Gazi University Journal of Science
Part A: Engineering And Innovation
GU J Sci Part:A
3(2):31-38 (2015)

Vehicle Parameter Identification Using Population Based Algorithms

Hakan GÖKDAĞ[1,♠]

[1]*Bursa Technical University, Mechanical Engineering Department*, 16190 *Osmangazi, Bursa, TURKEY*

Received: 22/09/2014 Accepted: 11/05/2015

ABSTRACT

This work deals with parameter identification of a vehicle using population based algorithms such as Particle Swarm Optimization (PSO), Artificial Bee Colony Optimization (ABC) and Genetic Algorithm (GA). Full vehicle model with seven degree of freedom (DoF) is employed, and two objective functions based on reference and computed responses are proposed. Solving the optimization problem vehicle mass, moments of inertia and vehicle center of gravity parameters, which are necessary for later applications such as vehicle control and performance analysis, are obtained. It is demonstrated the proposed approach achieves to determine unknown parameters with negligible relative errors in spite of noise interference.

Key Words: *Optimization; vehicle parameter identification; particle swarm, artificial bee colony*

1. INTRODUCTION

Vehicle dynamic parameters such as mass, moment of inertia, and center of gravity coordinates may change depending on loading conditions. These changes have significant impact on handling, ride, breaking and traction performance of vehicle [1]. In order to improve dynamic control of vehicle, accurate values of these properties are needed [2].

Various approaches mainly focused on vehicle's longitudinal and lateral motions are introduced for vehicle parameter identification. Venture et al. [2], for example, proposed a method based on multi-body model of the car. Furukawa and Dissanayake [3] introduced a multi-objective optimization based method to determine vehicle state parameters. By this way, a solution space rather than a single solution is obtained, which enables parametric study. Wesemeier and Isermann [4] used one track vehicle model to determine cornering stiffness and center of gravity parameters. The proposed method needs input variables such as vehicle forward speed, lateral acceleration, yaw rate and slip angle. Khaknejad et al [5] identified mass, yaw moment of inertia, the distance between center of mass and front axle, and velocity of a sedan car using bicycle model of vehicle and least square estimation with exponential forgetting factor. The authors state that estimated parameters can be used to develop adaptive control systems and to prepare benchmark tables. Wilhelm et al [6] proposed an objective function (OF) based on the difference of measured and simulated powers, and they used MATLAB optimization toolbox to minimize the function. By this way mass, rolling resistance and aerodynamic coefficients as well as efficiency of power train of an electric vehicle are determined. Recently, Kidambi et.al. [7] assessed the

♠Corresponding author, e-mail: hakan.gokdag@btu.edu.tr

accuracy and performance of four estimation methods, i.e. recursive least squares with multiple forgetting factors; extended Kalman filtering; a dynamic grade observer; and parallel mass and grade (PMG) estimation using a longitudinal accelerometer.

Rozyn and Zhang [1] state that longitudinal estimators require the input forces. If the input forces are not measured correctly the inertia parameters may be estimated with errors over 20 %. To measure these forces many sensors are necessary, which increases cost and complexity. Thus, the authors, different from the many works based on vehicle longitudinal and/or lateral dynamics, used vertical vibration model to predict mass, pitch and roll inertia moments. Their method is based on modal parameter estimation using free-decay responses of the vehicle and estimation of the system characteristic matrix. To this end the authors used simplified 3 degree of freedom (DoF) model of the 12 DoF vehicle which includes vehicle, body-seat, and engine sub-systems. Motivated by the authors' work, a different approach is proposed in the present study. Using 7 DoF full vehicle vibration model, vertical inputs from four tyres are applied, and four time responses from predetermined locations on the vehicle are recorded. Then, OFs based on the difference of recorded and computed responses are minimized to determine vehicle properties. To this end popular non-gradient based methods such as GA, PSO, and ABC are tested and compared. It is shown that the present method gives accurate results.

2. MATERIAL AND METHOD

2.1. Vehicle Model

In this study no model reduction such as in Ref. [1] is applied, instead the full car model depicted in Figure 1 is considered [8]. This model includes DoFs such as the body bounce x, body roll φ and pitch θ, wheels hop x_1, x_2, x_3, x_4. y_i (i=1 to 4) denotes independent road excitations, m is vehicle body mass, I_x and I_y are mass moments of inertia with respect to longitudinal and lateral axes, respectively. k_f represents front suspension stiffness while k_r stands for the rear suspension. c_f and c_r are the corresponding suspension dampings. k_R is the stiffness of antiroll bar. k_{tf} and k_{tr} denote tire vertical stiffnesses, m_i (i = f, r) indicates wheel masses. a_i and b_i (i=1,2) are body center of gravity coordinates along the vehicle axes.

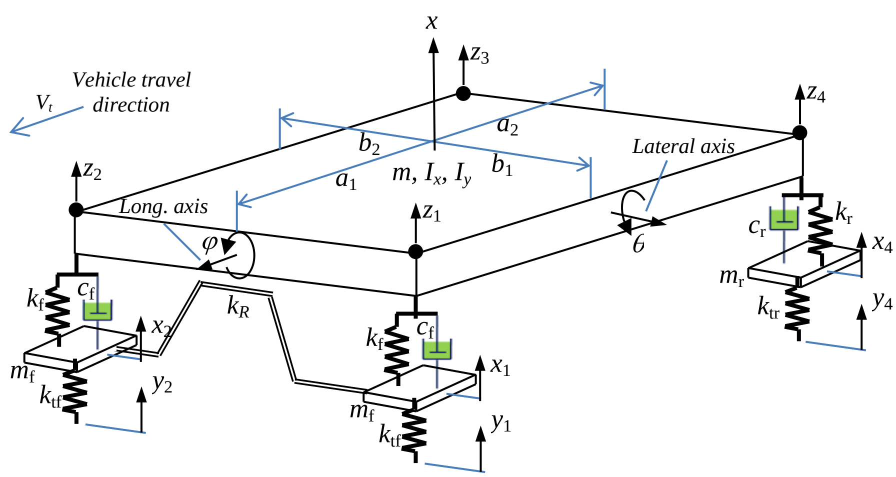

Figure 1. The vehicle full car model

Applying Lagrange method the equations of motion are obtained in matrix form as follows (see Ref. [8] for details):

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{F\} \qquad (1)$$

where

$$[M]=\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_f & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_r \end{bmatrix}, \quad \{F\}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ y_1 k_{tf} \\ y_2 k_{tf} \\ y_3 k_{tr} \\ y_4 k_{tr} \end{bmatrix} \tag{2}$$

$$[C]=\begin{bmatrix} c_{11} & c_{12} & c_{13} & -c_f & -c_f & -c_r & -c_r \\ c_{21} & c_{22} & c_{23} & -b_1 c_f & b_2 c_f & b_1 c_r & -b_2 c_r \\ c_{31} & c_{32} & c_{33} & a_1 c_f & a_1 c_f & -a_2 c_r & -a_2 c_r \\ -c_f & -b_1 c_f & a_1 c_f & c_f & 0 & 0 & 0 \\ c_f & b_2 c_f & a_1 c_f & 0 & c_f & 0 & 0 \\ -c_r & b_1 c_r & -a_2 c_r & 0 & 0 & c_r & 0 \\ -c_r & -b_2 c_r & -a_2 c_r & 0 & 0 & 0 & c_r \end{bmatrix} \tag{3}$$

$$\begin{aligned} & c_{11}=2(c_f+c_r),\ c_{21}=c_{12}=b_1(c_f-c_r)+b_2(c_r-c_f) \\ & c_{31}=c_{13}=2(a_2 c_r-a_1 c_f),\ c_{22}=(c_f+c_r)(b_1^2+b_2^2) \\ & c_{32}=c_{23}=c_f(a_1 b_2-a_1 b_1)+c_r(a_2 b_2-a_2 b_1) \\ & c_{33}=2(c_f a_1^2+c_r a_2^2) \end{aligned} \tag{4}$$

$$[K]=\begin{bmatrix} k_{11} & k_{12} & k_{13} & -k_f & -k_f & -k_r & -k_r \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & b_1 k_r & -b_2 k_r \\ k_{31} & k_{32} & k_{33} & a_1 k_f & a_1 k_f & -a_2 k_r & -a_2 k_r \\ -k_f & k_{42} & a_1 k_f & k_{44} & \frac{-k_R}{w^2} & 0 & 0 \\ k_f & k_{52} & a_1 k_f & \frac{-k_R}{w^2} & k_{55} & 0 & 0 \\ -k_r & b_1 k_r & -a_2 k_r & 0 & 0 & k_r+k_{tr} & 0 \\ -k_r & -b_2 k_r & -a_2 k_r & 0 & 0 & 0 & k_r+k_{tr} \end{bmatrix} \tag{5}$$

$$\begin{aligned} & k_{11}=2(k_f+k_r),\ k_{21}=k_{12}=k_f(b_1-b_2)+k_r(b_2-b_1) \\ & k_{31}=k_{13}=2(a_2 k_r-a_1 k_f),\ k_{22}=k_R+(k_f+k_r)(b_1^2+b_2^2) \\ & k_{32}=k_{23}=k_f(a_1 b_2-a_1 b_1)+k_r(a_2 b_2-a_2 b_1) \\ & k_{42}=k_{24}=-b_1 k_f-\frac{k_R}{w},\ k_{52}=k_{25}=b_2 k_f+\frac{k_R}{w} \\ & k_{33}=2(k_f a_1^2+k_r a_2^2),\ k_{44}=k_f+k_{tf}+\frac{k_R}{w^2},\ k_{55}=k_{44},\ w=b_1+b_2 \end{aligned} \tag{6}$$

In this work Newmark Beta method is applied to solve Eq.(1).

2.2. The Proposed Approach

It is assumed all vehicle parameters except for m, I_x, I_y, a_1, b_1 are known. As in many parameter identification works some inputs are applied and response of the system to these inputs are recorded. In this work, it is assumed while the vehicle moving with constant speed V_t, the tires are subject to step like vertical inputs. Hence, the value of V_t will determine the width of the power spectrum. That is, as Figure 2 depicts, the wider a time signal in time domain the narrower its frequency spectrum. Larger values of V_t will lead to narrower input signals in time domain. The aim is to excite all vibration modes of vehicle in order to gather enough data. In general, natural frequencies of a vehicle are in the range 0 to 15 Hz, so there is no need to consider higher speeds. In this work, the value V_t = 20 km/h is experienced to be sufficient for the vehicle considered. On the other hand, tires at the right and left sides should not be subject to step inputs with the same height. Otherwise roll mode (φ) of the vehicle may not be observed.

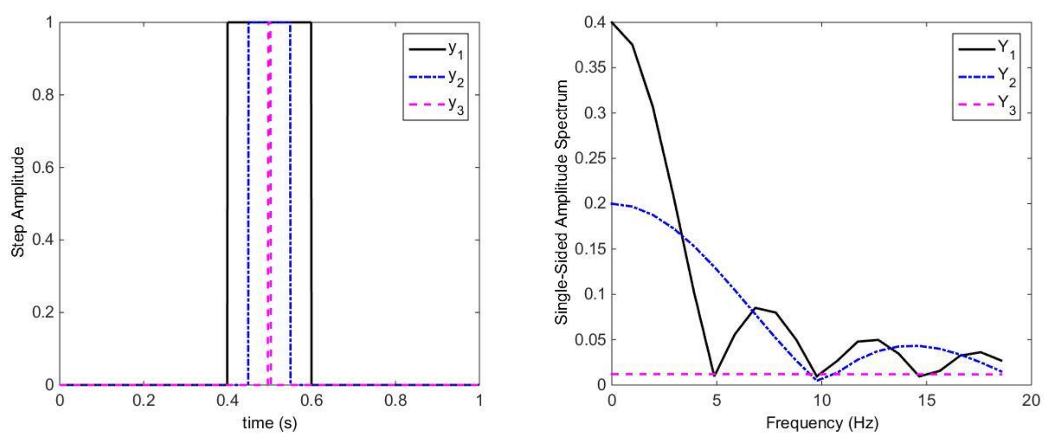

Figure 2. Time signals and their power spectrums.

Under these conditions displacement and acceleration response of the vehicle are recorded for specific inputs. In Ref. [1] it is advised if only accelerometers are used to measure response then they should be spaced as far as possible to maximize signal to noise ratio of the phase and amplitude information. Hence, the authors propose to place sensors at the outer edges of the vehicle. In accordance with this advice the response points, i.e. z_i, (i=1,2,3,4), are determined as shown in Figure 1. These are computed as follows:

$$\left.\begin{aligned} z_1 &= x+\varphi b_1-\theta a_1 \\ z_2 &= x-\varphi b_2-\theta a_1 \\ z_3 &= x-\varphi b_2+\theta a_2 \\ z_4 &= x+\varphi b_1+\theta a_2 \end{aligned}\right\} \tag{7}$$

Depending on acceleration or displacement response, the following OFs can be introduced:

$$f_1(\{x\})=\frac{\sum_{i=1}^{4}\left\|z_i^r-z_i^c\right\|}{\sum_{i=1}^{4}\left\|z_i^r\right\|},\quad f_2(\{x\})=\frac{\sum_{i=1}^{4}\left\|\ddot{z}_i^r-\ddot{z}_i^c\right\|}{\sum_{i=1}^{4}\left\|\ddot{z}_i^r\right\|} \tag{8}$$

where $\{x\} = [m, I_x, I_y, a_1, b_1]$ is the vector including unknown vehicle parameters, $\|\cdot\|$ is the norm of variable. Upper scripts r and c indicate reference and computed data, respectively. Reference data is the vehicle response recorded for specific input. In practice this is measured by sensors for a vehicle state. But in this work, because of lack of experimental data, this is produced using mathematical model, and contaminated by random numbers to represent measurement noise. To determine the element of $\{x\}$ the following optimization problem is formed:

$$\min(f) \quad \text{subject to } \{\text{LB}\}<\{x\}<\{\text{UB}\} \tag{9}$$

where f is f_1 or f_2 in Eq.(8), {LB} and {UB} denote lower and upper boundary vectors determining the range in which unknown parameters can take value. Generally, to solve the problems like this, population based algorithms are implemented to avoid local solutions. Thus, two popular and easily applicable algorithms, i.e. PSO and ABC, are applied in this work as well as GA toolbox of MATLAB (R2014a).

2.3. The Optimization Algorithms

PSO is developed by Kennedy and Eberhart [9]. It is a stochastic optimization method inspired by swarm behavior of birds and insects. The algorithm is initialized with a "swarm" including randomly created particles. Particles refer to the candidate points in the search space of the problem. To obtain the best solution each particle adjusts its trajectory towards its own previous best position and towards the previous best position of the swarm. By this way, each particle moves in the search space with an adaptive velocity, and stores the best position of the search space. Various versions of PSO are available in the literature. In this work, the one called contemporary PSO (CPSO) is employed because of its higher convergence speed. In CPSO location (x) and velocity (v) of a particle are updated with the following equations [10,11].

$$v_{ij}^{k+1} = \chi\left(v_{ij}^k + c_1R_1(p_{ij}^k - x_{ij}^k) + c_2R_2(p_{gj}^k - x_{ij}^k)\right) \tag{10}$$

$$x_{ij}^{k+1} = x_{ij}^k + v_{ij}^{k+1},\quad i=1,2,..,N,\ \ j=1,2,...,m,\ \ k=1,2,...,K_{\max} \tag{11}$$

where k is the iteration counter, $K_{\max}$ denotes the maximum number of iterations, m is dimension of the problem, p_{ij}^k and p_{gj}^k are, respectively, the best positions of the ith particle and the swarm found until the k^{th} iteration. R_1 and $R_2 \in$ U(0,1), where U means uniform random distribution. c_1 and c_2 are positive weighting constants called cognitive and social coefficients, respectively. χ is the constriction factor defined as $\chi = 2\left(\left|2 - \phi - \sqrt{\phi^2 - 4\phi}\right|\right)^{-1}$, where $\phi = c_1 + c_2$, and $\phi > 4$. Common values of acceleration coefficients are $c_1 = c_2 = 2.05$, thus $\chi = 0.7298$ [11]. The pseudo code of the algorithm is as follows

1. Generate random population
2. Repeat
   2.1 Evaluate fitness values
   2.2 Modify the best particles in the swarm
   2.3 Choose the best particle
   2.4 Calculate the velocities by Eq.(10)
   2.5 Update particle positions by Eq.(11)
3. Until requirements are met

Another population based method to be tested is the ABC algorithm. This is based on the intelligent foraging behavior of honey bees [12]. The bees, i.e. the individuals in the swarm, are classified into three types: Employed, onlooker, and scout bees. Each employed bee is associated with a food source, which it exploits currently. A bee waiting in the hive to choose a food source is an onlooker bee. The employed bees share information about the food sources with onlookers. A scout bee carries out a random search to discover new food sources [13]. At each iteration the employed and onlooker bees improve the solutions by a neighborhood search. A new solution (v_i) in the neighborhood of an existing one (x_i) is produced as follows:

$$v_{i,j} = x_{i,j} + V(x_{i,j} - x_{s,j}),\qquad s\neq i \tag{12}$$

where s $\in$ [1,N] is an integer, and V is a uniform random number in the interval [-1,1]. s and the dimension parameter j are chosen randomly. Then, greedy selection is performed between x_i and v_i. The onlooker bees are placed on food sources by the roulette wheel selection method. Thus, an onlooker bee chooses a food source at position x_i with a probability p_i calculated as follows:

$$p_i = \frac{\text{fit}_i}{\sum_{n=1}^{SN}\text{fit}_n} \tag{13}$$

where *SN* denotes the number of employed bees, and it is equal to half of the population; *SN*=*N*/2, [13]. "fit" means the fitness value calculated by the following equation

$$\mathrm{fit}_i = \begin{cases} \dfrac{1}{1+\mathrm{fit}_i} & \text{if } \mathrm{fit}_i \le 0 \\ 1+\mathrm{abs}(\mathrm{fit}_i) & \text{if } \mathrm{fit}_i > 0 \end{cases} \qquad (14)$$

A solution representing a food source is abandoned by an employed bee if it cannot be improved for a predetermined number of trials indicated by the parameter "limit". The employed bee then becomes a scout bee and randomly produces a new solution replacing the existing solution. The value of limit is determined as limit=*SN* x *D*, where D is dimension of the problem [14]. The pseudo code of the algorithms is as follows:

1. Generate random population; x_i, i=1,..,*SN*
2. Evaluate the population
3. Repeat
   2.1 Produce new solution by Eq.(12) and apply greedy selection
   2.3 Calculate p_i values by Eq.(13)
   2.4 Place the onlookers on the food sources
   2.5 Determine the abandoned solution (if any);
   2.6 Record the best solution
3. Until requirements are met

Computer codes written in MATLAB environment are used. The code for CPSO is written by the author of this work, and that for the ABC algorithm is downloaded from the owner's website [15].

3. RESULTS AND DISCUSSION

As an application the following vehicle properties [1] are considered: m = 1300 kg, m_f = 40 kg, m_r = 65 kg, I_x = 500 kgm^2, I_y = 2500 kgm^2, $a_1 = a_2$ = 1.3m, $b_1 = b_2$ = 0.75m, k_f = 45000 N/m, k_r = 80000 N/m, $k_{tr} = k_{tf}$ = 200000 N/m, k_R = 50000 N/m, c_f = 2800 Ns/m, c_r = 3500 Ns/m. While vehicle speed is V_t = 20 km/h tires are subject to inputs as in Figure 3. Input heights are different, as stated before. That is, front right and rear right tires are subject to step displacement with height 5cm (y_1 and y_2) while the input height is 4 cm for the left tires. To obtain reference data, displacement and acceleration responses to these inputs are computed, and noise is added as follows:

$$z^r = z^c + std(z^c) * Np * randn(size(z^c)) \qquad (15)$$

where N_p noise percentage, std(·) is the standard deviation, *randn* means uniform random distribution with zero mean and unit standard deviation. Noise is assumed to represent errors due to measurement and instrumentation. In practice, reference data is measured for a state of the vehicle. However, in this work it is produced by the mathematical model of the vehicle and noise is added as in Eq.(15). Noisy displacements and accelerations are as shown in Figure 4.

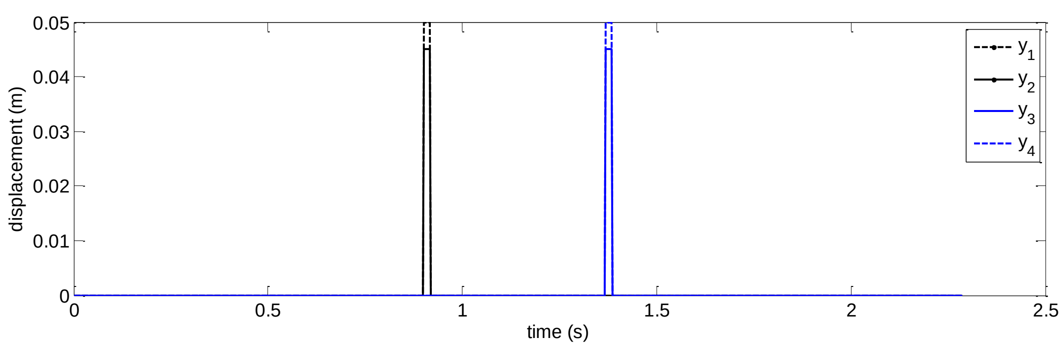

Figure 3. Inputs to the tires

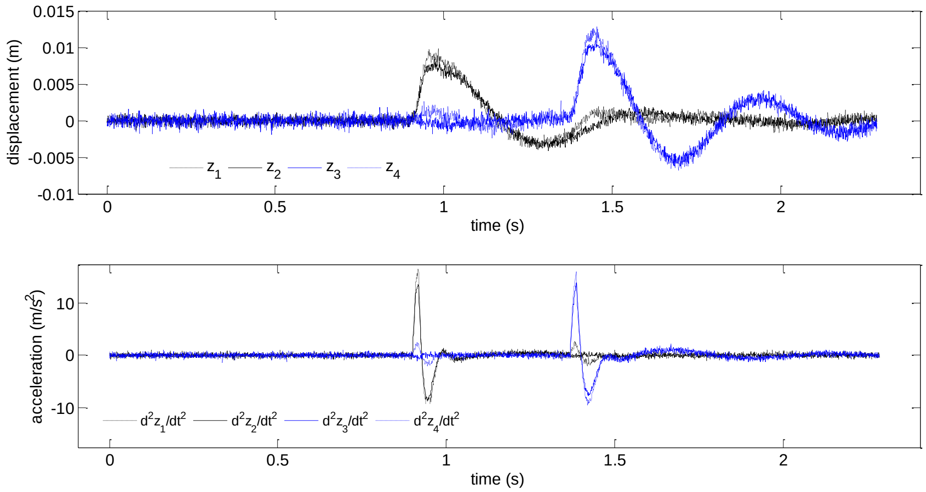

Figure 4. Vehicle response to the inputs

The optimization algorithms were run with the following parameters: Population size 20; number of iterations 50. The search space is determined in the range between 30% lower and upper of the optimum values ({OV}), i.e. {LB} = 0.7{OV}, {UB} = 1.3{OV}, that is the range of search space is wider compared to some relevant works in the literature. In GA default values are retained for the other parameters. For example, crossover fraction is 0.8, mutation function type is Gaussian, method of reproduction is Elite Count with the value equal to 0.05x(Population Size). Besides, the value of TolFun is set to zero to avoid early convergence before maximum number of generations is reached. Each algorithm was run thirty times to obtain statistical results. The mean of estimated values and their closeness to the real results are given in Table 1 and 2, depending on the noise amount in the reference data, and the OFs employed. In the Tables RE means relative error, NRE is the norm of relative errors, which is a measure of the closeness of solution vector to {OV}. Bold numbers correspond to the best results. According to the Tables, ABC finds the closest solution to the real at each case. The other two, generally, exhibits similar performance and gives acceptable results as well. However, in ABC the only parameter is the "limit" number while the other two, especially GA, have more parameters needing adjustment, and they are generally problem dependent. On the other hand, f_2 seems to be superior to f_1, thus in practice acceleration response may be more useful. Acquisition of acceleration response is more feasible considering current instruments. Besides, relative errors, when compared with the relevant literature, indicate that the proposed approach yields accurate results in spite of noise interference.

Table 1. Estimated values for f_1 in Eq.(8). (Est: Estimated, RE: Relative error, NRE: Norm of Relative Errors.)

N_p = 10%

		GA		CPSO		ABC	
	{OV}	Est	RE (%)	Est	RE (%)	Est	RE (%)
m	1300	1321	1.589	1299.95	**-0.003**	1299.81	-0.015
I_x	500	517	3.299	510	1.930	497	**-0.700**
I_y	2500	2499	**-0.024**	2499	-0.052	2497	-0.117
a_1	1.3	1.296	-0.337	1.299	**-0.039**	1.302	0.165
b_1	0.75	0.755	0.725	0.751	0.155	0.7502	**0.023**
NRE (%)	---		3.75		1.94		**0.729**

N_p = 20%

		GA		CPSO		ABC	
	{OV}	Est	RE (%)	Est	RE (%)	Est	RE (%)
m	1300	1312	0.923	1297	-0.259	1297	**-0.211**
I_x	500	527	5.412	505	0.997	498	**-0.430**
I_y	2500	2462	-1.511	2495	-0.218	2495	**-0.196**
a_1	1.3	1.312	0.894	1.304	0.326	1.304	**0.285**
b_1	0.75	0.759	1.235	0.7502	**0.030**	0.7503	0.044
NRE (%)	---		5.894		1.103		**0.593**

Table 2. Estimated values for f_2 in Eq.(8)

N_p = 10%

		GA		CPSO		ABC	
	{OV}		RE (%)		RE (%)		RE (%)
m	1300	1287	-0.933	1300.22	0.017	1300.06	**0.004**
I_x	500	517	3.482	535	7.067	500.91	**0.182**
I_y	2500	2493	-0.284	2498	-0.072	2498	**-0.067**
a_1	1.3	1.3298	2.289	1.2969	-0.234	1.3003	**0.024**
b_1	0.75	0.7588	1.171	0.7504	0.048	0.7503	**0.037**
NRE (%)	---		4.44		7.07		**0.199**

N_p = 20%

		GA		CPSO		ABC	
	{OV}		RE (%)		RE (%)		RE (%)
m	1300	1301	0.117	1298	**-0.124**	1296	-0.284
I_x	500	542	8.422	536	7.250	507	**1.445**
I_y	2500	2562	1.040	2503	**0.111**	2504	0.177
a_1	1.3	1.326	1.969	1.301	**0.083**	1.301	0.106
b_1	0.75	0.755	0.642	0.751	0.116	0.751	**0.087**
NRE (%)	---		8.74		7.25		**1.49**

Figure 5 shows the body center of gravity displacement (x), pitch (θ) and roll (φ) angle responses of the system for true (m=1300 kg, I_x=500 kgm^2, I_y=2500 kgm^2,a_1=1.3 m, b_1=0.75 m) and estimated parameters by ABC in Table 2 (m=1296 kg, I_x=507 kgm^2, I_y=2504 kgm^2,a_1=1.301 m, b_1=0.751 m). It is clear both responses are in well agreement.

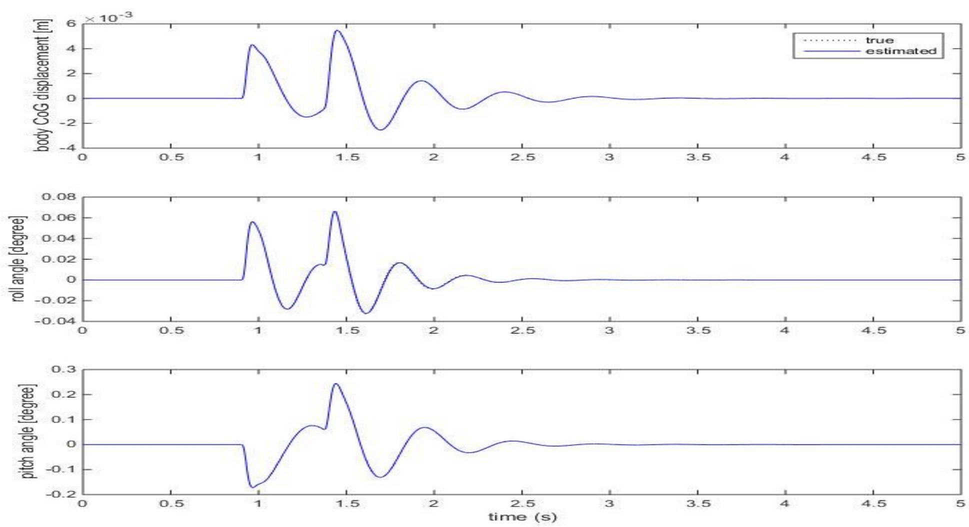

Figure 5. Vehicle time responses.

4. CONCLUSION

In this work an approach to determine some vehicle parameters is introduced. Using full vehicle model vertical response is obtained for predetermined vehicle speed and bump geometry. Then objective functions based on the difference of previously recorded responses and vehicle mathematical model output are defined and minimized by non-gradient algorithms such as GA, PSO and ABC. It is observed when acceleration response based objective function is minimized by ABC algorithm, the most accurate results are obtained in spite of noise interference. The results show that the present method has the potential to be employed in vehicle parameter identification practices.

REFERENCES

[1] Rozyn, M. and Zhang, N., “A method for estimation of vehicle inertial parameters”, Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, 48:5, 547-565, (2010).

[2] Venture, G., Bodson, P., Gautier, M., and Khalil, W., “Identification of the dynamic parameters of a car”, SAE Technical Paper, doi:10.4271/2003-01-1283.

[3] Furukawa, T, and Dissanayake, G., "Parameter identification of autonomous vehicles using multi-objective optimization", Engineering Optimization, 34:4, 369-395, (2002).

[4] Wesemeier, D., and Isermann, R., "Identification of vehicle parameters using stationary maneuvers", Control Engineering Practice, 17, 1426-1431, (2009).

[5] Khaknejad, M. B., Kazemi, R., Azadi, Sh., and Keshavaraz, A., "Identification of vehicle parameters using modified least square method in ADAMS/Car", Proceedings of 2011 International Conference on Modelling, Identification and Control, Shanghai, China, 98-103, (2011).

[6] Wilhelm, E., Bornatico, R., Widmer, R., Rodgers, L., and Soh, G.S., "Electric vehicle parameter identification", EVS26 International Battery, Hybrid and Fuel Cell Electric Vehicle Symposium, Los Angeles, California, 1-10, (2012).

[7] Kidambi, N., Harne, R.L., Fuji, Y., and Pietron, G.M. "Methods in vehicle mass and road grade estimation", SAE International Journal of Passanger Cars-Mechanical Systems doi:10.4271/2014-01-0111.

[8] Jazar, R.N. Vehicle Dynamics Theory and Application, Springer, New York, (2008).

[9] Kennedy, J., and Eberhart, R. "Particle swarm optimization", Proceedings of the 4th IEEE International Conference on Neural Networks, 4, 1942-1948, (1995).

[10] Clerc, M., and Kennedy, J., "The particle swarm-explosion, stability, and convergence in a multidimensional complex space", IEEE Transactions on Evolutionary Computation 6(1), 58-73, (2002).

[11] Parsopoulos, K. E., and Vrahatis, M. N., Particle Swarm Optimization and Intelligence: Advances and Applications. IGI Global, Hershey PA, (2010).

[12] Baştürk, B., and Karaboğa, D., "An artificial bee colony (ABC) algorithm for numeric function optimization", Proceedings of the IEEE Swarm Intelligence Symposium, Indianapolis (USA), May (2006).

[13] Narasimhan, H., "Parallel artificial bee colony (PABC) algorithm", IEEE World Congress on Nature & Biologically Inspired Computing, Coimbatore, 306-311, (2009).

[14] Karaboğa, D., and Akay, B. "A comparative study of artificial bee colony algorithm", Applied Mathematics and Computation, 214, 108-132, (2008).

[15] http://mf.erciyes.edu.tr/abc/software.htm.