m(k)-1}, \xi_{n(k)-1}) = \varepsilon \tag{12}$$

Similarly, one can show that

$$\lim_{k\to\infty} N(\xi_{m(k)-1}, \xi_{n(k)-1}) = \varepsilon \tag{13}$$

Now taking the upper limit when $k \to \infty$ in (11) and using (9) and (12), we have

$$\psi(\varepsilon) \le \psi(\varepsilon) - \lim_{k\to\infty} \inf \phi\left(N(\xi_{m(k)-1}, \xi_{n(k)-1})\right) \le \psi(\varepsilon) - \phi(\varepsilon) < \psi(\varepsilon),$$

which is a contradiction. Therefore, we have $\lim_{n,m\to\infty} \left\|\xi_n - \xi_m\right\|_{E_0} = 0$ and hence the sequence $\{\xi_n\}$ is a Cauchy sequence in E_0. Since E_0 is a Banach space, we can find $\xi^* \in E_0$ such that $\lim_{n\to\infty} \xi_n = \xi^*$.

As $\xi_n \in \Re_c$ and $\Re_c$ is topologically closed, we have $\xi^* \in \Re_c$.

Next, we will show that ξ^* is a PPF dependent fixed point of T.

From (1) and taking account (2), we have

$$\psi\left(\left\|T\xi_{n-1} - T\xi^*\right\|_E\right) \le \psi\left(\max\left\{ \frac{\left\|\xi^*(c) - T\xi^*\right\|_E \left\|\xi_{n-1}(c) - T\xi_{n-1}\right\|_E}{1+\left\|\xi_{n-1} - \xi^*\right\|_{E_0}}, \frac{\left\|\xi_{n-1}(c) - T\xi_{n-1}\right\|_E}{1+\left\|\xi_{n-1} - \xi^*\right\|_{E_0}}, \left\|\left\|\xi_{n-1} - \xi^*\right\|_{E_0}\right\| \right\} \right)$$

$$-\phi\left(\max\left\{ \frac{\left\|\xi^*(c) - T\xi^*\right\|_E \left\|\xi_{n-1}(c) - T\xi_{n-1}\right\|_E}{1+\left\|\xi_{n-1} - \xi^*\right\|_{E_0}}, \left\|\left\|\xi_{n-1} - \xi^*\right\|_{E_0}\right\| \right\} \right)$$

$$= \psi\left(\max\left\{ \frac{\left\|\xi^*(c) - T\xi^*\right\|_E \left\|\xi_{n-1}(c) - \xi_n(c)\right\|_E}{1+\left\|\xi_{n-1} - \xi^*\right\|_{E_0}}, \frac{\left\|\xi_{n-1}(c) - \xi_n(c)\right\|_E}{1+\left\|\xi_{n-1} - \xi^*\right\|_{E_0}}, \left\|\left\|\xi_{n-1} - \xi^*\right\|_{E_0}\right\| \right\} \right)$$
$$-\phi\left(\max\left\{ \frac{\left\|\xi^*(c) - T\xi^*\right\|_E \left\|\xi_{n-1}(c) - \xi_n(c)\right\|_E}{1+\left\|\xi_{n-1} - \xi^*\right\|_{E_0}}, \left\|\left\|\xi_{n-1} - \xi^*\right\|_{E_0}\right\| \right\} \right)$$

Taking limit as $n \to \infty$ in the above inequality regarding property of ϕ, ψ and since $\lim_{n\to\infty} \|\xi_{n-1}(c) - \xi_n(c)\|_E = 0$ we have

$$\psi\left(\|\xi^*(c) - T\xi^*\|_E\right) \le \psi\left(\|\xi^*(c) - T\xi^*\|_E\right) - \phi\left(\|\xi^*(c) - T\xi^*\|_E\right) < \psi\left(\|\xi^*(c) - T\xi^*\|_E\right)$$

which is a contradiction. Hence we deduce that $T\xi^* = \xi^*(c)$ and therefore ξ^* is a PPF dependent fixed point of T in $\Re_c$.

To prove uniqueness of PPF dependent fixed point of T in $\Re_c$, let η^* is another PPF dependent fixed point of T in $\Re_c$. Since $\Re_c$ is algebraically closed, we have

$$\begin{aligned} \|\xi^* - \eta^*\|_{E_0} &= \|\xi^*(c) - \eta^*(c)\|_E \\ &= \|T\xi^* - T\eta^*\|_E \end{aligned} \tag{14}$$

Now using contractive condition (1), we have

$$\psi\left(\|\xi^* - \eta^*\|_{E_0}\right) = \psi\left(\|T\xi^* - T\eta^*\|_E\right)$$

$$\le \psi\left(\max\left\{\frac{\|\eta^*(c) - T\eta^*\|_E \|\xi^*(c) - T\xi^*\|_E}{1 + \|\xi^* - \eta^*\|_{E_0}}, \frac{\|\xi^*(c) - T\xi^*\|_E}{1 + \|\xi^* - \eta^*\|_{E_0}}, \left\|\|\xi^* - \eta^*\|_{E_0}\right\|\right\}\right)$$
$$-\phi\left(\max\left\{\frac{\|\eta^*(c) - T\eta^*\|_E \|\xi^*(c) - T\xi^*\|_E}{1 + \|\xi^* - \eta^*\|_{E_0}}, \left\|\|\xi^* - \eta^*\|_{E_0}\right\|\right\}\right)$$

implying thereby that

$$\psi\left(\|\xi^* - \eta^*\|_{E_0}\right) \le \psi\left(\|\xi^* - \eta^*\|_{E_0}\right) - \phi\left(\|\xi^* - \phi^*\|_{E_0}\right) < \psi\left(\|\xi^* - \eta^*\|_{E_0}\right).$$

It is a contradiction. Hence $\xi^* = \eta^*$.

Corollary: 3.2. Let $T : E_0 \to E$ be a mapping satisfying

$$\psi\left(\|T\xi - T\eta\|_E\right) \le \psi\left(N(\xi, \eta)\right) - \phi\left(N(\xi, \eta)\right)$$

for all $\xi, \eta \in E_0$, where $\psi \in \Psi$ and $\phi \in \Phi$ and

$$N(\xi, \eta) = \max\left\{\frac{\|\eta(c) - T\eta\|_E \|\xi(c) - T\xi\|_E}{1 + \|\xi - \eta\|_{E_0}}, \|\xi - \eta\|_{E_0}\right\}$$

If $\Re_c$ is topologically closed and algebraicallyclosed with respect to difference, then *T*has a unique PPF dependent fixed point in $\Re_c$.

Taking ψ to be identity mapping and $\phi(t)=(1-k)t$ for all $t \geq 0, k(0,1)$, we have the following result as a consequence of our main theorem.

Corollary: 3.3. Let $T: E_0 \rightarrow E$ be a mapping satisfying

$$\|T\xi - T\eta\|_E = k \max\left\{\frac{\|\eta(c)-T\eta\|_E \|\xi(c)-T\xi\|_E}{1+\|\xi-\eta\|_{E_0}}, \|\xi-\eta\|_{E_0}\right\}$$

for all $\xi, \eta \in E_0$.

If $\Re_c$ is topologically closed and algebraicallyclosed with respect to difference, then *T*has a unique PPF dependent fixed point in $\Re_c$.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] V. Berinde, “Approximating fixed points of weak contractionsusing the Picard iteration”,*Nonlinear Analysis Forum*, vol. 9(1)(2004), pp. 43–53.

[2] S. R. Bernfeld, V. Lakshmikantham, and Y. M. Reddy, “Fixedpoint theorems of operators with PPF dependence in Banachspaces”,*Applicable Analysis*, vol. 6(4)(1997), pp. 271–280.

[3] S. K. Chatterjea, “Fixed-point theorems”,*Comptes Rendus de l’Acad´emieBulgare des Sciences*, vol. 25(1972), pp. 727–730.

[4] Lj. B. Ciri´c, “A generalization of Banach’s contraction principle”, *Proceedings of the American Mathematical Society*, vol. 45(1974), pp. 267–273.

[5] B. K. Dass and S. Gupta, “An extension of Banach contraction principle through rational expression”,*Indian Journal of Pureand Applied Mathematics*, vol. 6(12)(1975), pp. 1455–1458.

[6] B.C.Dhage, “On some common fixed point theorems with PPF dependence in Banach spaces”, *Journal of Nonlinear Science andIts Applications*, vol. 5(3)(2012), pp. 220–232.

[7] Z. Drici, F. A. McRae, and J. Vasundhara Devi, “Fixed-point theorems in partially ordered metric spaces for operatorswith PPF dependence”,*Nonlinear Analysis: Theory, Methods & Applications A*, vol. 67(2)(2007), , pp. 641–647.

[8] M. A. Geraghty, “On contractive mappings”,*Proceedings of the American Mathematical Society*, vol. 40(1973), pp. 604–608.

[9] D. S. Jaggi, “Some unique fixed point theorems”,*Indian Journal of Pure and AppliedMathematics*, vol. 8(2)(1977), pp. 223–230.

[10] M. Jleli, V.C.Rajie, B.Samet and C.Vetro, “Fixed point theorems on ordered metric spaces and application to nonlinear elastic beam equations”,*J. Fixed Point Theory and applications*, vol.12(1-2)(2012), 175-192.

[11]R. Kannan, “Some results on fixed points—II”,*The American Mathematical Monthly*, vol. 76(1969), pp. 405–408.

[12]M.A. Kutbi and W. Sintunavarat “On sufficient conditions for the existence of past-present-future dependent fixed point in the Razumikhin class and application”,*Abstract and Applied Analysis*,vol.2014,Article ID342687,8 pages.

[13] A. Meir and E. Keeler, “A theorem on contraction mappings”, *Journal of Mathematical Analysis and Applications*, vol. 28(1969), pp.326–329.

[14] W. Sintunavarat and P. Kumam, “PPF dependent fixed point theorems for rational type contraction mappings in Banach spaces”, *Journal of Nonlinear Analysis and Optimization*, vol. 4(2)(2013), pp. 157–162.

[15] T. Suzuki, “A generalized Banach contraction principle that characterizes metric completeness”, *Proceedings of the AmericanMathematical Society*, vol.136(5)(2008), pp.1861–1869.