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A finite element procedure for sliding contact problems involving heterogeneous coefficient of friction

Heterojen sürtünme katsayılı kayma temas problemleri için bir sonlu elemanlar çözümü

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A Finite Element Procedure for Sliding Contact Problems Involving Heterogeneous Coefficient of Friction

Araştırma Makalesi / Research Article

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ABSTRACT

A new finite element procedure is developed for the analysis of sliding contact problems involving spatially varying coefficient of friction. The problem is implemented using APDL (ANSYS Parametric Design Language) considering the Augmented Lagrange method as the contact solver. Upon discretization of the contact interface into multiple contact pairs, a sequence of steps is followed to evaluate the resultant friction force required for the sliding contact. As a case study, heterogeneous-friction contact problem between an orthotropic laterally graded half-plane and a rigid flat stamp is investigated under plane strain assumption. The proposed iterative procedure is proved reliable by comparing the results to those generated by a SIE (Singular Integral Equation) approach for isotropic laterally graded half-planes. Extra results are presented to reveal the effects of problem parameters on the contact stresses and the friction force. The paper outlines a convenient numerical solution for an advance sliding contact problem, and the results can be used in validation purposes of experimental and analytical studies.

Keywords: Heterogeneous friction coefficient, sliding frictional contact, laterally graded materials, finite element method.

Heterojen Sürtünme Katsayılı Kayma Temas Problemleri için bir Sonlu Elemanlar Çözümü

ÖZ

Yatay eksende değişkenlik gösteren sürtünme katsayısının var olduğu kayma temas problemleri için yeni bir sonlu elemanlar yöntemi geliştirilmiştir. Problem için "Augmented Lagrange" yöntemi temel temas problemi çözücüsü olarak seçilmiş ve modellemeler APDL (ANSYS Parametrik Tasarım Dili) ortamında yapılmıştır. Temas ara-yüzeyinin birçok temas çiftine bölünmesiyle kaymalı temas için gerekli olan sürtünme kuvveti, geliştirilen yinelemeli bir algoritma ile hesaplanmıştır. Durum incelemesi olarak bir rijit düz zımba ile enine derecelendirilmiş ortotropik yarı-düzlem arasındaki heterojen-sürtünmeli kayma temas problemi düzlem gerinimi varsayımı ile ele alınmıştır. Bu çalışmada ortaya konulan prosedürün güvenilirliği ve geçerliliği, sonuçlarının literatürde var olan (Tekil integral denklemleri kullanılarak izotropik malzemeler için elde edilmiş) sonuçlarla karşılaştırılarak ispatlanmıştır. Yanı sıra bu çalışmada çeşitli problem parametrelerinin temas gerilmeleri ve sürtünme kuvveti üzerine olan etkileri gösterilmiştir. Bu çalışma ileri seviye bir temas probleminin çözümü için kolay uygulanabilir yeni bir sayısal yöntem ortaya koymaktadır. Elde edilen sonuçlar analitik ve deneysel çalışmaların yorumlanması ve doğrulanmasında kullanılabilecektir.

Anahtar Kelimeler: Heterojen sürtünme katsayısı, sürtünmeli temas, enine derecelendirilmiş malzemeler, sonlu elemanlar yöntemi.

1. INTRODUCTION

In literature, contact mechanics analyses between mating components have been performed to be able to predict and restrain damages triggered by contact stresses. From this aspect, optimization of problem parameters that provide mitigation of contact stresses becomes essential for the purpose of service life extension. The prominent failure type induced by the frictional contact loadings is the formation of surface crackings, the risks of which can be alleviated by introducing spatial material gradations through the elastic medium [1-2]. The graded structures employed in contacting bodies macroscopically acquire smooth spatial transitions from brittle to ductile materials via special production techniques, such as electron beam physical vapor deposition (EBPVD) and thermal spraying [3-4]. Investigations on the microstructure of the deposited structures reveal that they have anisotropic material characteristics. For instance, the coatings manufactured through the plasma spray technique are observed to be disposed in thin plates possessing direction-dependent material properties [5]. Column-like forms are shown in the microstructure of EBPVD coatings [6]. Therefore, it becomes physically rational to consider a deposited graded structure as an orthotropic graded elastic material. There are a vast amount of

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studies regarding frictional/frictionless contact mechanics problems of orthotropic homogeneous/graded structures. Shi et al. [7] have solved the contact problem between an orthotropic half-plane and a punch of an ellipsoidal profile. A similar problem has been investigated by Swanson [8] employing a point load solution. A contact mechanics model for an orthotropic viscoelastic-half plane has been proposed by Rodriguez et al. [9]. Dong et al. [10] have performed a solution for the frictionless contact problem of an orthotropic homogeneous half-plane loaded by collinear stamps. A study examining the dynamic effects of the frictional sliding contact on an orthotropic homogeneous half-space has been put forward by Zhou et al. [11]. Zhou and Lee [12] have developed closed-form solutions for the contact problems of piezoelectric orthotropic homogeneous half-planes. A SIE (singular integral equation) based analytical solution for the frictional contact problem between an orthotropic homogeneous half-plane and a flat punch has been carried out by Guler [13]. Kucuksucu et al. [14] have outlined a semi-analytical SIE solution on the frictional sliding contact mechanics problem of an orthotropic graded half-plane. Guler et al. [15] have examined the circular punch contact on an orthotropic graded half-plane employing both semi-analytical SIE approach and finite element method. Arslan and Dag [16] have put forward a dual solution for the frictional contact mechanics problem of an orthotropic graded coating loaded through flat and triangular rigid punches. Both finite element method and SIE approach have been employed in that study.

In the aforementioned articles material gradations are introduced through thickness direction. Gradation of material properties in lateral direction are also considered in many studies. Dynamics of laterally graded beams [17], elastic wave propagation in laterally graded waveguides [18], decay of Saint-Venant end effect in laterally graded inhomogeneous solids [19] and frictional sliding contact analysis of laterally graded half-planes [20-23] has been investigated in literature.

Studies related to sliding contact mechanics analyses usually consider constant friction coefficient through contact interfaces. However, the formation of surface crackings due to frictional contact forces inevitably leads to fretting fatigue [24] and; in fretted contact interfaces spatial variation of the friction coefficient has been revealed experimentally [25-26]. Moreover, it has been claimed that change in material constituents through the lateral direction inherently causes spatial variation of the friction coefficient [22]. Hence, interpretations on the influences of friction variation upon contact stresses can be useful before conducting experiments such as fretting fatigue [25] and sliding contact tests [27]. In literature only a few studies consider heterogeneous friction coefficient in contact problems utilizing analytical techniques. Dag [22] has outlined a SIE based study on the contact problem of an isotropic laterally graded material pressed against a rigid flat stamp under plane strain assumption. Exponential spatial variation of the friction coefficient at the contact interface is assumed to prevail in the mentioned study. Ballard [28] has studied a plane contact problem between an isotropic homogeneous elastic half-space and a rigid punch of an arbitrary profile, where the friction coefficient is a step function through a spatial coordinate axis.

Although analytical studies have many potential merits, they are generally toilsome to handle. Hence computational procedures focusing on different contact problems should be developed for the purpose of validation of analytical studies and, to conveniently figure out contact behavior of materials under various contact conditions. In this paper, a new finite element procedure is proposed for the solution of heterogeneous-friction contact problems. The study is conducted utilizing ANSYS Parametric Design Language (APDL) regarding plane strain assumption. The augmented Lagrange algorithm is selected as a contact solver. Upon discretization of the contact interface into multiple contact pairs, friction coefficient of each contact pair is computed using the position of its centroid. The resultant friction force which is required for the sliding contact analysis is evaluated through a successfully converging iterative set of steps. The heterogeneous-friction contact problem of an orthotropic laterally graded half-plane loaded through a rigid flat stamp is selected as the case study, which has not been investigated in any work published so far. Exponential spatial variations for the orthotropic stiffness coefficients and the friction coefficient are introduced through the lateral direction. The procedure is validated referring the comparisons of the results to those computed by a SIE approach for isotropic laterally graded materials [22]. Extra results are provided to reveal the effects of the friction variation, degree of orthotropy and non-homogeneity parameter upon the contact stress curves and the friction force. The procedure presented in this study is shown to be effective in solution of advance contact problems with spatially varying physical properties at contact interface. As a prominent conclusion of the case study, one can infer that the contact stresses can be mitigated remarkably upon increasing the degree of orthotropy.

2. SOLUTION PROCEDURE

The problem geometry is depicted in Fig. 1. A heterogeneous-friction contact problem between an orthotropic laterally graded half-plane and a flat rigid punch is investigated. The friction force *Q* and the contact force *P* are transferred through a rigid flat punch. Orthotropic stiffness coefficients of the elastic medium are stated in the reduced constitutive relations for plane strain assumption:

$$\begin{Bmatrix} \sigma_{xx}(x,y) \\ \sigma_{yy}(x,y) \\ \sigma_{xy}(x,y) \end{Bmatrix} = \begin{bmatrix} c_{11}(y) & c_{12}(y) & 0 \\ c_{12}(y) & c_{22}(y) & 0 \\ 0 & 0 & c_{66}(y) \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}(x,y) \\ \varepsilon_{yy}(x,y) \\ 2\varepsilon_{xy}(x,y) \end{Bmatrix} \quad (1)$$

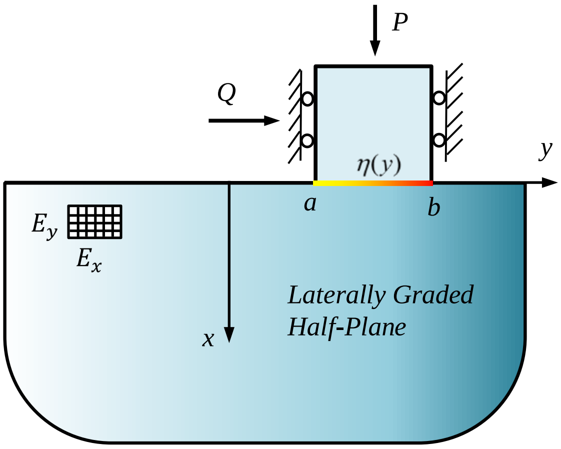

Figure 1. Problem Configuration

where $\varepsilon_{ij}(x,y)$ $(i,j=x,y)$ and $\sigma_{ij}(x,y)$ $(i,j=x,y)$ are the strain and stress components, respectively. $c_{11}(y)$, $c_{22}(y)$, $c_{12}(y)$ and $c_{66}(y)$ are the orthotropic stiffness coefficients, each of which exponentially varies through the lateral y-direction [16]:

$$c_{11}(y)=c_{110}e^{\alpha y}, \qquad c_{22}(y)=c_{220}e^{\alpha y}. \qquad \text{(2a-b)}$$

$$c_{12}(y)=c_{120}e^{\alpha y}, \qquad c_{66}(y)=c_{660}e^{\alpha y}. \qquad \text{(2c-d)}$$

here α represents the non-homogeneity parameter. c_{110}, c_{220}, c_{120} and c_{660} are the orthotropic stiffness constants defined in terms of the engineering parameters at *y=0*:

$$c_{110}=\frac{E_x^2\left(v_{yz}^2 v_{zx}E_x-v_{xz}E_y\right)}{\Delta} \qquad \text{(3a)}$$

$$c_{220}=\frac{v_{xz}E_xE_y^2\left(v_{xz}v_{zx}-1\right)}{\Delta} \qquad \text{(3b)}$$

$$c_{120}=-\frac{v_{xz}E_xE_y\left(v_{yz}v_{zx}E_x+v_{xy}E_y\right)}{\Delta} \qquad \text{(3c)}$$

$$c_{660}=\mu_{xy} \qquad \text{(3d)}$$

$$\Delta=v_{yz}^2v_{zx}E_x^2+v_{xy}^2v_{xz}E_y^2+ +v_{xz}E_xE_y\left(v_{xz}v_{zx}+2v_{xy}v_{yz}v_{zx}-1\right) \qquad \text{(3e)}$$

A spatially varying friction coefficient prevails at the contact interface, which is expressed as follows [22]:

$$\eta(y)=\eta_a\exp\left\{\ln\left(\frac{\eta_b}{\eta_a}\right)\frac{y-a}{b-a}\right\} \qquad a<y<b \qquad (4)$$

where *a* and *b* stand for the locations of the punch edges as seen in Fig. 1 and, $\eta_a=\eta(a)$ and $\eta_b=\eta(b)$.

Solution of the problem is carried out utilizing APDL. The finite element model used can be seen in Fig. 2. Dimensions of the rectangular finite element model are selected in such a way that they have no effect on the stresses in the vicinity of the contact region. A total of 94883 quadrilateral and triangular finite elements are employed in the discretization. Note that a high degree of mesh refinement is arranged in the model in order to capture the elastic gradation better. The variations of the orthotropic stiffness coefficients through the half-plane are imposed by using the homogeneous finite element approach which is integrated into APDL code. In the homogeneous finite element method, the material properties of finite elements are defined at their centroids.

Since heterogeneous friction coefficient prevails between the medium surface and the flat punch surface, the contact region is needed to be discretized into multiple contact pairs for each of which different friction coefficient can be assigned. Illustration of the contact pairs used can be seen in Fig. 3. Equally sized 300 contact

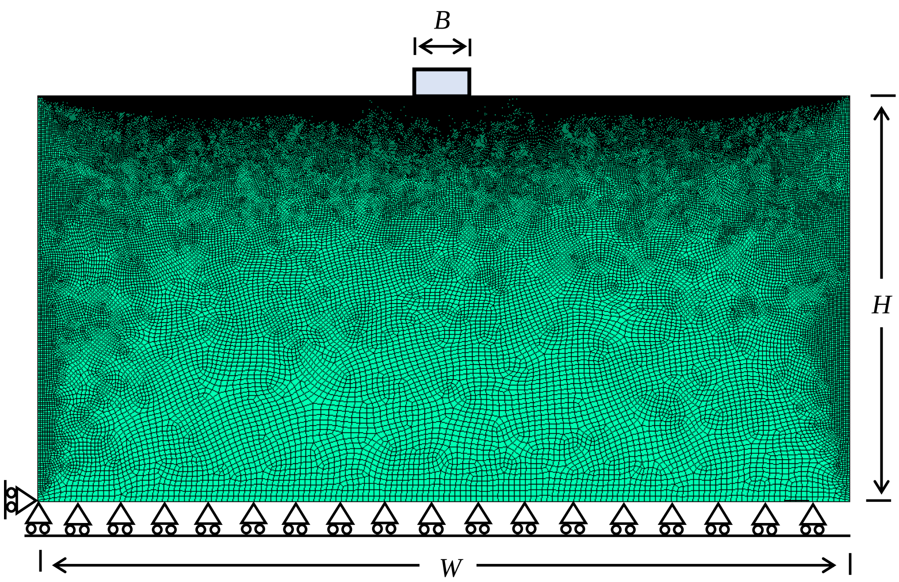

Figure 2. Finite element model; $B/H=2/15$; $B/W=1/15$.

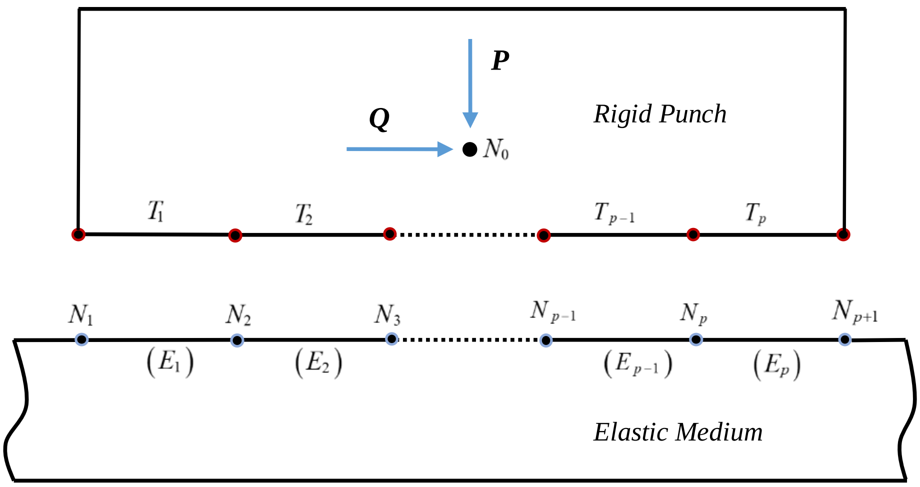

Figure 3. Demonstration of contact pairs

pairs are defined in order to impose a smooth spatial variation of the friction coefficient through the contact region. Hence 300 target surfaces (T_i) and 300 corresponding contact surfaces (E_i) are employed in the model. Note that each of the contact surfaces (E_i) is represented by a single contact element CONTA172.

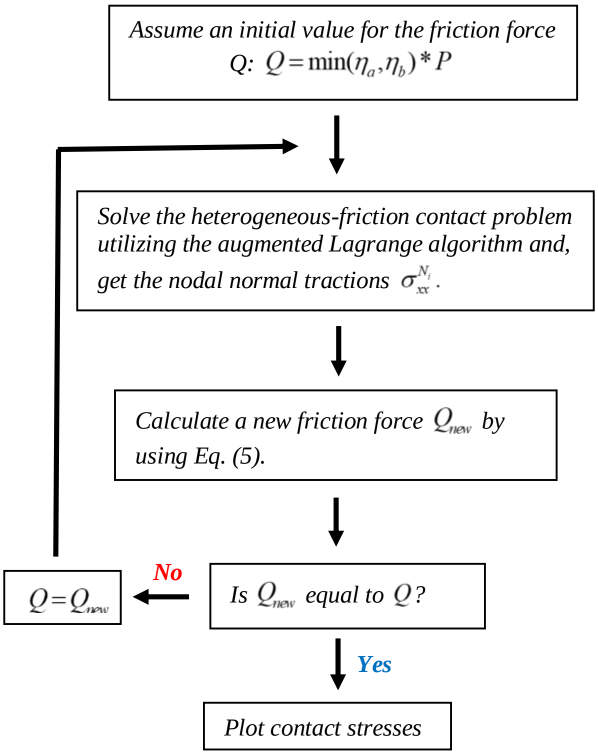

Figure 4. Computation of the friction force Q.

Similarly, each of the target surfaces (T_i) is represented by a single rigid target element TARGE169. The values of the friction coefficient to be assigned for the contact pairs are computed using their centroidal locations and Eq. (4). A mutual pivot node N_0 is identified for all the contact pairs, at which the contact forces Q and P are exerted (see Fig. 3). Note that rotation of the pivot node N_0 is fixed to zero.

The friction force Q required for the frictional sliding contact is dependent on the distribution of normal traction $\sigma_{xx}(0, y)$ through the contact interface, hence cannot be determined directly. In conjunction with a successfully converging iterative set of steps (see Fig. 4), the friction force Q is computed as the summation of friction forces generating at the contact pairs:

$$Q = \sum_{i=1}^{p} \frac{b-a}{p} \eta(r_i) \sigma^{E_i} \tag{5}$$

where σ^{E_i}, p, r_i, $\eta(r_i)$ represent the elementary normal tractions, total number of contact pairs, centroidal positions of the contact surfaces E_i, respectively. σ^{E_i} and r_i are computed as follows:

$$\sigma^{E_i} = \frac{\left(\sigma_{xx}^{N_i} + \sigma_{xx}^{N_{i+1}}\right)}{2} \qquad (i = 1, ..., p) \tag{6}$$

$$r_i = \frac{(2i-1)(b-a)}{2p} + a \qquad (i = 1, ..., p) \tag{7}$$

$\sigma_{xx}^{N_i}$ $(i = 1, ..., p+1)$ here stands for the normal tractions on the nodes N_i that are illustrated in Fig. 3. Note that extrapolation of the traction values found at gauss integration points to nodes yields very accurate results and, does not create any convergence difficulties as can be observed in the following section.

3. CASE STUDY

As seen in Fig. 1, a complete heterogeneous-friction contact prevails between the elastic surface and the flat rigid stamp, whose trailing and leading ends are located at $y=a$ and $y=b$, respectively. P and Q represents the normal and frictional contact forces acting on the stamp.

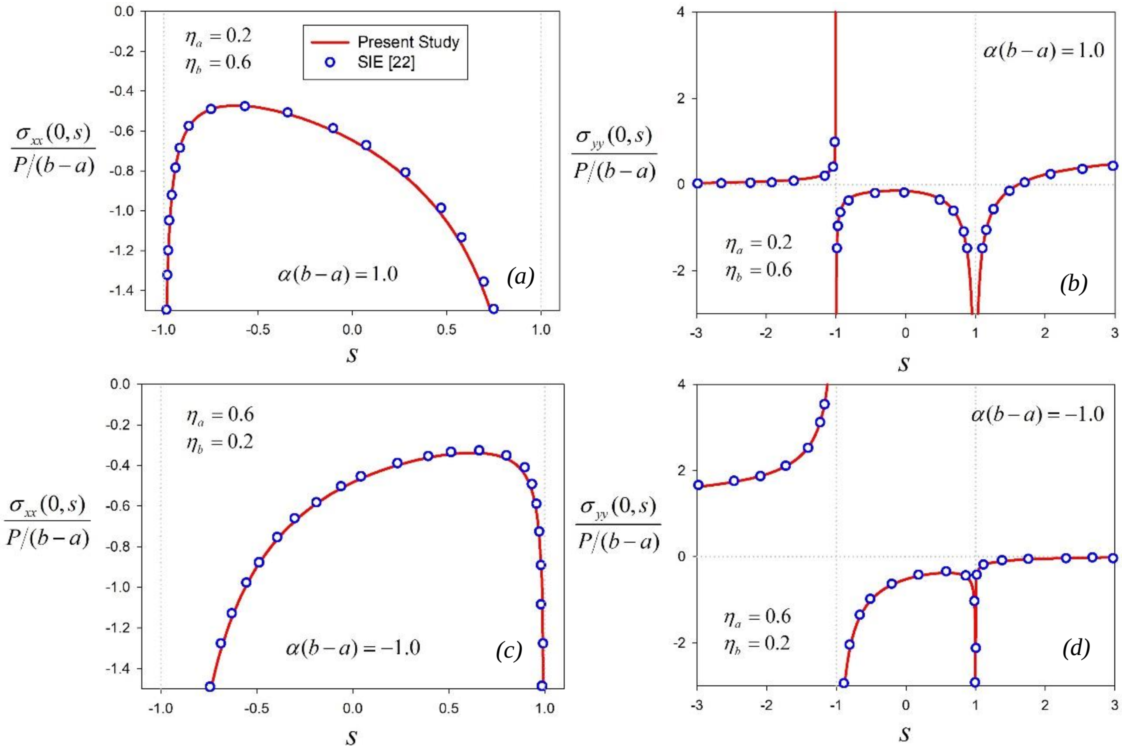

Figure 5. Comparisons of the normalized contact stresses to those generated by a SIE [22] approach for isotropic laterally graded half-planes: (a-b) Normal and lateral stresses for $\alpha(b-a)=1.0,\ \eta_a=0.2,\ \eta_b=0.6$; (c-d) Normal and lateral stresses for $\alpha(b-a)=-1.0,\ \eta_a=0.6,\ \eta_b=0.2$; $\nu_{xy}=\nu_{xz}=\nu_{zx}=\nu_{yz}=0.25$; $E_x=E_y=E_z=2(1+\nu_{xy})\mu_{xy}$.

$\eta(y)$ stands for the friction coefficient which is an exponential function of y-coordinate. In the first section of the parametric analyses, comparison results are presented to reveal the validity of the computational procedure. After all, effects of problem parameters on the results are demonstrated. To be able to evaluate results independent of scaling, the problem parameters must be represented in their normalized forms. Hence, the stiffness gradation and stresses are normalized with respect to the normal contact force *P* and the punch size *(b-a)*.

All the normalizations considered in the present work are taken consistent with the SIE based study performed by Dag [22]. The non-homogeneity parameter α is normalized with respect to the contact length as $\alpha(b-a)$. Moreover, $\alpha(b+a)=0$ in all the computations, which indicates that x-axis passes through the centerline of the stamp. It is worthy of notice that the elastic medium stiffens in positive y-direction when $\alpha(b-a)>0$ and softens when $\alpha(b-a)<0$.

Table 1. Comparisons of Q/P results to those generated by a SIE [22] approach for isotropic laterally graded half-planes subjected to heterogeneous-friction contact; $\nu_{xy}=\nu_{xz}=\nu_{zx}=\nu_{yz}=\nu=0.25$; $\mu_{xy}=\mu$; $E_x=E_y=E_z=2(1+\nu)\mu$.

	Q/P							
	$\alpha(b-a)=1.0,\ \eta_a=0.2.$				$\alpha(b-a)=-1.0,\ \eta_b=0.2.$			
	$\eta_b=0.2$	$\eta_b=0.4$	$\eta_b=0.6$	$\eta_b=0.8$	$\eta_a=0.2$	$\eta_a=0.4$	$\eta_a=0.6$	$\eta_a=0.8$
SIE [22]	0.200	0.324	0.434	0.532	0.200	0.339	0.473	0.606
Present	0.199	0.322	0.431	0.529	0.199	0.336	0.467	0.599
Diff. %	0.50	0.62	0.69	0.56	0.50	0.88	1.20	1.16

The contact stress curves are presented in normalized forms with respect to the nominal contact force $P/(b-a)$. The plots for the normalized normal stress $\sigma_{xx}(x,y)/(P/(b-a))$ and lateral stress $\sigma_{yy}(x,y)/(P/(b-a))$ are generated versus the non-dimensional y-coordinate:

$$s=\frac{2y-(b+a)}{b-a} \tag{8}$$

Note that $s=+1$ at the leading end and, $s=-1$ at the trailing end of the flat punch. Plasma-sprayed Alumina is utilized as the reference orthotropic material at $s=0$, for which the mechanical properties read [16]:

$$E_x = 116.36\,GPa\,, \qquad E_y = 90.43\,GPa\,. \tag{9a-b}$$

$$\mu_{xy} = 38.21\,GPa \tag{9c}$$

$$\nu_{xy} = 0.28\,, \quad \nu_{xz} = 0.27\,, \quad \nu_{zx} = 0.21\,, \quad \nu_{zy} = 0.14\,. \tag{9d-g}$$

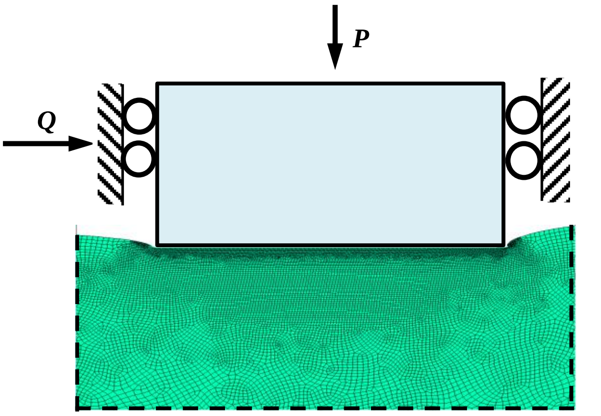

Figure 6. Deformed contact zone of an orthotropic laterally graded half-plane; $\alpha(b-a)=1.0$; $\eta_a = 0.2$; $\eta_b = 0.6$.

The other mechanical properties are computed through:

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y}, \qquad \frac{\nu_{xz}}{E_x} = \frac{\nu_{zx}}{E_z}, \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z}. \tag{10a-c}$$

Additionally, an orthotropic material must obey the following restrictions [16]:

$$1-\nu_{xy}\nu_{yx} > 0, \qquad 1-\nu_{xz}\nu_{zx} > 0, \qquad 1-\nu_{yz}\nu_{zy} > 0. \tag{11a-c}$$

$$1-\nu_{xy}\nu_{yx}-\nu_{xz}\nu_{zx}-\nu_{yz}\nu_{zy}-2\nu_{xy}\nu_{yz}\nu_{zx} > 0 \tag{11d}$$

The proposed finite element procedure is developed considering an orthotropic laterally graded material model. By using the same procedure, one can also get results for isotropic laterally graded materials in which only the shear modulus is graded. Hence, parametric analysis for isotropic laterally graded materials can be performed employing the reductions:

$$E_x = E_y = E_z = 2(1+\nu)\mu \tag{12}$$

where ν and μ represent the Poisson's ratio and shear modulus for isotropic materials, respectively. Fig. 5 illustrates some comparisons of the normalized stress results to those evaluated in a study based on the SIE approach [22] for isotropic laterally graded half-planes. Table. 1 tabulates the contact force ratio Q/P evaluated by the present procedure and a SIE approach for isotropic laterally graded materials. These results are computed for 2 different non-homogeneity parameters and 4 different friction coefficients. Note that the friction coefficient is assumed to increase in positive y-direction when $\alpha(b-a)>0$ and decrease when $\alpha(b-a)<0$ in all the computations. Also note that when the difference between η_a and η_b is increased, the degree of variation in the friction coefficient increases through the contact interface. As can be observed in Fig. 5 and Table.1, excellent agreement of the results with those generated by a SIE approach is attained for various combinations of the problem parameters. Hence, the proposed procedure seems highly feasible in the examination of heterogeneous-friction sliding contact problems.

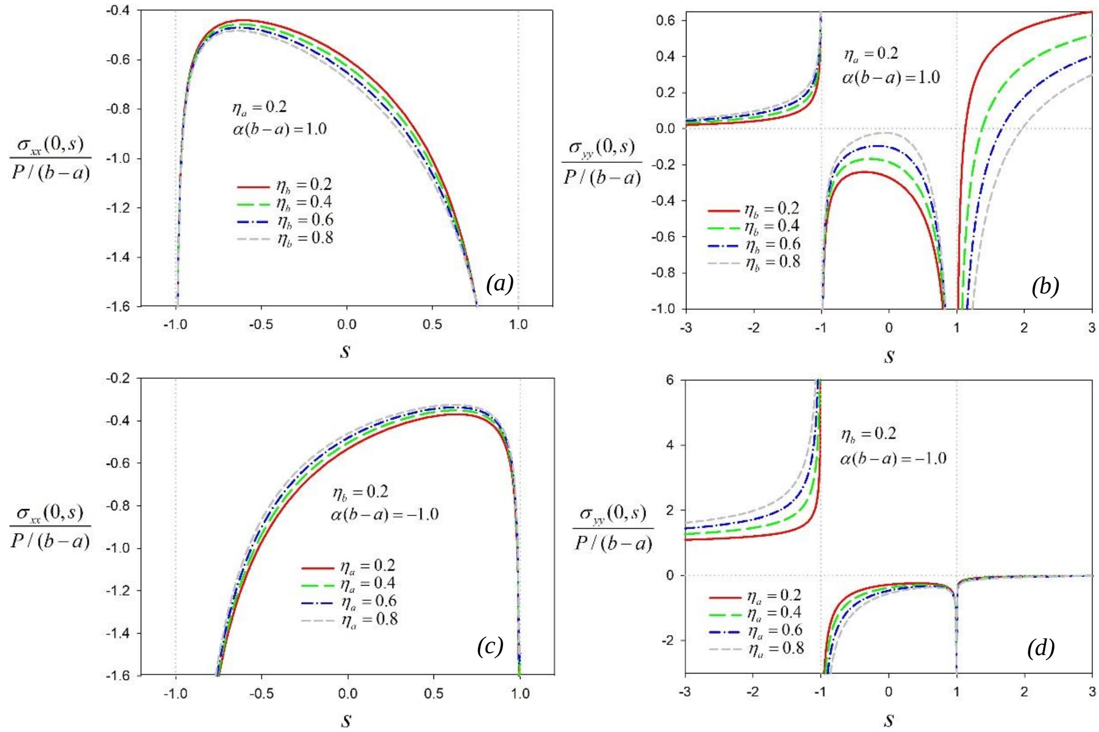

Figure 7. Effect of the friction coefficient variation on the normalized contact stress distributions for orthotropic laterally graded half-planes: (a, c) Normal stresses; (b, d) Lateral stresses.

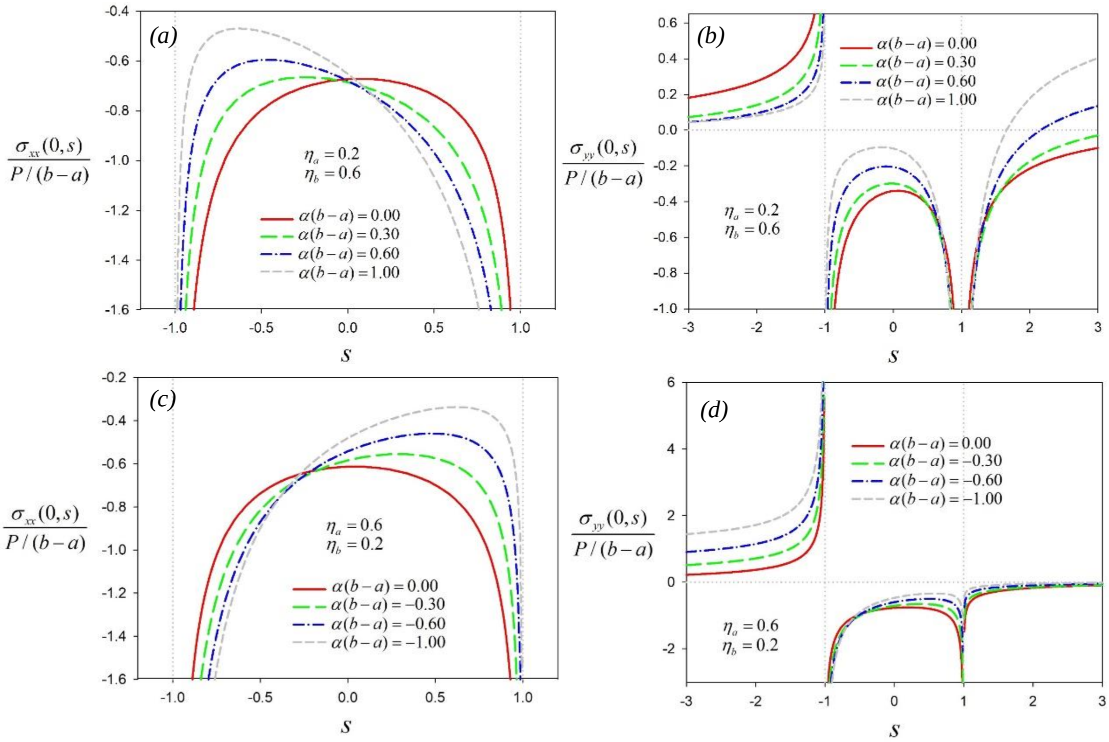

Figure 8. Effect of the lateral gradation on the normalized contact stress distributions for orthotropic laterally graded half-planes: (a, c) Normal stresses; (b, d) Lateral stresses.

Fig. 6 depicts the deformed contact zone of an orthotropic laterally graded half-plane for $\alpha(b-a)=1.0$, $\eta_a = 0.2$, $\eta_b = 0.6$. Fig. 7 plots the effects of the friction coefficient variation on the normalized normal stress $\sigma_{xx}(x,y)/\left(P/(b-a)\right)$ and lateral stress $\sigma_{yy}(x,y)/\left(P/(b-a)\right)$ for orthotropic laterally graded half-planes. When the friction coefficient at the leading end η_b is increased from 0.2 to 0.8 for $\alpha(b-a)=1.0$

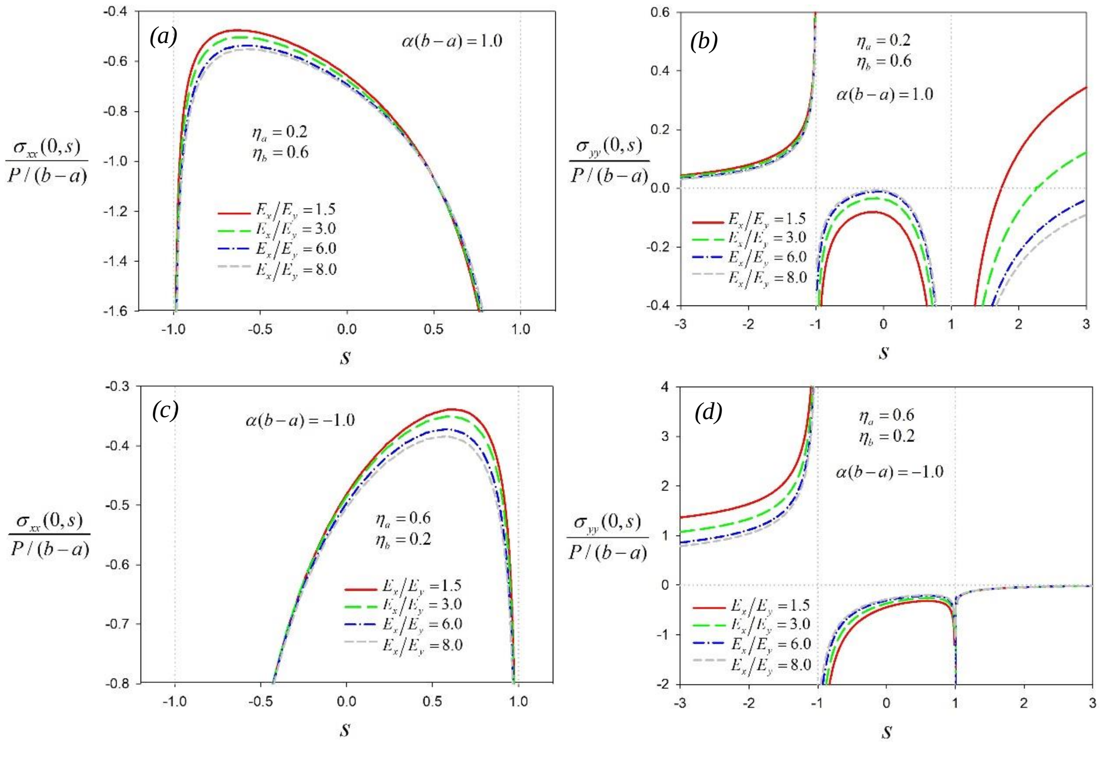

Figure 9. Effect of the elastic modulus ratio on the normalized contact stress distributions for orthotropic laterally graded half-planes: (a, c) Normal stresses; (b, d) Lateral stresses.

and $\eta_a = 0.2,$ magnitude of the normalized normal stress $\sigma_{xx}(x,y)/(P/(b-a))$ elevates. Experimental studies reveal that the lateral tensile stresses occurring due to the sliding frictional contact loadings play a prominent role in the surface crack initiation near trailing ends [1-2]. $\alpha(b-a) = -1.0,$ magnitude of the normalized normal stress increases and, the normalized lateral tensile stress decreases remarkably as seen in Fig. (9d). Table. 2 tabulates the force ratio Q/P evaluated by considering orthotropic laterally graded materials for various combinations of the problem parameters. When $\alpha(b-a) < 0,$ the force ratio Q/P becomes larger relative to the case $\alpha(b-a) > 0.$ When E_x/E_y is increased from 1.5 to 6.0, a slight drop is observed in the force ratio Q/P for all the cases.

Table 2. *Q/P* results for orthotropic laterally graded materials subjected to heterogeneous friction contact.

E_x/E_y	*Q/P*							
	$\alpha(b-a)=1.0,\ \eta_a=0.2.$				$\alpha(b-a)=-1.0,\ \eta_b=0.2.$			
	$\eta_b=0.2$	$\eta_b=0.4$	$\eta_b=0.6$	$\eta_b=0.8$	$\eta_a=0.2$	$\eta_a=0.4$	$\eta_a=0.6$	$\eta_a=0.8$
1.5	0.199	0.322	0.430	0.528	0.199	0.336	0.468	0.600
3.0	0.199	0.320	0.425	0.520	0.199	0.335	0.467	0.598
6.0	0.200	0.317	0.419	0.510	0.200	0.333	0.463	0.592

When η_b is raised from 0.2 to 0.8 for $\alpha(b-a) = 1.0$ and $\eta_a = 0.2,$ the normalized lateral tensile stress also increases. When the friction coefficient at the trailing end η_a is increased from 0.2 to 0.8 for $\alpha(b-a) = -1.0$ and $\eta_b = 0.2,$ magnitude of the normalized normal stress decreases, whereas the normalized lateral tensile stress increases significantly. In Fig. 8, influences of the lateral gradation on the normalized normal stress $\sigma_{xx}(x,y)/(P/(b-a))$ and lateral stress $\sigma_{yy}(x,y)/(P/(b-a))$ are demonstrated for orthotropic laterally graded half-planes. As the normalized non-homogeneity parameter $\alpha(b-a)$ is increased from 0.00 to 1.00 for $\eta_a = 0.2$ and $\eta_b = 0.6,$ the normalized normal stress curve slants to the left decreasing in magnitude near the trailing end. Considering the same alteration on $\alpha(b-a),$ the normalized lateral stress decreases remarkably as seen in Fig. 8(b). When the normalized non-homogeneity parameter $\alpha(b-a)$ is decreased from 0.00 to -1.00 for $\eta_a = 0.6$ and $\eta_b = 0.2,$ the normalized normal stress curve slants to the right decreasing in magnitude near the leading end. The normalized lateral stress increases significantly when the same alteration is employed in $\alpha(b-a).$

Fig. 9 depicts effects of degree of orthotropy on the normalized normal stress $\sigma_{xx}(x,y)/(P/(b-a))$ and lateral stress $\sigma_{yy}(x,y)/(P/(b-a))$ for orthotropic laterally graded half-planes. To be able obtain orthotropic materials possessing different E_x/E_y ratios, E_x of the Plasma-sprayed Alumina is altered taking the restrictions in Eqs. (10) and (11) into account. As the ratio E_x/E_y is increased from 1.5 to 8.0 for $\eta_a = 0.2,$ $\eta_b = 0.6$ and $\alpha(b-a) = 1.0,$ magnitude of the normalized normal stress increases and, the normalized lateral tensile stress is almost not effected. When the ratio E_x/E_y is increased from 1.5 to 8.0 for $\eta_a = 0.6,$ $\eta_b = 0.2$ and

4. CONCLUSIONS

In this paper an iterative computational procedure is developed to investigate heterogeneous-friction sliding contact problems. As the case study, the contact mechanics problem between a flat rigid punch and a laterally graded orthotropic medium is examined considering exponentially varying friction coefficient at the interface. The problem is handled under plane strain assumption via APDL. In the first stage of the parametric analyses, comparisons of the numerical results to those evaluated by a SIE approach is given for isotropic laterally graded materials. Excellent agreement between two methods reveals the reliability of the proposed finite element procedure. Effects of the problem parameters are also illustrated. When positive lateral gradation $(\alpha > 0)$ is introduced through the medium, the risks of failure due to surface crack initiations can be alleviated. However, negative gradation $(\alpha < 0)$ through the orthotropic medium increases the surface cracking risks. As the degree of orthotropy is increased, failure risks due to lateral tensile stress can be mitigated remarkably regardless of the sign of gradation. Also note that the surface cracking risks may increase dramatically with the change of the friction coefficient at the trailing end. As well as presenting an effective computational approach for heterogeneous-friction contact mechanics problems, this study provides results that can be useful in the validation of analytical studies and, in the prediction of contact behaviors of advanced materials before performing experiments.

REFERENCES

[1] Surresh S., "Graded materials for resistance to contact deformation and damage", ***Science***, 292: 2447–2451, (2001)

[2] Zhang Y., "Overview: Damage resistance of graded ceramic restorative materials", ***Journal of the European Ceramic Society***, 32: 2623-2632, (2012)

[3] Wolfe D. E. and Singh J., "Titanium carbide coatings deposited by reactive ion beam-assisted, electron beam–physical vapor deposition", ***Surface and Coating Technology***, 124: 142-153, (2000)

[4] Khor, K. A., Dong, Z. L. and Gu, Y. W., "Plasma sprayed functionally graded thermal barrier coatings", ***Material Letters***, 38: 437–444 (1999)

[5] Sampath S., Herman H., Shimoda N. and Saito T., "Thermal spray processing of FGMs", ***MRS Bulletin***, 20: 27-31 (1995)

[6] Kaysser W.A. and Ilschner B., "FGM research activities in Europe", ***MRS Bulletin***, 20: 22-26, (1995)

[7] Shi D., Lin Y. and Ovaert T.C., "Indentation of an orthotropic half-space by a rigid ellipsoidal indenter", ***Journal of Tribology-Transactions of the ASME***, 125: 223–231, (2003)

[8] Swanson S.R., "Hertzian contact of orthotropic materials", ***International Journal of Solids and Structures***, 41: 1945–1959, (2004)

[9] Rodriguez N.V., Masen M.A. and Schipper D.J., "A contact model for orthotropic viscoelastic materials", ***International Journal of Mechanical Sciences***, 74: 91–98, (2013)

[10] Dong X.Q., Zhou Y.T., Wang L.M., Ding S.H. and Park, J.B., "Stress state of two collinear stamps over the surface of orthotropic materials", ***Archive of Applied Mechanics***, 84: 639–656, (2014)

[11] Zhou Y.T., Lee K.Y. and Jang Y.H., "Indentation theory on orthotropic materials subjected to a frictional moving punch", ***Archives of Mechanics***, 66: 71–94, (2014)

[12] Zhou Y.T. and Lee K.Y., "Exact solutions of a new 2D frictionless contact model for orthotropic piezoelectric materials indented by a rigid sliding punch", ***Philosophical Magazine***, 92: 1937–1965, (2012)

[13] Guler M.A., "Closed-form solution of the two-dimensional sliding frictional contact problem for an orthotropic medium", ***International Journal of Mechanical Sciences***, 87:72–88, (2014)

[14] Kucuksucu A., Guler M.A. and Avci A., "Mechanics of sliding frictional contact for a graded orthotropic half-plane", ***Acta Mechanica***, 226: 3333–3374 (2015)

[15] Guler M.A., Kucuksucu A., Yilmaz K.B. and Yildirim B., "On the analytical and finite element solution of plane contact problem of a rigid cylindrical punch sliding over a functionally graded orthotropic medium", ***International Journal of Mechanical Sciences***, 120: 12-29, (2017)

[16] Arslan O. and Dag S., "Contact mechanics problem between an orthotropic graded coating and a rigid punch of an arbitrary profile", ***International Journal of Mechanical Sciences***, 135: 541-554, (2018)

[17] Babilio E., "Dynamics of an axially functionally graded beam under axial load", ***The European Physical Journal Special Topics***, 222: 1519–1539, (2013)

[18] Baron C. and Naili S., "Propagation of elastic waves in a fluid-loaded anisotropic functionally graded waveguide: Application to ultrasound characterization", ***The Journal of the Acoustical Society of America***, 127: 1307-1317, (2010)

[19] Borrelli A., Horgan C. and Patria M. C., "Exponential decay of end effects in anti-plane shear for functionally graded piezoelectric materials", ***Proceedings of the Royal Society of London A-Mathematical and Physical Sciences***, 460: 1193-1212, (2004)

[20] Arslan O., "Computational contact mechanics analysis of laterally graded orthotropic half-planes", ***World Journal of Engineering***, 14: 145-154, (2017)

[21] Dag S., Guler M.A., Yildirim B. and Ozatag A.C., "Frictional Hertzian contact between a laterally graded elastic medium and a rigid circular stamp", ***Acta Mechanica***, 224, 1773-1789, (2013)

[22] Dag S., "Consideration of spatial variation of the friction coefficient in contact mechanics analysis of laterally graded materials", ***ZAMM-Journal of Applied Mathematics and Mechanics***, 96: 121-36, (2016)

[23] Khajehtourian R., Adibnazari S. and Tash, S., "The influence of grain size and grain size distribution on sliding frictional contact in laterally graded materials", ***Applied Mechanics and Materials***, 157: 964-969, (2012)

[24] Dag S. and Erdogan F., "A surface crack in a graded medium loaded by a sliding rigid stamp", ***Engineering Fracture Mechanics***, 69: 1729-1751, (2002)

[25] Hills D.A. and Nowell D., "Mechanics of fretting fatigue", ***Kluwer Academic Publishers***, Netherlands, (1994)

[26] Vadivuchezhian K., Sundar S. and Murthy H., "Effect of variable friction coefficient on contact tractions", ***Tribology International***, 44: 1433-1442, (2011)

[27] Ren L. and Zhang Y., "Sliding contact fracture of dental ceramics: Principles and validation", ***Acta Biomaterialia***, 10: 3243–3253, (2014)

[28] Ballard P., "Steady sliding frictional contact problem for a 2d elastic half-space with a discontinuous friction coefficient and related stress singularities" ***Journal of the Mechanics and Physics of Solids***, 97: 225-259, (2016