



# Common Fixed Points for $PD$ – Operator on Probabilistic Metric Space

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## ABSTRACT

In this paper, we obtain some common fixed point theorems for recently introduced notion of on a set  $X$  equipped with the function  $F: X \times X \rightarrow \Delta$  without using the triangle inequality besides relaxing symmetric condition. Our results extend the results of Pathak and Rai. [Common fixed points for  $PD$ -operator pairs under relaxed conditions with applications, Journal of Computational and Applied Mathematics 239(1)(2013) 103-113], Hussain et al.[Common fixed points for  $JH$ -operators and occasionally weakly biased pairs under relaxed conditions, Nonlinear Anal.74(2011) 2133-2140], Bhatt et al.[Common fixed point theorems for occasionally weakly compatible mappings under relaxed conditions, Non-linear analysis Theory, Methods and appl. 73(2010) 176-182] and several others.

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## 1. INTRODUCTION

The theory of probabilistic metric spaces was introduced by Menger [2] in connection with some measurements in Physics. The first effort in this direction was made by Sehgal [4], who in his doctoral dissertation initiated the study of contraction mapping theorems in probabilistic metric spaces. Since then, Sehgal and Bharucha - Reid [8, 9] obtained a generalization of Banach Contraction Principle on a complete Menger space which is an important step in the development of fixed point theorems in Menger space. Over the years, the theory has found several important applications in the investigation of physical quantities in quantum particle physics and string theory as studied by El.Naschie [19, 20]. The area of probabilistic metric spaces is also of fundamental importance in probabilistic functional analysis.

In 1976, Jungck [10] initiated a study of common fixed points of commuting maps. On the other hand in 1982 Sessa [11] initiated the tradition of improving commutativity in fixed- point theorems by introducing the notion of weakly commuting maps in metric spaces. Jungck [13] soon enlarged this concept to compatible maps. The notion of compatible mappings in a Menger space has been introduced by Mishra [15]. After this, Jungck and Rhoades [17] gave the concept of weakly compatible maps. Aamri and El Moutawakil [23] introduced the  $(E.A.)$  property and thus generalized the concept of non-compatible maps. The results obtained in the metric fixed point theory by using the notion of non-compatible maps or the  $(E.A.)$  property is very interesting. Al-Thagafi and Shahzad [27] (see also, Jungck and Rhoades [25]) defined the concept of occasionally weakly compatible mappings which is more general than the concept of weakly compatible maps. Bhatt et al. [30] have given application of occasionally weakly compatible

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mappings in dynamical programming. Pathak and Hussain [31] defined the concept of  $P$ -operators. Hussain et.al [32] gave the concepts of  $JH$ -operators and occasionally weakly  $g$ -biased. Recently Pathak and Rai [34] proved some common fixed point theorems for more generalized non commuting notion, namely,  $PD$ -operators and gave some applications in variational inequalities and dynamical programming.

In this paper, we extend some common fixed point theorems for  $PD$ -operators under relaxed condition on probabilistic metric space. Our results extend the results of Pathak and Rai [34], Hussain et al. [32], Bhatt et al. [30] and others [5, 16, 21, 24, 26, 28, 29, 33, 35].

We begin with the following basic definitions of concepts relating to probabilistic metric spaces for ready reference and also for the sake of completeness.

**Definition 1.1.** [3, 12]. A distribution function (on  $[-\infty, +\infty]$ ) is a function  $F: [-\infty, +\infty] \rightarrow [0, 1]$  which is left-continuous on  $R$ , non-decreasing and  $F(-\infty) = 0$ ,  $F(+\infty) = 1$ . The Heaviside function  $H$  is a distribution function defined by,

$$H(t) = \begin{cases} 0, & \text{if } t \leq 0 \\ 1, & \text{if } t > 0. \end{cases}$$

**Definition 1.2.** [22]. A distance distribution function  $F: [-\infty, +\infty] \rightarrow [0, 1]$  is distribution function with support contained in  $[0, \infty]$ . The family of all distance distribution functions will be denoted by  $\Delta^+$ . We denote

$$D^+ = \left\{ F: F \in \Delta^+, \lim_{x \rightarrow \infty} F(x) = 1 \right\}.$$

**Definition 1.3.** [12]. A probabilistic metric space in the sense of Schweizer and Sklar is an ordered pair  $(X, F)$ , where  $X$  is a nonempty set and  $F: X \times X \rightarrow \Delta^+$ , if and only if the following conditions are satisfied ( $F(x, y) = F_{x,y}$  for every  $x, y \in X \times X$ ):

- (i) for every  $(x, y) \in X \times X$ ,  $F_{x,y}(0) = 0$ ;
- (ii) for every  $(x, y) \in X \times X$ ,  $F_{x,y} = F_{y,x}$ ;
- (iii)  $F_{x,y} = 1$ , for every  $t > 0 \Leftrightarrow x = y$ ;
- (iv) for every  $(x, y, z) \in X \times X \times X$  and for every  $t_1, t_2 > 0$ ,
- (v)

$$F_{x,y}(t_1) = 1, F_{y,z}(t_2) = 1, \Rightarrow F_{x,z}(t_1 + t_2) = 1.$$

For each  $x$  and  $y$  in  $X$  and for each real number  $t \geq 0$ ,  $F_{x,y}(t)$  is to be thought of as the probability that the distance between  $x$  and  $y$  is less than  $t$ . Indeed, if  $(X, d)$  is a metric space, then the distribution function  $F_{x,y}(t)$  defined by the relation  $F_{x,y}(t) = H(t - d(x, y))$  induces a probabilistic metric space.

**Definition 1.4.** Let an ordered pair  $(X, F)$ , where  $X$  is a nonempty set and  $F$  is a mapping from  $X \times X$  into  $\Delta^+$  satisfying the following condition:

$$(1) \quad F_{x,y}(t) = 1 \forall t > 0 \Leftrightarrow x = y.$$

Where  $F: X \times X \rightarrow \Delta^+$ , defined by  $F_{x,y}(t) = H(t - d(x, y))$  for all  $x, y \in X$  and  $d$  be a function  $d: X \times X \rightarrow [0, \infty)$  such that  $d(x, y) = 0$  iff  $x = y$ ,  $\forall x, y \in X$  (symmetric and triangle conditions are not required). A topology  $\tau(d)$  on  $X$  is given by  $U \in \tau(d)$  if and only if for each  $x \in U$ ,  $B(x, \epsilon) \subset U$  for some  $\epsilon > 0$ , where  $B(x, \epsilon) = \{y \in X : d(x, y) < \epsilon\}$ .

**Remark 1.1.** We note that every symmetric (semi-metric) space  $(X, d)$  [1] can be realized as a probabilistic semi - metric space by taking  $F: X \times X \rightarrow \Delta^+$ , defined by  $F_{x,y}(t) = H(t - d(x, y))$  for all  $x, y$  in  $X$ . So probabilistic semi metric spaces provide a wider framework than that of the symmetric spaces and are better suited in many situations. In this paper we have relaxed the symmetric condition from probabilistic semi metric space.

**Definition 1.5.** [6, 7]. Let  $(X, F)$  be a probabilistic metric space and  $A$  be a nonempty subset of  $X$ . The probabilistic diameter  $\delta_A: [0, +\infty) \rightarrow [0, 1]$  is defined by,

$$\delta_A(x) = \sup_{t < x} \inf_{p, q \in A} \{F_{p,q}(t)\}.$$

If  $(X, F)$  satisfies condition (1), the probabilistic diameter is defined by,

$$\delta_A(x) = \sup_{t < x} \inf_{p, q \in A} \{F_{p,q}(t), F_{q,p}(t)\}.$$

Let  $X$  be a non-empty set together with the function  $F: X \times X \rightarrow \Delta^+$  satisfying the condition (1). A point  $x$  in  $X$  is called a coincidence point of  $f$  and  $g$  iff  $fx = gx$ . In this case  $w = fx = gx$  is called a point of coincidence of  $f$  and  $g$ . Let  $C(f, g)$  and  $PC(f, g)$  denote the sets of coincidence points and points of coincidence, respectively, of the pair  $(f, g)$ .

**Definition 1.6.** Let  $X$  be a non-empty set together with the function  $F: X \times X \rightarrow \Delta^+$  satisfying the condition (1), two self maps  $f$  and  $g$  of a space  $(X, F)$  are called

- (i)  $P$ -operators iff for some  $t$ , there is a point  $x \in C(f, g)$  such that  $F_{x,gx}(t) \geq \delta_{C(f,g)}(t)$  and  $F_{gx,x}(t) \geq \delta_{C(f,g)}(t)$ .
- (ii)  $JH$ -operators iff for some  $t$ , there is a point  $w = fx = gx$  in  $PC(f, g)$  such that  $F_{x,gx}(t) \geq \delta_{C(f,g)}(t)$  and  $F_{gx,x}(t) \geq \delta_{C(f,g)}(t)$ .
- (iii) weakly  $g$ -biased, iff  $F_{fgx,gx}(t) \geq F_{fgx,fx}(t)$  whenever  $F_{fx,gx}(t) = 1$ . for some  $t$ .
- (iv) occasionally weakly  $g$ -biased, if and only if there exists some  $x \in X$  such that whenever  $F_{fx,gx}(t) = 1$  and  $F_{gfgx,gx}(t) \geq F_{fgx,fx}(t)$  for some  $t$ .
- (v)  $PD$  -operators iff for some  $t$ , there is a point  $x \in C(f, g)$  such that  $F_{fgx,gfx}(t) \geq \delta_{PC(f,g)}(t)$  and  $F_{gfgx,fgx}(t) \geq \delta_{PC(f,g)}(t)$ .

**Example 1.1.** Let  $X = [0, 1]$  and  $F_{x,y}(t) = H(t - d(x, y))$ , where,

$$d(x, y) = \begin{cases} e^{x-y} - 1, & \text{if } x \geq y \\ e^{y-x}, & \text{otherwise.} \end{cases}$$

Define  $f, g : X \rightarrow X$  by,

$$f(x) = \begin{cases} 1, & \text{if } x = 0 \\ x^2 & \text{if } 0 < x \leq \frac{1}{2} \\ 1 & \text{if } \frac{1}{2} < x \leq 1. \end{cases}, g(x) = \begin{cases} 1, & \text{if } x = 0 \\ x^2 & \text{if } 0 < x \leq 1. \end{cases}$$

Here  $C(f, g) = \{0, 1\}$  and  $PC(f, g) = \{1, \frac{1}{4}\}$ . In this example  $(f, g)$  is  $(PD)$  -operator but not commuting, not weakly compatible and not occasionally weakly compatible.

**Example 1.2.** Let  $X = [0, 1]$  and  $F_{x,y}(t) = H(t - d(x, y))$ , where,

$$d(x, y) = \begin{cases} e^{x-y} - 1, & \text{if } x \geq y \\ e^{y-x}, & \text{otherwise.} \end{cases}$$

Define  $f, g : X \rightarrow X$  by,  $f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}, g(x) = \begin{cases} 2x^2 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ .

Here  $C(f, g) = \{0, 1\}$  and  $PC(f, g) = \{1, 2\}$ . In this example  $(f, g)$  is  $P$ -operator and  $JH$ -operator pair but not  $(PD)$  -operator pair. We also observe that if  $(f, g)$  is  $PD$ -operator then it is not necessary that  $(f, g)$  be  $P$ -operator and  $JH$ -operator pair.

## 2. MAIN RESULTS

In this section, we prove some fixed point theorems for a pair of  $PD$ -operators on space  $(X, F)$  without imposing the restriction of the triangle inequality or symmetry on  $F$ . In this section, we also prove several fixed point theorems for four self-mappings on  $(X, F)$ , where  $F: X \times X \rightarrow \Delta$  satisfying the condition (1) (without imposing the restriction of the triangle inequality and symmetry only on point of coincidence). We begin with the following theorems.

**Theorem 2.1.** Let  $X$  be a non-empty set together with the function  $F: X \times X \rightarrow \Delta$  satisfying the condition (1). Suppose  $f$  and  $g$  are  $PD$ -operators on  $X$  satisfying the following condition:

$$(2) \quad F_{fx,fy}(t) \geq F_{gx,gy}\left(\frac{t}{a}\right) + \min\left\{F_{fx,gx}\left(\frac{t}{b}\right), F_{fy,gy}\left(\frac{t}{b}\right)\right\} + \min\left\{F_{gx,gy}\left(\frac{t}{c}\right), F_{gx,fx}\left(\frac{t}{c}\right), F_{gy,fy}\left(\frac{t}{c}\right)\right\},$$

for all  $x, y \in X$  with  $f(x) \neq fy$  and  $t > 0$  where  $0 < a < 1, 0 < b < 1$  and  $0 < c < 1$ . Then  $f$  and  $g$  have a unique fixed point.

**Proof.** Since  $(f, g)$  is  $PD$ -operator pair there exist a point  $u$  in  $x$  such that

$$fu = gu \text{ and}$$

$$(3) \quad F_{fgu,fgu}(t) \geq \delta_{PC(f,g)}(t) \text{ and } F_{gfu,gfu}(t) \geq \delta_{PC(f,g)}(t)$$

First, we claim that  $PC(f, g)$  is singleton. If possible, suppose  $w$  and  $w_1$  be two distinct points in  $X$  such that  $fu = gu = w$  and  $fv = gv = w_1$  for some  $u, v \in C(f, g)$ . Then from (2), we get

$$F_{w,\omega_1}(t) = F_{fu,fv}(t) \geq F_{gu,gv}\left(\frac{t}{a}\right) + 1 + \min\left\{F_{gu,gv}\left(\frac{t}{c}\right), F_{gu,fu}\left(\frac{t}{c}\right), F_{gv,fu}\left(\frac{t}{c}\right)\right\} = F_{gu,fu}\left(\frac{t}{a}\right) + 1 + F_{gu,fu}\left(\frac{t}{c}\right) > 1.$$

This is a contradiction. Hence,  $w = w_1$ . Thus  $PC(f, g)$  is singleton and  $w$  is the unique point of coincidence. This further implies.  $\delta(PC(f, g)) = 1$ . Using (3)  $fgu = gfu$  for some  $u$  in  $X$ . Now by (2), we have,

$$F_{fu,ffu}(t) \geq F_{gu,gfu}\left(\frac{t}{a}\right) + 1 + \min\left\{F_{gu,gfu}\left(\frac{t}{c}\right), F_{gu,fu}\left(\frac{t}{c}\right), F_{gfu,ffu}\left(\frac{t}{c}\right)\right\} = F_{gu,ffu}\left(\frac{t}{a}\right) + 1 + F_{gu,ffu}\left(\frac{t}{c}\right) > 1.$$

This is a contradiction. Hence,  $fu = ffu = gfu$  and  $fu$  is a common fixed point  $f$  and  $g$ . Uniqueness follows from (2). Let a function  $\emptyset$  be defined by  $\emptyset: [0, 1] \rightarrow [0, 1]$  satisfying the condition  $\emptyset(q) > q$ , for all  $0 \leq q < 1$ .

**Theorem 2.2.** Let  $X$  be a non-empty set together with the function  $F: X \times X \rightarrow \Delta$  satisfying the condition (1). If  $(f, g)$  is  $PD$ -operator pair. Suppose

$$(4) \quad F_{fx,fy}(t) \geq \emptyset \left[ \min \{F_{gx,gy}(t), F_{gx,fy}(t), F_{fx,gy}(t), F_{gy,fy}(t)\} \right],$$

for all  $x, y \in X$  and  $t > 0$ . Then  $f$  and  $g$  have a unique common fixed point.

**Proof.** Since  $(f, g)$  is  $PD$ -operator pair there exist a point  $u$  in  $X$  such that  $fu = gu$  and

$$(5) \quad F_{fgu,gfu}(t) \geq \delta_{PC(f,g)}(t) \text{ and } F_{gfu,gfu}(t) \geq \delta_{PC(f,g)}(t).$$

First, we claim that  $PC(f, g)$  is singleton. If possible, suppose  $w$  and  $w_1$  be two distinct points in  $X$  such that  $fu = gu = w$  and  $fv = gv = w_1$  for some  $u, v \in C(f, g)$ . Then from (4), we can easily get,  $w = w_1$ , i.e.  $w = fu = gu = fv = gv = w_1$ . Therefore  $PC(f, g)$  is singleton i.e.,  $w = fu = gu$  is the unique point of coincidence.  $\delta(PC(f, g)) = 1$ . From (5),  $fgu = gfu$ , for some  $u, v \in C(f, g)$ . Now, by (4), we have,

$$\begin{aligned} F_{ffu,fu}(t) &\geq \emptyset \left[ \min \{F_{gfu,gu}(t), F_{gfu,fu}(t), F_{ffu,gu}(t), F_{gu,fu}(t)\} \right], \\ &= \emptyset \left[ \min \{F_{ffu,fu}(t), F_{ffu,fu}(t), F_{ffu,fu}(t), 1\} \right], \\ &= \emptyset \left[ F_{ffu,fu}(t) \right]. \end{aligned}$$

Since  $\emptyset: [0, 1] \rightarrow [0, 1]$  satisfying the condition  $\emptyset(q) > q$ , for all  $0 \leq q < 1$ . Therefore,  $F_{ffu,fu}(t) > F_{ffu,fu}(t)$  which is a contradiction. Therefore  $ffu = fu = gfu$ ,  $f$  and  $g$  have a common fixed point. Uniqueness is obvious. Therefore,  $f$  and  $g$  have a unique common fixed point. This completes the proof of the theorem.

**Corollary 2.1.** Let  $X$  be a non-empty set together with the function

$$F: X \times X \rightarrow \Delta \text{ satisfying the condition (1). If } f \text{ and } g \text{ are } PD \text{ -operator on } X. \text{ Suppose}$$

$$(6) \quad F_{fx,fy}(t) \geq \emptyset[F_{gx,gy}(t)],$$

for some  $x, y \in X$  and  $t > 0$ . Then  $f$  and  $g$  have a unique fixed point.

The proof of the following theorem can be easily obtained by replacing condition (4) by condition (7) the proof of Theorem 2.2.

**Theorem 2.3.** Let  $X$  be a non-empty set together with the function

$F: X \times X \rightarrow \Delta$  satisfying the condition (1). If  $f$  and  $g$  are PD-operator on  $X$ . Suppose

$$(7) \quad F_{fx,fy}(t) \geq \min \{F_{gx,gy}(t), F_{gx,fy}(t), F_{fx,gy}(t), F_{gy,fy}(t)\}$$

for some  $x, y \in X$  and  $t > 0$ . Then  $f$  and  $g$  have a unique common fixed point.

**Theorem 2.4.** Let  $X$  be a non-empty set together and  $F: X \times X \rightarrow \Delta$  satisfying the condition (1). If  $f$  and  $g$  are PD-operator on  $X$ . Suppose that  $f, g, S, T$  are self - mappings on  $X$ . If

$$(8) \quad F_{z,w}(t) = F_{w,t}(t),$$

whenever  $w$  and  $z$  are points of coincidence of  $\{f, S\}$  and  $\{g, T\}$  respectively, and

$$(9) \quad F_{fx,gy} > \min\{F_{Sx,Ty}(t), F_{Sx,fx}(t), F_{Ty,gy}(t), F_{Sx,gy}(t), F_{Ty,fx}(t)\},$$

for each  $x, y \in X$  for which  $fx \neq gy$ , then  $f; g; S$  and  $T$  have a unique common fixed point.

**Proof.** By hypothesis there exists points  $x, y \in X$  such that  $fx = Sx$  and  $gy = Ty$ .

Suppose that  $F_{fx,gy}(t) \neq 1 \forall t > 0$ . Then from equation (9),

$$F_{fx,gy}(t) > \min\{F_{fx,gy}(t), F_{fx,fx}(t), F_{gy,gy}(t), F_{fx,gy}(t), F_{gy,fx}(t)\},$$

$$> F_{fx,gy}(t).$$

This is a contradiction. Hence  $F_{fx,gy}(t) = 1$  for all  $t > 0$ . This implies that  $fx = gy$ . So  $fx = Sx = gy = Ty$ . Moreover, if there is another point  $z$  such that  $fz = Sz$ , then, using (9) it follows that  $fz = Sz = gy = Ty$ , or  $fx = fz$  and  $w = fx = Sx$  is the unique point of coincidence of  $f$  and  $S$ . Thus  $\delta(PC(f, S)) = 1$  and  $\delta(PC(g, T)) = 1$ . Then by definition of PD-Operator  $F_{fSx,Sfx}(t) = 1$  and hence  $fSx = Sfx$ : Similarly  $F_{gTy,Tgx}(t) = 1$  and  $gTy = Tgx$ . Hence from the repeated use of condition (8) and (9) we can easily show that,  $f, g, S$  and  $T$  have a unique common fixed point.

Let the control function  $\emptyset: R^+ \rightarrow R^+$  be a continuous non decreasing function such that  $\emptyset(2t) \geq 2\emptyset(t)$  and,  $\emptyset(1) = 1$ . Let a function  $\psi$  be defined by  $\psi: [0,1] \rightarrow [0,1]$  satisfying the condition  $\psi(q) > q$ , for all  $0 \leq q < 1$ .

**Theorem 2.5.** Let  $X$  be a non-empty set together and  $F: X \times X \rightarrow \Delta$  satisfying the condition (1). Suppose  $f, g, S, T$  are self-mappings of  $X$  and that the pairs  $\{f, S\}$  and  $\{g, T\}$  are each PD-operators on  $X$ . If

$$(10) \quad F_{z,w}(t) = F_{w,t}(t),$$

whenever  $w$  and  $z$  are points of coincidence of  $\{f, S\}$  and  $\{g, T\}$  respectively, and

$$(11) \quad \psi(F_{fx,gy}(t)) \geq \psi(M_\emptyset(x, y)), \text{ where,}$$

$$M_\emptyset(x, y) = \min \left\{ \begin{array}{l} \emptyset(F_{Sx,Ty}(t)), \emptyset(F_{Sx,fx}(t)), \emptyset(F_{gy,Ty}(t)), \\ \frac{1}{2} [\emptyset(F_{Sx,gy}(t)) + \emptyset(F_{fx,Ty}(t))] \end{array} \right\}$$

for each  $x, y \in X$  for which  $fx \neq gy$ , then  $f, g, S$  and  $T$  have a unique common fixed point.

**Proof.** By hypothesis there exist points  $x, y$  in  $X$  such that  $w = fx = Sx$  and  $z = gy = Ty$ . We claim that  $fx = gy$ . Suppose that  $fx \neq gy$ : Then (10) and (11) we get,  $\psi(F_{fx,gy}(t)) \geq \psi(M_\emptyset(x, y)) > \psi(F_{fx,gy}(t))$ , which is a contradiction. Therefore  $\varphi(F_{fx,gy}(t)) = 1$ , which further implies that  $F_{fx,gy}(t) = 1$ . Hence the claim follows i.e.  $w = fx = gy = z$ . Now the repeated use of condition (11), we can show that  $f, g, S$  and  $T$  have a unique common fixed point.

Define  $G = \{\emptyset: \mathbb{R}^+ \rightarrow \mathbb{R}^+\}$  such that,  $(g_1)$  If  $u \in \mathbb{R}^+$  is such that,

$$u \geq \emptyset(u, 1, 1, u, u) \text{ or } u \geq \emptyset(1, u, 1, u, u) \text{ or } u \geq \emptyset(1, 1, u, u, u), \text{ then } u = 1.$$

**Theorem 2.6.** Let  $X$  be a non-empty set together and  $F: X \times X \rightarrow \Delta$  satisfying the condition (1). Suppose  $f, g, S, T$  are self-mappings of  $X$  and that the pairs  $\{f, S\}$  and  $\{g, T\}$  are each PD-operators on  $X$ . If

$$(12) \quad F_{z,w}(t) = F_{w,t}(t),$$

whenever  $w$  and  $z$  are points of coincidence of  $\{f, S\}$  and  $\{g, T\}$  respectively, and

$$(13) \quad F_{fx,gy}(t) \geq (F_{Sx,Ty}(t), (F_{fx,Sx}(t)), (F_{gy,Ty}(t)), (F_{fx,Ty}(t)), (F_{gy,Sx}(t))),$$

for each  $x, y \in X$  for which  $fx \neq gy$ , then  $f, g, S$  and  $T$  have a unique common fixed point.

**Proof.** It follows from the given assumptions that there exists a point  $x \in X$  such that  $fx = Sx$  and there exists another point  $y \in X$  for which  $gy = Ty$ .

Suppose that  $fx \neq gy$ . Then, from (13) we have,

$$F_{fx,gy}(t) \geq \emptyset(F_{fx,gy}(t), 0, 0, F_{fx,gy}(t)F_{gy,fx}(t)).$$

Since  $fx$  and  $gy$  are points of coincidence of  $\{f, S\}$  and  $\{g, T\}$  respectively. Hence, from (13) we get,  $F_{fx,gy}(t) \geq \emptyset(F_{fx,gy}(t), 0, 0, F_{fx,gy}(t)F_{gy,fx}(t))$ . Therefore, from  $(g1)\delta(PC(f, S)) = 1$  and  $\delta(PC(g, T)) = 1$ . This implies that  $F_{fx,gy}(t) = 1$  and hence  $fx = gy$ . Similarly,  $F_{gTx, Tgx}(t) = 1$  and hence  $gTx = Tgx$ . Hence from the repeated use of condition (12) and (13) we can easily show that  $f, g, S$  and  $T$  have a unique common fixed point.

**Remark 2.1.** As an application of Corollary 2.1, the existence and uniqueness of a common solution of the functional equations arising in dynamic programming can be established which extends Theorem 4.1 [30].

## CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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