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Three group classification problem approach based on fuzzy goal programming

Bulanık hedef programlama tabanlı üç gruplu sınıflandırma problemi yaklaşımı

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Three Group Classification Problem Approach Based on Fuzzy Goal Programming

Highlights

- *A new models proposed has been made for the classification problem.*
- *The models created are based on fuzzy logic and mathematical programming.*
- *To examine performance of the proposed models, comparison was made with the Fisher's Linear Discriminant Function and some mathematical programming approaches in the literature.*

Graphical Abstract

In this study, two-step method based on fuzzy linear programming was developed to solve the three-group classification problems. In the first step of the proposed method, the classification score of each unit as well as misclassified observations are identified. In the second step, incorrectly classified units are assigned to the correct group, while the status of the correctly classified units is maintained.

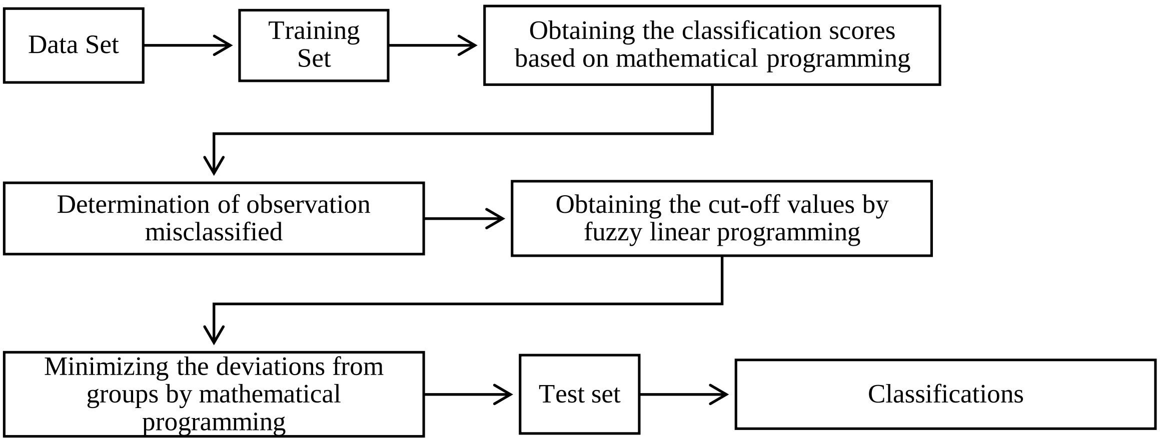

Figure. Flow chart of proposed method

Aim

In this study, a new fuzzy logic and mathematical programming based model was proposed to solve three-group classification problem.

Design & Methodology

The proposed method combines the Fuzzy logic and mathematical programming method.

Originality

By considering the literature,, it is noteworthy that the number of methods based on fuzzy logic and mathematical programming is insufficient.

Findings

The models proposed ultimately displayed better performance than other models in terms of correct classification rate. Fuzzy number type for which correct classification rate was the highest was triangle fuzzy number.

Conclusion

With the proposed method, it is possible to achieve high correct classification success in the three group classification problems.

Declaration of Ethical Standards

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

Three Group Classification Problem Approach Based on Fuzzy Goal Programming

Araştırma Makalesi / Research Article

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ABSTRACT

In this study, a new fuzzy logic and mathematical programming based model was proposed to solve three-group classification problem. Determination of cut-off value, which corresponds to discrimination axis in classification problems, has importance. Status of the cut-off value such as asymmetric triangle fuzzy number, trapezoid fuzzy number and gauss fuzzy number was examined. The proposed approach displayed better performance when compared to Fisher's Linear Discriminant Function and some mathematical programming-based models by using three group data sets used frequently in the literature.

Keywords: Three group classification problem, fuzzy logic, mathematical programming, goal programming.

Bulanık Hedef Programlama Tabanlı Üç Gruplu Sınıflandırma Problemi Yaklaşımı

ÖZ

Bu çalışmada, üç gruplu sınıflandırma probleminin çözümü için bulanık mantık ve matematiksel programlamaya dayalı yeni bir model önerilmiştir. Sınıflandırma problemlerinde ayırma eksenine karşılık gelen kesme değerinin belirlenmesi önem arz etmektedir. Kesme değerinin; asimetrik üçgen bulanık sayı, yamuk bulanık sayı ve gauss bulanık sayı olması durumları incelenmiştir. Önerilen yaklaşım, literatürde sıkça kullanılan 3 gruplu veri setleri kullanılarak Fisher'in Doğrusal Diskriminant Fonksiyonu ve bazı matematiksel programlama yöntemleri ile karşılaştırıldığında daha iyi performans göstermiştir.

Anahtar Kelimeler: Üç gruplu sınıflandırma problemi, bulanık mantık, matematiksel programlama, hedef programlama.

1. INTRODUCTION

One of the most important problems in data analysis is classification problem that includes designation of observations to one of the predetermined groups by basing on characteristics in relevant variables set. Classification problems are frequently encountered on social sciences, finance, marketing, medicine, science and engineering areas. Parametric and nonparametric approaches can be applied to solve a classification problem. The first frequently used parametric method is linear discriminant function (FLDF) proposed by Fisher (1936) [1], the second one is logistic regression analysis and the third one is quadratic discriminant function proposed by Smith (1947) [2]. In classification studies, when parametric test assumptions were not provided, many linear programming-based mathematical programming (MP) model was developed. Linear Programming-based classification model was proposed by Freed and Glover [3] in 1980's, MSD (Minimum sum of deviations model which was developed by Stam and Ragsdale [4] in the early 1990's was based and many MP-based classification models were developed. The purpose of these models is to determine cut-off value allowing discrimination between the groups.

In the study, Goal Programming Problem (GPP) is discussed with fuzzy targets. In addition to theoretical interest, decision-making in a fuzzy environment also attracts attention in application due to the fact that both targets and their significance levels are not exactly stated. The purpose of this study is to develop a new model for three group classification problem when right constant in linear programming model is a fuzzy number. Developed model is based on fuzzy goal programming having different membership functions. In applications, there is no limitation for membership function type. It is based on decision maker's demand and experience. The model was compared with FLDF and examined multi-group models via real data set.

In second section of the study, Fisher's Linear Discriminant Function and some multi-group mathematical programming-based classification models are discussed and proposed model, fuzzy logic and fuzzy numbers shall be introduced briefly in third section. In fourth section, proposed classification models and classification of models are discussed and in the last section there is conclusions and discussion part.

2. LITERATURE REVIEW

The Linear Discriminant Analysis (LDA) is the most important of linear classification techniques that are

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commonly used in classification. Testing whether there is important difference between averages of two or many group variables that were determined before the analysis, determining contribution of each variable to difference between the groups, and determining combination of predictor variables maximizing discrimination between groups in proportion to in-group change can be counted among the purposes of linear discriminant analysis.

The Linear Discriminant Analysis was suggested by Fisher (1936) [1]. Function achieves this by reflecting variable points on vector which separates variance between its classes at maximum level and minimizes in class variances. Statistical linear discriminant function proposed for two groups is commonly used method. Supposing there is p dimensional random vector corresponding to measures to be taken over $\underline{X}'=(X_1, X_2,\ldots,X_p)$ observation. Observation value of $\underline{X}$ shall vary by groups. $Y=\underline{l}'_{px1}\underline{X}_{px1}$ linear combination means Fisher's Linear Discriminant Function (FLDF).

When hypothesis of statistical techniques were not provided, many mathematical programming models were developed to solve classification problems. Application studies in classification problems occurred firstly in the later 1960's and in the early 1970's ((Rosen, 1965) [5], (Mangasarian, 1965) [6], (Smith, 1968) [7] and (Grinold, 1972) [8]). The interest for mathematical programming approaches to develop classification model increased with classification model which was proposed by Freed and Glover (1981) [3] and which basis on minimization of deviations total (MDS) for two-group classification problem. In addition to that d_i: represents deviation of i unit from discrimination function, if observation is classified correctly $d_i=0$, but if it is misclassified it becomes $d_i> 0$. In case two groups are separated linearly, minimum of deviations total is zero. Basic linear programming model is given as follows to create MSD discriminant function:

$$\min \sum_{i \in G_k} d_i$$

subject to:

$$\sum_{j=1}^{p} \alpha_j Z_{ij} \leq b + d_i \;\; i \in G_1$$

$$\sum_{j=1}^{p} \alpha_j Z_{ij} > b - d_i \;\; i \in G_2 \qquad (1)$$

where $Z_{ij} \geq 0$, α_j (j=1,2,…,p) and b are unrestricted variables and $d_i \geq 0$ (i=1,2,…,n). To solve model, α_j and b models and classification score of any model are obtained. When observations classification score is S_j, $S_j \geq b$ is classified to G_1, otherwise it is classified to G_2.

Mathematical programming-based models were proposed by many authors such as Bajgier and Hill (1982) [9], Lam and Moy (1997) [10], Glen (1999) [11]. Stam and Ragsdale (1992) [4] proposed a two-stage method to find discrimination function. In this method, observations that may coincide, that is to say, the observations that are difficult to be classified are defined in the first phase and these observations are reexamined in the second phase. Lam et al. (1996) [12] divided two-group classification operation into two phases. In the first stage, the classification function of the units is approximated to the average score of the group using the objective function, which minimizes the sum of deviations from the group average classification scores. From this step the cut-off values are obtained. In the second stage, the classification scores of the units are obtained. Bal et al. (2006) [14] proposed a linear programming model for two-group classification problems based on model of Lam et al. Approach in model that they proposed is based on minimizing total of deviations between classification score of all observations and group median scores. They solved classification problem in a single phase by transforming two-phase model into priority goal programming model in Bal et al. (2006) [14], Bal et al. (2006) [13] studies. Classification studies in the literature are generally based on two-group studies. Nevertheless, number of studies proposing multi-group mathematical programming approach is limited. According to Freed and Glover (1981) [3], the way for expanding two-group status into multi-group status is to use two-group combinations. However, this situation does not always give exact information about correct classification rate. For multi-group classification problems, a truer way is to use general single-function classification model (GSFC) proposed by Gehrlein (1986) [15] or general multiple function model (GMFC) proposed by Gehrlein (1986) [15] and Choo and Wedley (1985) [16]. When there are many outliers in data, GSFC model cannot guarantee that observations will be classified correctly. Gehrlein (1986) [15] and Choo and Wedley (1985) [16] proposed a general multi-function classification model (GMFC) to solve this problem. GMFC is defined in Eq. (2):

$$\min \sum_{i=1}^{n} y_i$$

subject to:

$$\alpha_{r0} + \sum_{j=1}^{p} \alpha_{rj} Z_{ij} - \alpha_{t0} - \sum_{j=1}^{p} \alpha_{tj} Z_{ij} + My_i \geq e, \forall\, i \in G_r, r = 1,2,\ldots,k, i = 1,2,\ldots,n, r \neq t \qquad (2)$$

where Z_{ij} means observation value for j. value of i. observation, k: group number; M:a big positive constant, p: variable number, n:total observation number, α_{rj}:weight assigned to Z_{ij} for i. observation in G_r group, α_{r0}:slippage constant for G_r group (threshold value for G_r group), α_{rj}: unrestricted variable, y_i: a binary variable showing whether i. observation was misclassified or not.

Lam and Moy (1996) [17] expanded two-group classification model developed by Lam et al. (1996) [18] to a multi-group classification model. In model that they called as MLM (The model developed by Lam and Moy), k group number is u=1,…,k-1, v=u+1,…,k, and it is given in Eq. (3) for each (u,v) pairs.

$$\min \sum_{i\in G_u, i\in G_v}^{n} d_i$$

subject to:

$$\sum_{j=1}^{p} w_j\,(X_{ij}-\bar{x}_j^u) + d_i \geq 0, j \in G_u$$
$$\sum_{j=1}^{p} w_j\,(X_{ij}-\bar{x}_j^v) - d_i \leq 0, j \in G_v$$
$$\sum_{j=1}^{p} w_j\,(\bar{x}_j^u - \bar{x}_j^v) \geq 1 \quad (3)$$

Total observation number is $n=n_1+n_2+\ldots+n_k$ for the observation number i.; n_r: G_r group and average regarding j. variable is defined as $\bar{x}_i^r = \frac{1}{n_r}\sum_{i\in G_r} X_{ij}$, (r=1,2,…,k). In the model, w_j (j=1,…,p) is unrestricted variable and is $d_i \geq 0$ for $i \in G_u$ ve $i \in G_v$. S_i classification scores of observations in are G_u and G_v found via w_j obtained for each (u,v) group pair after solving Eq. (4). In the second phase, c_{uv} values to be benefited in discrimination of groups are obtained with solving of Eq. (4) model. c_{uv} values are unrestricted variables and $d_{iuv} \geq 0$.

$$\min \sum_{i\in G_u}\sum_{u=1}^{k-1}\sum_{v=u+1}^{k} d_{iuv} + \sum_{i\in G_v}\sum_{u=1}^{k-1}\sum_{v=u+1}^{k} d_{iuv}$$

subject to:

$$S_{iuv} + d_{iuv} \geq c_{uv}, u = 1,\ldots,k-1, v=u+1,\ldots,k, i\in G_u$$
$$S_{iuv} - d_{iuv} \leq c_{uv}, u = 1,\ldots,k-1, v=u+1,\ldots,k, i\in G_v \quad (4)$$

Örkcü and Bal (2011) [19] combined two different models by benefiting from artificial neural network for multi-group classification problem and they created a hybrid model. Youssef et al. (2011) [20] developed a new two-phase technique which bases on linear programming model to solve three-group classification problems. Smaoui and Aouni (2017) [21] proposed a new approach for the classification problems for which cut-off value basing on Goal Programming and corresponding to discrimination axis is fuzzy. They discussed fuzziness with different membership functions. Doğan et al. (2019) [22] developed a method basing on regression analysis and mathematical programming for solving multi-group classification problems.

3. PROPOSED CLASSIFICATION MODEL

Many of real life problems are included in an environment in which objective function, some coefficients of constraints and right constants cannot be defined completely, in other words, in which they are indefinite. Fuzziness concept was firstly included in article "Fuzzy Sets" by A. Zadeh (1965) [23]. Many problems encountered in daily life can be defined with Linear Programming (LP). However, in most instances it may not be possible to exactly determine constraints or objective functions or both in Linear Programming. Fuzzy Linear Programming (FLP) may be applied to examine uncertainty seen in any of LP model parameters. Uncertainty hypothesis should be also provided in addition to valid proportionality, additivity and severance hypothesis for LP problems to evaluate any problem as FLP Problem. To provide uncertainty hypothesis, all or a part of objective function coefficient, right constant and weight coefficients parameters should not be known exactly but membership degrees of these parameters should be known. Right hand side parameters, which show maximum source amount in LP models, may not be defined exactly, that is to say, they may be fuzzy. Source constraints to be created in this case are defined as fuzzy source constraints.

Goal Programming (GP) is a technique that aims minimization of deviations of goals within them instead of making them directly maximum and minimum under determined constraints. There is no uncertainty in determining objective function and creating parameters. For this reason, fuzzy logic is suitable for the Goal Programming and it was benefited frequently in the literature. Fuzzy Goal Programming (FGP) created with application of fuzzy logic theory into GP is a technique that is used when there are indefinite goals and purposes. The first studies with which GP was processed with fuzzy number theory are studies of Narasimhan (1980) [24] and Hannan (1981) [25]. Fuzzy Goal Programming started to be used in classification studies in recent years. Li et al. (2006) [26] proposed flexible constrained fuzzy linear programming classification method. Hosseinzadeh et al. (2007) [27], Hosseinzadeh and Mansouri (2008) [28] and Ben Youssef and Rebai (2008) [29] developed classification model by taking decision making parameters as fuzzy parameters. Asymmetric triangle fuzzy number defined in real number line is defined with $\tilde{c}=(c_1, c, c_2)$. h ($h \in [0,1]$) level of a fuzzy set is defined as ordered set of all members whose membership value is h or higher. h level of $\tilde{c}$ fuzzy number is defined with $\tilde{c} = [c-(1-h)\eta_1, c+(1-h)\eta_2]$ interval. Here, η_1= $c-c_1$ and $\eta_2=c_2-c$, c_1 were determined as lower limit of $\tilde{c}$ fuzzy number and c_2 was determined as upper limit of $\tilde{c}$ fuzzy number and c center.

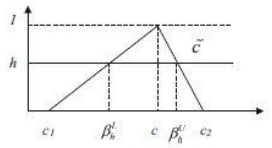
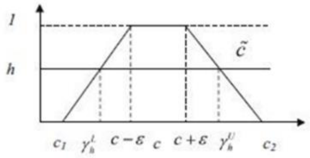
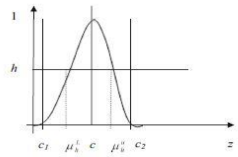
If fuzzy number is given by $\tilde{c}=(c_1, c-\varepsilon, c+\varepsilon, c_2)$ membership function, it becomes a trapezoid fuzzy number. h level of $\tilde{c}$ fuzzy number is defined as $\tilde{c}=[c_1+h\alpha, c_2-h\beta]$. h level of $\tilde{c}$ fuzzy number is defined with $\tilde{c}=[h(c-\varepsilon)+(1-h)c_1, h(c+\varepsilon)+(1-h)c_2]$ interval as $\alpha=c-\varepsilon-c_1$ and $\beta=c_2-c-\varepsilon$.

z model value and left and right fuzziness is given equally to σ with a Gauss fuzzy number $\tilde{c}=(z, \sigma, \sigma)$ because Gauss membership is symmetric around center value c. Gauss fuzzy cut-off value $\tilde{c}$ is defined by lower limit c_1 and center c and upper limit c_2. Alternatively, h-level of $\tilde{c}$ is calculated as interval as follows:

$$\tilde{c} = \left[c-\sigma\sqrt{-Log\,h}, c+\sigma\sqrt{-Log\,h}\right] = [\mu_h^L, \mu_h^U]$$

Besides, 68% of values are included within 1 standard deviation, 95% in 2 standard deviations and 99.7% in 3 standard deviations in normal distribution. By basing on this opinion, fuzzy number $\tilde{c}$ belongs to $[\mu-3\sigma, \mu+3\sigma]$ interval with the rate of 99.7%. And when it is considered that average μ is equal to $\frac{c_1+c_2}{2}$, then $\sigma = \frac{c_2-c_1}{6}$ value of the standard deviation may be removed. This value is used in Gauss fuzzy number approach.

Table 1. Types of Fuzzy Numbers

Fuzzy Number Type	**Asymmetric Triangle**	**Trapezoid**	**Gauss**
Membership Functions	$\begin{cases} 0, & z < c_1 \\ \frac{z-c_1}{c-c_1}, & c_1 \le z \le c \\ \frac{c_2-z}{c_2-c}, & c \le z \le c_2 \\ 0, & z > c_2 \end{cases}$	$\begin{cases} \frac{z-c_1}{c-\varepsilon-c_1}, & c_1 \le z \le c-\varepsilon \\ 1, & c-\varepsilon \le z \le c+\varepsilon \\ \frac{c_2-z}{c_2-c-\varepsilon}, & c+\varepsilon \le z \le c_2 \\ 0, & \text{otherwise} \end{cases}$	$exp\left\{-\frac{1}{\sigma}(z-c)^2\right\}$
Figure of fuzzy numbers			

The purpose of this study is to propose models for three-group classification problems having fuzzy cut-off value. Developed models are based on Fuzzy Goal Programming having different membership functions. A new three-group classification model was proposed by combining three-group classification model that was proposed by Youssef et al. (2011) [20] with model proposed by Smaoui and Aouni (2017) [21] for two-group classification problem. A single model can be created with combination of more than one classification model.

The proposed classification model is based on the Fuzzy Goal Programming. In this model, the cut-off value $\tilde{c}$ and $\tilde{s}$ were firstly considered as asymmetric triangle fuzzy number, secondly as trapezoid fuzzy number and thirdly as Gauss fuzzy number. Membership functions of these cut-off values can be submitted with $\mu_{\tilde{c}}$ and $\mu_{\tilde{s}}$ fuzzy sets. The aim is to determine cut-off values $\tilde{c}$ and $\tilde{s}$ accepted as fuzzy number by using different membership functions so as to minimize observation number that was misclassified.

In the model, assume that k is n_k observation belonging to k=1,2,3 group with each observation defined by *p* variable. Assume that j is X_{ij} which is value of variable j=1,2,…,p for i. observation of k group. It was assumed that second group was among the first and third group, in other words, that coincidence was between G_1 *and* G_2 groups and G_2 *and* G_3 groups. The model consists of two interrelated phases. In the first phase, it was aimed at defining observations that were misclassified, in the second phase it was aimed at assigning the observations that were misclassified to correct groups. c_1 and c_2 parameters in the models are reference points of $\tilde{c}$ fuzzy number and s_1 and s_2 parameters are reference points of $\tilde{s}$ fuzzy number. h was defined as height of fuzzy numbers. The first phase is given in Eq. (5):

$$\min \sum_{i \in G_k} d_{1i}^- + d_{2i}^+ + e_{2i}^- + d_{3i}^+$$

s.t.

$$\sum_{j=1}^{p} w_j x_{ij} - d_{1i}^+ + d_{1i}^- = c_2 , \quad i \in G_1$$

$$\sum_{j=1}^{p} w_j x_{ij} - d_{2i}^+ + d_{2i}^- = c_1 , \quad i \in G_2$$

$$\sum_{j=1}^{p} w_j x_{ij} - e_{2i}^+ + e_{2i}^- = s_2 , \quad i \in G_2$$

$$\sum_{j=1}^{p} w_j x_{ij} - d_{3i}^+ + d_{3i}^- = s_1 , \quad i \in G_3$$

$$c_2 - c_1 \ge 1$$

$$s_2 - s_1 \ge 1$$

$$c_1 - s_2 > 1$$

$$\sum_{j=1}^{p} w_j = 1$$

$$d_{ki}, e_{ki} \ge 0$$

$$w_j, c_1, c_2, s_1, s_2 \text{ unrestricted in sign} \qquad (5)$$

$c_1^*, c_2^*, s_1^*, s_2^*$ and w_j^* which are optimal values of w_j are obtained respectively for c_1, c_2, s_1, s_2 and j variable with solution of the first phase. E_1: the set of observations classified as Group 1 (G_1), E_2= the set of observations classified as Group 2 (G_2), E_3= the set of observations classified as Group 3 (G_3), E_0= the set of observations in the first classification gap (Group 1 and Group 2), and E_0'= the set of observations in the second classification gap (Group 2 and Group 3). Accordingly, the observations can be classified to sub sets as follows:

$$E_1 = \{i \in G / \sum_{j=1}^{p} w_j^* x_{ij} \geq c_2^*$$

$$E_2 = \{i \in G / s_2^* \leq \sum_{j=1}^{p} w_j^* x_{ij} \leq c_1^*$$

$$E_3 = \{i \in G / \sum_{j=1}^{p} w_j^* x_{ij} \leq s_1^*$$

$$E_0 = \{i \in G / c_1^* < \sum_{j=1}^{p} w_j^* x_{ij} < c_2^*$$

$$E_0' = \{i \in G / s_1^* < \sum_{j=1}^{p} w_j^* x_{ij} < s_2^*$$

Since some observations were misclassified, other sub sets are defined as follows:

$C_k=G_k \cap E_k$, k=1,2,3, the set of observations correctly classified into G_k

$D_1=G_1 \cap (E_2 \cup E_0)$: the set of observations from G_1 that are misclassified or in the first classification gap

$D_2=G_2 \cap (E_1 \cup E_0)$: the set of observations from G_2 that are misclassified or in the first classification gap

$D_2'=G_2 \cap (E_3 \cup E_0')$: the set of observations from G_2 that are misclassified or in the second classification gap

$D_3'==G_3 \cap (E_2 \cup E_0')$: the set of observations from G_3 that are misclassified or in the second classification gap.

The second phase is used to determine w_j^{**}(j=1,2, …,p), $c^*$and s^* constant terms. Here, the observation wanted to be classified is classified to any of three groups. The aim of second phase is bilateral. Firstly, it is aimed at creating a new function separating observations in D_1 and D_2 as far as possible while protecting correct classification of the observations in C_1 and C_2. Secondly, it is aimed at using new created function to separate observations in D_2 and D_3 while protecting correct classification of the observations in C_2 and C_3.

Models are given in Table 2 according to different fuzzy number types belonging to second phase of the model. In the models, w_j, c, s unrestricted variables, $d_{ki}, e_{ki} \geq 0$. w_j^{**}, $c^*$and s^* are optimal values obtained by the second model of respectively w, c and s to use a classification function that is a discrimination function obtained from the second phase.

Z_j new observation classification rule is as follows:

$\sum_{j=1}^{p} w_j^{**} Z_j \geq c^*$: the observation is classified into G_1.

$s < \sum_{j=1}^{p} w_j^{**} Z_j < c^*$: the observation is classified into G_2.

$\sum_{j=1}^{p} w_j^{**} Z_j \leq s^*$: the observation is classified into G_3.

4. APPLICATION

In the application phase, two different data sets, frequently used in the literature, were used. Data sets were taken from internet data base of University of California-Irvine (UCI), and they include real life data sets such as IRIS and Wine (Wine Identification) data set.

FISHER's IRIS Data Set: In this data set, there are 3 different decorative flowers such as setosa, versicolor and virginica. Data set consists of 150 flowers in total, in each of three groups there are 50 flowers. 4 different variables were observed from each flower: sepal length, sepal width, petal length and petal width.

Wine Data Set: This is data set obtained from 13 different chemical analyses performed in three wine types created from grape wines produced in Italia and grown by different people in a definite region. The data set consists of 178 observations in total, there are 59 observations in the first group, 71 observations in the second group and 48 observations in the third group.

FLDF: Fisher's Linear Discriminant Function

GMFC: Gehrlein's multi-function classification model

MLM: The model developed by Lam and Moy

Results were compared by separating 70% of the observations regarding data set as training and 30% as test sample. Comparison of correct classification rates are given in Table 3 according to the Fisher's Linear Discriminant Function, some mathematical programming approaches in the literature and fuzzy number height of the proposed method. For IRIS data set, correct classification rate was obtained at the rate of 0.94 in Fisher's Linear Discriminant Function and of 0.95 in Gehrlein's model and in the model developed by Lam and Moy. In the proposed model, when triangle fuzzy number was used the highest correct classification rate was obtained, and when height (h) of the fuzzy number was 0.1, 0.2, 0.3, 0.4, 0.5, the rate was 0.953. When trapezoid fuzzy number was used and the height (h) of the fuzzy number was 0.3, 0.4, 0.9, the best correct classification rate which was 0.947 was obtained. When Gauss fuzzy number was used and the height (h) of the fuzzy number was0.2, 0.3, 0.4, 0.5, the highest correct classification rate which was 0.953 was obtained

Table 2. Models of the Second Stage of the Proposed Method

Asymmetric Triangle Fuzzy Number Model	**Trapezoid Fuzzy Number Model**	**Gauss Fuzzy Number Model**
$\min \sum_{i \in G_k} d_{1i}^- + d_{2i}^+ + e_{2i}^- + d_{3i}^+$	$\min \sum_{i \in G_k} d_{1i}^- + d_{2i}^+ + e_{2i}^- + d_{3i}^+$	$\min \sum_{i \in G_k} d_{1i}^- + d_{2i}^+ + e_{2i}^- + d_{3i}^+$
s.t.	s.t.	s.t.
$\sum_{j=1}^{p} w_j x_{ij} \geq c_2^*\ , i \in\ C_1$	$\sum_{j=1}^{p} w_j x_{ij} \geq c_2^*\ , i \in\ C_1$	$\sum_{j=1}^{p} w_j x_{ij} \geq c_2^*\ , i \in\ C_1$
$\sum_{j=1}^{p} w_j x_{ij} \leq c_1^*\ , i \in\ C_2$	$\sum_{j=1}^{p} w_j x_{ij} \leq c_1^*\ , i \in\ C_2$	$\sum_{j=1}^{p} w_j x_{ij} \leq c_1^*\ , i \in\ C_2$
$\sum_{j=1}^{p} w_j x_{ij} \geq s_2^*\ , i \in\ C_2$	$\sum_{j=1}^{p} w_j x_{ij} \geq s_2^*\ , i \in\ C_2$	$\sum_{j=1}^{p} w_j x_{ij} \geq s_2^*\ , i \in\ C_2$
$\sum_{j=1}^{p} w_j x_{ij} \leq s_1^*, i \in\ C_3$	$\sum_{j=1}^{p} w_j x_{ij} \leq s_1^*, i \in\ C_3$	$\sum_{j=1}^{p} w_j x_{ij} \leq s_1^*, i \in\ C_3$
$\sum_{j=1}^{p} w_j x_{ij} - d_{1i}^+ + d_{1i}^- = c + (1-h)\eta_2, i \in D_1$	$s - c < 0$	$s - c < 0$
$\sum_{j=1}^{p} w_j x_{ij} - d_{2i}^+ + d_{2i}^- = c - (1-h)\eta_1, i \in D_2$	$c - c_1^* > 0$	$c - c_1^* > 0$
$\sum_{j=1}^{p} w_j x_{ij} - e_{2i}^+ + e_{2i}^- = s + (1-h)\theta_2, i \in D_2'$	$c_2^* - c > 0$	$c_2^* - c > 0$
$\sum_{j=1}^{p} w_j x_{ij} - d_{3i}^+ + d_{3i}^- = s - (1-h)\theta_1, i \in D_3'$	$s - s_1^* > 0$	$s - s_1^* > 0$
$c - c_1^* = \eta_1$	$s_2^* - s > 0$	$s_2^* - s > 0$
$c_2^* - c = \eta_2$	$\sum_{j=1}^{p} w_j x_{ij} - d_{1i}^+ + d_{1i}^- = h(c+\varepsilon) + (1-h)c_2^*, i \in D_1$	$\sum_{j=1}^{p} w_j x_{ij} - d_{1i}^+ + d_{1i}^- = c + \sigma_1\sqrt{-\log h},\ i \in D_1$
$s - s_1^* = \theta_1$	$\sum_{j=1}^{p} w_j x_{ij} - d_{2i}^+ + d_{2i}^- = h(c-\varepsilon) + (1-h)c_1^*, i \in D_2$	$\sum_{j=1}^{p} w_j x_{ij} - d_{2i}^+ + d_{2i}^- = c - \sigma_1\sqrt{-\log h},\ i \in D_2$
$s_2^* - s = \theta_2$	$\sum_{j=1}^{p} w_j x_{ij} - e_{2i}^+ + e_{2i}^- = h(s+\varepsilon) + (1-h)s_2^*, i \in D_2'$	$\sum_{j=1}^{p} w_j x_{ij} - e_{2i}^+ + e_{2i}^- = s + \sigma_2\sqrt{-\log h},\ i \in D_2'$
$s - c < 0$	$\sum_{j=1}^{p} w_j x_{ij} - d_{3i}^+ + d_{3i}^- = h(s-\varepsilon) + (1-h)s_1^*, i \in D_3'$	$\sum_{j=1}^{p} w_j x_{ij} - d_{3i}^+ + d_{3i}^- = s - \sigma_2\sqrt{-\log h},\ i \in D_3'$

Table 3. The Correctly Classification Rate of The Different Models

Other Models	**Data Set**					
	IRIS			**WINE**		
FLDF	0.94			0.95		
GMFC	0.95			0.95		
MLM	0.95			0.935		
Proposed Models for several h values	*Asymmetric Triangle*	*Trapezoid*	*Gauss*	*Asymmetric Triangle*	*Trapezoid*	*Gauss*
	0.953	0.94	0.947	0.961	0.961	0.949
	0.953	0.94	0.953	0.961	0.961	0.955
	0.953	0.947	0.953	0.961	0.961	0.955
	0.953	0.947	0.953	0.955	0.955	0.955
	0.953	0.94	0.953	0.949	0.955	0.955
	0.947	0.933	0.947	0.949	0.955	0.955
	0.94	0.94	0.947	0.955	0.949	0.955
	0.94	0.94	0.947	0.955	0.949	0.955
	0.94	0.947	0.947	0.955	0.949	0.955

For WINE data set, correct classification rate was obtained at the rate of 0.95 in the Fisher's Linear Discriminant Function and Gehrlein's classification model and of 0.935 in the model developed by Lam and Moy. In the proposed model, when Triangle and Trapezoid fuzzy number was used the highest correct classification rate which was 0.961 was obtained while fuzzy number height (h) was 0.1, 0.2, 0.3. When Gauss fuzzy number was used, the highest correct classification rate was obtained as 0.955 while the height (h) of fuzzy number was 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9.

5. CONCLUSION and DISCUSSION

In this study, a hybrid model is developed based on studies of previous researchers and fuzzy goal programming is used to in solving three-group classification problems. To examine performance of the proposed model, comparison was made with the Fisher's Linear Discriminant Function and some mathematical programming approaches in the literature. The models proposed ultimately displayed better performance than other models in terms of correct classification rate (Table 3). Fuzzy number type for which correct classification rate was the highest was triangle fuzzy number. In this article, classification performances of four different models were compared by basing on two real life data. The same comparison may be provided by making a simulation study.

DECLARATION OF ETHICAL STANDARDS

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission..

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