



A MODELING TECHNIQUE IN BUSINESS MANAGEMENT: MINIMIZATION OF DISTRIBUTION COSTS WITH INTEGER LINEAR PROGRAMMING MODEL

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Abstract

In this study, which aims at building a model that helps to minimize the firm's distribution costs, first of all, the distribution strategies of the chosen firm as an example are determined. In line with the existing plan, the total distribution cost of the firm is determined, then the model is established on the basis of the same distribution system and the data set related to the firm. This study shows that the distribution cost is minimized once the distribution cost obtained from the model is compared with the actual distribution costs.

The strategies of distribution, primarily, are concerned with the firm's own talents belonging to its own structure; product or service distribution is needed to be determined within these strategies against the firm's rivals.

After that, it is required to develop policies which increase the level of competitiveness of the firm by determining a list of existing or possible opportunities and threats related to outer environment.

Such issues like improvements in transportation infrastructure, which is the important element of making distribution strategies (such as recently constructions of new roads, attempts to make rail roads, sea lines and airways more attractive in Turkey); advantages and disadvantages provided in fuel by changing alternative fuel prices and/or taxes; security in the preferred route; economic activity in the region; political activity are some of the examples of opportunity and threats considered in outer environment analysis. Firm has to develop defense strategies to protect itself from existing or possible threat and attacks in the fields that it is weak and evaluate existing or possible opportunities in the fields that is strong. A closer look to inner and outer environment analysis employed in determining the

strategies will indicate that it is important to obtain route based distribution costs, which is one of the fundamental data employed in determination of true strategies, systematically, correctly, as reliable as possible.

In this study, the integer linear programming (ILP) model is employed in the minimization of the product distribution costs to determine route based distribution costs systematically, correctly and reliably and to provide competition advantages for firms. The results indicated that the integer linear programming model used in the empirical analysis provides the total of 32384.013 YTL savings for the firm compared to the distribution costs occurred when the firm sticks to the distribution plan prepared by its logistic department.

KeyWords: Decision Making Theory, Linear-Integer Programming, Distribution Costs.

İŞLETME YÖNETİMİNDE MODELLEME TEKNİĞİ: TAMSAYILI DOĞRUSAL PROGRAMLAMA MODELİYLE DAĞITIM MALİYETİNİN MİNİMİZE EDİLMESİ

Özet

Firmanın dağıtım maliyetini minimize etmeyi sağlayacak model oluşturmayı hedefleyen bu çalışmada, öncelikle örnek olarak seçilen firmanın dağıtım stratejileri belirlenmiştir. Mevcut plan doğrultusunda firmanın toplam dağıtım maliyeti tespit edilip, aynı dağıtım sistemi ve firma değerleri esas alınarak model kurulmuştur. Bu çalışma, modelle yapılan dağıtım maliyetinin gerçekleşen dağıtım maliyeti ile karşılaştırıldığında, dağıtım maliyetinin minimize edildiğini göstermektedir.

Dağıtım stratejileri her şeyden önce işletmelerin kendi iç bünyesine ait yetenekleri ile alakalı olup, ürün veya hizmet dağıtımını bu stratejiler çerçevesinde rakiplerine karşı net bir şekilde ortaya konulması gereken politikalarlardır.

Daha sonra, dış çevreye ait varolan veya varolması düşünülen fırsat ve tehditleri bir liste halinde belirleyerek işletmenin rekabet edebilirliğini artırıcı politikalar geliştirmek gerekir.

Dağıtım stratejilerinin oluşturulması bazında ulaşım altyapısındaki gelişmeler (Türkiye’de son zamanlardaki yeni yol yapımları; demir, deniz ve hava yollarının daha cazip hale getirilmeye çalışılması gibi), alternatif yakıt fiyatlarındaki ve/veya vergilerindeki değişiklikler sonucu oluşabilecek yakıt bazındaki avantajlar/dezavantajlar, oluşturulacak güzergahtaki güvenlik, bölgedeki ekonomik hareketlilik, siyasi hareketlilik, vb. gibi konular fırsat ve tehditleri oluşturabilecek dış çevre analizine ait yalnızca birkaç örnektir. Firma, rekabet üstünlüğü olan alanlarda varolan veya olası fırsatları değerlendirecek ve aynı zamanda zayıf olduğu alanlarda varolan veya olası tehdit ve saldırılardan kendisini koruyacak savunma stratejileri geliştirmelidir. Strateji belirlenmesine yönelik gerek iç ve gerekse dış çevre analizine yakından bakıldığında, doğru stratejilerin belirlenmesine esas teşkil edecek temel verilerden biri olan güzergah bazlı dağıtım maliyetlerinin sistematik, metodolojik, doğru ve güvenilir olarak temin edilebilmesi son derece önemlidir.

Bu çalışmada işletmeler için rekabet üstünlüğü sağlamak, güzergah bazlı dağıtım maliyetlerini sistematik, doğru ve güvenilir olarak tayin etmek amacıyla Tamsayı Doğrusal Programlama (TDP) modeliyle ürün dağıtım maliyetlerinin minimize-yonu yapıldı.

Uygulama sırasında, bir firmaya ait lojistik bölümünün dağıtım planında, firmaya tasarruf sağlayacak ve firmanın dağıtım maliyetlerine karşılık, ürünlere ait tablolar

kullanılarak (Türkiye genelinde) Tamsayı Doğrusal Programlama (TDP) modeli uygulanarak firmaya 32384.013YTL. tasarruf sağlandı.

Anahtar Sözcükler: Karar verme Teorisi, Tamsayı-Doğrusal Programlama, Dağıtım Maliyetleri.

I. Introduction

Today that the period of the classical ways of the decision makers at the management of the firms is behind, leading people understand that quantitative methods from management methods are very important. All firms have to take decisions at the short or long time period. These decisions may be stock politics and price politics of a small firm or politics like production, stocking, price and intercommunication of big firms. Quantitative techniques play an important role while taking these decisions.

A linear programming model for integrated steel production and distribution planning (Balakrishnan, A., Natarajan, H.P. and Pangburn, M.S., 2000), Optimizing Delivery Fees For a Network of Distributors (Ergülen, A., 2005), Creating Firms' Distribution Strategies Model: An Application on East and North Cities in the Turkey where distribution conditions are hard. Moreover, linear programming technics have wide attractiveness. It is used successfully in lots of the industries including all types of production, transport, energy and telex communication (Stapleton, Hanna, Markussen, 2003). Durhan and frie (1996), imply that minimizing transportation cost is important due to developing variate price strategies against rivals, and also general financial structure of the firm is one of the fundamental factors for reaching more medical structure.

Ulucan and Tarım (1997), in their study on the minimization of costs in the transportation of petroleum products with sea lines and Kalender (2003), in his work on AVGs' design problem employed complex integer programming methods. In addition, Tunçbilek (2003) carried out a study to find out the most efficient way of transportation. However, Özel (2000) formulates the distribution problem as an application of matrix equations in two dimension plane distribution problems. Şafak (2000) examines the optimality conditions in a distribution problem, which involves m number of depart and n number of destinations, making use of the properties of Lagrange function and Hessian matrix.

These supply chain networks (SCNs) are considered as a solution for effectively meeting customer requirements such as low costs, high product variety, high quality, and short lead times Busby, J.S. and Fan, I.S. (1993), Byrne, A.J. (1993), Goldman, L.S. (1994), Snow, C.C, Miles, R.E. and Coleman, H.J. (1992). The main purpose of transportation is to improve lower cost and high speed the quality and productivity.

This study shows that the distribution cost of products could be minimized with Integer Linear Programming model, using tables of products, which would provide savings to the firm and be in return to the distribution cost of a firm, in the distribution plan of the logistic department of a firm (business) during the marketing activities in which competition is important to all firms. The literature in this area can broadly be categorized into conceptual, empirical, and quantitative frameworks for designing and operating SCNs. It is important to note that in any area of study the conceptual models provide a strong foundation for developing rigorous quantitative frameworks.

Miles and Snow (1984) performed the initial work in the area of conceptual network models. They introduced the concept of external groups, which they termed as dynamic

networks. According to them, a dynamic network is a combination of independent business processes with each contributing what it does best to the network. Later, Lawless and Moore (1989) studied the application of dynamic networks in private and public industries.

Managerial processes for designing, operating, and care-taking a network were suggested by Snow and Thomas (1992). Talluri, S., Baker, R.C. and Sarkis, J. (2002). *European Journal of Operational Research* Snow, C.C, Miles, R.E. and Coleman, H.J.(1992) illustrated three types of network organizations: they are internal, stable, and dynamic. In an internal network, firms own most of their assets in the business, and they do not become involved in outsourcing.

In the literature review, which is carried out, it has been observed that quantitative methods related to decision-making used in relation to distribution problems. Some of these studies done in this field are: A linear programming model for integrated steel production and distribution planning Chen, M. and Wang, W. (1997).

In this study integer linear programming model it is shown that products' allocation cost can be minimized by using tables of products against firm's allocation cost at the allocation plan belong to a firm's logistic part.

II. ILP Model For Solving Allocation Problems

Integer programming is related to the optimization (maximization or minimization) of a linear function satisfying linear equality or inequality (Bazaraa, 1990). With linear programming, possible solutions to the minimization of a linear function, which is related to a group of variables, can be determined (Briend, A. and N. Darmon, 2000). Linear programming is a deterministic mathematical technique which involves optimal distribution of scarce resources subject to given optimality conditions (Trueman, 1981). Linear programming is an analysis of problems which involves minimization or maximization of a number of variables subject to linear inequality restrictions (Dorfman, 1958). Integer programming is a special extension of linear programming developed to provide optimum integer solution to linear programming problems (Lee, 1988). ILP model has a relation with linear programming (LP) model; moreover it is part of LP (Özgüven, 2003).

Evaluating today's conditions, minimizing transportation costs have some uses (benefits):

1. Providing cost and time based rival superiority at the allocation.
2. Providing elasticity or movement ability at the developed price politics
3. Making logistic and fiscal plans better.

These studies in the firms mean something only when there is a need or it solves a problem. Problem is defined as a difference between the current and last situation (Groebner, D.F., 1985). In another way, it is defined as determining the necessary needs from current situation to target situation. Solution is the activities that defeat problems or provide determined needs.

Problems relate to the integer linear programming involves many subject such as scale, programming, construction site, vehicle route. Problem includes an optimization of a linear function subject to linear restrictions and presentation of integer variables. If all variables are integers, the problem is called pure integer but if some variables are

continuous variables, then the problem is called as mixed integer. A general formula for pure integer linear programming can be written as (Pedroso, 2002):

$$\text{Max}_x \{cx : Ax \leq b, x \in Z_+^n\}$$

Z_+^n non-negative n dimensional vector integer set

A mxn matrix involves m number of restriction

X all of its variables are integers

In this study, ILP model for minimizing firms' distribution cost can be written as following:

$$Z_{\min} = \sum_{i=1}^m \sum_{j=1}^n (d_{ij}X_{ij} + r_iY_i) \quad i = 1, 2, \dots, m \text{ (m: number of vehicles types)}$$

$$j = 1, 2, \dots, n \text{ (n: number of region)}$$

Number of voyage constraint;

$$\sum_{j=1}^n (a_{ij}X_{ij} - c_iY_i) \leq t_i \quad i = 1, 2, \dots, m \text{ (m: number of vehicles types)}$$

$$j = 1, 2, \dots, n \text{ (n: number of region)} \quad [1]$$

Good's load constraint;

$$\sum_{i=1}^m f_i X_{ij} \geq h_k \quad i = 1, 2, \dots, m \text{ (m: number of vehicles types)}$$

$$j = 1, 2, \dots, n \text{ (n: number of region)}$$

$$k = 1, 2, \dots, n \text{ (n: load values belong to regions)} \quad [2]$$

Condition of Positively

$X_{ij} \geq 0$ and integer number $Y_i \geq 0$ and integer number

Here; I shows (vehicle type) and j shows region that vehicles make the expedition.

In the objective function;

Used parameters;

d_{ij} : voyage cost of i. vehicle to j. region.

r_i : renting cost of i type vehicle for 10 days.

Decision variables,

X_{ij} : number of voyage of i. vehicle to j. region

Y_i : number of vehicle of i type that is rented

At the [1] constraint;

Used parameters;

a_{ij} : Time needed for one voyage for i type vehicle to j region.

c_i : Working period of rented i type vehicle (in about 10 days)

t_i : Working period of firm's i type vehicle (in about 10 days)

At the [2] constraint;

Used parameters;

f_i : Tonnage of i type vehicle

h_k : Amount of load sent to k region.

At the general (ILP) model given above, plans can be done due to special data's belong to firm. Similarly, problem can be used as solution method for other suitable firms. Here, while solving for special data's different constraints for firms also change the general model.

The importance of this study is to make the firms in Turkey, from production to consumption, use scientific techniques tending to the planning.

In this decision problem, firms' distribution costs are shown as variables by using tables of price kg per carriage and region load data's belong to products of firms' logistic department.

Moreover, opposite to the firms distribution cost, by using tables belongs to distributed products, new distribution cost is determined for products with ILP model. After this, by comparing distribution costs, we see that distribution cost is minimized.

III. Application in a Nourishment Firm

The firm whose data's are used is one of the biggest ones in Turkey. This firm, also, distribute different food products from production to consumption.

There were lots of problems of distributing these products to consumption points. Because of this reason, to solve these problems, ILP modeling technique is used. This application is done for finding solutions against to the problems, transporting the taken orders by current done modeling technique and carrying the products from storage to logistic department without losing time.

For finding the distribution costs belong to firm's goods;

- 1- Making the tables of yearly (annually) distribution of quantity of the goods that are distributed to distributor,
- 2- Making the tables of kg price and km price of each voyage of the distributed goods that is given to distributors,
- 3- Making the tables of overall voyage numbers, according to first and second.

The distribution of firm's goods to 13 distributors is made between January and December as being 1th.10 days, 2th.10 days and 3th.10 days of the months for the whole year. In this study, we employed the data that belongs to the 13 distributors for the 1th.10 days, 2th.10 days and 3th.10 days of each month from January to December. The information about firm's total yearly loads is provided in Appendix-1 and the information related to per voyage kg prices and km prices are provided in Appendix-1.

a) Optimum Solution Plan

Optimum distribution plan belongs to products is made according to the distribution of the firm's first 10 days, second 10 days and third 10 days by using integer linear programming method.

1. Defining the Parameters and Decision Variables

In the model, X_{ij} variables indicate voyage numbers, $i:1,2,3$ indices indicate three vehicles type (13 Tonnes, 20 Tonnes, 25 Tonnes), $j:1,2,3,\dots,11$ indicate eleven places that vehicles go. X_{ij} indicates voyage number of i^{th} vehicle to j^{th} region. If, also, vehicles are not sufficient, Y_i variable will be defined. Here, i indicate the number of rented vehicles. Moreover, d_{ij} indicates voyage cost of i^{th} type vehicle to j^{th} region, and r_i shows renting cost of i^{th} type vehicle for 10 days.

There are three types of vehicles in the model;

X_{1j} : Voyage number of 13 tonnes vehicles to j region (normal vehicle)

X_{2j} : Voyage number of 20 tonnes vehicles to j region

X_{3j} : Voyage number of 25 tonnes vehicles to j region

If vehicles are not sufficient;

Y_{1j} : Rented 13 tonnes vehicle number

Y_{2j} : Rented 20 tonnes vehicle number

Y_{3j} : Rented 25 tonnes vehicle number

The places of distributors are 1. Afyon, 2. Ankara, 3. Antalya, 4. Balıkesir, 5. Burdur, 6. Bursa, 7. Edirne, 8. Eskişehir, 9. İstanbul, 10. İzmir, 11. Kayseri, 12. Kırşehir, 13.Konya

After determining the types of vehicles and places of voyages, variables for each distributor and employed vehicles are defined, in line with the distribution plan, as follows:

Ek-1: Explanations for variables of Types of Vehicle and of regions to where products delivered.

Variables**Explaining Variables**

- X_{11} : Total numbers of delivery to Afyon by vehicles with 13 tons of loading capacity
 X_{12} : Total numbers of delivery to Ankara by vehicles with 13 tons of loading capacity
 X_{13} : Total numbers of delivery to Antalya by vehicles with 13 tons of loading capacity

Variables**Explaining Variables**

- X_{14} : Total numbers of delivery to Balıkesir by vehicles with 13 tons of loading capacity
 X_{15} : Total numbers of delivery to Burdur by vehicles with 13 tons of loading capacity
 X_{16} : Total numbers of delivery to Bursa by vehicles with 13 tons of loading capacity
 X_{17} : Total numbers of delivery to Edirne by vehicles with 13 tons of loading capacity
 X_{18} : Total numbers of delivery to Eskişehir by vehicles with 13 tons of loading capacity
 X_{19} : Total numbers of delivery to İstanbul by vehicles with 13 tons of loading capacity
 X_{110} : Total numbers of delivery to İzmir by vehicles with 13 tons of loading capacity
 X_{111} : Total numbers of delivery to Kayseri by vehicles with 13 tons of loading capacity
 X_{112} : Total numbers of delivery to Kırşehir by vehicles with 13 tons of loading capacity
 X_{113} : Total numbers of delivery to Konya by vehicles with 13 tons of loading capacity
 Y_1 : The numbers of vehicles with 13 tons of loading capacity, which will be leased
 X_{21} : Total numbers of delivery to Afyon by vehicles with 20 tons of loading capacity
 X_{22} : Total numbers of delivery to Ankara by vehicles with 20 tons of loading capacity
 X_{23} : Total numbers of delivery to Antalya by vehicles with 20 tons of loading capacity
 X_{24} : Total numbers of delivery to Balıkesir by vehicles with 20 tons of loading capacity
 X_{25} : Total numbers of delivery to Burdur by vehicles with 20 tons of loading capacity
 X_{26} : Total numbers of delivery to Bursa by vehicles with 20 tons of loading capacity
 X_{27} : Total numbers of delivery to Edirne by vehicles with 20 tons of loading capacity
 X_{28} : Total numbers of delivery to Eskişehir by vehicles with 20 tons of loading capacity
 X_{29} : Total numbers of delivery to İstanbul by vehicles with 20 tons of loading capacity
 X_{210} : Total numbers of delivery to İzmir by vehicles with 20 tons of loading capacity

- X_{211} : Total numbers of delivery to Kayseri by vehicles with 20 tons of loading capacity
- X_{212} : Total numbers of delivery to Kırşehir by vehicles with 20 tons of loading capacity
- X_{213} : Total numbers of delivery to Konya by vehicles with 20 tons of loading capacity
- Y_2 : The numbers of vehicles with 20 tons of loading capacity, which will be leased
- X_{31} : Total numbers of delivery to Afyon by vehicles with 25 tons of loading capacity
- X_{32} : Total numbers of delivery to Ankara by vehicles with 25 tons of loading capacity
- X_{33} : Total numbers of delivery to Antalya by vehicles with 25 tons of loading capacity
- X_{34} : Total numbers of delivery to Balıkesir by vehicles with 25 tons of loading capacity
- X_{35} : Total numbers of delivery to Burdur by vehicles with 25 tons of loading capacity
- X_{36} : Total numbers of delivery to Bursa by vehicles with 25 tons of loading capacity
- X_{37} : Total numbers of delivery to Edirne by vehicles with 25 tons of loading capacity
- X_{38} : Total numbers of delivery to Eskişehir by vehicles with 25 tons of loading capacity
- X_{39} : Total numbers of delivery to İstanbul by vehicles with 25 tons of loading capacity
- X_{310} : Total numbers of delivery to İzmir by vehicles with 25 tons of loading capacity
- X_{311} : Total numbers of delivery to Kayseri by vehicles with 25 tons of loading capacity

Variables	Explaining Variables
------------------	-----------------------------

- | | |
|-----------|--|
| X_{312} | : Total numbers of delivery to Kırşehir by vehicles with 25 tons of loading capacity |
| X_{313} | : Total numbers of delivery to Konya by vehicles with 25 tons of loading capacity |
| Y_3 | : The numbers of vehicles with 25 tons of loading capacity, which will be leased |

2. Formulating the Constraint Conditions

Here constraints, according to defined vehicle and place that is gone, are made by going maximum 90 Km/h and 3 hours with 30 minutes rest. The vehicles Km/h velocities are affected by the route conditions between distribution center and distributors. In the model we use 14 units 13tonnes vehicles, 8 units 20tonnes vehicles, 3 unit's 25tonnes vehicles. Right hand side of the constraints is the possible voyage periods. $X_{ij} \geq 0$ and $Y_i \geq 0$ are integers.

Number of voyage constraint;

$$16X_{11} + 12X_{12} + 16X_{13} + 24X_{14} + 18,46X_{15} + 21,81X_{16} + 34,28X_{17} + 18,46X_{18} \\ + 26,66X_{19} + 24X_{110} + 10,43X_{111} + 10,90X_{112} + 10,90X_{113} - 240Y_1 \leq 3360$$

(10days x 24 hours=240 hours; 14vehicle x 10days x 24hours= 3360 hours)

$$16X_{21} + 12X_{22} + 16X_{23} + 24X_{24} + 18,46X_{25} + 21,81X_{26} + 34,28X_{27} + 18,46X_{28} \\ + 26,66X_{29} + 24X_{210} + 10,43X_{211} + 10,90X_{212} + 10,90X_{213} - 240Y_2 \leq 1920$$

(8 vehicle x 10days x 24hours= 1920 hours)

$$16X_{31} + 12X_{32} + 16X_{33} + 24X_{34} + 18,46X_{35} + 21,81X_{36} + 34,28X_{37} + 18,46X_{38} \\ + 26,66X_{39} + 24X_{310} + 10,43X_{311} + 10,90X_{312} + 10,90X_{313} - 240Y_3 \leq 720$$

(3 vehicle x 10daysx 24hours=720 hours)

(In these equations, the coefficients on decision variables are calculated as the total time period spent for the voyage between firm and the distributor under regular road conditions.)

Load, according to the variables which is used in the defining the decision variable, is the quantity of the good of each voyage at the first 10 days, second 10 days and third 10 days. According to these load constraints, load values at the Table 1 are used for each model one by one.

Good's load constraint;

$$13X_{11} + 20X_{21} + 25X_{31} \geq \text{Goods load}, 13X_{12} + 20X_{22} + 25X_{32} \geq \text{Goods load} \\ 13X_{13} + 20X_{23} + 25X_{33} \geq \text{Goods load}, 13X_{14} + 20X_{24} + 25X_{34} \geq \text{Goods load} \\ 13X_{15} + 20X_{25} + 25X_{35} \geq \text{Goods load}, 13X_{16} + 20X_{26} + 25X_{36} \geq \text{Goods load} \\ 13X_{17} + 20X_{27} + 25X_{37} \geq \text{Goods load}, 13X_{18} + 20X_{28} + 25X_{38} \geq \text{Goods load} \\ 13X_{19} + 20X_{29} + 25X_{39} \geq \text{Goods load}, 13X_{110} + 20X_{210} + 25X_{310} \geq \text{Goods load} \\ 13X_{111} + 20X_{211} + 25X_{311} \geq \text{Goods load}, \\ 13X_{112} + 20X_{212} + 25X_{312} \geq \text{Goods load} \\ 13X_{113} + 20X_{213} + 25X_{313} \geq \text{Goods load}$$

3. Formulating the Objective function

Computation of coefficients of decisions variables, at the target equation in the model, is like that: According to the tonnage of the vehicles and months which vehicles go to which regions are computed.

This change is starting from each of January, March, June, July, August, and November. This situation is shown at Table 2.

By using these values, the target equations in the model are formulated by first 10 days, second 10 days and third 10 days.

January-February (first 10 days, second 10 days and third 10 days)

$$\begin{aligned}
 Z_{\min} = & 143845X_{11} + 139490X_{12} + 143845X_{13} + 171990X_{14} + 145990X_{15} \\
 & + 157742X_{16} + 220142X_{17} + 146497X_{18} + 187291X_{19} + 171990X_{110} \\
 & + 132145X_{111} + 135993X_{112} + 132145X_{113} + 221300X_{21} + 214600X_{22} \\
 & + 221300X_{23} + 264600X_{24} + 224600X_{25} + 242680X_{26} + 338680X_{27} \\
 & + 225380X_{28} + 288140X_{29} + 264600X_{210} + 203300X_{211} + 209220X_{212} \\
 & + 203300X_{213} + 276625X_{31} + 268250X_{32} + 276625X_{33} + 330750X_{34} \\
 & + 280750X_{35} + 303350X_{36} + 423350X_{37} + 281725X_{38} + 360175X_{39} \\
 & + 330750X_{310} + 254125X_{311} + 261525X_{312} + 254125X_{313} + 13000000Y_1 \\
 & + 20000000Y_2 + 25000000Y_3
 \end{aligned}$$

(The loading cost of a journey to Afyon for the firm is calculated as follows: the capacity of the first type of vehicle (13 ton) x per kg price of journey to the first region (11065) = 143845.)

Similarly, March, April, May, June, July, August, September, October, November and December target equations, a new mathematical modeling is done.

To see the validity of the constructed model, first period's result must be suitable to the given problem's result. If the results are suitable, solution of the model is accepted positive. This situation is also supported by the applications at the literature.

According to Riggs (1975), if results are suitable, solution of the model will be accepted positive.

While products are distributed to distributors, decision variables' costs are determined, and then by using winqsb package program, the period of constructing target equation with mathematical modeling will be ready. According to the gained results, we will have distribution costs belong to optimum solution plan with models.

Distribution cost belongs to model is given at Table 3.

Table 3.

Total cost yearly belong to Model	558678 YTL. 076 Ykrş.
Total loading yearly belong to Model	36.020.919
Total mobilization numbers yearly belong to Model	2083

The amount of loading was taken as a ton, and the cost was taken as YTL.

b) Firm's Plan

It is seen that the orders are changed time to time at the first 10 days, second 10 days and third 10 days. Especially the orders are piled at the third 10 days because: Firm determines the price at the third 10 days, Firm's budget is done at the third 10 days.

While the firm determines the distribution cost, distribution cost belongs to the firm's distribution plan is found according to the 13 distributor's orders from January to December at the first 10 days, second 10 days and third 10 days.

Table 4.

Total cost yearly belong to firm	591062 YTL. 089 Ykrş
Total loading yearly belong to firm	36.020.919
Total mobilization numbers yearly belong to firm	2938

The amount of loading was taken a ton, the cost was taken YTL

According to this, when we compare both optimum solution plan and firm's plan; there is a difference between the optimum solution plan's distribution cost and the firm's distribution cost. This is shown in Table 5.

Table 5.

Total data of the firm	
Annual Total Cost =	591062 YTL.089 Ykrş.
Annual Total Load =	36.020.919
Annual Total Delivery Numbers =	2938
Total Data of the Model	
(Annual Total Cost) =	558678 YTL.076 Ykrş.
(Annual Total Load) =	36.020.919
Annual Total Delivery Numbers) =	2083
Annual Total Savings = Annual Total Cost of the Firm Total Cost - (Annual Total Cost of the Model)	
=	591062.089 – 558678.076
=	32384.013

The amount of loading was taken a ton, the cost was taken YTL

When we look to Table 5, we see that annually total saving is 32384.013YTL. This means distribution cost with model have an advantage of %5,48 to the firm's distribution cost.

This model is done as like the firm's distribution system and like distributing the products to the consumption points from distributors. According to the computed results, we can make savings by minimizing the distribution cost of the firms by means of constructing mathematical models with integer linear programming. Moreover, it will be possible to predict the distribution cost of the next produced products.

IV. Result

After interviewing the firm, mathematical models are constructed by looking to the firm's distribution system. ILP modeling technique is used to arrange the firm's distribution problem. This application is arranged by carrying the products to logistic department without losing time and using the current modeling for the taken orders, and solving the problems when it comes out.

These models are done as like the firm's distribution system and like distributing the products from distributors to the consumption points.

We show the superiority of the modeling by using the firm's data counter to the current distribution system of the firm.

That is, modeling results for distribution have minimum solution according to the current system.

It is shown that we can make savings at important points by constructing the integer linear programming. This also shows we have an advantage of 5,48% of modeling than the current system of the firm.

Moreover, it is shown that it will be possible to predict the distribution cost of the next produced products.

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Appendix-I:

Table 1a: The amount of demands by wholesalers

Months	Afyon				Ankara			
	1.10	2.10	3.10	T	1.10	2.10	3.10	T
January	14266	32423	39772	86460	79601	180911	221917	482428
February	12440	28273	34681	75394	82967	188561	231302	502830
March	11733	26666	32710	71108	59031	134162	164572	357766
April	13426	30515	37431	81372	15718	35723	43820	95260
May	9991	22707	27854	60552	27007	61380	75293	163680
June	11779	26771	32839	71389	40500	92046	112909	245455
July	12420	28226	34624	75270	33196	75446	92546	201188
August	16440	37364	45833	99638	28133	63939	78432	170504
Septem.	15341	34865	42768	92974	32547	73970	90737	197254
October	21365	48557	59564	129486	59227	134606	165117	358950
Novem.	21038	47814	58652	127504	57907	131608	161439	350954
Decem.	14895	33853	41526	90274	33042	75095	92116	200252
Total	175134	398033	488254	1061421	548876	1247445	1530200	3326521

Note: Values were taken by being tons. Vehicles have air conditioner and 13 tons.

Table 1b: The amount of demands by wholesalers

Months	Antalya				Balıkesir			
	1.10	2.10	3.10	T	1.10	2.10	3.10	T
January	32403	73643	90335	196380	2625	5966	7319	15910
February	53067	120608	147945	321620	9029	20522	25173	54724
March	51365	116738	143198	311300	7288	16565	20319	44172
April	49204	111827	137174	298204	5767	13106	16077	34950
May	44920	102092	125232	272244	7024	15963	19581	42568
June	34322	78005	95686	208014	6081	13820	16953	36854
July	61050	138749	170199	369998	4300	9773	11989	26062
August	34216	77763	95389	207368	13604	30918	37926	82448
Septem.	47211	107297	131618	286126	8923	20280	24877	54080
October	55869	126975	155756	338600	6883	15644	19190	41718
Novem.	66557	151267	185554	403378	13794	31350	38456	83601
Decem.	29656	67399	82676	179731	5490	12476	15304	33270
Total	559839	1272361	1560763	3392963	90809	206384	253164	550357

See the note in the Table 1a.

Table 1c: The amount of demands by wholesalers

Months	Burdur			Bursa				T
	1.10	.10	.10	.10	.10	3.10	T	
January	7404	16826	20640	44870	31164	70827	86881	188872
February	16616	37763	46323	100702	32857	74675	91602	199134
March	7025	15967	19586	42578	31812	72301	88689	192802
April	14043	31916	39151	85110	32450	73751	90467	196668
May	11456	26036	31938	69430	20188	45883	56283	122354
June	7771	17661	21664	47096	27123	61643	75615	164380
July	9613	21848	26800	58260	23399	53180	65234	141812
August	16314	37076	45480	98870	36792	83619	102573	222984
September	6113	13894	17043	37050	22319	50726	62223	135268
October	16493	37485	45982	99960	43174	98122	120363	261658
November	13358	30360	37242	80960	58588	133155	163337	355080
December	14141	32138	39422	85700	26373	59939	73525	159837
Total	140347	318970	391270	850586	386240	877818	1076791	2340849

See the note in the Table 1a.

Table 1d: The amount of demands by wholesalers

Months	Edirne				Eskişehir			
	1.10	2.10	3.10	T	1.10	2.10	3.10	T
January	13813	31394	38510	83718	15446	35104	43061	93610
February	12931	29389	36050	78370	11294	25668	31486	68448
March	8440	19181	23529	51150	15662	35595	43663	94920
April	5288	12019	14743	32050	13657	31039	38074	82770
May	12062	27415	33629	73106	14185	32239	39546	85970
June	9648	21928	26898	58474	12664	28781	35305	76750
July	12602	28642	35134	76378	10149	23066	28295	61510
August	15643	35553	43612	94808	10669	24248	29745	64662
September	11429	25976	31863	69268	17805	40466	49638	107909
October	14218	32314	39638	86170	15439	35088	43041	93568
November	8213	18667	22898	49778	22811	51843	63594	138248
December	12726	28922	35478	77125	18805	42740	52427	113972
Total	137015	311398	381982	830395	178586	405876	497875	1082337

See the note in the Table 1a.

Table 1e: The amount of demands by wholesalers

Months	Istanbul				Izmir			
	1.10	2.10	3.10	T	1.10	2.10	3.10	T
January	168781	383593	470541	1022915	41030	93249	114385	248664
February	175665	399238	489732	1064634	40794	92714	113729	247238
March	155755	353990	434227	943972	45498	103406	126844	275748
April	147997	336357	412598	896952	33038	75086	92106	200230
May	154320	350728	430226	935274	36367	82652	101387	220406
June	135583	308144	377990	821718	32395	73626	90315	196336
July	105209	239112	293311	637632	49026	111424	136680	297130
August	155593	353621	433774	942988	33485	76102	93351	202938
September	11147	457152	560773	1219072	46756	106264	130350	283370
October	188982	429505	526859	1145346	32841	74639	91557	199038
November	211516	480719	589682	1281918	25646	58287	71499	155432
December	161079	366088	449068	976235	43625	99147	121620	264392
Total	1961628	4458246	5468782	11888656	460502	1046596	1283824	2790922

See the note in the Table 1a.

Table 1f: The amount of demands by wholesalers

Months	Kayseri				Kırşehir			
	1.10	2.10	3.10	T	1.10	2.10	3.10	T
January	50265	114239	140133	304638	11446	26015	31911	69372
February	56004	127283	156133	339420	18525	42103	51646	112274
March	50610	115022	141094	306726	14649	33293	40839	88780
April	45919	104361	128016	278295	7144	16236	19916	43295
May	53462	121504	149045	324010	14466	32876	40328	87670
June	53503	121598	149160	324260	16066	36514	44790	97370
July	44423	100962	123847	269232	14019	31862	39084	84966
August	57514	130714	160342	348570	17059	38771	47558	103388
September	49352	112163	137586	299100	15675	35626	43701	95002
October	66419	150953	185168	402540	13358	30359	37240	80956
November	74009	168203	206328	448540	21304	48419	59393	129116
December	56698	128860	158068	343626	16478	37451	45939	99868
Total	658178	1495859	1834920	3988957	180189	409521	502346	1092057

See the note in the Table 1a.

Table 1g: The amount of demands by wholesalers

Konya				
Months	1.10 gün	2.10 gün	3.10 gün	T
Janua.	33751	76706	94093	204550
Febru.	42686	97013	119002	258700
March	39618	90041	110451	240110
April	35412	80483	98725	214620
May	38178	86768	106435	231380
June	37080	84273	103375	224728
July	41479	94270	115638	251386
Augus.	42740	97136	119153	259028
Septem.	54858	124677	152937	332472
Octob.	38646	87833	107741	234220
Novem.	66489	151112	185363	402964
Decem.	42828	97337	119400	259566
Total	513764	1167647	1432313	3113724

See the note in the Table 1a.

Appendix-2:

Table 2a: The Delivery cost of goods per Kg/Krş and per Km/Krş

Afyon			Ankara			Antalya		
Months	Price.of kğ.	Price.of km.	Months	Price.of kğ.	Price.of km	Months	Price.of kğ.	Price.of km
January	11065	251486	January	10730	285276	January	11065	258723
February	11065	251486	February	10730	285276	February	11065	258723
March	11895	270347	March	11535	306672	March	11895	278127
April	11895	270347	April	11535	306672	April	11895	278127
May	11895	270347	May	11535	306672	May	11895	278127
June	12787	290623	June	12400	329672	June	12787	298987
July	13746	312420	July	13330	354398	July	13746	321411
August	14777	335851	August	14330	380978	August	14777	345516
September	14777	335851	September	14330	380978	September	14777	345516
October	14777	335851	October	14330	380978	October	14777	345516
November	15885	361040	November	15405	409551	November	15885	371430
December	15885	361040	December	15405	409551	December	15885	371430

Table 2b: The cost for per delivery by being Kg and Km coming from demand centers

Balıkesir			Burdur			Bursa		
Months	Price.of kg.	Price.of km.	Months	Price.of kg.	Price.of km	Months	Price.of kg.	Price.of km
January	13230	191964	January	11230	217261	January	12134	189375
February	13230	191964	February	11230	217261	February	12134	189375
March	14222	206361	March	12072	233556	March	13044	203578
April	14222	206361	April	12072	233556	April	13044	203578
May	14222	206361	May	12072	233556	May	13044	203578
June	15289	221838	June	12977	251073	June	14022	218846
July	16435	238476	July	13951	269903	July	15074	235260
August	17668	256362	August	14997	290146	August	16205	252904
September	17668	256362	September	14997	290146	September	16205	252904
October	17668	256362	October	14997	290146	October	16205	252904
November	18993	275589	November	16122	311907	November	17420	271872
December	18993	275589	December	16122	311907	December	17420	271872

Table 2c: The cost for per delivery by being Kg and Km coming from demand centers

Edirne			Eskişehir			İstanbul		
Months	Price.of kg.	Price.of km.	Months	Price.of kg.	Price.of km	Months	Price.of kg.	Price.of km
January	16934	188646	January	11269	214181	January	14407	199467
February	16934	188646	February	11269	214181	February	14407	199467
March	18204	202794	March	12114	230245	March	15488	214427
April	18204	202794	April	12114	230245	April	15488	214427
May	18204	202794	May	12114	230245	May	15488	214427
June	19569	218004	June	13023	247513	June	16650	230509
July	21037	234354	July	13999	266077	July	17898	247797
August	22615	251930	August	15049	286033	August	19241	266382
September	22615	251930	September	15049	286033	September	19241	266382
October	22615	251930	October	15049	286033	October	19241	266382
November	24311	270825	November	16178	307485	November	20684	286361
December	24311	270825	December	16178	307485	December	20684	286361

Table 2d: The cost for per delivery by being Kg and Km coming from demand centers

Izmir			Kayseri			Kirşehir		
Months	Price.of kğ.	Price.of km.	Months	Price.of kğ.	Price.of km	Months	Price.of kğ.	Price.of km
January	13230	192178	January	10165	396831	January	10461	363636
February	13230	192178	February	10165	396831	February	10461	363636
March	14222	206591	March	10927	426593	March	11246	390909
April	14222	206591	April	10927	426593	April	11246	390909
May	14222	206591	May	10927	426593	May	11246	390909
June	15289	222085	June	11747	458587	June	12089	420227
July	16435	238742	July	12628	492982	July	12996	451744
August	17668	256647	August	13575	529955	August	13971	485625
September	17668	256647	September	13575	529955	September	13971	485625
October	17668	256647	October	13575	529955	October	13971	485625
November	18993	275896	November	14593	569702	November	15019	522047
December	18993	275896	December	13575	569702	December	15019	522047

Table 2e: The cost for per delivery by being Kg and Km coming from demand centers

Konya		
Months	Price.of kğ.	Price.of km.
January	10165	371193
February	10165	371193
March	10927	399032
April	10927	399032
May	10927	399032
June	11747	428959
July	12628	461131
August	13575	495716
September	13575	495716
October	13575	495716
November	14593	532895
December	13575	532895