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Investigation of Optimist and Pessimist Situations via DEA with Fuzzified Data: Banking Example

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ABSTRACT

In this study Data Envelopment Analysis (DEA) is applied to sample banking data when the data is fuzzified. In market conditions, due to some reasons, data is not always obvious and specific. Herewith, it is focused on the fuzziness caused by the diversification of goods and services; examines the applicability of α-cut approach and investigates the optimistic and pessimistic case of DEA from customers' and firms' perspective. Then, an application scheme is proposed when the number of options is excessive for classical methods.

Key Words: *Fuzzy DEA, Banking, Customer, Optimist and Pessimist Approaches*

1. INTRODUCTION

The purpose of this study is to investigate the efficiency of customers and firms in terms of optimist and pessimist Data Envelopment Analysis (DEA) approach in fuzzified environment which is known to be opposite to each other. DEA is a classical performance measurement tool introduced by Charnes et al. [1] and is a data-driven technique that uses observed data to measure the relative efficiency of Decision Making Units (DMUs) with various models and applications for different purposes and requirements. The lack of any data point is tedious and causes problems in use and hence it is encountered frequently in real life. The data of one or more DMUs might be lost, incorrect or fuzzified. The solution includes the use of fuzzy programming techniques. Therefore, through the use of the most optimistic, most pessimistic and most possible values, the probability distribution of the missing data is obtained and then fuzzy programming algorithms are used in the DEA.

Fuzzy mathematical programming proposed by Zadeh [2] is used for a long time in the literature and surveyed by Rommelfanger [3], Herrera and Verdegay [4] and Zimmermann [5] in different dates. In the context of fuzzy DEA, the most comprehensive study of the subject is Hatami-Marbini et al. [6] and Emrouznejad and Tavana [7].

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In literature, different aspects of banking sector have been studied through DEA in abundance. In this regard, Popovici [8], Liu et al. [9] and Paradi and Zhu [10] are the most recent literature surveys. DEA has been used to study abundantly for measuring the service quality or operating efficiency of bank branches or firm level [11-15]. Chen et al. [16], taking into account the market risk, evaluated the banks in a fuzzy environment. Bal and Gölcükcü [17] looked into the sector from the customers' perspective.

Due to the effects of globalization, any country or a region cannot be isolated from global events. Concurrently, global society is also influenced by countries or regions. The most striking and recent example is the world crisis in 2008 caused by some financial actors. The actors of the market manage the crisis by different pricing policies or product diversification due to the expectations of customers. Consequently, the product and/or price options that serve customers through diversification reaches such a large quantity that it becomes impossible to make direct comparison. This is a kind of fuzzification applied by firms for increasing profit. Banking could also be considered in this context. Although the sample data of this study is about the banking sector, other service or production sectors could also face similar circumstances. In this context, companies create a fuzzy environment by providing multiple options through product or service diversification in order to satisfy different expectations of their customers in some way changing the appearance or use of the product.

In the following sections, firstly, DEA and Fuzzy DEA methodology is given with a brief and α-cut approach of Kao and Liu [18], which is classical for the aforementioned type of problems denoted with an example. Secondly, the data variables and an alternative solution is introduced with an application scheme and explained why it aims to present a new proposal against Kao and Liu model [18]. The analyses and test results are given with tables in third section. Finally, it is discussed and concluded.

2. METHODS AND THE MATERIALS

Data Envelopment Analysis is a classic method used to measure the relative efficiency of similarly operating DMUs which have common inputs and outputs. It is a non-parametric technique and based on Linear Programming (LP). Due to its practicality and versatility, DEA is accepted and spread a lot with new models and wide application areas since the emerging work of Charnes et al. [1]. The ratio model, which depicts the idea of relative efficiency measurement of DMUs, is transformed to LP form [1];

$$\begin{aligned}
&\max \quad w_0 = \sum_r m_r y_{r0} \\
&\text{s.t.} \\
&\sum_i v_i x_{i0} = 1 \\
&\sum_r m_r y_{rj} - \sum_i v_i x_{ij} \le 0 \\
&m_r; v_i \ge \varepsilon \\
&j = 1,\dots,n;\ r = 1,\dots,s;\ i = 1,\dots,t
\end{aligned}$$

(1)

General explanations could be found in textbooks [19-21]. Recent developments and the extent of subject could be found in surveys and reviews [6, 8-10, 22-26].

2.1. Fuzzy DEA

The decision-makers are forced to choose between different options in many aspects of life. The differences between these options could be minuscule, or sometimes each option caters to our different needs. In such cases, any decision maker has fine-tune, rearrange or waive certain requirements according to priority, thus, a fuzzy environment appears naturally. At this point, Zadeh's [2] contribution to the literature produces a solution. Fuzzy logic and fuzzy set concept is used in a variety of areas, the use of mathematical programming is also quite common [3-5, 7, 27, 28].

In mathematical programming fuzziness can be seen in;

- Technology matrix
- Right hand side coefficients
- Objective function
- Double or triple combinations of above.

In DEA, fuzziness can be observed primarily in data, namely technology matrix. If all data is fuzzy, the objective function will inherently be fuzzy, otherwise only the DMUs with fuzzy data have fuzzy objective function. The efficiency scores may also be fuzzy if at least a fuzzy DMU is on efficient frontier. Right hand side coefficients are taken as 0 due to model so it is awkward to be fuzzy in DEA. Sengupta introduced the concept of fuzziness into DEA [29, 30]. Besides that, a few more solutions exist for various types of fuzziness. Hatami-Marbini et al. [6] classified these proposals into four main types;

- Tolerance approach
- α-level based approach
- Fuzzy ranking approach
- The possibility approach

Some recent works other than these approaches are classified as an addition [31] under the heading;

- Fuzzy arithmetic

- The Fuzzy Random/Type-2 Fuzzy Set
- Other developments of Fuzzy DEA

These approaches include many different applications from each other.

2.2. Classical Method (α-Cut Approach)

The best representing following fuzzy DEA example was given by Kao and Liu [18]. There are four hypothetical decision making units named A, B, C, D and producing 5, (5, 6, 8, 9), 9, 15 units of outputs using 1, 2, 3, 5 units of inputs respectively. Here, (5, 6, 8, 9) is a trapezoidal number as represented in Figure 1.

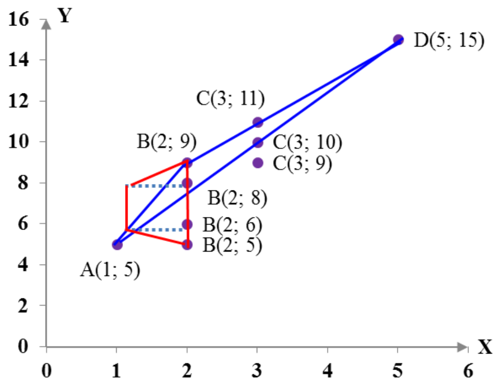

Figure 1: Graphical representation of the example of Kao and Liu

As indicated in Figure 1, as long as the output of DMU B is less than 7.5 the efficient frontier is the connecting line between A and D. Since the efficiency of a decision making unit is measured as the distance between the unit and the efficient frontier the efficiency score of A, C, and D could be attained as 1.0, 0.9, and 1.0 respectively. On the other hand the efficiency of B differs from 5/7.5=0.666 to 7.5/7.5=1 while its output differs from 5 to 7.5. If the output of B exceeds 7.5, the efficient frontier is broken into two pieces as shown in figure 1. Moreover, the efficiency of DMU C is evaluated according to this frontier.

As indicated above example if any observation is not exact, the efficiency of corresponding DMU is not exact. Furthermore, if this DMU is on efficient frontier the efficiency of DMUs which are referencing this unit are also inexact. The classical method related to our study is α-cut approach proposed by Kao and Liu [18].

If the values of inputs $\{x_{ij}\}$ and outputs $\{y_{ij}\}$could approximately be known, they could be represented as fuzzy sets $\tilde{X}_{ij}$ and $\tilde{Y}_{ik}$ which have a membership function of $\mu_{\tilde{X}_{ij}}$ and $\mu_{\tilde{Y}_{ik}}$ respectively as follows;

$$\tilde{X}_{ij} = \left\{ x_{ij}, \mu_{\tilde{X}_{ij}}\left(X_{ij}\right) \middle| x_{ij} \in S\left(\tilde{X}_{ij}\right) \right\}$$

$$\tilde{Y}_{ik} = \left\{ y_{ik}, \mu_{\tilde{Y}_{ik}}\left(Y_{ik}\right) \middle| y_{ik} \in S\left(\tilde{Y}_{ik}\right) \right\}$$

(2)

Wherein $S(\tilde{X}_{ij})$ and $S(\tilde{Y}_{ik})$ are the support set of $\tilde{X}_{ij}$ and $\tilde{Y}_{ik}$ as a universal set of k^th output and j^th input of i^th DMU. Consequently, the efficiency of DMU_0 could be evaluated by the following model;

$$\begin{aligned} &\max \quad w_0 = \sum_r m_r \tilde{Y}_{r0} \\ &\text{s.t;} \\ &\sum_i v_i \tilde{X}_{i0} = 1 \\ &\sum_r m_r \tilde{Y}_{rj} - \sum_i v_i \tilde{X}_{ij} \leq 0 \\ &m_r ; v_i \geq \varepsilon \\ &j = 1,\dots,n;\ r = 1,\dots,s;\ i = 1,\dots,t \end{aligned} \tag{3}$$

In the model above, all the values are taken as fuzzy. Mainly, if a value is fuzzy, based on the rest of the data, a triangular membership function is evaluated with minimum, average and maximum values. This values corresponds to most pessimist, most possible and most optimistic values respectively. Once membership functions are evaluated for all fuzzy values, the results could be obtained [18]. Basically, α-cut approach depends on obtaining a set of possible values. Suppose that;

$$\begin{aligned} \left(X_{ij}\right)_\alpha &= \left\{ x_{ij} \in S\left(\tilde{X}_{ij}\right) \middle| \mu_{\tilde{X}_{ij}} \geq \alpha \right\} = \left[\left(X_{ij}\right)_\alpha^L, \left(X_{ij}\right)_\alpha^U \right] \\ &= \left[\min_{x_{ij}} \left\{ x_{ij} \in S\left(\tilde{X}_{ij}\right) \middle| \mu_{\tilde{X}_{ij}} \geq \alpha \right\}, \max_{x_{ij}} \left\{ x_{ij} \in S\left(\tilde{X}_{ij}\right) \middle| \mu_{\tilde{X}_{ij}} \geq \alpha \right\} \right] \end{aligned} \tag{4a}$$

$$\begin{aligned} \left(Y_{ik}\right)_\alpha &= \left\{ y_{ik} \in S\left(\tilde{Y}_{ik}\right) \middle| \mu_{\tilde{Y}_{ik}} \geq \alpha \right\} = \left[\left(Y_{ik}\right)_\alpha^L, \left(Y_{ik}\right)_\alpha^U \right] \\ &= \left[\min_{y_{ik}} \left\{ y_{ik} \in S\left(\tilde{Y}_{ik}\right) \middle| \mu_{\tilde{Y}_{ik}} \geq \alpha \right\}, \max_{y_{ik}} \left\{ y_{ik} \in S\left(\tilde{Y}_{ik}\right) \middle| \mu_{\tilde{Y}_{ik}} \geq \alpha \right\} \right] \end{aligned} \tag{4b}$$

represents the α-cut set of $\tilde{X}_{ij}$ and $\tilde{Y}_{ik}$ and indicates the α-possibility of corresponding inputs and outputs. As a result Kao and Liu [18] gave the following single step DEA formulation.

$$\begin{aligned}
\max \quad & w_0 = \sum_r m_r \left(Y_{r0}\right)_\alpha^L \\
\text{s.t.} \quad & \\
& \sum_i v_i \left(X_{i0}\right)_\alpha^U = 1 \\
& \sum_r m_r \left(Y_{rj}\right)_\alpha^U - \sum_i v_i \left(X_{ij}\right)_\alpha^L \le 0, \quad \text{for KVB 0} \\
& \sum_r m_r \left(Y_{rj}\right)_\alpha^L - \sum_i v_i \left(X_{ij}\right)_\alpha^U \le 0, \quad j = 1,\dots,n;\ j \ne 0 \\
& m_r; v_i \ge \varepsilon \\
& r = 1,\dots,s;\ i = 1,\dots,t
\end{aligned} \tag{5a}$$

$$\begin{aligned}
\max \quad & w_0 = \sum_r m_r \left(Y_{r0}\right)_\alpha^U \\
\text{s.t.} \quad & \\
& \sum_i v_i \left(X_{i0}\right)_\alpha^L = 1 \\
& \sum_r m_r \left(Y_{rj}\right)_\alpha^L - \sum_i v_i \left(X_{ij}\right)_\alpha^U \le 0, \quad \text{for KVB 0} \\
& \sum_r m_r \left(Y_{rj}\right)_\alpha^U - \sum_i v_i \left(X_{ij}\right)_\alpha^L \le 0, \quad j = 1,\dots,n;\ j \ne 0 \\
& m_r; v_i \ge \varepsilon \\
& r = 1,\dots,s;\ i = 1,\dots,t
\end{aligned} \tag{5b}$$

Thereafter, the membership functions

$$\mu_{\tilde{w}_r} = \left[\left(w_r\right)_\alpha^L , \left(w_r\right)_\alpha^U \right]$$

(6)

for different α-levels than can be achieved by the solution of above (5a) and (5b) model for every DMU and every fuzzy values.

Data and Proposed Model

Data related to our study were collected separately for each bank over the internet. All banks that are operating and providing services to individual customers in Turkey are included in the data set but they are coded with numerals in order not to cause a conflict of interest. About 20 banks grants loans to individual customers under the headings of consumer credit (TKT), vehicle purchase loans (TST) and housing loans (KON). Furthermore, they accept deposits of Turkish Lira (TL), US Dollar (USD) and Euro (EUR) from individual customers. Here, loan rates are determined as input for customer and output for banks. But concerning the customers, they are outputs for customers and inputs for banks.

There is a vast number of loan/interest rates options offered to customers depending on terms or the amount of money. In such case, as could be seen in Table 1-2, the abundance of options may cause a fuzzy environment due to the number of comparisons that have to be made among choices.

Table 1: The number of options via date and amounts that are given by banks to customer according to variables

Bank No	Variables						Total	Number of Comparison
	TL	USD	EURO	TKT	KON	TST		
1	11	3	3	2	5	1	25	990
2	2	2	3	1	2	2	12	48
3	6	2	2	1	1	2	14	48
4	2	4	4	5	1	1	17	160
5	2	3	3	1			9	18
6	4	4	4	9	2	1	24	1152
7	8	8	8	1		1	26	512
8	6	2	2		7		17	168
9	4	3	2	1	2	2	14	96
10	2	3	3	1	1		10	18
11	3	4	4	1	2	1	15	96
12	8	7	7	1	1		24	392

13	3	2	2	1			8	12
14	2	1	1	1	1	1	7	2
15	5	4	4	1		4	18	320
16	12	7	7	3		4	33	7056
17	2	1	1	3	2	1	10	12
18	4	1	1	1	1	2	10	8
19	1	1	1	2	5	2	12	20
20	3	3	3	2	4	2	17	432
Total	90	65	65	38	37	27	322	**1,90E+39**

The exponential number on the lower right hand side corner of the table is the total numbers of comparisons that have to be made among choices. It reveals that the situation is extremely vague for customer. This can be considered as fuzzification in some way. Moreover, the empty cells colored dark represent the DMUs which are not giving corresponding service. E.g. Bank 5, 7, 13, 14 and 16 did not offer housing loans. Bank 4 also did not give vehicle purchase loans etc. There are eight banks with missing services likewise, namely banks 5, 7, 8, 10, 12, 13, 15 and 16. This is another source of fuzziness for customers who want to evaluate the best servicing bank. Under the assumption that disregarding the amount and term constraints will not pose a problem for customers, the number comparisons are shown in Table 2 which includes only the maximum and minimum value as a choice. The number on the lower right hand side is still exponential and keeps its vagueness. α-cut approach of Kao and Liu (17) cannot give a practical solution for customer so it is used by banks to improve their profitability against customer. Thus, insisting on this approach is useless.

Table 2: The number of options via date and amounts that are given by banks to customer according to variables when only max. and min. taken

Bank	Variables						Total	Number of Comparison
No	TL	USD	EURO	TKT	KON	TST		
1	2	2	2	2	2	1	25	32
2	2	2	2	1	2	2	12	32
3	2	2	2	1	1	2	14	16
4	2	2	2	2	1	1	17	16
5	2	2	2	1			9	8
6	2	2	2	2	2	1	24	32
7	2	2	2	1		1	26	8
8	2	2	2		2		17	16
9	4	2	2	1	2	2	14	64
10	2	2	2	1	1		10	8
11	2	2	2	1	2	1	15	16
12	2	2	2	2	1		24	16
13	2	2	2	1			8	8
14	2	1	1	1	1	1	7	2
15	2	2	2	1		2	18	16
16	2	2	2	2		2	33	32
17	2	1	1	2	2	1	10	8

18	2	1	1	1	1	2	10	4
19	1	1	1	2	2	2	12	8
20	2	2	2	2	2	2	17	64
Total	90	65	65	38	37	27	322	**1,5112E+23**

As mentioned above and as is seen in Table 1 and 2, data of some banks are not available depending upon the absence of corresponding service. Considering that the DEA depend on all the data observed, absence of data as mentioned creates an additional fuzziness which seems to affect the results of the analysis of all DMUs. Customers who want to choose a bank or a bank manager who wants to see the situation in the services sector will not reach the goal unless the missing data is restored. However, it is not possible to examine all the cases mentioned above, such as using the classical approach. In this instance, different from the approaches suggested in the literature [32-41] a method can be proposed is to investigate optimistic and pessimistic cases. It should be evaluated separately for customers and companies as indicated in Table 3.

Table 3: Method application scheme

	Explanation	Optimist	Pessimist
If data is available;			
Customer Oriented	Maximization Problem (profit / return)		
	Output: Deposit rate	Maximum	Minimum
	Input: Loan Rate	Minimum	Maximum
Firm Oriented	Maximization Problem (profit / return)		
	Output: Loan Rate	Maximum	Minimum
	Input: Deposit rate	Minimum	Maximum
If data is not available;			
Customer Oriented	Maximization Problem (profit / return)		
	Output: Deposit rate	Sector Maximum	Sector Minimum
	Input: Loan Rate	Sector Minimum	Sector Maximum
Firm Oriented	Maximization Problem (profit / return)		
	Output: Loan Rate	Sector Minimum	Sector Maximum
	Input: Deposit rate	Sector Maximum *	Sector Minimum
Averaging	To prevent the efficient frontier from non-servicing DMUs	Null cell replaced with relevant average of sector.	Null cell replaced with relevant average of sector.

* The DMU is attained to efficient frontier as relevant service is given by the bank

Even the missing values fuzzifies the efficiency of banks, the interest rates of loans cannot be lower than the sector minimum or more than the sector maximum. Therefore, the most pessimistic value could be taken as observed minimum; the most optimistic values could be taken as observed maximum and most possible values could be taken as sector average as mentioned in the method application scheme given in Table 3.

3. RESULTS

In our study, DEA (1) model is solved separately for each case according to the above application schema listed in Table 3. Results based on different expectations of customer and banks are given in Table 4. The dark cells of the tables represent the DMUs with missing values which are the cause of fuzziness. The noticeable points of the tables are the effect of fuzzy values. These values affect not only the efficiency of DMUs with missing/fuzzy values, but also the efficiency of other DMUs. The Kruskal-Wallis test results given in Table 5 also confirm these results.

Table 4: DEA results, according to the inclusion form of missing data into analysis

Bank No	Case 1				Case 2				Case 3			
	CP	**CO**	**FP**	**FO**	**CP**	**CO**	**FP**	**FO**	**CP**	**CO**	**FP**	**FO**
1	0,139	0,473	0,842	1	0,309	0,782	0,842	1	0,288	0,922	0,828	1
2	0,199	0,348	1	0,756	0,423	0,625	1	0,756	0,387	0,815	0,97	0,756
3	0,593	0,459	0,814	0,385	0,97	0,688	0,811	0,385	0,966	0,928	0,801	0,385
4	0,412	0,489	1	0,578	0,768	0,852	1	0,578	0,768	0,934	1	0,578
5	1	1	1	0,254	0,887	0,846	1	0,514	0,887	0,819	1	0,365
6	0,303	0,48	0,861	0,535	0,645	0,887	0,806	0,535	0,595	1	0,737	0,535
7	1	1	0,767	0,312	1	0,971	0,767	0,385	0,982	0,92	0,763	0,318
8	1	1	0,545	0,305	1	1	0,582	0,429	1	1	0,762	0,305
9	0,686	0,532	0,822	0,354	1	0,731	0,819	0,354	1	1	0,819	0,354
10	0,382	0,365	1	0,39	0,71	0,769	1	0,544	0,745	0,731	1	0,39
11	0,219	0,397	0,996	0,761	0,474	0,69	0,972	0,761	0,435	0,796	0,911	0,761
12	0,597	0,936	0,586	0,24	1	1	0,586	0,441	1	1	0,683	0,313
13	1	1	1	0,383	0,801	0,876	1	0,487	0,972	0,821	1	0,383
14	0,492	0,31	1	0,326	1	0,643	1	0,326	0,996	0,881	1	0,326
15	0,314	0,85	0,873	0,81	0,349	1	0,858	0,962	0,359	0,99	0,826	0,81
16	0,365	0,779	0,983	0,814	0,398	0,951	0,979	0,814	0,382	0,788	0,934	0,814
17	0,549	0,382	1	0,327	0,999	0,8	1	0,327	0,992	0,922	0,969	0,327
18	0,641	0,377	1	0,53	0,964	0,565	1	0,53	0,964	0,71	1	0,53
19	0,572	0,38	1	0,337	1	0,663	1	0,337	1	0,76	0,95	0,337
20	0,708	0,524	0,846	0,478	1	0,698	0,846	0,478	1	0,858	0,846	0,478

Case 1: The DEA results, when the sector maximum is used as missing values.

Case 2: The DEA results, when the sector minimum is used as missing values.

Case 3: The DEA results, when the sector averaging used as missing values.

CO: Customer Optimist; **CP:** Customer Pessimist; **FO:** Firm Optimist; **FP:** Firm Pessimist

Table 5: Kruskal-Wallis test results

	H_0	p	Decision
1	The distribution of customer pessimist DEA results over cases are the same	0,034	**H_0 rejected**
2	The distribution of customer optimist DEA results over cases are the same	0,004	**H_0 rejected**
3	The distribution of firm pessimist DEA results over cases are the same	0,587	H_0 not rejected
4	The distribution of firm optimist DEA results over cases are the same	0,406	H0 not rejected

Significance value taken as %5.

4. CONCLUSION AND DISCUSSION

Efficient operations of the economic actors in an economy are of vital importance for a country. Particularly, in the case of a fuzzy environment facing some difficulties in decision making is inevitable. Essentially, fuzzy DEA is a useful tool and guide that any decision maker can use in mentioned cases.

According to the results of Kruskal-Wallis test (Table 5), the DEA results obtained by the application of scheme given in Table 3 is statistically the same for banks, namely in almost every situation, the banks are efficient. Since the data used in this study are determined from banks, it would not be wise for the banks to offer services that would render them inefficient. Nevertheless, decisions implemented by the banks obviously affect the customers' efficiency. These results also indicate that, even if the variables are the same, evaluation of efficiency in terms of firms and customers is different. The perception that, in case firm is optimist the customer will be pessimist or in case the customer is optimist the firm will be pessimist is inaccurate. As a consequence, firms and customers evaluate their self-efficient frontiers independent from each other and if the two parts both aim to be efficient at the same time, they must come to an agreement. This study attempts and proposes a way of compromise that would make customers and firms efficient and satisfied. Evaluating a single model is left to the future research.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] Charnes, A., Cooper, W.W., and Rhodes, E., "Measuring the efficiency of decision making units", Eur. J. Oper. Res., 2(6): 429-444, (1978).

[2] Zadeh, L.A., "Fuzzy sets", Information and control, 8(3): 338-353, (1965).

[3] Rommelfanger, H., "Fuzzy linear programming and applications", Eur. J. Oper. Res., 92(3): 512-527, (1996).

[4] Herrera, F. and Verdegay, J., "Fuzzy sets and operations research: perspectives", Fuzzy Sets and Systems, 90(2): 207-218, (1997).

[5] Zimmermann, H.J., "Fuzzy set theory", Wiley Interdisciplinary Reviews: Computational Statistics, 2(3): 317-332, (2010).

[6] Hatami-Marbini, A., Emrouznejad, A., and Tavana, M., "A taxonomy and review of the fuzzy data envelopment analysis literature: Two decades in the making", Eur. J. Oper. Res., 214(3): 457-472, (2011).

[7] Emrouznejad, A. and Tavana, M., Performance Measurement with Fuzzy Data Envelopment Analysis, Springer, (2014).

[8] Popovici, M.-C., "A Survey on Bank Efficiency. Research With Data Envelopment Analysis and Stochastic Frontier Analysis", SEA–Practical Application of Science, (01): 134-142, (2013).

[9] Liu, J.S., et al., "A survey of DEA applications", Omega, 41(5): 893-902, (2013).

[10] Paradi, J.C. and Zhu, H., "A survey on bank branch efficiency and performance research with data envelopment analysis", Omega, 41(1): 61-79, (2013).

[11] Puri, J. and Yadav, S.P., "A fuzzy DEA model with undesirable fuzzy outputs and its application to the banking sector in India", Expert Systems with Applications, 41(14): 6419-6432, (2014).

[12] Puri, J. and Yadav, S.P., Fuzzy mix-efficiency in fuzzy data envelopment analysis and its application, in Performance Measurement with Fuzzy Data Envelopment Analysis, Springer, 117-155, (2014).

[13] Tavana, M. and Khalili-Damghani, K., "A new two-stage Stackelberg fuzzy data envelopment analysis model", Measurement, 53: 277-296, (2014).

[14] Wang, Y.-M. and Chin, K.-S., "Fuzzy data envelopment analysis: A fuzzy expected value approach", Expert Systems with Applications, 38(9): 11678-11685, (2011).

[15] Xia, Q., Liang, L., and Yang, F., Integrating Fuzzy Intermediate Factors in Supply Chain Efficiency Evaluation, in Performance Measurement with Fuzzy Data Envelopment Analysis, Springer, 243-254, (2014).

[16] Chen, Y.-C., et al., "The analysis of bank business performance and market risk—Applying Fuzzy DEA", Economic Modelling, 32: 225-232, (2013).

[17] Bal, H. and Gölcükcü, A., "Data Envelopment Analysis: an application to Turkish banking industry", Mathematical and Computational Applications, 7(1): 65-72, (2002).

[18] Kao, C. and Liu, S.-T., "Fuzzy efficiency measures in data envelopment analysis", Fuzzy sets and systems, 113(3): 427-437, (2000).

[19] Charnes, A., Data Envelopment Analysis: Theory, Methodology, and Applications: Theory, Methodology and Applications, Springer Science & Business Media, (1994).

[20] Cooper, W.W., Seiford, L.M., and Tone, K., Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software, Springer Science & Business Media, (2007).

[21] Zhu, J., Data Envelopment Analysis: A Handbook of Models and Methods, Springer, (2015).

[22] Cook, W.D. and Seiford, L.M., "Data envelopment analysis (DEA) - Thirty years on", Eur. J. Oper. Res., 192(1): 1-17, (2009).

[23] Cook, W.D., Seiford, L.M., and Zhu, J., "Data envelopment analysis: The research frontier", Omega, 41(1): 1-2, (2013).

[24] Emrouznejad, A., "Advances in data envelopment analysis", Ann. Oper. Res., 214(1): 1-4, (2014).

[25] Emrouznejad, A., Parker, B.R., and Tavares, G., "Evaluation of research in efficiency and productivity: A survey and analysis of the first 30 years of scholarly literature in DEA", Socio-Econ. Plann. Sci., 42(3): 151-157, (2008).

[26] Emrouznejad, A., Podinovski, V.V., and Thanassoulis, E., "Data envelopment analysis: Theory and applications", J.Oper.Res.Soc., 60(11): 1467-1468, (2009).

[27] Lai, Y.-J. and Hwang, C.-L., "Fuzzy mathematical programming(methods and applications)", Lecture notes in economics and mathematical systems, (1992).

[28] Zimmermann, H.-J., "Fuzzy programming and linear programming with several objective functions", Fuzzy sets and systems, 1(1): 45-55, (1978).

[29] Sengupta, J.K., "A fuzzy systems approach in data envelopment analysis", Computers & Mathematics with Applications, 24(8): 259-266, (1992).

[30] Sengupta, J.K., "Measuring efficiency by a fuzzy statistical approach", Fuzzy Sets and Systems, 46(1): 73-80, (1992).

[31] Emrouznejad, A., Tavana, M., and Hatami-Marbini, A., "The state of the art in fuzzy data envelopment analysis", in Studies in Fuzziness and Soft Computing, A. Emrouznejad and Tavana, M., Editors. 1-45,(2014).

[32] Kao, C., "Interval efficiency measures in data envelopment analysis with imprecise data", Eur. J. Oper. Res., 174(2): 1087-1099, (2006).

[33] Zhu, J., "Imprecise data envelopment analysis (IDEA): A review and improvement with an application", Eur. J. Oper. Res., 144(3): 513-529, (2003).

[34] Despotis, D.K. and Smirlis, Y.G., "Data envelopment analysis with imprecise data", Eur. J. Oper. Res., 140(1): 24-36, (2002).

[35] Wen, M., Uncertain Data Envelopment Analysis, Springer,

[36] Dyson, R.G. and Shale, E., "Data envelopment analysis, operational research and uncertainty", J.Oper.Res.Soc., 61(1): 25-34, (2010).

[37] Jahed, R., Amirteimoori, A., and Azizi, H., "Performance measurement of decision-making units under uncertainty conditions: An approach based on double frontier analysis", Measurement, 69: 264-279, (2015).

[38] Azizi, H., "DEA efficiency analysis: A DEA approach with double frontiers", International Journal of Systems Science, 45(11): 2289-2300, (2014).

[39] Inuiguchi, M. and Mizoshita, F., "Qualitative and quantitative data envelopment analysis with interval data", Ann. Oper. Res., 195(1): 189-220, (2012).

[40] Wang, Y.-M. and Lan, Y.-X., "Estimating most productive scale size with double frontiers data envelopment analysis", Economic Modelling, 33: 182-186, (2013).

[41] Entani, T., Maeda, Y., and Tanaka, H., "Dual models of interval DEA and its extension to interval data", Eur. J. Oper. Res., 136(1): 32-45, (2002).