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Designing and Manufacturing of a Modal Analysis Test Bench – Part one: Harmonic Shaker Development

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Abstract

Designing and manufacturing of an educational mode shape generator and analyzer is the main aim of this study. For this purpose, a group of undergraduate students were developed a test bench for the modal analysis of a cantilever beam. A harmonic exciter is designed and manufactured. Its name set as Ankara Yildirim Beyazit University first version or AYBU1. This exciter will used to excite a cantilever beam excite the example. The harmonic shaker is the most expensive part of such test bench. Therefore, an educational version of common harmonic shakers will be designed and produced. In this way the total price of final product shall be low as well.

Keywords: Modal analysis, harmonic shaker, vibration, design.

Modal Analiz Test Düzeneğinin Tasarımı ve İmalatı - Birinci Bölüm: Harmonik Titreştirici Geliştirilmesi

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Özet

Bu çalışma temel amacı, eğitimle ilgili bir mod şekil üreteci ve analizörünün tasarımı ve imalatıdır. Bu amaçla, bir grup lisans öğrencisi tarafından bir konsol kirişinin modal analizi için bir test düzeneği geliştirilmiştir. Bir harmonik uyarıcı tasarlanmış ve üretilmiştir. Deney setinin adı Ankara Yıldırım Beyazit Üniversitesi 'nin ilk versiyonu veya AYBU1 olarak belirlendi. Bu titreştirici ankastre bir kirişe uyarı vermek için kullanılacaktır. Harmonik titreştirici, bu test düzeneğinin en pahalı kısmıdır. Bu nedenle, ortak harmonik titreştiricilerin eğitimle ilgili bir versiyonu tasarlanacak ve üretilecektir. Bu şekilde nihai ürünün toplam fiyatı da düşük olacaktır.

Anahtar Kelimeler : Modal analiz, harmonik titreştirici, titreşim, tasarım.

1. Introduction

Modal analysis is a technique to detect the dynamical response of systems. It can be used to observe the resonance frequencies. This is very important in machine design and predictive maintained of structure when vibration is an important fact to be considered. In fact, the vibration of any structure or machine should be determined because this can cause destruction and decreasing efficiency. Rao [1] mentioned the vibration analysis of various systems and structures. Probably, the most classical example is cantilever beam vibration analysis. The mechanical engineering undergraduate students usually learn the basic of such vibration analysis in their third class, therefore they can use such knowledge for their final year projects as well. Shin et al. [2]

indicated that vibration analysis of a rotating cantilever beam is an important and peculiar subject of study in mechanical engineering. There are many engineering examples which can be idealized as rotating cantilever beams such as turbine blades or turbo engine blades and helicopter blades. For the proper design of the structures their vibration characteristics which are natural frequencies and mode shapes should be well identified. Compared to the vibration characteristics of nonrotating structures those of rotating structures often vary significantly. The variation results from the stretching induced by the centrifugal inertia force due to the rotational motion. The stretching causes the increment of the bending stiffness of the structure which naturally results in the variation of natural frequencies and mode shapes. Whitney [3] mentioned that resonance is that if excitation frequency comes across the structure's natural frequency, resonance event occurs. In many case resonances is undesirable and either excitation frequency or structure's natural frequency should be changed. The modal analysis of a structure or machine helps us to prevent resonance in diverse ways. Guenfoud et al. [4] presented an accurate procedure to determine free vibrations of beams and plates. The natural frequencies are exact solutions of governing vibration equations witch load to a nonlinear homogeny system. The bilinear and linear structures considered simulate a bridge. The dynamic behavior of this one is analyzed by using the theory of the orthotropic plate simply supported on two sides and free on the two others. Prasad [5] showed that modal analysis is a process of describing a structure in terms of its natural characteristics which are the frequency, damping and mode shapes on its dynamic properties. The change of modal characteristics directly provides an indication of structural condition based on changes in frequencies and mode shapes of vibration.

The dynamic behavior of structures has effect on the noise emission for structure. Therefore, knowledge of theoretical and experimental modal analysis is required for vibroacoustic deigns as well. Ranjbar et al. investigated various cases for vibroacoustic and noise reduction of various structures [6-27].

In next sections of this paper, the experimental setup for detection of dynamic behavior of a cantilever beam is shown. This project has been done by a group of undergraduate students.

2. Materials and Methods

Rao [1] presented the lateral vibration of a fix-free beam. He considered a cross-sectional element from the cantilever beam which is shown in Fig. 1.

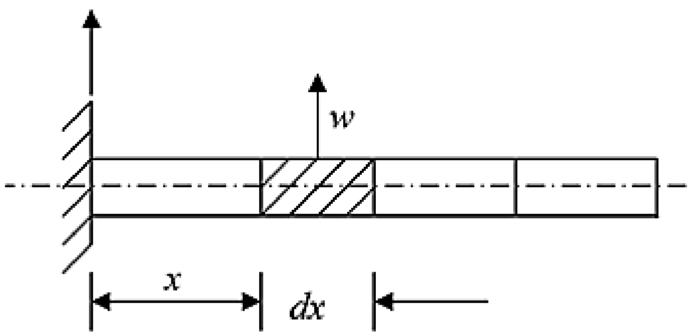

Figure 1.A cantilever beam under transverse vibration

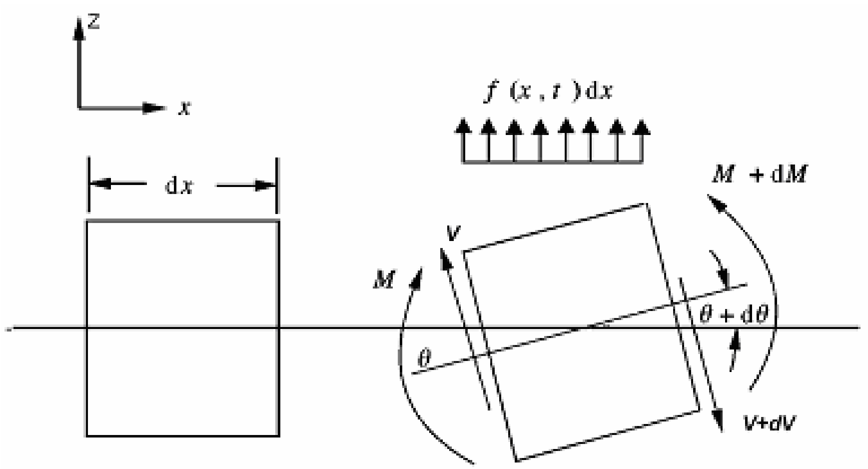

Figure 2. Freebody diagram of a section of a beam under transverse vibration

Consider the fix–free beam shown in Figure 1, where $M(x, t)$ is the bending moment; $V(x, t)$ is the shear force. Also, $f(x, t)$ is the external force per unit length of beam and the initial force acting on the element is

$$f(x,t) = \rho A(x) dx \frac{\partial^2 w}{\partial t^2} \tag{1}$$

The force equation of motion in the z direction gives

$$-(V + dV) + f(x,t)dx + V = \rho A(x) dx \frac{\partial^2 w}{\partial t^2} \tag{2}$$

Where ρ is the mass density and $A(x)$ is the cross-sectional area of the beam. The moment equation of motion neglecting rotary inertia about the y-axis passing through point O in Figure 2 leads to

$$(M + dM) - (V + dV)dx + f(x,t)dx \frac{dx}{2} - M = 0 \tag{3}$$

Writing

$$dV = \frac{\partial V}{\partial x} dx \text{ and } dM = \frac{\partial M}{\partial x} dx \tag{4}$$

and disregarding terms involving second powers in dx, Eq.(3) and (2) can be written as

$$-\frac{\partial V}{\partial x}(x,t) + f(x,t) = \rho A(x) \frac{\partial^2 w}{\partial t^2}(x,t) \tag{5}$$

$$dM = \frac{\partial M}{\partial x}(x,t) - V(x,t) = 0 \tag{6}$$

From Eq.(6), we will have

$$-\frac{\partial^2 M}{\partial x^2}(x,t)+f(x,t)=\rho A(x)\frac{\partial^2 w}{\partial t^2}(x,t) \tag{7}$$

By using the relation $V=\frac{\partial M}{\partial x}$ from above two equations from the beam theory or Euler Bernoulli, the relationship between the bending moment and deflection is

$$M(x,t)=EI(x)\frac{\partial^2 w(x,t)}{\partial t^2} \tag{8}$$

E is young modulus, *I* is the moment of inertia of beam. Furthermore, the equation of motion for a force lateral no uniform beams

$$\frac{\partial^2}{\partial x^2}\left\{EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right\}+\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2}=f(x,t) \tag{9}$$

For a uniform beam

$$\left\{EI(x)\frac{\partial^4 w(x,t)}{\partial x^4}\right\}+\rho A(x)\frac{\partial^2 w(x,t)}{\partial t^2}=f(x,t) \tag{10}$$

Which can be reduced for the case $f(x, t) = 0$ for vibration as

$$c^2\,\frac{\partial^4 w(x,t)}{\partial x^4}+\frac{\partial^2 w(x,t)}{\partial t^2}=0 \tag{11}$$

Where

$$c=\sqrt{\frac{EI(x)}{\rho A(x)}} \tag{12}$$

Since the equation of the motion involves a second order derivative with respect to time and a fourth order derivative with respect to *x*, two initial equations and four boundary conditions are needed for finding a unique solution for $w(x,t)$. Usually, the values of transverse displacement and velocity are specified as $w(x,0)$ and $\dot{w}(x,0)$ at t=0, so that the initial conditions becomes

$$w(x,0)=w_0(x) \tag{13}$$

$$\frac{\partial w}{\partial t}(x,0)=\dot{w}_0(x) \tag{14}$$

The free vibration solution is written using the method of separation of variable

$$w(x,t)=w(x).w(t) \tag{15}$$

Substituting Eq.(15) into Eq.(11) and arranging it again gives

$$\frac{c^2}{w(x)}\frac{\partial^4 w(x)}{\partial x^4} = -\frac{1}{w(t)}\frac{d^2 w(t)}{dt^2} = a = \omega^2 \tag{16}$$

Where "a" is a constant value equals. Then,

$$\frac{\partial^4 w(x)}{\partial x^4} - \beta^4 w(x) = 0 \tag{17}$$

Where

$$\beta^4 = \frac{\omega^2}{c^2} \tag{18}$$

$$\frac{\partial^2 w(t)}{\partial t^2} + \omega^2 . w(t) = 0 \tag{19}$$

The solution of Eq.(19) can be express as

$$w(t) = A.cos(\omega t) + B.sin(\omega t) \tag{20}$$

Where *A* and *B* are constant which can be found from the initial conditions. Let assume the solution of

$$w(x) = C.e^{sx} \tag{21}$$

Where *C* and *s* are constant, and derive the equation as

$$s^4 - \beta^4 = 0 \tag{22}$$

The roots of the equation are

$$s_{1,2} = \pm\beta \qquad s_{3,4} = \pm i\beta \tag{23}$$

Then the solution becomes

$$w(x) = C_1.e^{\beta x} + C_2.e^{-\beta x} + C_3.e^{i\beta x} + C_4.e^{-i\beta x} \tag{24}$$

Where C_1, C_2, C_3 and C_4 are constants which can also be express as

$$w(x) = C_1.cos(\beta x) + C_2.sin(\beta x) + C_3.cosh(\beta x) + C_4.sinh(\beta x) \tag{25}$$

Therefore, the natural frequency of the beam is computed by

$$\omega = \frac{\beta^2}{c^2} = \beta^2\sqrt{\frac{EI}{\rho A}} = (\beta l)^2\sqrt{\frac{EI}{\rho A l^4}} \tag{26}$$

The function $w(x)$ is known as the normal mode or characteristic function of the beam and ω is called the natural frequency of vibration. For any beam, there will be an infinite number of normal modes with one natural frequency associated with each normal mode. The unknown constants C_1 to C_2 in Eq. (24) can be determined from the boundary conditions of the beam as indicated:

At x=0, $w(0) = 0$ and $\left.\dfrac{\partial w}{\partial x}\right|_{x=0} = 0$ (27)

At x=l, $\left.\dfrac{\partial^2 w}{\partial x^2}\right|_{x=l} = 0$ and $\left.\dfrac{\partial^3 w}{\partial x^3}\right|_{x=l} = 0$ (28)

The analytical and numerical values of first four natural frequency of a fix-free beam have been calculated and reported in Table 1. Beam length L is 1m, cross-sectional width is b=0.01m and cross-sectional-height is h=0.001m, Also the density of beam is $\rho = 7850 \dfrac{kg}{m^3}$, the young modulus is $E = 2.1\times10^{11} \dfrac{N}{m^2}$, where $I = \dfrac{bh^3}{12}$ (29)

Furthermore, the numerical values of mode shape frequencies have been calculated to compare with the exact values by software.

3. Results and Discussion

Table 1 shows the values of numerical and exact natural frequencies of the considered cantilever beam under fix-free boundary condition.

Table 1. Natural frequencies of a fix-free beam

Mode	Exact Natural Frequency (Hz)	Numerical Natural Frequency (Hz)
1	0.835517	0,83535
2	5.236093	5,23521
3	14.66121	14,662
4	28.73013	28,7516

We completed analytical and numerical method to analysis for a cantilever beam. After that, we wanted to use experimental method. An experimental device was needed for modal analysis. The commercial version of such setup is quite expensive. Therefore, we decided to produce an experimental setup. Now, we are giving information about our device.

The name of our device is AYBU Vibration Generator Version 1 as shown in Figure 3. The purpose of this device is to produce vibration. This vibration is used for modal analysis. This device supplies observation of mode shapes of materials. If a vibration is given to a material, this material shows individual motion in specific frequencies. The mode shape helps to understand acting of material against vibration.

Design of AYBU Vibration Generator Version 1 is different as its working principle from the other vibration generators. The working principle is based on centrifugal force. The device consists of four main parts. These are eccentric load, DC motor, rod and spring. Centrifugal force is produced with eccentric load by a dc motor. This motor is transmitted to the plate. A rod is attached to the plate. This rod supplies vibration and then it transmit to a material with a spring. After that, we can observe mode shape of a material. We made two different designs. Firstly, we used a dc motor and an eccentric load. It is AYBU Vibration Generator Version 0 in drawing 1.

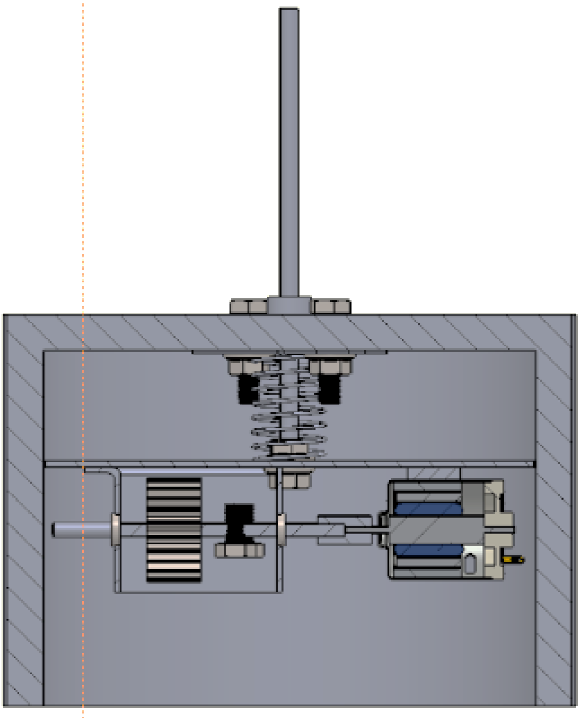

Figure 3. Vibration Generator AYBU version 1

We used our produced vibration generator to excite a cantilever beam which is shown in Figure 4. The setup was produced and tested.

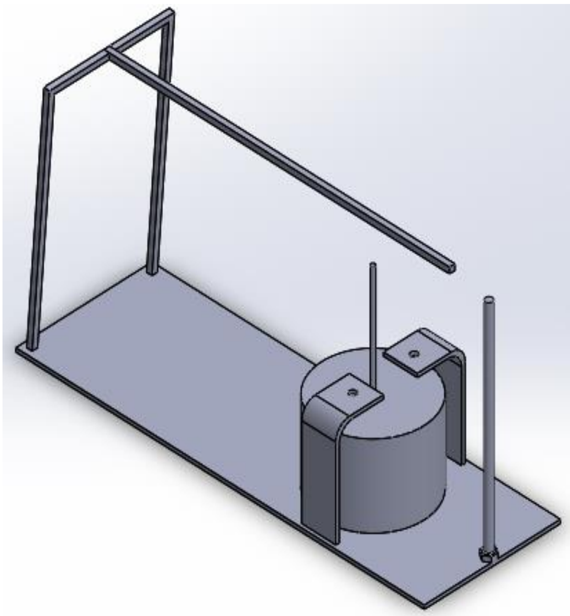

Figure 4. The general setup for modal analysis test-bench of a cantilever beam

4. Conclusions

A group of undergraduate students learned the basic of modal analysis theory by performing analytical, numerical and experimental studies for one year, as their graduation project. The built their own modal analysis test-bench. The most important part was to build a vibration shaker. Normally the commercial version of vibration shakers is very expensive, then it was beyond the afford to buy and use it. So, the students designed and manufactured their own vibration generator. It was good in practices. The new groups of students will continue this project to modify the initial design and improve it.

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